ENGINEERING VALUATION

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PREFACE

The goal toward which the authors have striven in writing "Engineering Valuation" has been to provide a comprehensive, reliable, and up-to-date treatise for practicing engineers, so arranged as to be suitable also for use as a college textbook. Its development has required sixteen years of intensive work, during which five previous editions (mimeographed) have been published (the last copyrighted in 1932); in each successive stage the manuscript has been tested thoroughly in actual class work for improvement wherever practicable. Since 1919, many court decisions in bitterly fought litigations have established and clarified fundamental valuation law. New researches have done much toward putting the mortality characteristics of industrial property on a sound scientific actuarial basis, corresponding to that of human mortality in life insurance. Engineering valuation has become as essential in private industrial enterprises as in public utilities. Endeavor has been made to treat all these new subjects adequately and fairly.

Throughout the development of the book use has been made of new material obtained by continuous valuation research, carried on by the Iowa Engineering Experiment Station; Dr. Marston was director of the Station till 1932, Dean Agg since. The condition-percent tables herein, and the methods of using mortality curves as aids in forecasting the probable service lives of industrial-property units, were developed in the course of these researches. Other examples in this book of valuation material believed by the authors to be new and unique include the methods for estimating and accounting for actual depreciation; the detailed analyses of 68 court decisions on valuation; the tables of approved or implied allowances in valuation decisions for overhead costs, preliminary-expense value, going value, and working capital.

The beginning valuation course which is being given at Iowa State College by the authors carries three "quarter credits" (equal to two "semester credits"); in this course, Chapter IX and parts of other chapters are used mainly for reference and Chapter VIII and parts of other chapters (notably Sections 323 to 331) are omitted. The omitted text, together with an actual valuation of some particular piece of property, forms the basis for an advanced course which has been offered by request since 1933. An extensive reference library on valuation subjects is required for graduate work; under the authors, graduate students, working on their own initiative, make intensive studies of special valuation questions and engage in individual valuation researches.

In preparing the present manuscript, primary responsibility was taken by Dean Marston for Chapters I to XV and Appendix B; by Dean Agg for the rest; both have worked on every part of the manuscript.

The authors wish to acknowledge their deep indebtedness to Professors Robley Winfrey and J. C. Hempstead, of Iowa State College; to Professor E. B. Kurtz, of the State University of Iowa; to Professor M. R. Good, of Montana State College; to Professor E. L. Grant, of Stanford University; all of these rendered valuable assistance at various times; acknowledgments are also due to the Interstate Commerce Commission and to many state utility commissions for furnishing reports and printed material. The authors have endeavored to make specific acknowledgments throughout the text of the treatise for assistance from every source on particular subjects, including permission to use published material. It is realized that inadvertently these acknowledgments may be incomplete.

Anson Marston.
Thomas R. Agg.

Ames, Iowa,
April, 1936.

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CHAPTER I

VALUATION AND VALUE

VALUATION

Individuals universally exercise the art of valuation without realizing the fact. Each exchange of property, of however trivial a character, involves an appraisal which is an elementary valuation. Each exchange of property, money included, involves an estimate of the relative worths of the items being exchanged. In ordinary trade, value is estimated by the almost instinctive decision that the price quoted is fair or unfair. In all these everyday affairs the exercise of the art of valuation is informal, intuitive, and inexact, as is well recognized; nevertheless, it is the basis of the exchange of property comprising the bulk of the world's commerce.

The more carefully ordered practices of the industrial world recognize the need for a systematic and theoretically correct procedure in estimating the exchange worths of industrial properties, whether it be in connection with sale, financial management, or the fixing of rates for services. In consequence, there has gradually developed what is known and recognized as a formal process to be employed in the art of valuation.

1.1. Valuation.—Valuation is defined as the art of estimating the fair exchange worths of specific properties.

For each property the exchange worth must be expressed in terms of some recognized medium of exchange; this usually will be the most stable monetary unit of the currency of the country in which the property is located. For international exchange purposes, the unit of worth adopted will generally be an internationally accepted money unit, such as the United States dollar or the English pound.

1.2. Engineering Valuation.—Engineering valuation is the art of estimating the fair exchange worths of specific properties in cases where professional engineering knowledge and judgment are essential. Examples of such enterprises are: mines, factories, buildings, engineering constructions of all kinds, and public utilities.

The art of engineering valuation has developed mainly since 1890. Prior to that date there was practically no such art, although more and more, by stress of developing need, engineers, economists, and the courts were being drawn into active participation in formal valuation work; this, however, was still mainly the province of the merchant, the account-

ant, and the financier. The literature of engineering valuation, though now quite extensive, dates almost entirely since 1900.

Engineering valuation is still in the formative stage. Engineers, accountants, economists, and lawyers are continually studying the various phases of the art and from time to time publishing their views in technical periodicals and in books. The federal government of the United States and many of the state governments have established commissions to regulate the various public utilities, and such commissions are publishing their opinions and decisions on valuation matters in official reports. The state courts, and the federal courts (which are the final arbiters in public-utility valuations), have handed down a mass of decisions; these show a gradual evolution toward the development of comprehensive and clearly formulated fundamental valuation principles.

Many of the methods employed in valuation work, and even some of the assumed fundamental underlying principles of engineering valuation, are still in controversy. The student of the art must recognize this situation and maintain an open mind when considering any controverted question relating to valuation.

1.3. Ordinary Valuation.—In ordinary exchanges of property, the value is determined by the judgment of the seller and the buyer, each taking into account his knowledge of the property, of prevailing exchange conditions for such property, and of his own exigencies and of those of the other party. By a process of barter, the seller and the buyer finally agree upon the actual sales value. In the majority of cases, only few and simple, if any, mathematical computations are needed. Such valuations are made authoritative and binding only through acceptance by both the owner and the prospective purchaser of the property.

1.4. Formal valuations of property are estimates of fair exchange worth, determined by the judgment of specially qualified valuators. Such valuations may be for use in actual property sales; or for many other purposes, such as taxation, securing loans, the determination of rents, and the establishment of fair commodity prices. The actual formal valuation is not computed by mathematical formula, but is fixed by expert judgment. Mathematical computations of varying degrees of complexity often are required, however, and these may be nontechnical or technical.

1.5. Valuation—Appraisal.—For the purposes of engineering valuation, the terms *valuation* and *appraisal* are synonymous.

THE FIELD OF ENGINEERING VALUATION

Engineering-valuation work entails a knowledge of the fundamental principles of valuation and of the cost, service lives, and operating characteristics of the components that comprise modern industrial properties. The field has broadened until valuation has become a part of the accepted

responsibility of management in many enterprises aside from those under the regulation of governmental agencies. Consequently, the engineer is likely to encounter valuation problems in connection with any sort of employment in which he may be engaged. Perhaps it will be of interest to point out a few special fields.

1.6. Engineering Executives.—Engineers frequently come eventually to fill responsible executive positions in manufacturing and other industrial undertakings; in all such cases a knowledge of the principles of valuation would seem essential.

1.7. Consulting Valuation Engineering.—Consulting engineering firms and individuals often find engineering-valuation work an important and remunerative part of their practice.

1.8. Appraisal companies devoted entirely to valuation work have multiplied; the need for their services is constantly growing because of the increasing needs of industries for correct valuation and depreciation data of their properties; without these, fair prices cannot be determined, true profits ascertained, or correct financial statements prepared.

1.9. Valuation Departments.—A large and increasing number of public utilities, private industrial enterprises, governmental commissions, and other agencies maintain organized valuation departments, with permanent staffs.

1.10. Salaried Valuation Engineers.—Many salaried valuation engineers are employed by consulting engineers, appraisal companies, and on the staffs of the valuation departments of public, private, and governmental enterprises and agencies.

1.11. Young Engineers.—While judgment and special skill of a character only attainable through experience in positions of responsibility are essential in making the important decisions in engineering-valuation work, nevertheless much of the extensive detail work required is of such character that it is particularly well adapted to the kind of service which young engineering graduates, under suitable direction and supervision, are qualified to render. Experience, for a time, in valuation work gives training extremely valuable to the young engineer, regardless of the branch of the profession he expects later to follow.

BASIC CHARACTERISTICS OF VALUE

1.12. Exchange Value.—Value is defined as[1] "worth estimated by any standard of purchasing power, especially by the market price, or the amount of money agreed upon as an equivalent to the utility and cost of anything."

[1] Webster's International Dictionary, Revision of 1890, p. 1593.

This is the sense in which the term *value* is used in valuation work. It is often called the *exchange value;* and may be contrasted with utility value, which is value purely from the standpoint of use.

1.13. The Fundamental Basis of Value.—The fundamental basis of the exchange value of any specific item of property is the actual present worth, to the present owner and to the would-be purchaser, of the probable future services of the item during its probable future life in service.

The future service may be of such character as to bring an annual money return to the owner, as in the case of real estate rented; or the future service may be of value to the owner because of his use of the item of property, as in the case of food consumed, or a house occupied; or the service may be of value to the owner mainly because of personal satisfaction in its ownership, as in the case of jewelry, or a painting.

Manifestly, the future life in service and the annual returns during the future service life cannot be determined with exactness and certainty. They are estimated on the basis of judgment and experience.

All values are of the nature of forecasts of future events and are subject to the uncertainties of all prophecies of the future. Values fluctuate with changes in prevailing opinions of what the future is likely to bring. They can never be determined by formulas or computations alone.

1.14. Factors Affecting Judgments in Establishing Value.—In reality, values are based upon the judgments, tacit or formal, of owners, would-be purchasers, courts, and valuation experts, of the present worths of future net returns. In ordinary exchange of property, however, no formal calculations or estimates are made of such future returns.

In formal valuations, certain factors must be taken into account and each given appropriate weight in arriving at an estimate of value. The courts hold that to each of these should be given "such weight as is just and right in each case"; but also hold that these weights must be decided by sound judgment, not by formula. This will be discussed more fully in another place. At this time it is sufficient to point out that the significant factors in estimating value are

1. The actual original cost of the property, corrected for depreciation and intangible elements to give the actual present investment in the property. This represents the price level of existing competitive investments in similar enterprises and affects near future net returns through the operation of existing competition.

2. The reproduction cost of the property, based on present prices and corrected for depreciation and intangible elements. This represents the price level at which both future replacements of the items of existing competitive properties and future competitive investments in similar properties must be made; it thus affects more remote future net returns, through the operation of competition in fixing prices.

3. The earning value of the property, which is the present worth of its probable future annual net earnings, as estimated mainly on the basis of actual past receipts and expenses. In properties other than public utilities, the earning value is often entitled to great weight in estimating the value.

4. The service-worth value of the property, or its value as determined by the "reasonable worth" of its services and/or commodities to customers. This is applicable especially to "fair rates" for public-utility services and "fair prices" for commodities; it is an indication of probable future returns in that it affords a basis for estimating whether present rates or prices are likely to continue in force.

5. The market value of the property, in comparison with the actual prevailing sales value of similar properties; or as represented by the market prices of the stocks and bonds of owner companies, called the "stock-and-bond value." This shows the present judgment of the investing public as to the present worth of the future net returns.

6. All other pertinent factors affecting value: These vary with different properties and include such things as the nature of the community served, type of product or service, materials and labor supply, transportation facilities, and all similar items.

1.15. The Process of Making an Engineering Valuation.—In ordinary exchange of property, the factors affecting value, enumerated in Sec. 1.14, are usually considered and given due weight in each case in an informal, more or less unconscious manner; in engineering valuation, as a result of many years of development in the art, a fairly well recognized formal valuation process has gradually been evolved. This formal process for making engineering valuations may be outlined as follows:

The General Method of Making an Engineering Valuation of an Industrial Property

I. PRELIMINARY AND PREPARATORY WORK

1. Make a preliminary examination of the property, and plan the valuation.

2. Make a complete detailed inventory of the present existing items of the property; by field work, aided by a thorough study of the book records, and carefully checked by final field examinations.

3. By the aid of the book records of the enterprise, study its inception, corporate structure, preliminary expenses, construction costs, annual incomes, annual operation costs, annual allowances for depreciation, annual dispositions of net returns, annual replacements, annual improvements and enlargements, past reorganizations (if any), and other historical data.

II. DETERMINATION OF THE FAIR PRESENT COST-VALUE OF THE ENTIRE PROPERTY

4. Determine the present fair values of all lands now owned and used.

5. Determine the present original-cost values of all present existing fixed-capital physical-property items except lands.

6. Determine the present reproduction-cost values corresponding to item 5, using the same inventory.

7. Determine the present fair cost-values of all present existing fixed-capital physical-property items, including lands; give such weights to the original costs and to the reproduction costs of the property units, other than lands, as are just and right in this case.

8. Determine the present fair values of all present existing intangible property items.

9. Determine the present average necessary working capital: (*a*) physical; (*b*) cash resources, and temporary investments in paid-up operation expenses.

10. Determine the present fair values of all other liquid funds wisely tied up in the enterprise.

11. Determine the total fair present cost-value of the entire property; equals 7 plus 8 plus 9 plus 10.

III. DETERMINATION OF EARNING VALUE; OF SERVICE-WORTH VALUE; OF STOCK-AND-BOND VALUE

12. Determine the earning value of the entire property; base it mainly on known average present and recent-past actual net earnings, but give reasonable weight to probable future changes in demand, supply, and general business conditions.

13. Determine the service-worth value of the entire property; this is what the earning value would be with prices for commodities and/or services equal to their "reasonable worths" to customers.

14. Determine the present stock-and-bond value of the entire property.

IV. FINAL DETERMINATION OF VALUE

15. Make the final determination of the present fair value of the entire property; give such weights as are just and right in this case to the values determined in items 11, 12, 13, and 14, and to all other pertinent factors affecting the value.

1.16. The Relation of Cost to Value.—Both the original cost and the reproduction cost of a property usually are important factors affecting its value, although in some cases the value may differ widely from either. The original costs new of the existing property items are the actual *original* investment in the existing property; the reproduction costs new are the *present* investment which new competitors in the business would have to make.

In the case of business enterprises of a private character, since the whole object of nonspeculative investments is to earn fair returns upon the sums invested, the amounts of competitive investments, in the long run, are bound to be just sufficient to cause such price adjustments as will insure fair net returns on investments which are equal to the cost-values of the properties which the enterprises operate; determined by a sound balancing of the original costs of existing and the reproduction costs of possible additional competitive properties. Hence, save in exceptional cases, both original cost and reproduction cost have much weight in determining property values, although probable future net returns are the real fundamental basis of value; and although, in rather exceptional individual cases, the values of particular properties may be widely different from their costs.

In the case of public utilities, which are business enterprises of a public character, the rates charged, and the resulting earnings, are not fixed by free competition but are subject to public regulation, down or up, to make them just sufficient to yield a "fair return" on "fair value." Hence the fair values of public-utility properties must be determined mainly from other considerations than their earnings, giving great weight

to original costs and to reproduction costs, save under exceptional circumstances.

NOTE on the "Vicious Circle of Reasoning."—Giving material weight to the earning values of public-utility properties in determining their fair values would be following the "vicious circle of reasoning" that: the higher the earnings the higher the value, and the higher the value the higher should be the earnings.

1.17. The Relation of Investment to Cost and to Value.—Under usual circumstances, the *original* investment in an existing property is equal to the sum of the original costs *new* of all the existing items of the property, tangible and intangible; this original investment is equal to what the value new of the existing property would have been, if all its existing items had been installed new at the same date. But, with the exception of some undepreciable items, the value of each item is inevitably decreased each year by depreciation; this the owners of the property are obligated to recoup by annual depreciation allowances, appropriated from annual income before calculating and distributing any net return. The depreciation allowances may be reinvested in new items, to replace old items and/or enlarge the property, thereby keeping it unwasted and the investment undiminished; or part of the investment may be paid back to the owners in the form of return larger than the true net return, thereby decreasing the investment.

It follows that the investment in the entire property at any date is equal to its present, depreciated, original-cost value at the date; obtained by deducting the total original-cost depreciations to date of all existing property items from their total original cost new.

1.18. Prudent Investment.—The prudent investment in an industrial property is the actual sum invested when all expenditures in connection with the enterprise were made in a careful, business-like, and competent manner.

1.19. The Rate Base.—The rate base is the "fair value" of a public-utility property, established as a basis for determining the charges to customers that will insure a "fair return."

1.20. The Relation of Time to Value.—A given sum of money in hand today is worth[1] more than the same sum to be received at some future date. In valuation work, it is therefore necessary to take account of the dates of all receipts and expenditures; all estimated future financial transactions must be reduced to a common date by the principles of compound interest. Figure 1.1 shows the present worth,[1] at different rates of compound interest, of $1.00 receivable 1 to 100 years in the

[1] The present worth of a given sum of money to be received at a given future date is the sum of money which, if invested today, will accumulate to the given sum at the given future date, if kept invested at the highest rate of compound interest which it can *surely* earn.

future. The rate of compound interest used should be that which long-time investments of the particular character being valued must earn to justify the investment, risks considered.

1.21. Estimates in Engineering-valuation Work.—The term *estimate* is used in engineering valuation in the sense in which it is employed in general engineering practice; that is, to indicate a carefully considered computation of some quantity the exact magnitude of which cannot be determined. The estimate represents the true magnitude as closely as

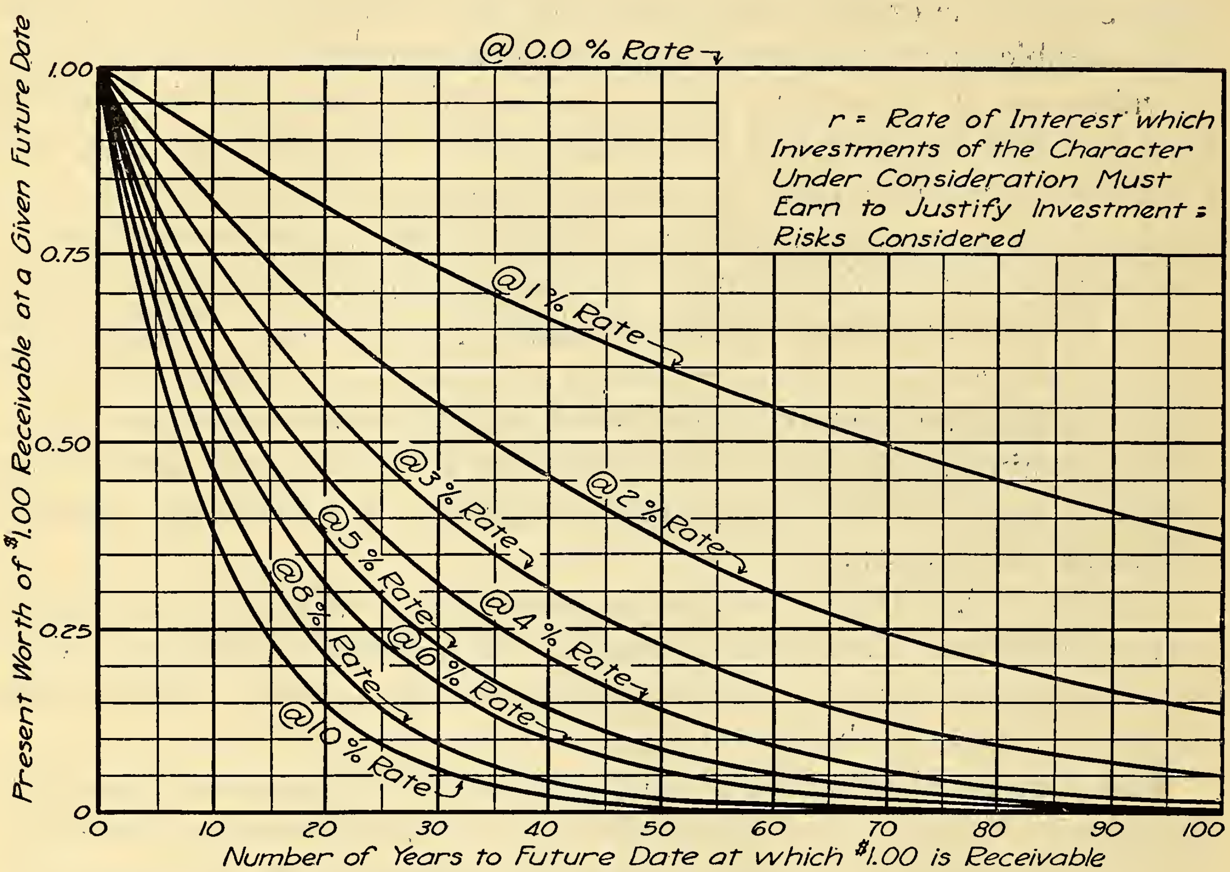


FIG. 1.1.—Diagram of present worth of $1 receivable at a given future date.

it can be determined by the exercise of sound judgment based on appropriate computations, and is not to be confused with offhand approximations that are little better than outright guesses.

THE NOMENCLATURE OF VALUE

The literature of engineering valuation abounds in terms that are used in a special technical sense, and which in consequence require a special definition to cover their meaning when used in valuation work. Those relating to value are accordingly defined in the following sections.

1.22. Fair Value.—Fair value is that estimate of the value of a property which is reasonable and fair to all concerned, every proper consideration having been given due weight. It is the value used as the rate base in public-utility rate regulation.

1.23. Book Value.—The book value of a unit of an industrial property is that estimate of its value which is carried on company books in which appropriate allowances for changes of value due to depreciation have been made.

1.24. Fair Book Value.—The fair book value of a unit of industrial property is its book value adjusted from time to time to allow fairly for changes in price levels.

1.25. Earning Value.—The earning value of a property is the present worth of its probable future net earnings, as prognosticated on the basis of recent and present expenses and earnings and the business outlook. When the net earnings are likely to continue at a uniform annual amount for a long future period, the earning value of the property is equal substantially to the capitalized annual net earnings; that is, annual net earnings divided by the rate of net return expressed in decimal form.

When the net earnings are likely to continue at a uniform rate for a limited number of years, they constitute a uniform year-end annuity, whose present worth, as computed by the proper formula (Sec. 5.6), is the earning value. In Fig. 1.1 it is shown that earnings of the distant future have very little present worth.

When the net earnings are likely to fluctuate in the future, separate computations can be made for each forecasted uniform period, or even for each year; such forecasts are usually too conjectural and fanciful to warrant giving them material weight in determining present earning values.

1.26. Service-worth Value.—The service-worth value of a property is what its "earning value" (Sec. 1.25) would be if the rates and/or prices charged were just equal to the "reasonable worths" to customers of the services and/or commodities sold.

The United States Supreme Court has ruled[1] that, in the case of public utilities: " . . . what the public is entitled to demand is that no more be exacted from it for the use of a public highway [or any other utility] than the services rendered by it are reasonably worth."

1.27. Market Value.—Market value is the value established in a public market, by actual exchanges between willing sellers and willing buyers.

The use of the term *market* suggests the idea of barter. When numerous sales occur on the market, the result is to establish fairly definite market prices as the basis of exchanges. Such market prices fluctuate to some extent and at any one time may be above or below the level which infallible judgment would adopt.

There are no markets on which the requisite exchanges occur in sufficient numbers to establish market values for entire industrial properties. However, the stocks and bonds of large industrial concerns frequently do

[1] See *Smyth v. Ames*, Sec. 8.3.

have regular market quotations, and their total value on the market is of the nature of the so-called market value (Sec. 1.28).

1.28. Stock-and-bond Value.—The stock-and-bond value of an industrial property is the sum of

1. The par values in dollars of the different issue of bonds multiplied by the corresponding current market prices per dollar.
2. The number of shares of each issue of stock multiplied by the corresponding current prices in dollars per share.

1.29. Capitalized Value.—The capitalized value of a uniform perpetual income is that sum of money whose annual interest, at the highest rate which it can certainly earn, is equal to the perpetual income in question. Annual incomes received at remote future dates do not affect the capitalized value greatly; if the income in question is reasonably certain to be received for a long period of years, it may often be treated as if it were perpetual.

1.30. Taxable Value.—The taxable value of a property is the value at which it should be assessed for taxation to make its taxes bear the same ratio to its true exchange value as do the taxes of other property in the same taxation unit. It is customary to assess property for taxation at only a fixed percentage of its full value. The assessment percentages vary in different communities; injustice can be avoided only by applying the same percentage to all properties within the same tax area.

PHYSICAL AND INTANGIBLE VALUE

1.31. Physical Value.—The physical value of a property is the value of that part of the property which has a physical existence, so that it can be apprehended by the senses. The physical property of an industrial enterprise consists of many physical units; lands, buildings, machines, poles, wires, ties, rails, roadbeds, culverts, bridges, and dams may be cited as examples.

Each physical-property unit has a value new, which is equal to its cost new; this may be actual original cost new, reproduction cost new, or book-value cost new (Sec. 4.19). The cost new of each physical-property unit includes

1. *The direct construction costs,* which are definitely incurred for the special unit (Sec. 11.10).
2. *The overhead construction costs,* which are incurred for all the units and are assigned to separate units on a percentage or other appropriate basis (Secs. 11.11 to 11.15).

The value new of each physical-property unit is inevitably reduced gradually during its service by depreciation (Sec. 4.1); at the time of any valuation, the depreciation must be subtracted from the new value to get the present value (Sec. 4.24).

1.32. Intangible Value.—The intangible value of a property is the value of that part of the property which does not have a physical existence. The intangible values include

1. *Preliminary-expense Value.*—This is the fair present capital allowance which should be made for the actual legitimate expenses incurred for the organization and promotion of the enterprise before construction of the property can begin (Secs. 12.1 to 12.8).

2. *Going Value.*—This is the "element of value in an assembled and established plant, doing business and earning money, over one not thus advanced" (Secs. 12.9 to 12.19).

3. *Good-will value,* equal to the present worth of future excess profits due to the good will of a clientele of customers (Secs. 13.1 to 13.7 and 13.15).

4. *Other intangible values,* such as those of patent rights, water rights, and contracts (Secs. 13.8 to 13.15).

NOTE.—The term *real* value includes intangible value; this is just as real as is physical value.

CHAPTER II

THE RELATION OF INDUSTRIAL-PROPERTY ACCOUNTANCY TO ENGINEERING VALUATION

Engineering-valuation work requires a clear and correct understanding of the distinctive features of industrial properties, to which its field is mainly confined; and of the methods and the underlying principles of industrial-property accountancy, whose records must supply many data essential for making valuations.

THE NATURE OF INDUSTRIAL PROPERTIES AND OF INDUSTRIAL ENTERPRISES

2.1. Industrial Properties.—Industrial properties are the plants and facilities constructed to serve the purposes of the industries; some examples are factories, office buildings, mines, quarries, hotels, railroads, telephone and telegraph systems, and power plants.

Private industrial enterprises are those which are of a private character. Their distinguishing characteristic is that their businesses are *not* "affected with a public interest"; hence, they are *not* subject to regulation by the public. Private industrial enterprises cannot obtain or exercise public rights, such as the right to condemn private property for public use. They are usually privately owned but in some instances may be owned by public corporations.

Private industrial properties are the properties constructed and operated by private industrial enterprises; they are employed in the manufacture of commodities sold on the open competitive market, or in furnishing services in open competition with similar enterprises.

Private industrial properties are most often of the *unlimited-service-life* type; in this, the enterprise, under normal conditions, is expected to continue indefinitely, although the separate units of the properties are retired and replaced whenever they become decrepit, inadequate, or obsolete. Examples are the familiar businesses such as steel manufacturing, milling, carpet making, food manufacturing, and furniture making.

A private industrial property may be of the *limited-service-life* type; this is the case when it depends upon exhaustible supplies of raw materials. Examples are mines, quarries, lumber mills, and some clay-working plants.

Public utilities are industrial enterprises whose business "holds such a peculiar relation to the public interests that there is superinduced upon it the right of public regulation."[1]

Public utilities are devoted mainly to the production of services indispensable to the public; to these it is essential that the public be accorded equal rights, on equal and equitable terms, regardless of whether the enterprises are "natural monopolies" or are competitive. Public utilities often require franchise grants of certain public rights; examples are the right to use streets, highways, or other public property, and the right to condemn private property for public use. It is owing to such characteristics that public utilities are "affected with a public interest," by reason of which the public has special regulatory rights over them which do not exist over private enterprises. Public utilities are sometimes privately and sometimes publicly owned.

Examples of public utilities include railways, highways, ferries, toll bridges, improved waterways, airways, telegraphs, telephones, waterworks, sewerage systems, gas plants, power and light plants, irrigation systems, heating plants, and sometimes grist mills, cotton gins, cableways, and insurance enterprises. The same type of enterprise may be a public utility under some conditions of circumstances and times, and not under others.

Public utilities are usually of the *unlimited-life* type; but those exclusively serving mines or other limited-life industries would be of the *limited-life* type.

2.2. Owners of Industrial Properties.—An industrial property may be owned by an individual, a business partnership, or a business corporation. In the most typical cases of engineering valuations, the properties are owned by business corporations. While the general principles of engineering valuation are the same for all classes of ownership, the preliminary costs of organizing and financing the enterprise may be different for individual and partnership owners than for corporations. Since individuals and partnerships do not issue stocks or sell bonds, their properties do not have stock-and-bond values.

2.3. Business Corporations.—A business corporation is a legally organized agency which, in the eyes of the law, is constituted "an artificial person created for preserving in perpetual succession certain rights which being conferred on natural persons only would fail in the process of time."[2] There are three classes of business corporations: public corporations, private non-stock corporations, and private stock corporations.

Public corporations are the governmental and special business corporations set up to transact all or any part of the business of such a governmental unit as a nation, state, county, township, city, or school district. Public corporations secure their original capital by taxation and/or by the sale of public bonds or other securities. The ownership of the prop-

[1] *Munn v. Illinois*, U.S. Sup. Ct., 1876. (Sec. 8.3.)

[2] This definition is usually credited to Blackstone, but the authors have been unable to find a reference to it.

erty of a public corporation is vested in the governmental unit which controls the enterprise. The citizens of the unit may be thought of as the owners of the property but they cannot sell or transfer their individual ownership rights, as do the stockholders of a private corporation.

Private non-stock corporations are the business corporations set up to carry on private non-profit-making enterprises, such as libraries or hospitals for instance. Their original capital is secured from donations and/or by the sale of bonds or other corporation obligations. The ownerships of their properties are vested in the corporations.

Private stock corporations are the private business corporations set up to carry on business enterprises for profit. Their original capital is secured by the sale of their stocks, bonds, and other corporation obligations. The ownership of their properties is vested in their stockholders.

The general principles of engineering valuation are the same for the properties of all three types of business corporations. Engineering valuation is not concerned with the controversy over public vs. private ownership.

2.4. Industrial-property Units.—Every industrial property, whatever may be its type, is made up of many separate units of physical property; each of these must be considered and accounted for separately in order to obtain complete and correct information to use as a basis of valuation of the property as a whole.

An industrial-property unit is any unit thereof that may be treated properly as an entity in estimating construction costs and depreciation charges. It follows, therefore, that for valuation purposes the unit must be of such a character that it renders service as a whole, and must be retired as a whole when it reaches the end of its service life.

The separation of the property into units and age-groups of like units for accountancy purposes is imperative, if the enterprise is to have a correct account of costs, values, and operating expenses, including depreciation. The rates of depreciation differ widely in different industrial-property units; no system of averaging depreciation gives correct results. Discretion is necessary in deciding which are separate units in any specific property.

Such things as boilers, motors, engines, lathes, concrete mixers, small buildings, and pumps are usually listed as units of physical property, either simple or composite (Sec. 2.6). Large buildings must in general be treated as composite (Sec. 2.6) units and subdivided into a number of subunits (Sec. 2.5), such as masonry, plumbing, millwork, the heating plant, radiators, elevators, and lighting system. Each of these may be replaced from time to time before the entire building is wrecked for salvage. Some units are so completely made up of separately renewable items that they remain continuously in service.

It is not practicable to treat small items of physical property (such as ties, rails, insulators, and poles) as separate accounting units; this is because of the great number in use. Property of this type should be accounted for as mortality age-groups of like units, in accordance with certain industrial-property mortality laws which are discussed in Chaps. III and VI.

In very large industrial properties, it may be practicable and desirable to form, for accounting purposes, mortality age-groups of like units, if sufficiently numerous, of greater size and cost than mentioned above. Lathes, motors, milling machines, and even locomotives may be so treated in some cases.

2.5. Industrial-property Subunits.—Many of the larger industrial-property units are composed of distinct parts, of substantial value, which require renewal from time to time before the retirement of the whole unit. An *industrial-property subunit* is a distinct subdivision, of substantial but comparatively minor value, constituting a part of one industrial-property unit and so related to it that the subunit requires retirement and renewal from time to time before the retirement of the whole unit. Examples of subunits: The feed pump, the automatic stoker, and the blower of a boiler; the roof, the plumbing, and the heating plant of a building; the floor of a bridge.

NOTE.—Care must be taken not to confuse the renewals of subunits with repairs. Replacements of small parts, such as bolts, valves, windows, minor wood and metal parts, paint and the like, are merely repair items.

2.6. Simple, Composite, and Continuous Physical-property Units.—A *simple physical-property unit* is a unit which renders service as an entity and is retired as an entity. A simple unit is not composed, wholly or in part, of separate subunits, which require separate retirement and renewal from time to time during the service life of the unit. Examples: Simple motors, lathes, pumps.

A *composite physical-property unit* is a unit which is composed, wholly or in part, of separate subunits, which require separate retirement and renewal from time to time during the service life of the unit. Examples: Large buildings, large bridges, some engines and boilers.

A *continuous physical-property unit* is a unit wholly composed of separate subunits, whose separate retirement and renewal from time to time enable the whole unit to be continued in service for as long as may be desired, without retirement. Examples: A fence, a railroad wooden snowshed, a wooden trestle bridge.

2.7. The Life History of an Industrial Property.—It is the general experience that an industrial enterprise must pass through several stages of development subsequent to the inception of the enterprise before

reaching the stage when it is an established and a profitable business. These stages may be described as follows:

1. *The preconstruction period,* during which the idea of establishing the enterprise is conceived and its promotion undertaken by one or more persons. Engineers, lawyers, and financiers may be called upon to report on the feasibility of the project, its probable cost, and the estimated returns. A company or partnership is formed; capital is secured; water, mineral, land, or other necessary rights optioned or bought; and, if required, a franchise is secured.

2. The *construction period,* during which the plants are designed and constructed and all facilities provided so that the plant is ready to begin operation.

3. The *operation period,* which follows the completion of construction. There may be some overlapping of the construction and operation periods due to the fact that parts of the plant are placed in operation while others are being completed.

3*a*. In the early part of the operation period the enterprise has to secure business, and it is customary to think of this as a subperiod, during which the business is being developed to the stage where it earns a fair return.

The various relationships during the life history of an industrial enterprise are outlined in Fig. 2.1.

Enlargements and Improvements.—The property of the typical successful industrial enterprise will undergo almost constant enlargement and improvement throughout the entire operation period, owing to the growth of the business and the progress of invention.

INDUSTRIAL-PROPERTY ACCOUNTANCY

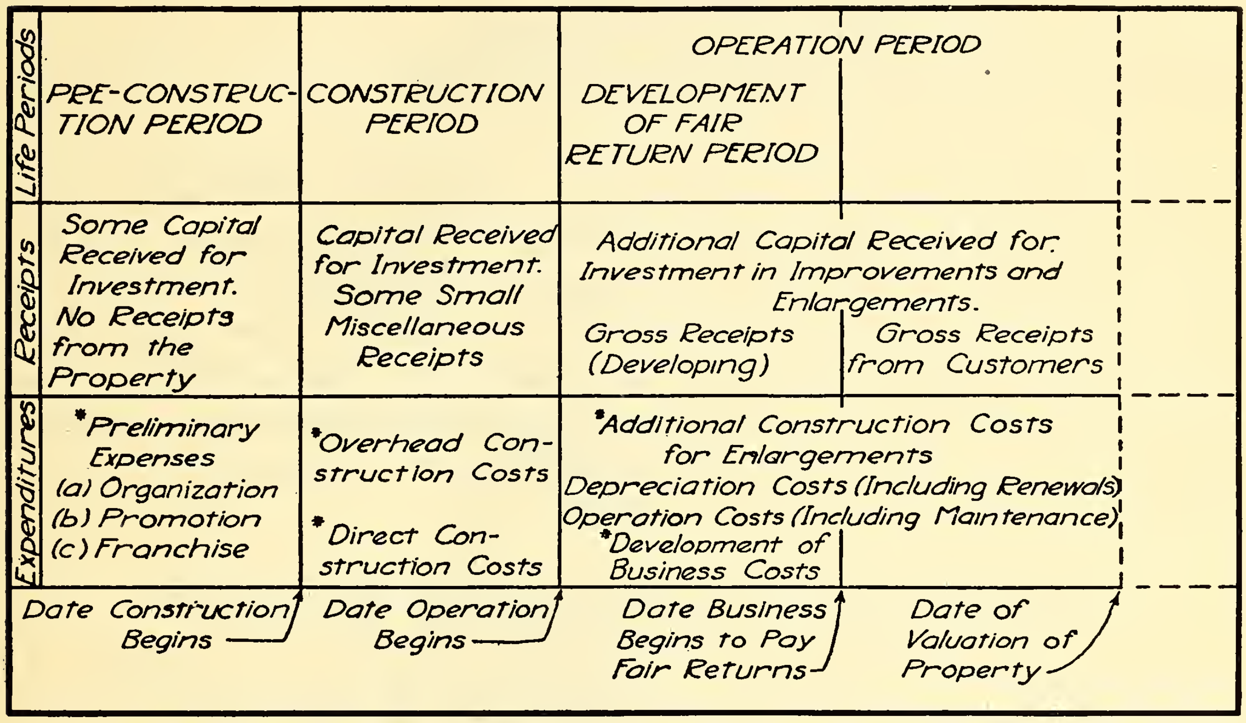
The making of a correct engineering valuation of an industrial property requires many data which can be ascertained accurately only from complete, properly kept book accounts of the enterprise.

Examples of what the valuation engineer needs include book records of the organization and promotion of the enterprise; its detailed capital structure; complete property ledger accounts of the costs and depreciations of all its existing property units; the detailed annual operation incomes and expenses; the annual depreciation appropriations; the annual net returns and their dispositions; the current assets and liabilities each year; the reserve accounts; the amortization accounts; the yearly balance sheets; the yearly income statements.

In those instances in which the records are incomplete or inaccessible, the valuator is much handicapped; he is compelled to substitute estimates for data not of record. This is frequently true of the early records of plants built some years ago. In more recent years, industrial enterprises have, as a rule, adopted much better accountancy methods, but satisfactory property ledger records of their separate property units are still all too often lacking.

It should be apparent that valuation engineers need training and experience in accountancy in order to interpret correctly the facts recorded in the accounts[1] of the properties they seek to value.

2.8. Uniform Classifications of Industrial-property Accounts.—The data of the business transactions of an industrial property are noted first in its special journals of original entry, from which they are posted to the ledgers of various primary accounts, adapted to the accounting requirements of the particular property.



Note:- Costs marked (*) occur but once, and are paid from Capital. The other costs occur yearly, and are paid from yearly receipts.

FIG. 2.1.—Diagram of the life history of an industrial property.

Uniform classifications of accounts should be adopted for the different types of industrial property. They already have been developed for a number of different kinds of public utilities (Secs. 2.10, 2.11); and are badly needed in all industries for the determination of fair prices. In such uniform classifications, the several primary accounts are standardized for each type of property; they are given distinctive account numbers, arranged systematically to correspond to six standard main groups, as follows.

THE SIX MAIN GROUPS OF INDUSTRIAL-PROPERTY PRIMARY ACCOUNTS

1. Balance sheet accounts
2. Fixed-capital accounts
3. Income accounts
4. Profit and loss accounts
5. Operating revenue accounts
6. Operating expense accounts

[1] This treatise on engineering valuation does not assume to take the place of texts on accountancy; to these the student must be referred. Only such of the most elementary principles as are essential in understanding the principles of engineering valuation are presented herein; and these only in the most general way.

Each of these main groups contains several, often many, standard numbered primary accounts.

Example: The uniform classifications of accounts prescribed by the Interstate Commerce Commission (1914) for steam railways call for 14 general and 424 standard numbered primary accounts, as follows:

....	63	Primary general balance sheet accounts
3 General, and	62	Primary fixed-capital accounts (of investment in road and equipment)
....	45	Primary income accounts
....	18	Primary profit and loss accounts
4 General, and	39	Primary operating revenue accounts
7 General, and	197	Primary operating expense accounts

2.9. Industrial-property-accountancy Periodic Statements.—The ledger accountancy data in the standard primary accounts (Sec. 2.8) are so extensive and complex, and change so continuously, that only by making up formal periodic statements (or accounts) based upon them can a comprehensive understanding be had of the true condition of the enterprise, its property and its operations, at definite dates and/or during definite periods. Four such periodic statements (or accounts) are customary, as follows:

The Four Periodic Statements (or Accounts) Customary in Industrial-property Accountancy

1. The balance sheet
2. The income statement (or account)
3. The operating statement
4. The profit and loss account

The *balance sheet*, which purports to show the assets and the liabilities at a definite date, corresponds most closely, though not precisely, to a summary engineering-valuation statement of the "fair value" of the property.

The *income statement* shows a summary of the financial results of operating the property for a particular year (or other period).

The *operating statement* gives summaries of the different operating revenues and expenses during the same year (or other period).

The *profit and loss account* shows the disposition of the net profit (or loss) during the same year (or other period).

Professor Herbert B. Dorau has prepared a chart, reproduced here (Fig. 2.2), which shows the general relations of the above periodic statements (or accounts) to each other, to the primary accounts ledgers, and to the original books of entry.

2.10. Interstate Commerce Commission's Uniform Classifications of Accounts.—The most authoritative and widely used uniform classifications of accounts yet adopted are those which the United States Interstate Commerce Commission has developed and prescribed for use by the various railway and telephone enterprises subject to its jurisdiction.

The classifications and three official interpretations of their provisions have been issued as follows:

Date	Title
Jan. 1, 1913	Uniform System of Accounts, Telephone Companies, Classes A, B
July 1, 1914	Classification of Income, Profit and Loss and General Balance Sheet Accounts for Steam Roads
July 1, 1914	Classification of Operating Revenues and Operating Expenses of Steam Roads
July 1, 1914	Classification of Investment in Road and Equipment of Steam Roads
July 1, 1914	Uniform System of Accounts for Electric Railways
Jan. 1, 1915	Uniform System of Accounts, Telephone Companies, Class C
July 1, 1916	Interpretations of Accounting Classifications for Telephone Companies
May 1, 1917	Interpretations of Accounting Classifications for Electric Railways
Jan. 1, 1918	Interpretations of Accounting Classifications for Steam Roads

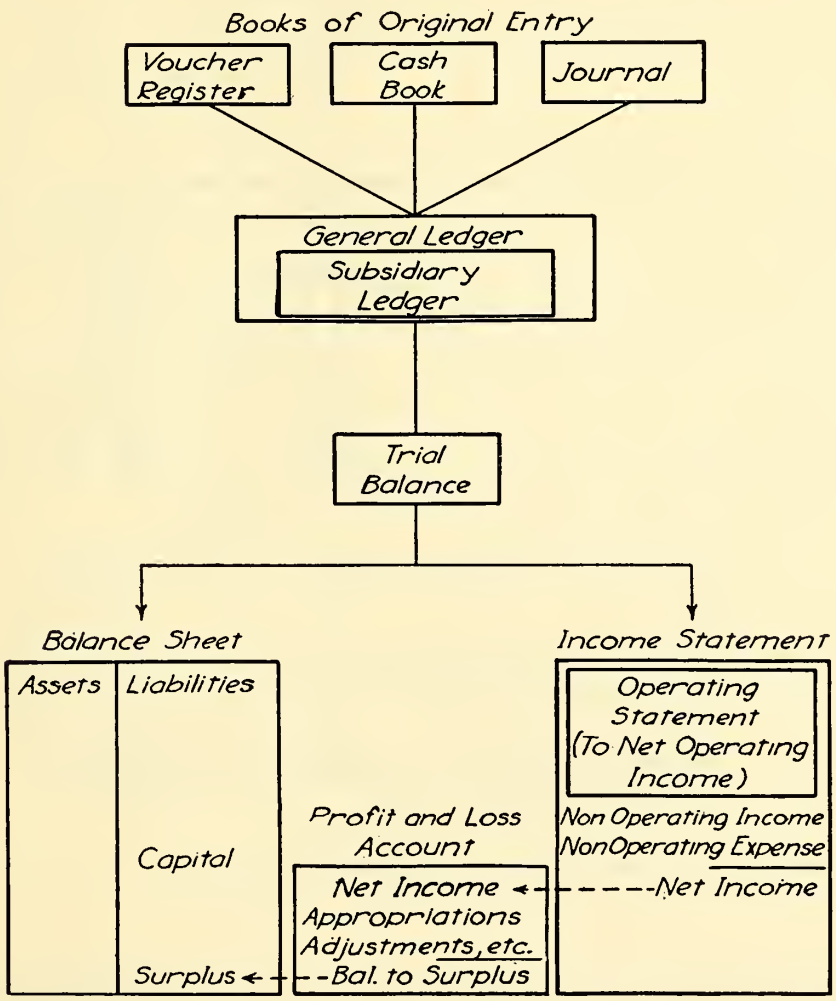


FIG. 2.2.—Diagram of the relations of the balance sheet, the income statement, the operating statement, and the profit and loss account to each other and to the books of original entry and the ledgers. (Reproduced by permission of The Macmillan Company from Chart 19, of "Material for the Study of Public Utility Economics," by Herbert B. Dorau, 1930.)

Compliance with the accounting classifications of the Interstate Commerce Commission is obligatory for all properties subject to supervision and regulation by the Commission. In general, this includes all steam and electric railways, express properties, and telephone properties oper-

ating in interstate business. Properties operating in intrastate business are under the authority of state legislatures, state railway and utility commissions, or sometimes local municipal governments or agencies.

One important characteristic of the Interstate Commerce Commission's uniform systems and classifications of accounts is that they provide for "depreciation reserve" (Sec. 6.23) and "current depreciation expense" (Sec. 6.8) accountancy. The Commission strongly favors making these compulsory but has temporarily (1934) suspended its order that they shall be compulsory for steam railways.

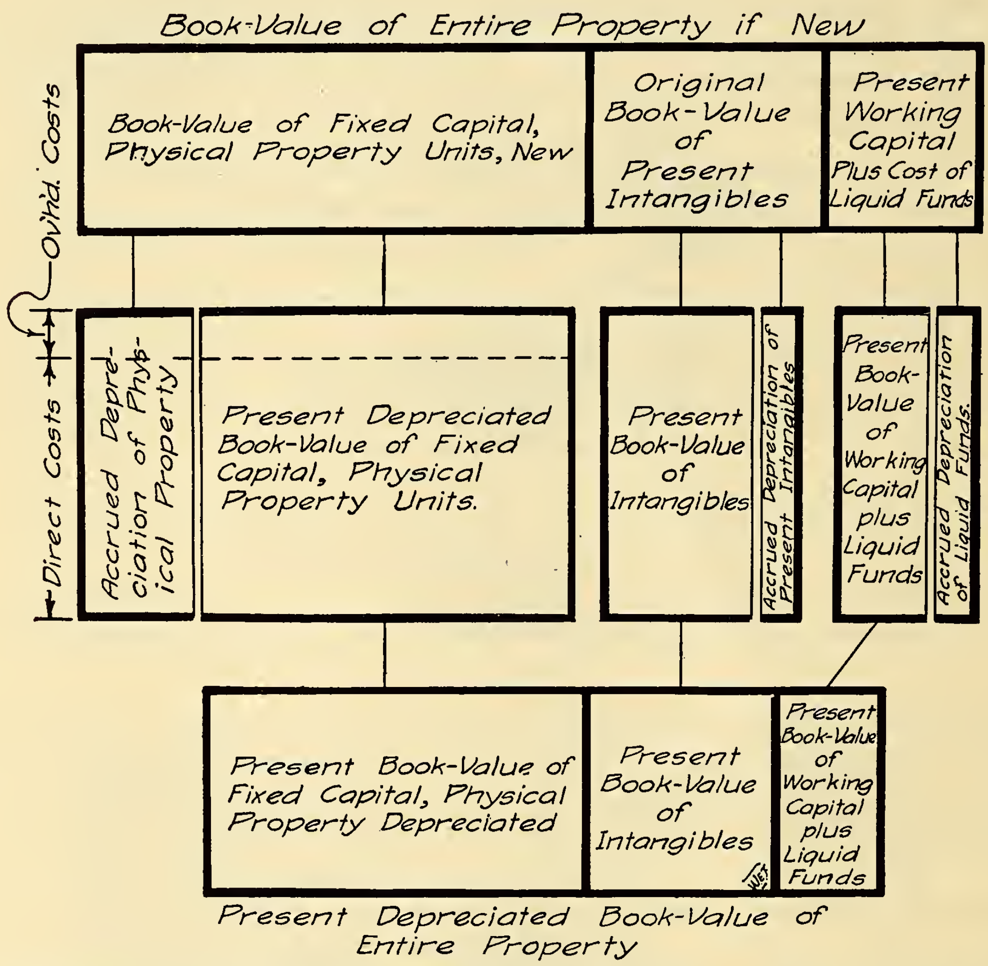


Fig. 2.3.—Diagram of the relations of the book-value accounts of an industrial property to its engineering valuation.

2.11. The National Association of Railway and Utilities Commissioners' Classifications of Accounts.—The National Association of Railway and Utilities Commissioners developed, and in 1922 *recommended* (it has no authority to *prescribe*) the following uniform classifications (Sec. 6.23) of utility accounts for general adoption by state utility commissioners and by utility owners:

Uniform Classification of Accounts for Water Utilities
Uniform Classification of Accounts for Gas Utilities
Uniform Classification of Accounts for Electrical Utilities

The above uniform classifications have been adopted in more than half the states. A number of state utility commissions and some utility owners depart from them by providing for "depreciation reserve" (Sec. 6.23) and "current depreciation expense" (Sec. 6.8) accountancy instead of the inadequate "retirement reserve" (Sec. 6.9) and "retirement expense" (Sec. 6.9) accounts in the Association's systems.

Apparently the retirement reserve and the retirement expense accounts were substituted by the Association for corresponding depreciation accounts upon urgent representations by those utility owners who are unwilling to admit the existence of depreciation of their properties, at least when considered as rate bases and as backing for issues of securities; and who wish freedom (Sec. 6.9) to manipulate their annual depreciation (or retirement) appropriations in accordance with their business policies, instead of making them equal every year to the actual depreciation during the year.

THE RELATIONS OF THE BOOK-VALUE ACCOUNTS OF AN INDUSTRIAL PROPERTY TO ITS ENGINEERING VALUATION

As industrial civilization constantly becomes more complex, it constantly becomes more necessary that industrial enterprises should keep, up to date, complete property ledger accounts of all their property units, so that such ledgers will constitute continuous inventories from which the present value of the property at any date and the true production-cost depreciation expense of each of its services and/or other products can be ascertained readily. Unfortunately such complete records are comparatively rare; it still is usually impracticable to determine "fair rates" and/or "fair prices" without long and expensive engineering valuations.

2.12. Book-value Accounts in Valuation.—In Fig. 2.3, the relations of the book-value accounts of an industrial property to its engineering valuation are shown by a diagram. The book values of the physical-property units when new are equal to their actual costs new, including both direct and overhead costs (Sec. 1.31). The "book values new" should be in view of present prices and the outlook for the future. The "actual" depreciations of the several units should be determined (by methods explained in Chap. V) every year and deducted from their book values new to get their "present depreciated book values."

The book values of the intangible (Chaps. XII, XIII) elements of the property, of its "working capital," (Chap. XIV) and of its "liquid funds," (Chap. XIV), wisely tied up in the enterprise, should be just equal to their actual present values, determined by methods explained later (Chaps. XII, XIII, XIV).

If all the above were done correctly, the present depreciated book value of the entire property would be its true "fair value," and the

production-cost depreciation expense accounts would be correct. In practice, however, the necessary skill and faithfulness to insure correctness cannot be assumed; the engineering valuator must himself determine the true fair values in detail. His work will be much facilitated and will be made much less costly by complete book-value accounts, faithfully kept.

2.13. Example of a Balance Sheet.—As already stated in Sec. 2.9, the balance sheet of an industrial enterprise corresponds most closely, though not precisely, to a summary engineering-valuation statement of the book value of its industrial property. An example (somewhat modified from the original) of a condensed balance sheet of a large power and light utility is presented on pages 26 and 27.

In comparing this balance sheet with Fig. 2.3, some real and some apparent differences will be noted as regards the depreciation reserve, the intangible values, the working capital, and the listing as assets of *claimed* rights to the future amortization of certain financing expenses.

The *depreciation reserve* (Sec. 6.23) is a valuation account which should show the total accrued depreciation of present existing property units; it is most often shown as an addition to the liabilities, as in this balance sheet; but it could just as well, perhaps better, be shown as a deduction from the assets, as indicated in Fig. 2.3. In this balance sheet, the fixed physical assets are shown on page 26 at their values new, instead of at their true present depreciated values.

Accountancy for Intangible Values.—It is a wise accountancy practice to be conservative in listing intangible values in the book assets of a business enterprise; none are listed in this balance sheet; nevertheless, preliminary expense value and going value would undoubtedly be allowed in an engineering valuation of this property. Good-will value would not be allowed, because the enterprise is a public utility.

Working capital is not shown separately in this balance sheet; it and more (Secs. 14.1 to 14.4) are included in the difference between the sum of current and deferred assets and the sum of current and deferred liabilities. Note that in this balance sheet, other liquid funds amount to only $65,750.55.

The *unamortized financing expense and other unamortized debits* items in this balance sheet would not be allowed (Sec. 12.7) in an engineering valuation of the property. Only a moderate preliminary-expense value would be allowed.

RELATIONS BETWEEN THE YEARLY INCOME AND EXPENDITURE ACCOUNTS OF AN INDUSTRIAL ENTERPRISE

The data for keeping the book-value capital accounts of an industrial enterprise constantly up to date and for preparing balance sheets whenever desired can be obtained only from comprehensive and detailed income and expense accounts; these are indispensable also for the efficient management of the yearly operation of the plant and enterprise.

2.14. The yearly financial operation of an industrial enterprise must, for efficient management, be accounted for by large numbers of special accounts; these are sometimes standardized to great advantage for

classes of enterprises of the same general character, as already discussed in Secs. 2.8 to 2.11.

There is presented in Fig. 2.4 a diagrammatic outline of the simplest and most general accountancy-valuation relationships of the financial operations of an industrial-property enterprise.

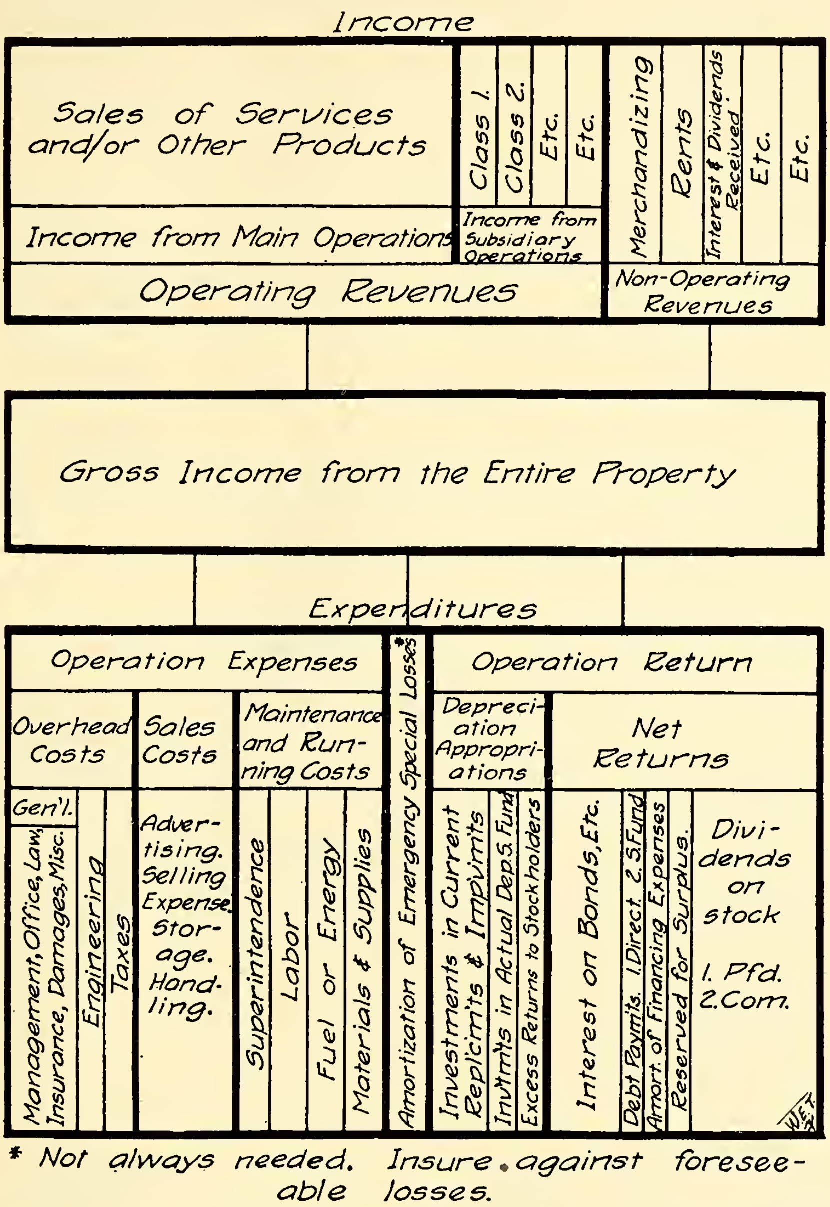


FIG. 2.4.—A diagram of the relations between the income and expenditure accounts of an industrial property.

The "gross income" is derived from various sources, each of which may require several special accounts. The view taken in this treatise is that gross income includes the receipts from all sources.

The "operation expenses" of the property come first in the annual expenditures; unless they are paid, operation stops. In addition to operation expenses, legitimate "amortization payments" to extinguish special emergency losses must be deducted from gross revenue before the owners of the property can be said to get any real "operation return" for the year.

Each year's operation return is the property of the owners; out of operation returns they must make a "depreciation appropriation" equal to the actual depreciation losses of value during the year. Unless the owners are repaid out of operation return

Condensed Balance Sheet of
December

Assets

Fixed Assets:			
Plant and Property:			
Electric Department		$61,450,255.78	
Steam-heating Department		2,483,999.51	
Water and Ice Department		438,707.04	
Coal-mining Property		891,389.40	
Construction in Progress		1,121,010.45	
Total Fixed Assets			$66,385,362.18
Current Assets:			
Materials and Supplies:			
For Construction (including replacements)		$ 221,134.71	
For Operation		468,847.32	
Merchandise		196,256.19	
Fuel Oil and Coal		171,206.62	
Other Materials and Supplies		80,865.27	
		1,138,310.11	
Other Current Assets:			
Cash		768,170.86	
Consumers' Accounts	$1,374,566.38		
Other Accounts	232,379.03		
	1,606,945.41		
Allowed for Losses	159,606.53	1,447,338.88	
Accrued Earnings (estimated)		583,172.69	
		2,798,682.43	
Total Current Assets			$ 3,936,992.54
Deferred Assets:			
Affiliated Companies' Notes and Accounts Receivable Deferred		$ 93,139.11	
Sundry Work in Progress		76,084.23	
Prepaid Taxes, Insurance, Rents, Interest, etc		189,597.02	
Total Deferred Assets			$ 358,820.36
Investments:			
Notes, Stocks, Bonds, Deposits, etc			$ 65,750.55
			$70,746,925.63
Unamortized Financing Expense:			
Bond Discounts and Expenses, 5 % issue		$ 2,464,286.02	
4½ % issue		317,106.86	
Brokerage on Preferred Stock of Predecessor Co		288,888.91	
		$ 3,070,281.79	
Other Unamortized Debits:			
Premiums and Expenses Retirement Funded Debts, Predecessor Co's		1,380,536.20	
Excess, Predecessor Co's. Securities over Book Value of Property Acquired Therefor		694,329.11	
		$ 2,074,865.31	$ 5,145,147.10
Grand Total Assets			$75,892,072.73

THE XYZ POWER AND LIGHT COMPANY
31, 1930

CAPITAL STOCK AND SURPLUS

Capital Stock:		
40,000 Shares Cumulative First Preferred 6 % Stock, Series B, No Par Value—Stated Value	$ 4,000,000.00	
525,000 Shares Common Stock, No Par Value—Stated Value	24,395,000.00	
	$28,395,000.00	
Surplus:		
Balance, Dec. 31, 1930	3,748,318.02	
Total Capital Stock and Surplus		$32,143,318.02
Liabilities:		
Fixed Liabilities:		
First Mortgage 30-year 5 % Gold Bonds Maturing Sept. 1, 1952	$25,000,000.00	
First Mortgage 30-year 4½ % Gold Bonds, Series B, Maturing Jan. 1, 1957	6,000,000.00	
Total Fixed Liabilities		$31,000,000.00
Current Liabilities:		
Accounts Payable:		
Purchases, Expenses, etc	$ 382,948.60	
Affiliated Companies	44,396.75	
Total Current Liabilities		$ 427,345.35
Deferred Liabilities:		
Income Taxes	$ 570,771.09	
General Taxes	252,743.87	
Interest	421,218.59	
Other Expenses	10,209.83	
	$ 1,254,943.38	
Consumers' Deposits	512,334.83	
Deferred Earnings	17,382.02	
Total Deferred Liabilities		$ 1,784,660.23
Reserves:		
For Injuries and Damages	$ 614,235.40	
Depreciation Reserve	9,922,513.73	
Total Reserves		$10,536,749.13
Grand Total Liabilities, Capital Stock, and Surplus		$75,892,072.73

for the depreciation losses of capital book value each year, the stability of the enterprise will be endangered by uncompensated wasting of the investment.

What is left from operation return, after the deduction of the yearly actual depreciation, is the true "net return." The first obligations against net return are the interest charges on funded debts. The remainder may be disposed of in different ways at the discretion of the owners or their representatives; if all goes well, the stockholders may receive some actual cash dividends in addition to their shares in the increase of the capital book value of the property.

2.15. Example of an Income and Expense Statement.—This is shown in tabular form (somewhat modified from the original) on page 29. The power and light utility in question is the same for which the condensed balance sheet is shown on pages 26 and 27. In the gross income statement, only the net income is shown for the merchandise sales. The bulk of the income was from sales of services produced by the plant, and the bulk of the operation expenses were for plant operation, including management and sales of services. Taxes constituted 17.74 percent of the total operation expenses. The operation expenses were 49.45 percent of the gross income, leaving 50.55 percent for operation return. The depreciation appropriation was 13.68 percent of the gross income, leaving 36.87 percent of gross income for the net return.

INDUSTRIAL-PROPERTY ACCOUNTANCY-VALUATION NOMENCLATURE

In the absence of an authoritative standard accountancy-valuation nomenclature, some commonly used terms are often given somewhat different meanings by different accountants and valuation engineers. Several of these accountancy-valuation terms will now be defined to indicate the meanings assigned them in this text.

2.16. Fixed Capital.—The fixed capital of an industrial enterprise is the capital kept invested in comparatively permanent "fixed" items of property, such as may reasonably be expected to continue in service at least one year. Fixed-capital items of property include:

1. All physical property except salable materials and supplies.
2. All intangible elements of property, including (*a*) preliminary-expense value, (*b*) going value, (*c*) good-will value, and (*d*) the values of "other intangibles" (such as contracts, patent rights, water rights).

2.17. Working Capital.—The working capital of an industrial property is the *average* amount of capital *wisely* kept invested in temporary items of property; normally used up in the operation of the plant in less than one year. Working capital items of property include

1. *Stocks of operation materials and supplies*, kept on hand to be used when needed; such as oil, waste, fuel, fittings, spare parts, small tools, and miscellaneous other small items.
2. *Operating Cash Resources, and Temporary Investments in Paid-up Operation Expenses.*—The operating cash resources should be sufficient to meet safely the

Condensed Income Statement of the XYZ Power and Light Co., 1930

Gross Income:			
Operation Earnings:			
Electric Sales		$13,618,962.82	
Steam Sales		450,220.86	
Water and Ice Sales		57,874.17	
Other Operation Earnings		376,562.38	
			$14,503,620.23
Other Income:			
Net Profit on Merchandise Sales		187,393.96	
Sundry Non-operation Income		54,535.93	
Interest Earned		79,097.28	
Discount Earned		25,782.29	
Dividends Received		33,221.22	
			$ 380,030.68
Gross Income			$14,883,650.91
Operation Expenses:			
Electric		$ 5,640,124.04	
Steam		322,157.72	
Water and Ice		35,524.78	
Extraordinary Advertising		56,169.66	
General Taxes		735,250.83	
Federal and State Income Taxes		570,771.09	
Operation Expenses			$ 7,359,998.12
Operation Return			$ 7,523,652.79
Depreciation Appropriation			$ 2,036,361.61
Net Return			$ 5,487,291.18
Disposition of Net Return:			
Interest Charges		$ 1,326,523.07	
Amortization of Financing Expenses:			
Funded Debt	$186,642.54		
Other Debits	45,379.29	232,021.83	
Addition to Surplus		550,746.28	
Preferred Stock Dividends		240,000.00	
Common Stock Dividends		3,138,000.00	
			$ 5,487,291.18

fluctuating needs from day to day for funds to pay operation-expense bills. The funds so paid out remain invested as working capital during the times which elapse before the company actually receives the earnings corresponding to the expenses. The paid-up operation expenses include the actual costs (including storage costs) of the materials and supplies withdrawn from stock for actual use.

Note.—Manufacturing enterprises should keep separate accounts of their working capital tied up in "raw manufacturing materials," "goods in process," and "finished

goods in stock"; trading enterprises should keep separate accounts of "trading goods in stock."

2.18. Gross Annual Income.—The gross annual income from an industrial property includes all receipts, from whatever sources. In Fig. 2.4, which applies best to a utility or manufacturing enterprise, the receipts are divided into two classes, operating and non-operating revenues. Each of these may usually be subdivided along the lines indicated in the diagram. These receipts are really earned by the individual units of the property, each contributing its share to the general result. Thus the boilers, the turbines, the pumps, and each of the other units in a power-generating plant really earn a part of the gross receipts; but it is not feasible or desirable to keep book accounts in which its share of the earning is credited to each unit. It is essential, however, to a correct understanding of engineering-valuation methods to recognize that in reality each unit has actually earned a share of the gross receipts.

2.19. Operation Expenses.—The operation expenses of an industrial property include all the expenditures for its operation. In Fig. 2.4, the operation expenses are divided into three classes, overhead costs, sales costs, and maintenance and running costs; each of these may be subdivided in various ways, somewhat as indicated in the diagram. Other classifications of operation expenses may be better adapted to other than utility or manufacturing enterprises. These operation costs are really incurred for the individual units of the property, a share for each; but it is not feasible or desirable to keep book accounts in which its full share of the operation expenses is charged to each unit. Accounts of some of the more significant operation expenses of some units may be kept, in order to test and control the efficiencies of operation.

2.20. Amortization payments are payments made from annual income into amortization funds maintained for the purpose of gradually extinguishing from the accounts such items as discounts on securities, other financing expenses, and various emergency losses too large to reimburse from one year's income. The amortization periods for emergency losses and financing expenses other than discounts must be decided by business judgment; those for discounts on securities should extend to their maturity dates.

As indicated in Fig. 2.4, the amortization payments to reimburse for special emergency losses, which could not be insured against in advance, should be deducted from income before estimating net return; amortization payments to extinguish discounts on securities and reimburse for other financing expenses should be paid by the owners, out of net return.

2.21. Operation Return.—The annual operation return is the amount remaining from gross annual receipts after paying the operation costs (and any amortization demands which happen to be current for the year

on account of special emergency losses which could not be insured against in advance).

The operation returns are the property of the owners; but from operation returns the property, if of the unlimited-life type, must be kept up by setting aside year by year such depreciation appropriations from income as will recoup all depreciation losses of value. In the case of limited-life properties, the operation returns are employed to retire the investment in the property and pay returns on the remaining investment. For such properties, the depreciation accountancy is most often conducted on the replacement basis, carrying no reserves for depreciation.

It must be quite apparent that each unit of the property employed in the enterprise earns its share of the operation returns; but just as it is not feasible or desirable to keep book accounts in which a share of the gross receipts is credited, and a share of the operation costs is charged, to each individual unit, it is likewise impracticable to credit each unit of the property with its share of the operation return. The contribution of each unit of property to the operation returns is definite and real, even though it is not segregated on the books.

2.22. Annual Depreciation Appropriations.—The annual depreciation appropriations are the yearly amounts set aside from income to recoup depreciation losses. They should be just equal to the total "actual" depreciation on all property units during the year.

As indicated in Fig. 2.4, the depreciation appropriation for any one year may be larger than the costs of replacements of units during that year (because of the depreciation accruing during the year on units remaining in service); in such case, the balance should be reinvested in the property, if possible, as indicated in Fig. 2.4. The depreciation during each year and the total depreciation to the end of each year should both be shown in the property ledgers for each individual property unit (or age-group of like units) (Secs. 6.15 to 6.20).

2.23. Net Return.—The net return from an industrial property is the balance left at the end of the year after deducting from the gross income the sum of operation expenses, plus amortization payments (if any) to reimburse special emergency losses, plus the annual depreciation during the year. If the annual depreciation appropriation is not equal to the true "actual" depreciation during the year, the net return shown by the accounts is not the *true* net return, and, in this respect, the accounts are not true accounts.

As indicated in Fig. 2.4, the first demand upon net-return earnings is to pay the annual interest on the funded debt; after this, at the option of the management, appropriations of net earnings may be made for such purposes as payment of the principal of the funded debt (or building up sinking funds for its payment), or for the amortization of discounts

and/or other financing expenses; it may be considered wise to make reservations for surplus, to be invested in securities or in the property; the remainder, if any, is available to pay dividends to the stockholders, the preferred stock coming first.

Each unit of the property earns a share of the net returns, but, as discussed in connection with gross income, operation expenses, and operation returns, it is not feasible or desirable to keep book accounts assigning to each unit of the property a definite share of the net returns. In considering certain aspects of depreciation, it is helpful to recognize that each unit of the property does, in effect, earn a definite share of the net returns, even though it is not credited to the unit in the book accounts.

CHAPTER III

INDUSTRIAL-PROPERTY MORTALITY CHARACTERISTICS[1]

A complete, correct knowledge of the mortality characteristics of the various kinds of industrial property is essential for the correct determination of the fair rates, and/or fair prices, necessary to the financial success of industrial enterprises. In spite of this fact, it is only recently that the true laws of industrial-property mortality have begun to be understood; ignorance on the subject still prevails widely; correct feasible methods for forecasting the probable service lives of particular property units, and of ascertaining correctly and providing wisely for the large industrial production expenses and losses of value due to depreciation, have only now become available in a form adapted to general use.

INDUSTRIAL-PROPERTY MORTALITY CURVES

The mortality-curve method of arranging and studying mortality data has been in use a long time, by insurance actuaries, to determine human life expectancy and corresponding premium rates; it is now coming into use by valuation engineers to assist in determining the probable lives of the existing property units of different kinds, still in service in particular industrial properties.

3.1. Industrial-property Mortality Curves and Tables.—Mortality or survivor curves of humans similar to those shown in Fig. 3.1 have been used for the determination of insurance rates for some 200 years. Such curves show the number of persons of the original group who survive the various ages of life.

It seems that the compilation of similar curves for physical property should have been equally as natural a development, but such was not the case. Only since 1902 have any such curves been compiled; the number now available must be considered as limited when compared with the multitudes of items and classes of property in service. Professor Kurtz

[1] The first four center divisions of this chapter are a condensation of Chaps. III and IV of the fourth (mimeographed) edition of this treatise. Chapter III was written by Prof. Edwin B. Kurtz and Chap. IV by Dr. Marston. Some of the material also appeared in "Expectancy of Physical Property" by Professor Kurtz (The Ronald Press, New York, 1930) and in "Life Characteristics of Physical Property" by Robley Winfrey and Edwin B. Kurtz, which appeared as *Bull.* 103, Iowa Eng. Exp. Sta., 1931.

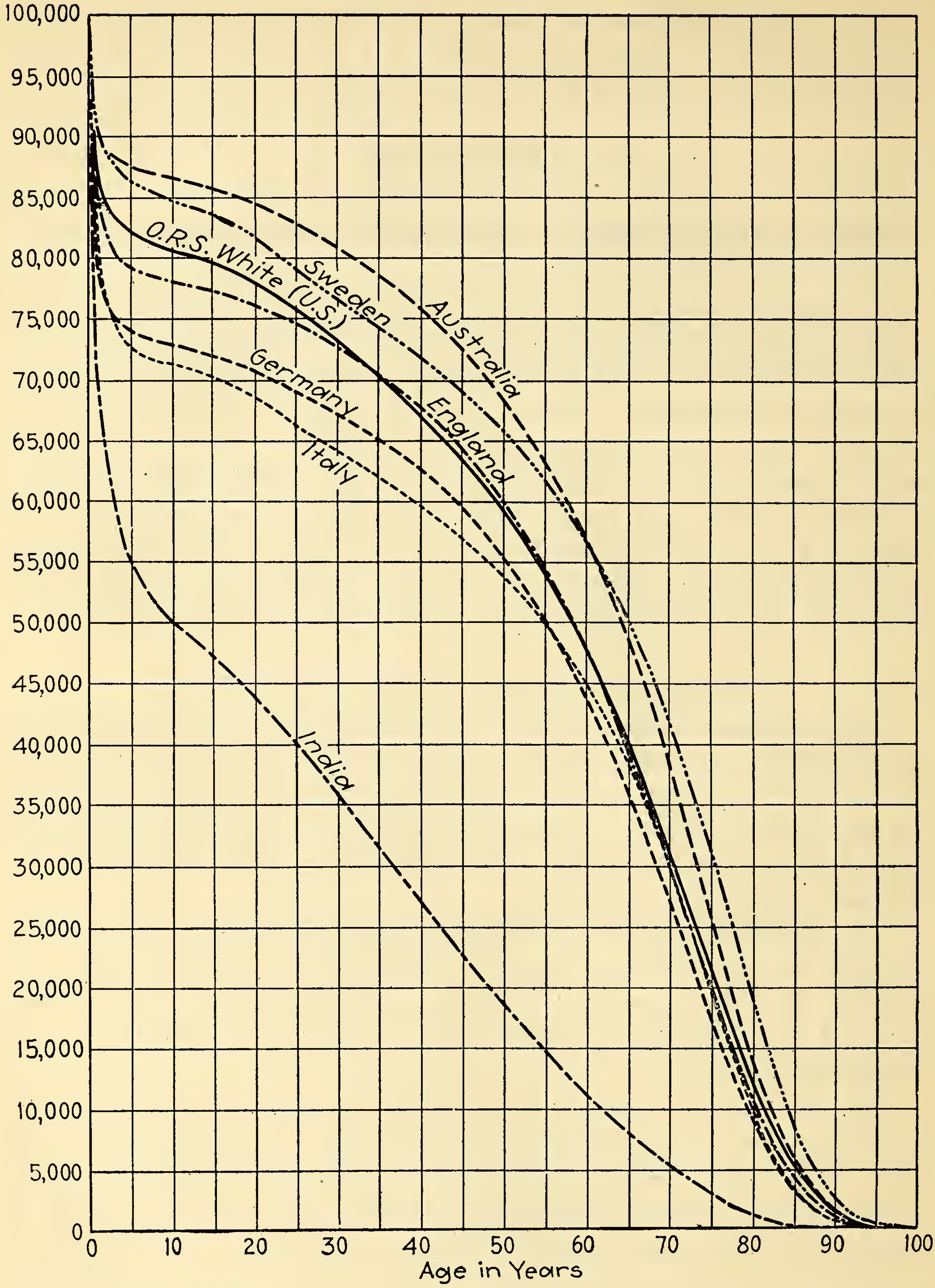


FIG. 3.1.—Human mortality curves for the United States and various foreign countries. (*Reprint from United States life tables.*)

and the authors have compiled the following list of pioneer contributions to mortality-curve data:

1903. Alvord, J. W.: 48 water works pumps, 32 boilers.

1905. Christiani, Geheimer Oberpostrat (chief engineer, postal system): 248,707 telegraph poles.

1912. National Electric Lamp Association: Incandescent lamps.

1914. Gruhl, Edwin: Street car wheels.

1916. Kurtz, Edwin B., and H. L. Baker: Chicago pumping engines.

1916. New York Telephone Co.: Several rate case studies of the mortality of aerial and underground cables.

1917. Kurtz, Edwin B.: 1,372 electric line poles in Missouri.

1917. American Society Civil Engineers, Valuation Committee: Mortality tables of a number of water-supply sources, pumping stations, railway stations.

1918. Thorne, Mabel E.: railroad ties, U.S. Forest Products Laboratory.

1918. Kates, E. J.: Large numbers of locomotives, passenger and box cars.

1921. Kurtz, Edwin B.: 309 electric line poles in Wisconsin.

1924. Great Northern Railway: Several mortality studies of particular types of railroad property.

1925. Wallace, Lew: 7 kinds of farm machinery, Iowa Engineering Experiment Station.

1926. Winfrey, Robley: Iowa automobiles in service in 1912 and in 1922, Iowa Engineering Experiment Station.

1928. Winfrey, Robley: Iowa automobiles in service in 1926, Iowa Engineering Experiment Station.

1928. American Telephone and Telegraph Company: Several classes of telephone property.

1931. Winfrey, Robley, and Edwin B. Kurtz: *Bull.* 103, Iowa Engineering Experiment Station (containing all the preceding items and more).

1936. Winfrey, Robley: *Bull.* 125, Iowa Engineering Experiment Station (containing all the preceding curves and 111 additional).

The Iowa Engineering Experiment Station.—In addition to the above, it may be said that the Iowa Engineering Experiment Station has been actively engaged in the collection and tabulation of industrial-property mortality-curve data since 1922; it expects to continue this and other industrial-property mortality researches indefinitely.

These mortality researches have been under the active direction of Dr. Marston throughout. Till 1924, the collection of mortality data was largely by Professor Kurtz and since then by Robley Winfrey. In 1931, the station made the first publication[1] of a large collection of industrial-property mortality curves and tables. The first 65 curves in Table B.1, Appendix B, are from *Bull.* 103; as also 13 of the mortality type curves. Data from the 111 additional mortality curves listed in Table B.1 have been collected and compiled by the station since 1930; Mr. Winfrey has done and is continuing notable research work on mortality type curves, including mortality renewals and condition-percent curves.

[1] In *Bull.* 103. Note that Professor Kurtz had included 17 mortality curves in 1917 in his unpublished thesis at the University of Wisconsin.

3.2. Nomenclature of Mortality Analyses.—Figure 4.1, Chap. IV, presents a diagrammatic outline of the fundamental depreciation relations of a single unit of industrial property, and of the changes in its present values and in the re-estimates of its probable life from year to year during its service life.

The *service life* of any physical unit of an industrial property is the period of time between the (past) date when it was first put into service new as a part of the property and the (also past) date when it was retired from service. The service life is ascertainable with certainty only when and after the unit has been retired; it is then an historical fact, determined once for all.

The *service age* of any physical unit of an industrial property is the period of time between the (past) date when it was first put into service new, as a part of the property, and the (present) date of making a valuation of the unit, or an estimate or a re-estimate of its depreciation. The service age is an historical fact; it should be ascertainable with certainty from the books of the company, or from other evidence.

The *expectancy* of any physical unit of an industrial property is the period of time between the (present) date when it is estimated and the (future) date when, in the considered opinion of persons who are well qualified to judge, and who are fully cognizant, through personal examination, of all of the pertinent facts involved, the unit will most probably be retired from service. The expectancy is always an estimate, made before the date of retirement; it must be reconsidered and re-estimated from time to time during the service life of the unit, in accordance with the development of its actual service-life history.

The *probable life* of any physical unit of an industrial property is the period of time between the (past) date when it was first put into service new, as a part of the property, and the (future) date when, in the considered opinion of persons who are well qualified to judge, and who are fully cognizant, through personal examination, of the pertinent facts involved, the unit will most probably be retired from service. The probable life is estimated by first estimating the expectancy and then adding the service age; consequently, it is always an estimate, made before the date of retirement. Hence it must be reconsidered and re-estimated from time to time during the service life of the unit, in accordance with the development of its actual service-life history.

A *mortality group* of like physical-property units is a group of such units whose *originals* were all put in service during the same year.

An *age-group* of like physical-property units is a mortality group all of whose *present* units were put into service during the same year.

The *age* of a mortality group of like physical-property units is the number of elapsed years since the year during which the original units

were put in service. The age of the group will be the same as the service ages of the original units, but not the same as the service ages of renewals.

The *average life* of an age-group of like physical units of industrial property is the average number of years of service rendered by the individual units of the group. The average life of an age-group is calculated by dividing the sum of the service-life years of all its individual units by the number of its units. The service lives of some individual units of an age-group will always be much shorter, and of some much longer than the average life of the group. Not infrequently, a unit of the group may be found to have a service life double the average for the group.

Renewals of physical units of industrial property are their replacements at retirement by substantially duplicate units. During a period of years equal to the service life of the longest survivor of the original age-group, part of the renewals are of the original units and part are renewals of renewals; after this, all renewals are renewals of renewals; these fluctuate at first from year to year but eventually would become constant.

The *annual renewals* occurring in a group of like units of industrial property are the total renewals made necessary during individual calendar years by the retirement of units of the groups.

Normal annual renewals are the annual renewals of the units of an age-group of like industrial-property units when they become constant.

The *annual rate of normal renewals* is equal to the quotient of 100 percent divided by the average life of the original age-group, in years.

The *condition-percent* of an industrial-property unit, or age-group of like units, is 100 times the ratio of its present depreciable (Sec. 4.22) value divided by its depreciable value when new. Four applications of the term condition-percent need to be considered:

1. The condition-percent of a single property unit.
2. The condition-percent of the average-survivor unit, at any service age, of an age-group of like units.
3. The average condition-percent of all survivors of an age-group, of like units, at any service age.
4. The average condition-percent of all the units in service at any date (including survivors of all service ages) in an age-group of like units kept at 100 percent numbers by continued renewals.

3.3. The Nomenclature of Mortality Curves.—*Mortality survivor curves* show the numbers, or percentages, of survivors of the original members of large age-groups of like industrial-property units, from date to date during the service life of the longest surviving unit.

Mortality-frequency curves show the rates of retirement of the original members of large age-groups of like units, from date to date during the service life of the longest surviving unit.

The *mode* is that point on the frequency curve which shows the highest rate of retirement.

Actual mortality curves are mortality curves constructed from the mortality data of the actual service lives of all the individual members of a large group of like industrial-property units.

Mortality probable-life curves are curves indicating the probable service lives (service ages *plus* expectancies) of the average survivors at different

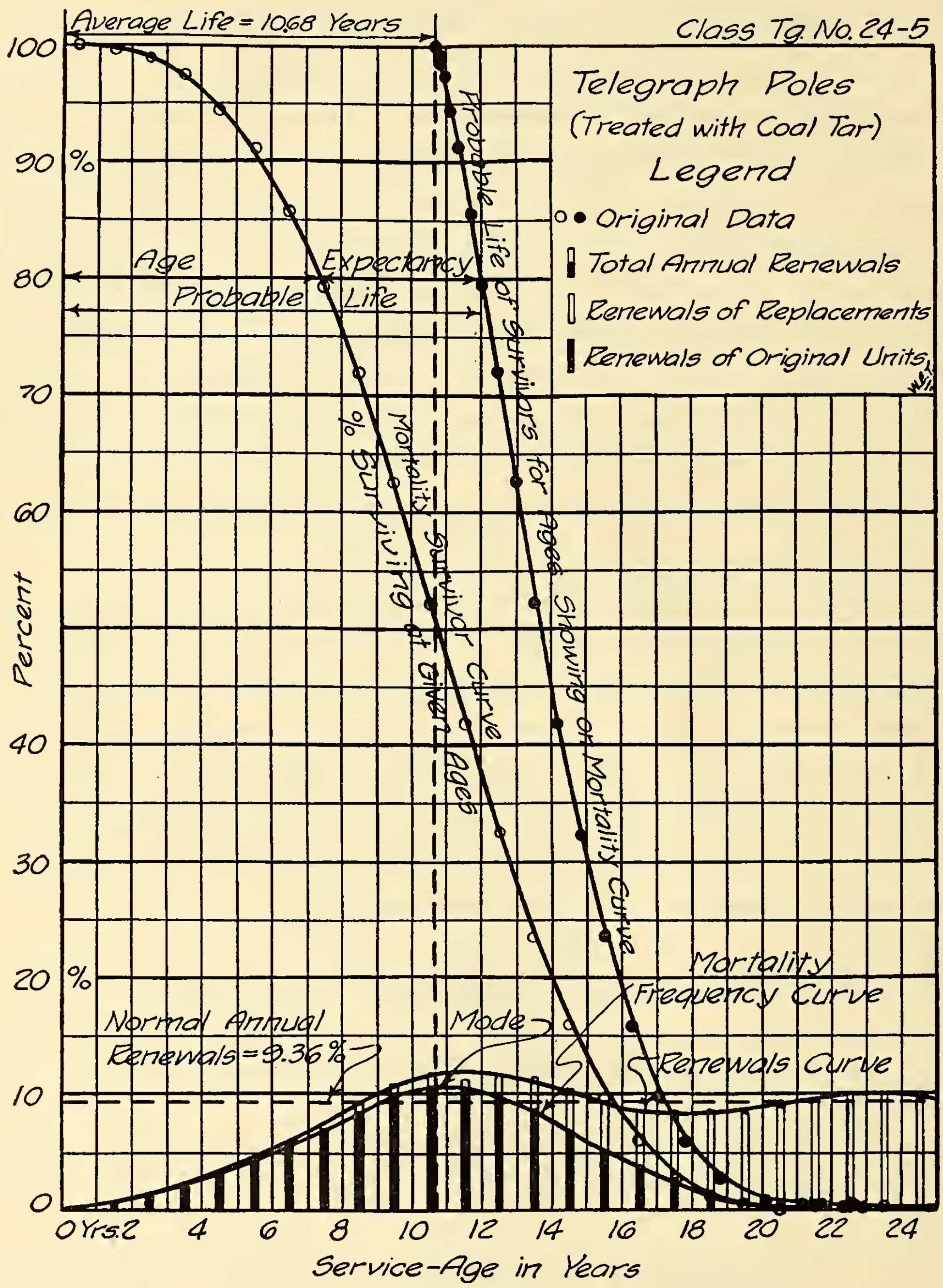


Fig. 3.2.—Mortality curve of 30,009 wooden telegraph poles.

service ages of the original members of large age-groups of like industrial-property units. In this treatise, the probable-life curves are plotted on, and as a part of, both actual and type mortality curves.

Mortality type curves are mortality curves (of the various kinds) showing different classes of mortality characteristics; each typical of the mortality data of several different kinds of industrial-property units.

Mortality renewals curves are curves which show the numbers, or percentages, of renewals which probably will be required from year to year in the future to keep at 100 percent the numbers of units in large mortality groups of like industrial-property units. The rate of annual renewals fluctuates materially for some years, often many, after the group starts service but would eventually become constant.

Mortality condition-percent curves are curves which show, from year to year, for a large age-group of industrial property like units:

1. The condition-percent of the average survivor of an age-group, or
2. The average condition-percent of all the survivors of an age-group, or
3. The average condition-percent of all the units in service at any date, including renewals and renewals of renewals in a mortality group.

3.4. Industrial-property Mortality Curves.—It is believed that knowledge and understanding of the characteristics of industrial-property mortality curves can be attained most readily by study of an actual curve. Figure 3.2 shows various actual mortality curves constructed from the mortality data (collected by Christiani) of 30,009 wooden telegraph poles in Germany, between 1852 and 1905.

The original mortality data for these curves were the numbers of poles retired at different service ages, stated to the nearest full year. These data were reduced to percents of 30,009 and arranged for computation purposes as shown in Table 3.1.

Column (1), Table 3.1, shows the age intervals during which the retirements, reported at ages of 1, 2, 3, etc., years, must be assumed to have occurred, namely, from one-half year before to one-half year after the age in years reported.

Note that if the original mortality data collected, as is sometimes the case, had been the retirements during each complete calendar year after the poles were first put in service (instead of the numbers retired at ages 1, 2, 3, etc.), the age intervals would have been 0 to 1, 1 to 2, 2 to 3, etc. (instead of ½ to 1½, 1½ to 2½, 2½ to 3½, etc.).

Column (2) shows (in percents of the entire original group) the retirements of units during each age interval. These are the ordinates of the frequency curve in Fig. 3.2.

Column (3) shows the percents of units surviving at the beginning of each age interval. These are the ordinates of the survivor curve in Fig. 3.2.

Column (4) shows the services (in percent-years) rendered during each age interval by the average number of poles in service during the interval. Note that each number in column (4) is the area of a vertical strip, one-half year wide for the first age interval and one year wide for all others, of the total area under the survivor curve.

It hence is apparent that the total area under the survivor curve measures the total services rendered by the full 100 percent of the original units in the entire group (30,009 in this case), during their entire respective service lives.

In this case, the total services rendered by the 100 percent of original units was 1,067.88 percent-years. Hence the average life of the group was

$$1{,}067.88 \div 100 = 10.68 \text{ years (Sec. 3.2).}$$

Column (5) shows the remaining services (in percent-years) at the beginning of each age interval, yet to be rendered by those units of the original group not yet

retired. Hence, each number in column (5) shows the area under the survivor curve to the right of the age ordinate at the beginning of the age interval.

TABLE 3.1.—COMPILATION OF THE MORTALITY DATA OF 30,009 WOODEN TELEGRAPH POLES TREATED WITH COAL TAR

Age interval, years	Units retired during age interval, %	Survivors at beginning of age interval, %	Service during age interval, %-years	Remaining service at beginning of age interval, %-years	Expectancy at beginning of age interval, years	Probable-life at beginning of age interval, years
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0 - ½	0.00	100.00	50.00	1,067.88	10.68	10.68
½- 1½	0.35	100.00	99.82	1,017.88	10.18	10.68
1½- 2½	0.74	99.65	99.28	918.06	9.21	10.71
2½- 3½	1.45	98.91	98.19	818.78	8.28	10.78
3½- 4½	3.19	97.46	95.86	720.59	7.39	10.89
4½- 5½	2.96	94.27	92.79	624.73	6.63	11.13
5½- 6½	5.68	91.31	88.47	531.94	5.83	11.33
6½- 7½	6.11	85.63	82.58	443.47	5.18	11.68
7½- 8½	7.28	79.52	75.88	360.89	4.54	12.04
8½- 9½	9.63	72.24	67.42	285.01	3.95	12.45
9½-10½	10.37	62.61	57.43	217.59	3.48	12.98
10½-11½	10.33	52.24	47.07	160.16	3.07	13.57
11½-12½	9.54	41.91	37.14	113.09	2.70	14.20
12½-13½	9.06	32.37	27.84	75.95	2.34	14.84
13½-14½	7.26	23.31	19.68	48.11	2.06	15.56
14½-15½	6.44	16.05	12.83	28.43	1.77	16.27
15½-16½	3.77	9.61	7.73	15.60	1.62	17.12
16½-17½	3.01	5.84	4.33	7.87	1.35	17.85
17½-18½	1.84	2.83	1.91	3.54	1.25	18.75
18½-19½	0.50	0.99	0.74	1.63	1.65	20.15
19½-20½	0.12	0.49	0.43	0.89	1.82	21.32
20½-21½	0.13	0.37	0.31	0.46	1.24	21.74
21½-22½	0.21	0.24	0.13	0.15	0.63	22.13
22½-23½	0.03	0.03	0.02	0.02	0.67	23.17
23½-24½	0.00	0.00	0.00	0.00	0.00	23.50
Total.......	100.00	......	1,067.88	Av. life = 1,067.88 ÷ 100 = 10.68 years		

Column (6), Table 3.1, shows the expectancy, at the beginning of each age interval, of the average-survivor unit at that age.

Since the numbers in column (5) show the areas under the survivor curve to the right of the age ordinates at the beginnings of the age intervals (which areas measure

the services yet to be rendered by the survivor units), while the percents of survivor units are shown in column (3), it is manifest that the expectancies in column (6) must be the quotients of the numbers in column (5) divided by the corresponding numbers in column (3). Hence, the expectancy equation is

$$\text{The expectancy at any age} = \frac{\text{area under mortality curve to right of age ordinate}}{\text{age ordinate } (= \text{survivors at the age})}.$$

Note that the expectancy at service age 0 is the average life of the group.

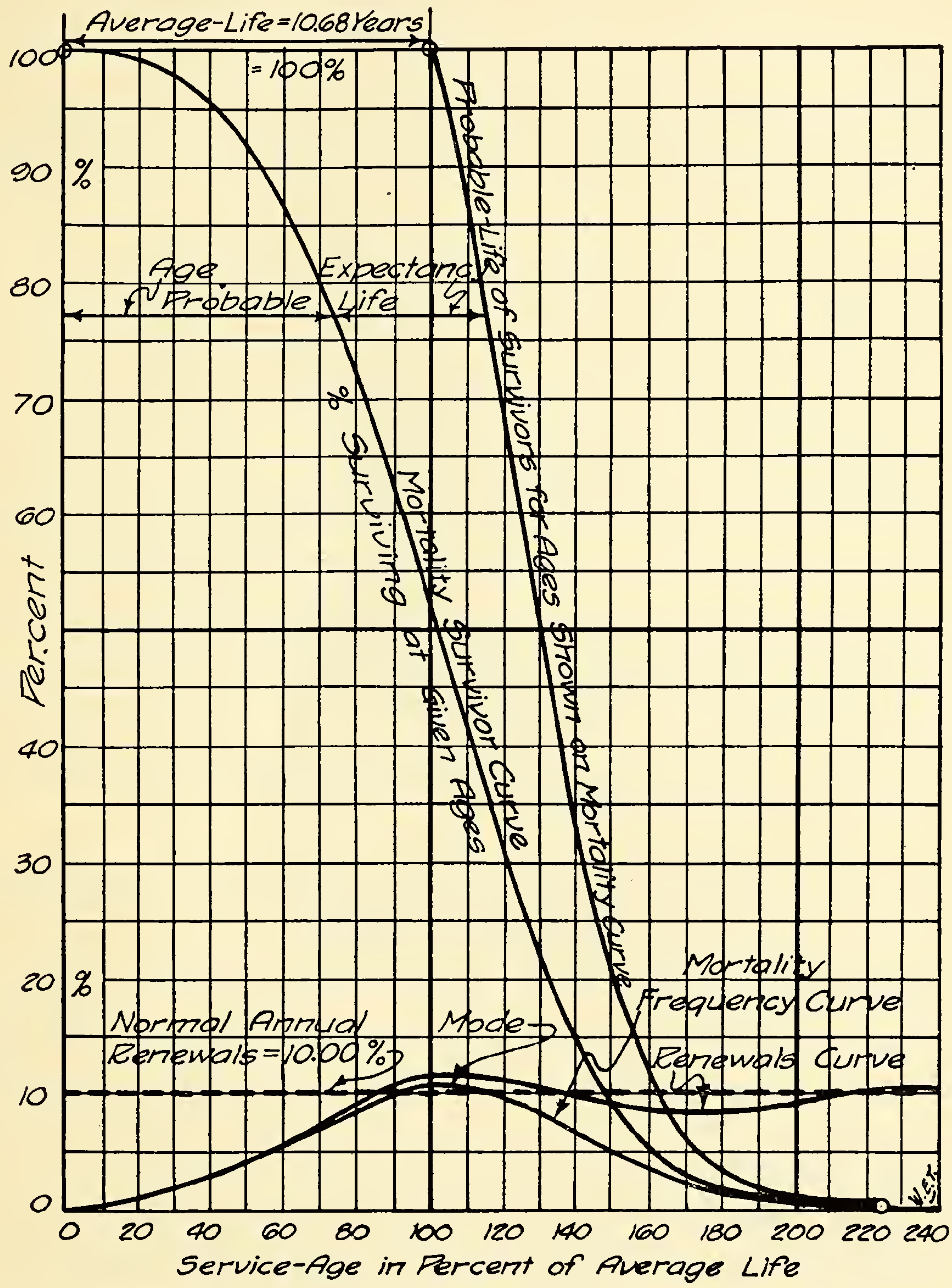


FIG. 3.3.—Mortality curve for wooden telegraph poles generalized in percents of average life.

Column (7) shows the probable lives of the average-survivor units at the beginnings of the respective age intervals. These probable lives are equal, in each case, to *expectancy plus age*. The numbers in column (7) are used to plot the *probable-life curve* in Fig. 3.2, in the manner indicated thereon.

NOTE.—The ordinates of the renewals curve in Fig. 3.2 are obtained by a tedious computation, simple in principle, as indicated in Sec. 3.30.

3.5. Generalized Mortality Curves.—It is obvious that the mortality curve of telegraph poles shown in Fig. 3.2 could not be used directly for any other group of poles having an average life different from that (10.68 years) of the particular group for which the retirement data were obtained. If redrawn, showing the ages in percents of average life, however, it could be used for any other group whose average life is known, or can be estimated with sufficient accuracy; provided, also, that the ordinates, as they do in Fig. 3.2, show the survivors in percents of the total group, instead of in numbers of survivors, as they are sometimes plotted.

Generalized mortality curves are mortality curves plotted with ordinates showing the survivors in percents of the total age-group of like units, and with abscissas showing the service ages in percents of the average life of the group. Figure 3.3 is Fig. 3.2 redrawn in generalized form.

3.6. Derivative Mortality Curves.—The mortality-frequency curve may properly be considered the base curve; from this, a number of derivative mortality curves may be built up; these include the survivor curve, the probable-life curve, the renewals curve, the condition-percent-of-average-survivors curve, the average-condition-percent-of-original-age-group-survivors curve, and the average-condition-percent-of-survivors-of-all-ages curve (this last for groups kept by renewals at 100 percent numbers). These are all shown in Fig. 3.4 (for mortality type curve S_3, see Appendix B). The condition-percent curves in Fig. 3.4 are for 10 years average life and the 6 percent present-worth method (Chap. V).

Note.—In Fig. 3.7, a condition-percent-of-original-age-groups average-survivor curve is shown in addition to the above.

METHODS OF COMPILING INDUSTRIAL-PROPERTY MORTALITY CURVES AND TABLES

Three systematic methods have been developed for compiling industrial-property mortality curves; the *annual-rate* method, the *original-group* method, and the *individual-unit* method. Of these, the annual-rate method is much the best, because it is based on the collection and compilation of the data of *all* units in service during a period of recent years, both those retired and those still in service; the original-group method uses only the data of one installation; the individual-unit method uses only the data of those units which have been retired.

3.7. Annual-rate Method.—The mortality data collected for use in the annual-rate method should be those for a recent normal period, of say 3 to 10 years, which will give retirement rates fairly representative of present and probable future policies and service conditions. The ideal period is one so short that it only reflects present policies and standards; and yet is long enough for sufficient retirements to have been made at

each age to give reliable average retirement rates, helpful in forecasting the future.

For any type of industrial property, the steps in the annual-rate method of compiling a mortality curve are as follows:

1. Determine the numbers of units (or their total values), grouped by ages, of the property retired each year of the mortality period adopted; and the average annual retirements at each age throughout the period. Table 3.2 illustrates the compilation of data of the annual retirements of the water stations of a large railroad system during a five-year mortality period, 1922 to 1926, inclusive.

2. Determine the numbers of units (or their total values), grouped by ages, of the property in service at the beginning of each year of the mortality period adopted; and the average annual total numbers of units (or their total values) of the property of

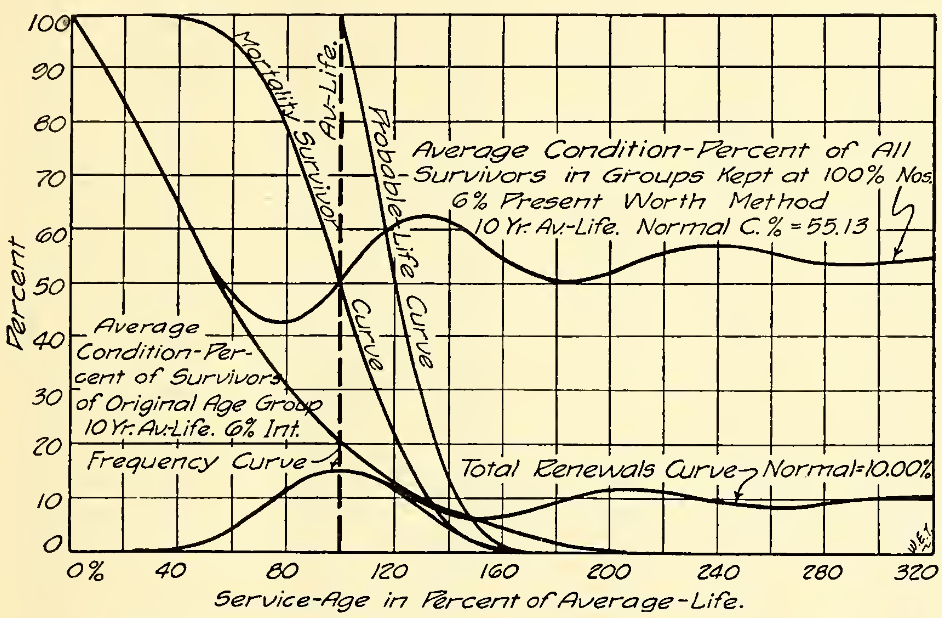


FIG. 3.4.—Mortality and other curves for type Curve S_3.

each age in service at the beginning of the average year. Table 3.3, corresponding to Table 3.2, illustrates the compilation of the data of the average values of the water stations of different ages in service, on the railroad studied, during each year, 1922 to 1926, inclusive.

3. Using the average annual retirements, at each age, determined as in 1, above; and the average amounts of property in service, of each age, determined as in 2, above; calculate:

a. The retirement rate of the property in service at each age.

b. The decrease during each service-age interval in the percent of survivors (by multiplying the retirement rate for that age by the percent of survivors at the beginning of the service-age interval).

c. The percent of survivors at the end of each age interval for which the necessary data were found.

Table 3.4, corresponding to Tables 3.2 and 3.3, illustrates the calculation of the retirement ratios and the percents of survivors, outlined herein. The retirement rates (in percents), in column (5), are equal to 100 × (3) ÷ (4).

The percents remaining in service, in column (6), are obtained by successively *subtracting* the products of the retirement rates in column (5) from the preceding percents of survivors in column (6).

Table 3.2.—Annual Retirements of Water Stations in a Large Railroad System

Age interval, years		Annual retirements					Total retirements, 1922–1926, incl.	Average retirements, 1922–1926, incl.
From	To	1922	1923	1924	1925	1926		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	½	....	....	....	....	....	0	0
½	1½	$ 100	$ 169	....	....	$ 105	$ 374	$ 75
1½	2½	....	1,207	$ 2,721	$ 1,453	473	5,854	1,171
2½	3½	1,401	1,067	1,450	601	3,240	7,759	1,552
3½	4½	40,407	8	2,213	1,807	5,558	49,993	9,998
4½	5½	307	3,575	294	1,281	2,727	8,184	1,637
5½	6½	1,216	309	6,755	2,239	1,500	12,019	2,404
6½	7½	....	174	535	24,421	16,808	41,938	8,388
7½	8½	87	602	589	3,890	4,612	9,780	1,956
8½	9½	1,694	3,150	1,526	16	305	6,691	1,338
9½	10½	25	....	....	1,011	1,829	2,865	573
10½	11½	1,167	....	2,424	20,094	767	24,452	4,890
11½	12½	2,218	145	5,604	3,663	19,543	31,173	6,235
12½	13½	....	1,925	....	14,848	931	17,704	3,541
13½	14½	392	7,321	2,438	2,293	17,803	30,247	6,049
14½	15½	1,718	239	1,765	2,025	7,416	13,163	2,633
15½	16½	1,756	4,968	....	74	5,352	12,150	2,430
16½	17½	550	609	1,591	4,773	224	7,747	1,549
17½	18½	416	....	182	13,299	2,287	16,184	3,237
18½	19½	....	....	4,473	4,005	....	8,478	1,696
19½	20½	14,871	1,743	1,388	1,140	28,378	47,520	9,504
20½	21½	2,813	1,045	49,380	1,431	8,216	62,885	12,577
21½	22½	2,046	1,981	3,803	2,826	1,194	11,850	2,370
22½	23½	4,598	3,820	16,154	....	....	24,572	4,914
23½	24½	3,515	1,002	10,694	18,561	2,473	36,245	7,249
24½	25½	....	1,123	....	12,729	2,979	16,831	3,366
25½	26½	1,818	....	1,137	1,630	7,917	12,502	2,500
26½	27½	....	....	....	1,773	701	2,474	495
27½	28½	....	1,668	....	....	5,214	6,882	1,376
28½	29½	413	....	3,919	....	185	4,517	903
29½	30½	....	558	....	3,189	....	3,747	749
30½	31½							
31½	32½	....	....	1,587	....	....	1,587	317
32½	33½	1,638	....	....	....	2,181	3,819	764
33½	34½	7,047	....	....	1,516	....	8,563	1,713
34½	35½	....	2,409	....	....	....	2,409	482
35½	36½							
36½	37½	....	....	....	....	3,563	3,563	713
37½	38½	....	....	....	3,001	....	3,001	600
38½	39½	1,782	....	....	....	1,913	3,695	739
39½	40½							
40½	41½	....	....	488	....	....	488	98
41½	42½							
		$93,995	$40,817	$123,110	$149,589	$156,394	$563,905	$112,781

Table 3.3.—Cost of Water Stations in Service at Each Age for Each Year of the Observation Period

Average age, years	Property in service each year					Total for 5-year period, 1922–1926	Average cost for 5-year period, 1922–1926
	1922	1923	1924	1925	1926		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
½	$ 97,382	$ 109,879	$ 167,720	$ 178,549	$ 92,532	$ 646,062	$ 129,212
1½	482,054	97,282	109,710	167,720	178,549	1,035,315	207,063
2½	361,632	482,054	96,075	106,989	166,267	1,213,017	242,604
3½	372,693	360,231	480,987	94,625	106,388	1,414,924	282,985
4½	193,924	332,250	360,223	478,774	92,818	1,457,989	291,598
5½	43,906	193,617	328,641	359,929	477,493	1,403,586	280,717
6½	344,386	42,690	193,263	321,886	357,690	1,259,915	251,983
7½	406,709	344,386	42,516	192,728	297,465	1,283,804	256,761
8½	95,360	406,622	343,784	41,927	188,838	1,076,531	215,306
9½	567,253	93,666	403,472	342,258	41,911	1,448,560	289,712
10½	45,407	567,228	92,704	403,472	341,247	1,450,058	290,012
11½	229,259	44,240	565,852	90,280	383,378	1,313,009	262,602
12½	84,747	227,041	44,095	560,248	86,617	1,002,748	200,550
13½	91,145	84,430	225,116	44,095	545,400	990,186	198,037
14½	249,157	90,753	77,109	222,678	41,802	681,499	136,300
15½	407,510	247,439	90,514	75,344	220,653	1,041,460	208,292
16½	204,050	405,754	242,471	90,514	75,270	1,018,059	203,612
17½	69,969	203,500	405,145	240,880	85,741	1,005,235	201,047
18½	131,413	69,553	203,500	404,963	227,581	1,037,010	207,402
19½	92,568	131,413	69,553	199,027	400,958	893,519	178,704
20½	142,238	77,697	129,670	68,165	197,887	615,657	123,131
21½	320,697	139,425	76,652	80,290	66,734	683,798	136,760
22½	51,158	317,742	137,444	72,849	77,464	656,657	131,331
23½	121,575	46,560	313,922	121,290	72,849	676,196	135,239
24½	15,516	118,060	45,558	303,228	102,729	585,091	117,018
25½	45,850	15,516	116,937	45,558	290,499	514,360	102,872
26½	62,377	44,032	15,516	115,800	43,928	281,653	56,331
27½	7,290	62,377	44,032	15,516	114,027	243,242	48,648
28½	44,036	7,290	60,709	44,032	15,516	171,583	34,317
29½	4,827	43,623	7,290	56,790	44,032	156,562	31,312
30½	9,597	4,827	43,065	7,290	53,601	118,380	23,676
31½	37,462	9,597	4,827	43,065	7,290	102,241	20,448
32½	9,635	37,462	9,597	3,240	43,065	102,999	20,600
33½	12,725	7,997	37,462	9,597	3,240	71,021	14,204
34½	5,397	5,678	7,997	37,462	8,081	64,615	12,923
35½	5,767	5,397	3,269	7,997	37,462	59,892	11,978
36½	....	5,767	5,397	3,269	7,997	22,430	4,486
37½	5,487	....	5,767	5,397	3,269	19,920	3,984
38½	2,754	5,487	....	5,767	2,396	16,404	3,281
39½	....	972	5,487	....	5,767	12,226	2,445
40½	....	....	972	5,487	....	6,459	1,292
41½	2,006	....	....	484	5,487	7,977	1,595
42½	....	2,006	....	....	484	2,490	498
43½	....	....	2,006	....	....	2,006	401
44½	....	....	....	2,006	....	2,006	401
45½	....	....	....	....	2,006	2,006	401
46½							
	$5,476,918	$5,491,540	$5,616,026	$5,671,465	$5,614,408	$27,870,357	$5,574,071

Table 3.4.—Method of Obtaining Annual Retirement Rates and Percentage of Property Remaining in Service

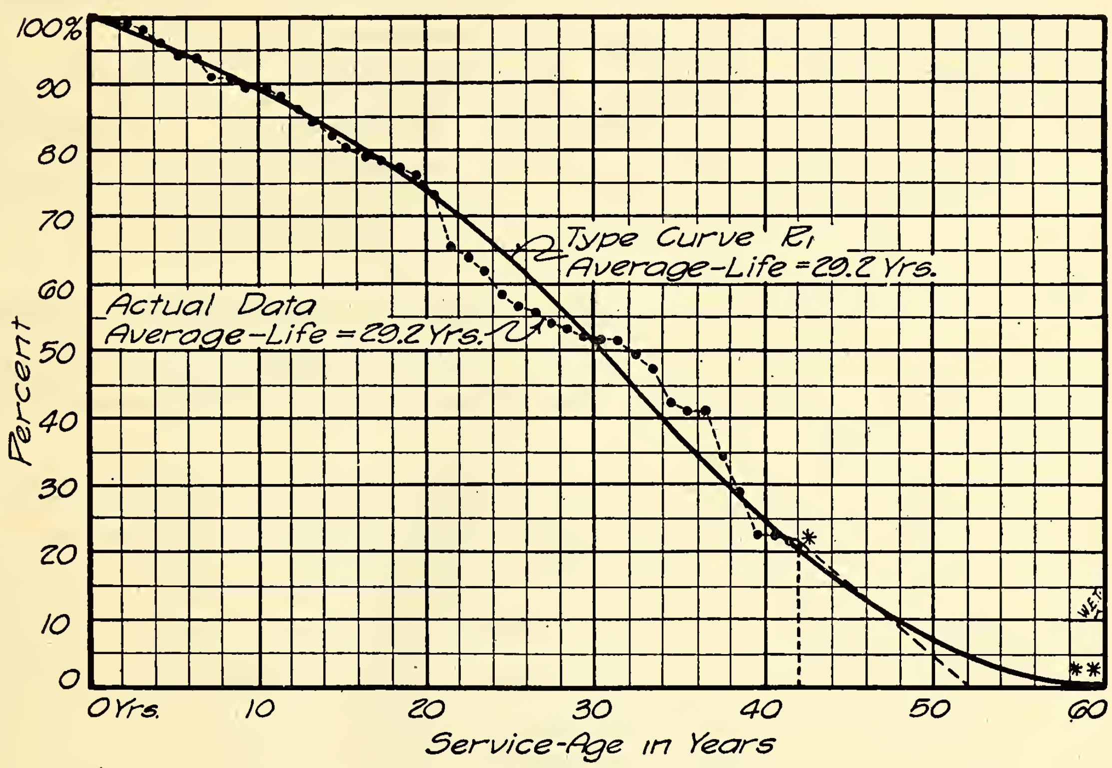
Age interval, years		Average retirement during interval	Average cost of property in service at age in column (1)	Retirement rates in percents, 100 × (3) ÷ (4)	Percents of survivors at age in column (2)
From	To				
(1)	(2)	(3)	(4)	(5)	(6)
0	½	0	....	......	100.000
½	1½	$ 75	$ 129,212	0.058	99.942
1½	2½	1,171	207,063	0.566	99.376
2½	3½	1,552	242,604	0.640	98.740
3½	4½	9,998	282,985	3.533	95.252
4½	5½	1,637	291,598	0.561	94.718
5½	6½	2,404	280,717	0.856	93.907
6½	7½	8,388	251,983	3.329	90.781
7½	8½	1,956	256,761	0.762	90.089
8½	9½	1,338	215,306	0.621	89.530
9½	10½	573	289,712	0.198	89.353
10½	11½	4,890	290,012	1.686	87.847
11½	12½	6,235	262,602	2.374	85.762
12½	13½	3,541	200,550	1.766	84.247
13½	14½	6,049	198,037	3.055	81.673
14½	15½	2,633	136,300	1.932	80.095
15½	16½	2,430	208,292	1.167	79.160
16½	17½	1,549	203,612	0.761	78.558
17½	18½	3,237	201,047	1.610	77.293
18½	19½	1,696	207,402	0.818	76.661
19½	20½	9,504	178,704	5.318	72.584
20½	21½	12,577	123,131	10.214	65.170
21½	22½	2,370	136,760	1.733	64.041
22½	23½	4,914	131,331	3.742	61.645
23½	24½	7,249	135,239	5.360	58.341
24½	25½	3,366	117,018	2.876	56.663
25½	26½	2,500	102,872	2.430	55.286
26½	27½	495	56,331	0.879	54.800
27½	28½	1,376	48,648	2.828	53.250
28½	29½	903	34,317	2.631	51.849
29½	30½	749	31,312	2.392	50.609
30½	31½	....	23,676	......	50.609
31½	32½	317	20,448	1.550	49.825
32½	33½	764	20,600	3.709	47.977
33½	34½	1,713	14,204	12.060	42.191
34½	35½	482	12,923	3.730	40.617
35½	36½	....	11,978	......	40.617
36½	37½	713	4,486	15.894	34.161
37½	38½	600	3,984	15.060	29.016
38½	39½	739	3,281	22.524	22.480
39½	40½	....	2,445	......	22.480
40½	41½	98	1,292	7.585	20.775*
41½	42½	....	1,595	......	20.775*
42½	43½	....	498	......	20.775*
43½	44½	....	401	......	20.775*
44½	45½	....	401	......	20.775*
45½	46½	....	401	......	20.775*
46½	47½	....	0	......	20.775*
		$112,781	$5,574,071		

* No record of retirements at ages over 41.

4. Determine the average life and the mortality type curve corresponding to the survivor percents found in 3, above. This can be done by plotting the survivor-percent data, so far as they extend, to give points on an actual mortality curve for the property units studied, all as shown in Fig. 3.5 herewith, and then proceeding as follows:

   a. Make an approximate extension of the actual mortality curve to completion, freehand, as in Fig. 3.5, keeping in mind the shapes of the mortality type curves in Appendix B.

   b. Compute the approximate average life by dividing the total area under the approximate actual mortality curve by 100. See column (4), Table 3.1, for the detailed procedure.



*No record of later retirements. Broken line from 42 to 52 years used in computating the average-life.
**Mortality type curve continues farther.

FIG. 3.5.—Mortality curve of railway water stations by annual rate method. (See Table 3.4.)

   c. Select and plot the mortality curve best fitting the actual mortality data by direct comparison with the mortality type curves given in Appendix B, using first the approximate average life computed, but varying it to get a better fit if necessary.

   d. Adopt the average life indicated by the best-fitting type curve.

   e. The computations of expectancies and probable lives are the same for the annual-rate method as shown in columns (4), (5), and (6), Table 3.1, for the individual-unit method.

NOTE.—Study of many complete mortality curves has shown that the correct mortality type curve to use can almost always be selected with small error when the mortality data collected include sufficient retirements to reduce the survivor percents in column (6), Table 3.4, to 75 percent or less.

3.8. Original-group Method.—In this method, which is applicable only to large original age-groups of like units, the actual mortality

curve of the group is built up gradually during the service lives of the original-age-group survivors; complete retirement-data records are kept on a suitable age-group property ledger and are plotted from year to year on a suitable blank form thereon; the mortality type curve best fitting the data is determined from time to time.

Forecasts of future retirements and other future mortality data are made by the use of the mortality type curves found to fit the data best. At the start the type curve is chosen by judgment; as the data of the actual retirements of the property units originally in the group accumulate from year to year, the right type curve to use is found by comparison; it may be changed as often as the actual retirements require.

3.9. Individual-unit Method.—In this method, only data of units already retired are used in compiling the mortality curve. The mortality data collected are tabulated by groups; each group shows the number of units (or their total value) retired at a given age, usually stated to the nearest full year.

Actually, the units in each such group have been retired at different times scattered over an age interval of one year, beginning one-half year before and ending one-half year after the given age of retirement, stated to the nearest full year.

For convenience in the use of the mortality curve, it is customary to reduce the numbers of units retired in the different age intervals to percents of the total units retired at all ages; from these the percents of survivors at the beginning of each age interval are readily computed. The mortality data are then in the form shown in columns (1), (2), (3), Table 3.1, which see. Note that the mortality data shown in columns (1), (3), Table 3.1, correspond precisely to those shown in columns (1), (2), (6), Table 3.4, for the annual-rate method. The computations of expectancies and probable lives, shown in columns (4), (5), (6), Table 3.1, would be of the same character for both the annual-rate and the individual-unit methods.

Figure 3.2, for 30,009 wooden telegraph poles, treated with coal tar, retired in Germany between 1852 and 1905, is a good example of a mortality curve compiled by the individual-unit method but based on adequate retirement data.

However, the individual-unit ("grave-yard") method is not satisfactory for compiling mortality curves which are to be used as aids in forecasting future service lives, or other future mortality data, of industrial-property units:

1. When the retirement data, as in the case of those for Fig. 3.2, extend over a period at least as long as the service life of the oldest survivor, they are too old to reflect advances in the arts and sciences and changes in management policies.

2. When the retirement data, as is often the case, extend only over a period shorter than the service life of the longest survivor, it is manifest that the mortality curve fails

to include the longest lived units; hence, the true average life of like units under like conditions will be longer than the average life computed from the curve.

Figure 3.6 shows annual-rate and individual-unit mortality curves for the alternating-current electric current meters in a large utility plant.

Mortality data were obtained for each year, 1925 to 1932, inclusive; a total of about 44,000 meters served during all or part of the eight years; of these, 1,043 were retired during the eight years.

The annual-rate curve data, from the entire 44,000 meters, closely fit an R_4 mortality type curve with an average service life of 30 years. This gives the best indication obtainable of the probable future of the 43,000 meters still in service in 1932 unless new inventions or some material change of policy intervene.

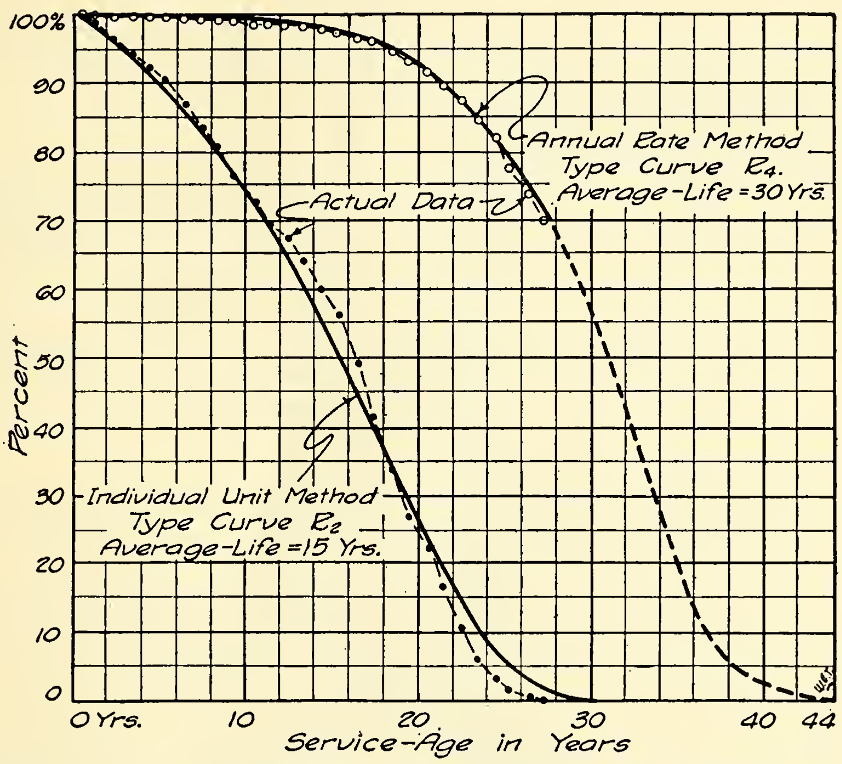


FIG. 3.6.—Annual rate and individual unit mortality curves of the alternating-current electric current meters in a large utility plant, 1925–1932, inclusive. About 44,000 meters, 1,043 retirements, 1925–1932.

On the other hand, the individual-unit mortality curve, made from the data of only the 1,043 meters actually retired during 1925 to 1932, obviously indicates altogether too short an average life. If retirement data are kept of the 43,000 meters yet in service until the last one is retired, there is every reason to expect that the resulting individual-unit curve would not greatly differ from the annual-rate curve in Fig. 3.5, but, by then (say 1975), the retirement data are very likely to be mainly of historical interest only.

INDUSTRIAL-PROPERTY MORTALITY TYPE CURVES

The researches of the Iowa Engineering Experiment Station have shown that the almost numberless different kinds of industrial property in service can be classified, so far as their mortality characteristics are concerned, in a limited number of mortality types. The station has

developed and published[1] 18 mortality type curves, which are presented in Appendix B. This number is likely to be increased later to 20.

3.10. The 18 Industrial-property Mortality-type-frequency Curves.—The bases of the classification of the 18 mortality type curves are the locations and heights of the modes (Sec. 3.3) of their frequency curves. As shown in Figs. B.1 to B.3, Appendix B, six of the frequency curves, designated L_0, L_1, L_2, L_3, L_4, L_5, are skewed to the left, having their modes to the left of their average-life age ordinates; seven, designated S_0, S_1, S_2,

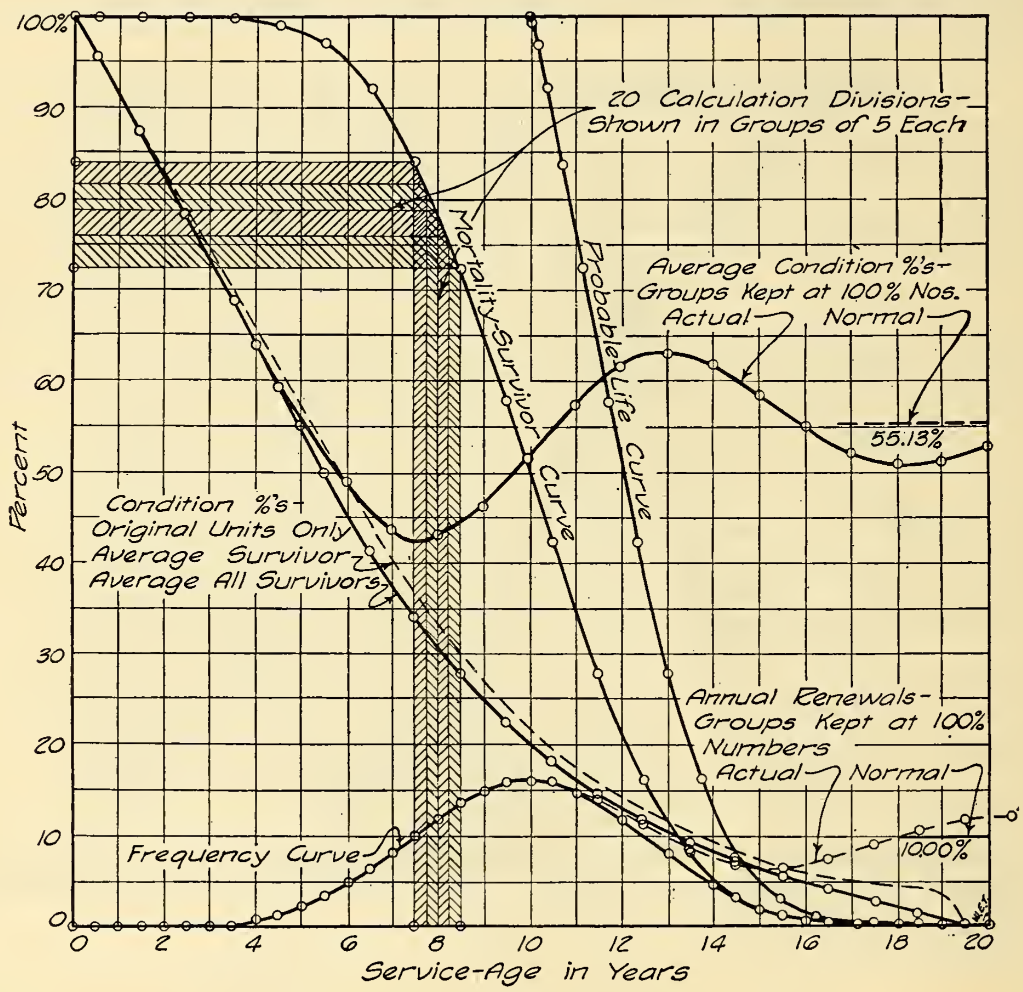


FIG. 3.7.—Mortality curve for S_3 type group of property units. To illustrate the computations for 10 years average life and 6 % return.

S_3, S_4, S_5, S_6, are symmetrical, having their modes on their average-life age ordinates; and five, designated R_1, R_2, R_3, R_4, R_5, are skewed to the right, having their modes to the right of their average-life age ordinates.

In all three groups, the respective curves are given subscript numbers increasing with the heights of their modes; in the L group the mode of L_0 is farthest left, while in the R group the mode of R_1 is farthest right.

3.11. The 18 Industrial-property Mortality-type-survivor and Probable-life Curves.—The 18 mortality type curves are presented in Figs. B.4

[1] WINFREY, ROBLEY, Statistical Analyses of Industrial Property Retirements, *Bull.* 125, *Iowa Eng. Exp. Sta.*, 1936.

to B.6, Appendix B. Each of them includes both a survivor curve and a probable-life curve. Note that, to permit the general use of these curves, it has been necessary to "generalize" (Sec. 3.5) them by plotting the service ages in percents of average life. Hence, to apply these type curves to any particular kind of industrial-property units, it is necessary first to ascertain the average life of like units under like service conditions. The particular type curve to use must then be selected by methods to be explained in Secs. 3.15 and 3.16.

The six "left-mode" type curves, shown in Fig. B.4, apply to kinds of property whose distinguishing mortality characteristic is that materially large retirements begin quite early in the service life; property units of the kinds whose mortality characteristics correspond to the "right-mode" curves, of Fig. B.6, remain in service comparatively long, in percentages of average life, before high retirement rates begin.

3.12. The 18 Industrial-property Renewals Type Curves.—Three examples of these are shown in Fig. B.8, Appendix B. Like the survivor curves and the probable-life curves, their coordinates are plotted in percents of survivors and percents of average life. See Sec. 3.30 for further discussion. Table B.3, Appendix B, gives the results of the renewals calculations for all of the 18 type curves.

THE SELECTION OF THE PARTICULAR TYPE CURVES TO USE FOR DIFFERENT KINDS OF INDUSTRIAL PROPERTY

Industrial-property mortality curves are so new that no adequate publication of mortality data has yet been made which will enable the valuation engineer to select the right type curves to use for many kinds of property without first collecting mortality data of the property and then comparing its mortality characteristics with those of the different curves.

To do this, and to make use of any type curve selected, it is first necessary to ascertain the average life in service of the units of the particular kind of industrial property under consideration. Two methods are available for estimating the average life during service of any particular kind of industrial-property units:

1. *From Actual Mortality-curve Data.*—This is much the better method whenever it is practicable to get the necessary data.
2. *By the aid of average-life tables* supplemented by any other retirement data possessed or readily obtainable. This method should be used only when it is impracticable to get data for an actual mortality curve for the particular kind of industrial property under consideration.

3.13. Use of Mortality-curve Data.—Average lives of different kinds of industrial property should always be determined from mortality-curve data. The only good reason for tolerating estimates by any other

method is lack of the necessary mortality data, combined with the impossibility of obtaining them without unwarranted effort and cost. Other methods substitute mere guesses for facts.

The mortality-curve data should always be compiled by the annual-rate or the original-group method in forecasting average lives (Secs. 3.7, 3.9, and Fig. 3.5).

The method of calculating average life from the data of a mortality curve (constructed by any method) has already been described in Sec. 3.4 and is illustrated in column (4), Table 3.1. The average-life equation is

$$\text{Average life} = \frac{\text{total area under the mortality curve}}{100(\text{when survivor ordinates are in percents})}.$$

Numerical Example: The average life of the railroad water station, whose annual-rate mortality curve is given in Table 3.4 and Fig. 3.5, is calculated as follows:

Area under curve, age 0–½	50.000
Area under curve, age ½–1	49.985
Area under curve, age 1–42 (sum of ordinates 1½–41½)	2,717.970
Area of triangle, age 42–52 (20.775)(1%)	103.875
Total area under mortality curve	2,921.830

$$\text{Average life of the water station} = \frac{2{,}921.83}{100} = 29.2 \text{ years.}$$

3.14. Use of Average-life Tables.—It has long been the custom for valuation engineers, utility and other industrial-property owners, regulatory commissions, and income-tax administrators, to make extensive use of published tables[1] of the average lives of different kinds of industrial property.

An extensive average-life table is presented in Appendix A; it may be referred to as an aid to the valuator (especially when supplemented by additional data of his own) in estimating average lives in those cases in which it is really impracticable to get actual mortality-curve data.

Numerical Example: The average life of units similar to a steam turbine engine, of which data obtained by personal examination are

1. Service age, 15 years.
2. Present physical condition, good for its age.
3. Past service conditions, favorable.
4. Probable future service conditions, mild.

Referring to the average-life table in Appendix A, we find estimates of the average lives of steam turbines ranging from 15 to 30 years, with a majority around 20 years.

In view of the favorable past and probable mild future service conditions of this engine, it would not be unreasonable to estimate the average life of similar engines under similar service conditions at 25 years.

NOTE.—Estimates of average life when actual mortality-curve data cannot be had will be much more worthy of respect when the valuator has personally collected mor-

[1] Critical study of the various published average-life tables indicates that in considerable degree they have been compiled from each other; and that in general they indicate the practice of valuators rather than actual mortality data.

tality data of his own and uses published life tables merely to supplement his data and help his judgment.

3.15. Selection of Mortality Type Curves.—The best way to select the right mortality type curves to use in forecasting the service lives of industrial-property units is by direct comparison with actual mortality-curve data; especially until continued research overcomes the present lack of actual mortality curves for more than a limited fraction of the myriad different kinds of industrial property.

When actual mortality-curve data for the particular kind of property under consideration are available (or can be collected with reasonable effort and cost), the process of making the selection is one of "cut and try," until the most satisfactory fit of curve and data is secured.

Usually, the approximate average life is first calculated from the data, so that the service-age abscissas, as well as the survivor ordinates, can be plotted in percents to the same scale as the type curves; the first average life tried should be changed, if necessary, as may be required to get the best fit.

Note.—The Iowa Engineering Experiment Station is preparing, and doubtless will publish later, sets of curves of each mortality type plotted in years (for a series of different average lives); this is to facilitate the selection of the right curve without a preliminary calculation of the approximate average life.

3.16. Table of Mortality Type Curves.—Table B.1, Appendix B, shows the mortality type curves which, by direct comparison as described in Sec. 3.15, some 176 actual mortality curves compiled by the Iowa Engineering Experiment Station have been found to fit best. In the absence of an actual mortality curve of the particular property units he is considering, the valuator can often select the right mortality type curve to use by referring to Table B.1.

3.17. Tentative Type Curves and Average Lives.—Sometimes the kind of property is not listed in Table B.1, and actual mortality-curve data for the particular kind of industrial property under consideration are unobtainable at reasonable effort and cost.

The valuator may then use the two, or more, tentative type curves (and, if necessary, the two, or more, tentative average lives) judged necessary to forecast retirement dates within the range of the probable uncertainty which they indicate. This range will often be found to be surprisingly small.

3.18. Use of Mortality Type Curves and Probable Lives.—The method of using mortality type curves to assist the valuator in forecasting retirement dates is clearly described on pages 516 to 519 of Appendix B. It must always be remembered that mortality curves cannot take the place of the valuator's judgment. However, they enable him to base his judgments on facts instead of guesses.

THE ESTIMATION OF THE PROBABLE LIVES, IN SERVICE, OF INDUSTRIAL-PROPERTY UNITS

It was stated (Sec. 1.13) that the fundamental basis of value is prevailing opinion as to the present worth of probable future returns from a property. One of the first steps, therefore, in estimating the present value and the total and annual depreciation of industrial property must be to forecast for each unit, or group of like units, its probable number of future years in service; adding this to its service age gives its probable life, in service.

3.19. Necessity of Estimates of Probable Lives.—It is becoming increasingly important and increasingly feasible to keep the records of total and annual actual depreciation and the book-value accounts of industrial properties constantly up to date. This is accomplished by re-estimating the probable life and the total actual depreciation, and thereby the actual annual depreciation, every year, for every separate unit or age-group of like units of physical property. Such yearly estimates of actual depreciation are necessary in order to determine the true costs of each commodity and/or service produced, the true net return from the property each year, and its true fair value at any given date.

Correct accounts of all of these are required for sound rate and price making, for the correct determination of income and other taxes, for the correct determination of true profit or loss each year, and for the wise management and conservation of the property.

3.20. Method of Estimating Probable Lives.—The estimation and re-estimation of the probable lives of the separate physical units of an industrial property require examinations from time to time of every unit, or age-group of like units, of the property by well-qualified persons, who ascertain the changes in physical condition and in service conditions throughout actual service life. The steps in re-estimating the probable lives of the units or age-groups of like units should be about as follows:

1. Make actual direct examination of each unit or age-group of like units, ascertaining and recording its service age. With the aid of personal judgment, determine and record:

   a. The present physical condition, FOR ITS AGE.

   b. The past service conditions.

   c. The probable future service conditions.

Note.—In addition, it will generally be necessary to make a numerical estimate of the "probable future operation-return ratio," to permit compensating for lowered operating efficiency or lessened value of service due to age (Sec. 4.28).

Rating Scales.—In deciding and in recording the physical condition and the service conditions, use the following (or similar) rating scales:

Physical Condition.—Excellent (E), good (G), average (A), poor (P), bad (B), FOR THE AGE.

Service Condition.—Mild (M), favorable (F), average (A), unfavorable (U), severe (S).

2. Estimate the average life of similar units under similar service conditions (Secs. 3.13, 3.14). One object of estimating the average life is to permit the use of the mortality type curves in Appendix B, which are plotted in percents of average life.

3. Select the mortality and probable-life curve, in Appendix B, which best fits the mortality characteristics of similar property (for methods see Secs. 3.15, 3.16, 3.17, and pages 516 to 519 in Appendix B).

4. Estimate the probable life of the property unit, or age-group of similar units, using the mortality type curve selected, and making appropriate allowances for the physical condition, for the age, and for the probable future service conditions found in this case (for methods see Sec. 3.18, and page 518 in Appendix B).

3.21. Estimation of the Probable Life of a Water Station.—The method described in Sec. 3.20, for estimating the probable life, in service, of an industrial-property unit, will be applied to the case of a railroad water station on a certain railroad, as follows:

An actual mortality curve for the water stations on this railroad has already been compiled by the annual-rate method in Sec. 3.7; the resulting curve is shown in Fig. 3.5. The process of estimating the probable life of the particular water station in question would work out about as follows:

1. By personal examination the data of this station are found to be: service age, 35 years; physical condition, good, for its age, probable future service conditions, favorable.

2. The average life of water stations on this railroad has already been calculated in Sec. 3.13 from the actual mortality curve of water stations to be 29.2 years.

3. The selection of the mortality type curve to use might proceed as follows:

From the actual mortality curve of water stations on this railroad, it is noted that the percentages of survivors at different service ages (in percents of average life) are

% of average life	Service age, years	% surviving
25	7.3	92
50	14.6	81
75	21.9	70
100	29.2	52
125	36.5	34
150	43.8	17

Comparing the above coordinates with the mortality type curves in Appendix B, it is at once apparent that, without any doubt, the correct type curve to select is R_1.

4. To estimate the probable life of this particular water station, it is found, by reference to mortality type curve R_1, that the average-survivor water station 35 years old would have a probable life of 148 percent of the average life of water stations on this railroad; and, further, that only 2½ percent of stations have probable lives greater than 180 percent of average life.

This particular station has a better physical condition and will serve under better probable future service conditions than the average survivor. Its probable life is evidently somewhere between 148 and 180 percent and may reasonably be estimated at 160 percent of average life.

Answer: The probable life of this water station is 47 years. Its expectancy, of future service is 47 − 35 = 12 years.

3.22. Compilation and Use of a Mortality Curve for Grain Binders.—By personal canvass of Iowa farmers in 1925, Lew Wallace obtained the mortality data[1] given in Table 3.5 of some 91 grain binders, of which 36 had been retired and 55 were still in service. The student may be required to compile these data and construct mortality curves by the annual-rate method, the individual-unit method, or both. He may further be required actually to use the annual-rate-method curve to estimate the probable lives of different individual binders; with different service ages, different physical conditions and different probable future service conditions.

Table 3.5.—Mortality Data of 91 Iowa Grain Binders

Age interval, years	No. still in use	No. retired	Age interval, years	No. still in use	No. retired
0 – ½					
			15½–16½	2	1
1 – 1½			16½–17½	1	1
1½– 2½			17½–18½	1	2
2½– 3½	1		18½–19½		
3½– 4½	1	1	19½–20½		2
4½– 5½	8	2			
			20½–21½	1	2
5½– 6½	3	2	21½–22½		
6½– 7½	2		22½–23½		
7½– 8½	6		23½–24½		
8½– 9½	3	1	24½–25½		
9½–10½	7	6			1
10½–11½	6		25½–26½	1	1
11½–12½	3	3	26½–27½	1	1
12½–13½	2	2	27½–28½		1
13½–14½	2	3	28½–29½		
14½–15½	3	3	29½–30½	1	1
			30½–31½		
			Total	55	36

MORTALITY-PROBABILITY THEORIES AS APPLIED TO INDUSTRIAL PROPERTY. CALCULATIONS OF CONDITION-PERCENTS AND OF RENEWALS

Sections 3.1 to 3.22 have been written for the main purpose of giving the reader correct and readily understandable descriptions and

[1] Wallace, Lew E., Depreciation of Farm Machinery, *Trans. Am. Soc. Agr. Eng.*, **19,** 139 (1925).

explanations, first, of frequency, survivor, and probable-life mortality curves of industrial-property units; second, of the actual construction of such curves from suitable collections of mortality facts; third, of the use of such curves to guide and assist the judgment of valuators in forecasting the probable lives, in service, of particular industrial-property units, and/or groups of like units, not yet retired.

This part will be devoted, first, to consideration of the relation of mortality curves based on mortality facts to actuarial theories of mathematical mortality probabilities; second, to an explanation of the somewhat tedious calculations required to determine the average condition-percents, at different ages, of the survivors of the original units of an original age-group of like units, and/or the future renewals and average condition-percents of all the units in an age-group maintained at 100 percent numbers by continued renewals.

3.23. Basis of the Mortality Curves Presented Herein.—As must be apparent from the descriptions and explanations in the preceding parts of this chapter, all mortality curves herein, including the type curves, are based entirely on collections of actual mortality facts. The collected data have been plotted to large scales, and the curves have been hand-graduated to fit the facts. The mortality facts have been compiled in forms consistent with probability laws, but points on the curves have not been determined by mere calculation from general equations; restriction to use of such equations would undoubtedly impair that close connection between theory and facts without which theories are worse than useless.

3.24. Mortality-frequency-curve Equations.—Section 3.24 will be devoted to a summary of the mathematical theory of mortality-frequency curves as developed by actuarial mathematicians. For any really comprehensive study of the subject the reader must be referred to its extensive and none-too-easy literature.[1]

[1] The following treatises may be mentioned:

KING, GEORGE, "Institute of Actuaries' Textbook of the Principles of Interest, Life Annuities, and Assurances," Pt. II, Life Contingencies, 2d ed., Charles and Edwin Layton, London, 1902, pp. 1–104.

THOMPSON, JOHN S., A Determination of the Constants in Makeham's Formula by the Method of Least Squares, *Trans. Actuarial Soc. Am.*, **12**, pp. 225–240, 1911.

HENDERSON, ROBERT, "Mortality Laws and Statistics," Mathematical Monographs, No. 15, 1st ed., John Wiley & Sons, Inc., New York, 1915.

ELDERTON, W. PALIN, and S. J. ROWLAND, Graduation by Makeham's Hypotheses, *Jour. Inst. Actuaries*, **50**, pp. 251–265, 1917.

FISHER, ARNE, "The Mathematical Theory of Probabilities and Its Application to Frequency Curves and Statistical Methods," 2d ed., The Macmillan Company, New York, 1922, pp. 215–260.

GERHARD, F. BRUCE, A Graphic Method of Applying Makeham's Formula to Mortality Experience, *Trans. Actuarial Soc. Am.*, **24**, pp. 398–413, 1923.

When chance alone operates to determine the relative frequency of variations of different magnitudes from a normal mean, the mathematical theory of probabilities gives the equation $y = k\frac{1}{e^{h^2x^2}}$ for the frequency curve, where y equals frequency, x is the magnitude of variation from the normal mean, k and h are constants, and e the base of natural logarithms.

Besides mere chance, various other causes (such as the lessening of vitality with increase of age) affect human mortality, of which human mortality curves and tables to guide actuaries have been in course of development since De Moivre's hypothesis was first published in 1725.

Gompertz, in 1825, developed another hypothesis to allow for other mortality factors than chance; Makeham modified the Gompertz frequency equation in 1860. Then Prof. Karl Pearson developed 12 type frequency curves[1] which ever since have been quite notable in actuarial literature. In 1906, W. Palin Elderton[2] published "a thorough discussion of the basis and practical application of the Pearsonian frequency curves to actuarial data."

The Gram-Charlier and the Poisson-Charlier series expressions for frequency curves are discussed by many writers, notably by Arne Fisher. These expressions are well known in the biological sciences and are quite satisfactory for the graduation of certain types of frequency observations. They are perhaps not so well known as the Gompertz-Makeham and Pearsonian formulas.

The general equation[3] for all the Pearsonian frequency types is

$$\frac{dy}{dx} = \frac{y(x + a)}{f(x)},$$

RIETZ, H. L. (editor), "Handbook of Mathematical Statistics," Houghton Mifflin Company, Boston, 1924.

ELDERTON, W. PALIN, "Frequency Curves and Correlation," 2d ed., Charles and Edwin Layton, London, 1927.

"Depreciation Charges of Telephone Companies," Docket 14,700, Interstate Comm. Comm., March 19–21, 1928, Washington, D.C. (Testimony of the witnesses for the Bell System Companies, privately printed by the Am. T. & T. Co.)

KURTZ, EDWIN B., "Life Expectancy of Physical Property Based on Mortality Laws," The Ronald Press, New York, 1930. (The original manuscript of this book was a doctorate dissertation on engineering valuation developed under Dr. Marston at Iowa State College.)

WINFREY, ROBLEY and EDWIN B. KURTZ, Life Characteristics of Physical Property, *Bull.* 103, *Iowa Eng. Exp. Sta.*, 1931.

WINFREY, ROBLEY, Statistical Analyses of Industrial Property Retirements, *Bull.* 125, *Iowa Eng. Exp. Sta.*, 1936.

The nomenclature used herein is that adopted in the book by Prof. Edwin B. Kurtz, and *Bull.* 125 by Robley Winfrey. In the book especial reference is made to pages 82 to 125 and 154 to 157. However, none of Professor Kurtz's seven type curves correspond to any of the type curves of this text. These were developed later; for them reference must be made to *Bull.* 125, which was published later than the book.

[1] ELDERTON, W. PALIN, "Frequency Curves and Correlation," 2d ed., Charles and Edwin Layton, London, 1927.

[2] *Ibid.*

[3] KURTZ, *op. cit.*, pp. 88, 90.

in which y = frequency.

x = age, measured from the origin of coordinates.

$f(x)$ = any function of x (usually $b_0 + b_1x + b_2x^2$).

a, b_0, b_1, b_2 are empirical constants.

Note that, in reality, all type frequency curves known, including the Gompertz-Makeham and the Pearsonian, are empirical. Their only sound basis is their "goodness of fit" to actual mortality facts. Some actuarial writers seem to think, erroneously, that mathematical equations possess in themselves some mysterious authority.

By comparison with collections of the actual mortality facts of different classes of industrial property, it has been found[1] that the Pearsonian types II and VII fit with exceptional closeness the "symmetrical-mode" type-frequency curves herein (Appendix B); but that none of his 12 types fit any of the left-mode or right-mode curves. For these, Robley Winfrey, in 1932, developed new type frequency equations, given below.

General Type Forms for Frequency Equations[2]

LEFT MODE NOS. 0 AND 1

$$Y_x = Y_0\left(1 - \frac{(x \pm d_m)^2}{a^2}\right)^m \quad \text{For } x \text{ values to left of mode}$$

$$Y_x = Y_0\left(1 - \frac{(x \pm D_m)}{A^2}\right)^M \quad \text{For } x \text{ values to right of mode}$$

LEFT MODES NOS. 2, 3, 4, AND 5 AND RIGHT MODES NOS. 1, 2, 3, 4 AND 5

$$Y_x = Y_e\left(1 + \frac{x \pm D_m}{A_1}\right)^{M_1}\left(1 - \frac{x \pm D_m}{A_2}\right)^{M_2} + y_e\left(1 + \frac{x \pm d_m}{a_1}\right)^{m_1}\left(1 - \frac{x \pm d_m}{a_2}\right)^{m_2}$$

SYMMETRICAL NOS. 0, 1, 2, 3, 4, 5, AND 6

$$Y_x = Y_0\left(1 - \frac{x^2}{a^2}\right)^m$$

in which Y_x = ordinate to the frequency curve at age x.

Y_0 = ordinate to the frequency curve at its mode.

Y_e = ordinate to the major constituent curve at its mean.

y_e = ordinate to the minor constituent curve at its mean.

NOTE.—The x of the curve's mean ordinate is determined by moments.

x = age (in units equal to 10 percent of average life), measured from the average-life ordinate.

D_m, d_m = distances from the respective means of the major and the minor constituent curves to the average-life ordinate.

$A, A_1, A_2, a, a_1, a_2, M, M_1, M_2, m, m_1, m_2$ are empirical constants.

For the numerical values of the above coefficients and constants see the equations below.

Note further, that the modes of the respective mortality-type-frequency curves are located as shown in Table B.2, Appendix B.

[1] WINFREY and KURTZ, *op. cit.*, p. 33.

[2] WINFREY, *op. cit.*

FINAL EQUATIONS OF THE 18 TYPE FREQUENCY CURVES[1]

In the following 18 equations, x is measured from the mean, or average life; negative values of x being to the left of 100 percent of average life and positive values to the right. An age interval of 10 percent of average life is equal to x. Therefore, if $x = -2.5$, the equivalent age is 75 percent of average life and when $x = +4.2$ the equivalent is 142 percent of average life.

LEFT MODE NO. 0

$$Y_x = 6.24256418\left(1 - \frac{(x + 5.06)^2}{24.60758105}\right)^{0.4411811}$$ For x values to left of 49.4% of average life

$$Y_x = 6.24256418\left(1 - \frac{(x + 5.06)^2}{1569.183739}\right)^{7.75906308}$$ For x values to right of 49.4% of average life

LEFT MODE NO. 1

$$Y_x = 7.45095687\left(1 - \frac{(x + 4)^2}{85.49500000}\right)^{4.77742941}$$ For x values to left of 60% of average life

$$Y_x = 7.45095687\left(1 - \frac{(x + 4)^2}{697.8983268}\right)^{4.74147112}$$ For x values to right of 60% of average life

LEFT MODE NO. 2

$$Y_x = 6.2\left(1 + \frac{x - 0.56632298}{10.56632298}\right)^{2.00691507}\left(1 - \frac{x - 0.56632298}{18.11962398}\right)^{4.15639835} + 4.03141046\left(1 + \frac{x + 1.98831766}{4.90258200}\right)^{2.73360830}\left(1 - \frac{x + 1.98831766}{12.07825433}\right)^{8.19831032}$$

LEFT MODE NO. 3

$$Y_x = 6.12\left(1 + \frac{x - 0.69997304}{9.94997304}\right)^{2.51767682}\left(1 - \frac{x - 0.69997304}{13.35543784}\right)^{3.72163230} + 8.19722280\left(1 + \frac{x + 1.22119072}{6.98766177}\right)^{10.15754029}\left(1 - \frac{x + 1.22119072}{16.85048078}\right)^{25.90598437}$$

LEFT MODE NO. 4

$$Y_x = 9.98024081\left(1 + \frac{x - 0.15}{7.90137257}\right)^{3.93835786}\left(1 - \frac{x - 0.15}{12.56942216}\right)^{6.85588885} + 10.65547182\left(1 + \frac{x + 0.37}{3.20813494}\right)^{5.41629880}\left(1 - \frac{x + 0.37}{7.67537928}\right)^{14.35075856}$$

LEFT MODE NO. 5

$$Y_x = 12.95508249\left(1 + \frac{x - 0.035}{5.97529460}\right)^{4.80007778}\left(1 - \frac{x - 0.035}{10.56942216}\right)^{9.25948944} + 16.62209606\left(1 + \frac{x + 0.0665}{2.09813494}\right)^{3.87938359}\left(1 - \frac{x + 0.0665}{5.70548783}\right)^{12.26857634}$$

SYMMETRICAL NO. 0

$$Y_x = 6.95219904\left(1 - \frac{x^2}{100}\right)^{0.74857140}$$

[1] WINFREY, *op. cit.*

SYMMETRICAL NO. 1

$$Y_x = 9.08025966\left(1 - \frac{x^2}{100}\right)^{1.82839970}$$

SYMMETRICAL NO. 2

$$Y_x = 11.91103882\left(1 - \frac{x^2}{100}\right)^{3.70009374}$$

SYMMETRICAL NO. 3

$$Y_x = 15.61048797\left(1 - \frac{x^2}{100}\right)^{6.9015918}$$

SYMMETRICAL NO. 4

$$Y_x = 22.32936082\left(1 - \frac{x^2}{81}\right)^{11.93537940}$$

SYMMETRICAL NO. 5

$$Y_x = 33.22051575\left(1 - \frac{x^2}{64}\right)^{21.43782170}$$

SYMMETRICAL NO. 6

$$Y_x = 52.47259169\left(1 - \frac{x^2}{49}\right)^{41.63414220}$$

RIGHT MODE NO. 1

$$\begin{aligned} Y_x = {} & 4.87234751\left(1 + \frac{x + 2.1173}{19.08200310}\right)^{2.16036988}\left(1 - \frac{x + 2.1173}{12.2}\right)^{1.02056945} \\ & + 2.95921394\left(1 + \frac{x - 2.03848}{9.25013197}\right)^{2.69374074}\left(1 - \frac{x - 2.03848}{6.76380495}\right)^{1.69831583} \end{aligned}$$

RIGHT MODE NO. 2

$$\begin{aligned} Y_x = {} & 7.12332684\left(1 + \frac{x + 0.47}{30.05448169}\right)^{9.16816044}\left(1 - \frac{x + 0.47}{9.05171312}\right)^{2.062414192} \\ & + 3.131138251\left(1 + \frac{x - 0.56}{20.83194842}\right)^{15.02457571}\left(1 - \frac{x - 0.56}{5.45698291}\right)^{3.197679162} \end{aligned}$$

RIGHT MODE NO. 3

$$\begin{aligned} Y_x = {} & 9.05940676\left(1 + \frac{x + 0.224}{12.38788195}\right)^{4.12077843}\left(1 - \frac{x + 0.224}{6.67527345}\right)^{1.75935760} \\ & + 6.06359763\left(1 + \frac{x - 0.69961}{5.59381606}\right)^{4.19664858}\left(1 - \frac{x - 0.69961}{3.45678267}\right)^{2.21134707} \end{aligned}$$

RIGHT MODE NO. 4

$$\begin{aligned} Y_x = {} & 15.20129316\left(1 + \frac{x + 0.11}{17.92683200}\right)^{14.05850860}\left(1 - \frac{x + 0.11}{5.41801100}\right)^{3.55112010} \\ & + 5.85667821\left(1 + \frac{x - 0.70}{2.56783700}\right)^{3.66879450}\left(1 - \frac{x - 0.70}{3.45398750}\right)^{5.27997721} \end{aligned}$$

RIGHT MODE NO. 5

$$Y_x = 14.99330391\left(1 + \frac{x + 0.12869}{7.0000000}\right)^{5.79473520}\left(1 - \frac{x + 0.12869}{3.8764409}\right)^{2.76276990}$$
$$+ 15.44614441\left(1 + \frac{x - 0.2086}{4.2350000}\right)^{6.05833400}\left(1 - \frac{x - 0.2086}{2.4150000}\right)^{3.02500040}$$

3.25. Characteristics of 18 Type Frequency Curves.—These are shown in Table B.2, Appendix B. In cases where actual mortality curves of particular kinds of industrial property are lacking, an experienced valuator, who has personally examined the property and made himself familiar with its history, may be able, from his knowledge of the causes and characteristics of its retirements, to select two or more tentative mortality type curves (from the 18, herein) which will enable him to determine the probable range in the dates of future retirements.

The causes of retirements affect the rates of retirements in different ways as follows:

1. Retirements from accidents to which all units of a given class are equally exposed should decrease with decrease in survivors as age increases but remain constant in percentage of survivors.
2. Retirements from physical decrepitude would seem likely to increase continuously with age in percentage of survivors but eventually to decrease in numbers on account of decrease in number of survivors.
3. Retirements for obsolescence, inadequacy, and supersession are likely to cause sudden large retirements which often can be foreseen some years in advance by competent valuators. Example: The change from direct-current to alternating-current equipment which occurred after 1900 in the large utility to which Fig. 3.5 applies.

Retirements may also be affected irregularly by changes in administrative policies; as when retirements are hastened by decision to improve the property, or retarded by financial exigencies during business depressions.

The mortality-type-frequency curves, herein (see Appendix B), are based on actual mortality data which include all retirements from all causes combined.

3.26. Mortality-survivor-curve Equations.—Manifestly, the mathematical equations of mortality survivor curves are simply the integrals or the summations of their frequency-curve equations.

The ordinates of all mortality survivor curves in this treatise have been obtained from their frequency curves by summation; it is believed that summation from frequency curves made to fit actual mortality data are of much greater value than curves restricted to theoretical mathematical equations. In the case of all type curves, herein, Simpson's quadrature formula $\Sigma_0^1 y\,dx = \frac{1}{6}(y_0 + 4y_{\frac{1}{2}} + y_1)$ has been used, wherever required for accuracy, because of curvature of the frequency curve between the summation ordinates.

It is possible to develop approximate survivor-curve equations, of the Gompertz-Makeham form, by a process of approximation which is described by Kurtz.[1] These survivor-curve equations are in the form $y = ks^x g^{c^x}$, in which y equals survivors at age x, and k, s, g, and c are constants.

[1] *Op. cit.*, pp. 113–125.

The degree of approximation obtained by Professor Kurtz is shown by his Figs. 58 to 64, but note that none of his seven type curves are identical with any of the type curves herein. His survivor-curve equations are in reality empirical.

It has not been found practicable to fit any Gompertz-Makeham type curves to the mortality-type-survivor curves given in Appendix B; the authors believe the use of such equations in accountancy for industrial-property units to be not only unwarranted but incorrect. Actual mortality facts should be used instead of such equations.

3.27. Probable-life-curve Equations.—All probable-life curves given in this treatise are calculated from their frequency and survivor curves in the manner illustrated in Table 3.1, columns (4), (5), (6), and (7). The probable-life curves are plotted on the same figure with their survivor curves; this is for convenience in use (see Figs. 3.2, 3.3, 3.4, and B.4, B.5, B.6).

It is manifest that theoretical mortality-probability equations could hardly be expected to be possible of derivation for probable-life curves constructed in this manner. However, Professor Kurtz has found[1] them to fit reasonably close to empirical parabolic equations of the form

$$\text{Probable life} = 100 + ax^b,$$

in which x equals age, and a and b are constants.

3.28. Average Survivor Units and Their Condition-percents.—From the manner of compiling expectancies and probable lives (Table 3.1), it is apparent (and should be clearly understood) that

1. The probable life of the average survivor at any age in an age-group of like industrial-property units is the probable average life of all survivors of that age.
2. Some of the individual survivors reach the end of their service lives before, and the rest after, the probable life indicated by mortality tables and/or curves.

The *average survivor* unit at any age is defined as a survivor having an actual future-service life just equal to the average actual future-service life of all survivor units at the age.

The condition-percent of an average survivor unit is usually found by first ascertaining its service age and estimating its probable life, each to the nearest year, by the methods described in Secs. 3.20 and 3.21 and then looking up its condition-percent in the condition-percent table in Appendix C. The probable life can only be forecasted; it would be absurd to use fractional years in the service ages and probable lives used.

For smooth mortality-type-curve computations, the condition-percent should be computed; it is equal to

$$(100)\frac{(1+r)^n - (1+r)^x}{(1+r)^n - 1} \text{ [Eqs. (5–7) to (5–10)]},$$

where r is the rate of return, and x and n are the exact service ages and

[1] *Op. cit.*, pp. 154–157.

probable lives, respectively, both stated in years (including decimals of years). Use logarithms where needed.

It must be clearly understood that the *condition-percent of the average survivor at any service age is not* the same as the *average condition-percent of all survivors* at that age. The latter can be computed correctly only by an intricate process of averaging which is described in Sec. 3.29.

3.29. Average Condition-percents of Survivor Units.—The method of calculating the average condition-percent of the survivor units of a mortality age-group at different service ages will be illustrated by the calculations for mortality type curve S_3 (Appendix B). The curve is shown in Fig. 3.7; its data are given in Table 3.6.

Table 3.6.—Data of Mortality Type Curve S_3, Appendix B
For 10 years average life and 6% rate of return

Age interval, years	Retirements during age interval, %	Age, years	Survivors at age, %	Average condition-percent of survivors* at age, %	Average survivor	
					Probable life, years	Condition-percent,† %
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0– ½	0.00000	0	100.00000	100.0000	10.00	100.00
½– 1½	0.00045	½	100.00000	95.9066	10.00	96.26
1½– 2½	0.01797	1½	99.99955	87.3366	10.00	88.45
2½– 3½	0.16722	2½	99.98158	78.2748	10.00	80.17
3½– 4½	0.75366	3½	99.81436	69.7947	10.01	71.44
4½– 5½	2.1916	4½	99.06070	59.1759	10.06	62.39
5½– 6½	4.7260	5½	96.8691	49.8764	10.17	53.28
6½– 7½	8.1505	6½	92.1431	41.3580	10.39	44.65
7½– 8½	11.7419	7½	83.9926	33.9167	10.72	36.83
8½– 9½	14.4902	8½	72.2507	27.6337	11.16	30.03
9½–10½	15.5210	9½	57.7605	22.4346	11.70	24.34
10½–11½	14.4902	10½	42.2395	18.1674	12.32	19.65
11½–12½	11.7419	11½	27.7493	14.6636	13.01	15.85
12½–13½	8.1505	12½	16.0074	11.7692	13.76	12.83
13½–14½	4.7260	13½	7.8569	9.3545	14.54	10.29
14½–15½	2.1916	14½	3.1309	7.3167	15.37	8.36
15½–16½	0.75366	15½	0.9393	5.5742	16.22	6.72
16½–17½	0.16722	16½	0.18564	4.0647	17.10	5.44
17½–18½	0.01797	17½	0.01842	2.7370	18.02	4.59
18½–19½	0.00045	18½	0.00045	1.5558	19.00	4.29
19½–20	0.00000	19½	0.00000	0.0000		

* The average condition-percent of the survivors at any service age was computed by the method described below, and illustrated in Table 3.7.

† The condition-percent of the average survivors at any service age was computed by the formula as described in Sec. 3.28.

The average condition-percents of all the survivors at different ages of an S_3 mortality type age-group of industrial-property units having an average service life

of 10 years are shown in column (5), Table 3.6; they may be compared with the condition-percents of the corresponding average survivors, as shown in column (7). The same data are shown graphically in Fig. 3.7. The average survivors have higher condition-percents than the true average condition-percents of all survivors.

The computations of the average condition-percents in column (5), Table 3.6, are shown in Table 3.7; they may be described as follows:

Step 1.—Compute the probable lives of the retirements during each age interval. The results are shown in column (2), Table 3.7.

The process is to divide each service year into a sufficient number of equal parts (20 in Fig. 3.7, of which each of the four vertical strips for the year 7½ to 8½ represents five); find from the frequency curve, by computation, the retirements during each one-twentieth year, of which the actual average service lives are the respective mid-ages; then multiply each one-twentieth-year retirements by its actual average service life, add the products, and divide by the total retirement during the year to get the probable life of the total year's retirements, treated as a unit.

Step 2.—Compute the condition-percents, at each service age, of each service-year's retirements, with results shown in columns (5) to (25), Table 3.7.

Each of these condition-percents was computed as the value of

$$(100)\frac{(1+r)^n - (1+r)^x}{(1+r)^n - 1},$$

in which n equals probable life and x equals service age.

Step 3.—Compute the average condition-percent of all the survivors at each service age (with results shown for each age in the horizontal line C).

The process was to get and sum up the products of each year's retirements by their condition-percent at the age (with results shown for each age in the horizontal line A); then divide the sum of the products by the total survivors at the age, to get the weighted mean; this is their average condition-percent.

NOTE.—The method illustrated in Table 3.7 was developed by the Iowa Engineering Experiment Station, which is proceeding with the tedious computations required to determine the average condition-percents of all survivors, at each service age, for each mortality type curve.

3.30. Renewals; Calculations and Curves.—In Secs. 3.28 and 3.29 only original-unit survivors of original age-groups were considered. In the normal operation of industrial properties, each unit retired must be renewed immediately (unless replaced by a different kind of unit); it is desirable to study the retirements, renewals, and average condition-percents, at different service ages (measured from the date of installation of the original age-group), in groups of units constantly maintained at 100 percent numbers by continued renewals.

In such a group, the *first renewal* is that of one of the *original units.* Throughout the service life of the longest survivor, the renewals are partly *renewals of original units* and partly are *renewals of renewals;* after the retirement of the longest survivor of the original units, the renewals are entirely renewals of renewals. Both the renewals of original units and the renewals of renewals can be calculated from the frequency-curve data of the groups; the computations, while simple in principle,

TABLE 3.7.—COMPUTATIONS OF AVERAGE CONDITION-PERCENTS OF SURVIVORS
Mortality type curve S_3, 10 years average life, 6% return

Age interval, years	Probable life (n)	$(1+r)^n - 1$	Frequency	Condition-percent of each frequency group of units remaining in service at indicated ages x								
				$x = 0$	$x = \frac{1}{2}$	$x = 1\frac{1}{2}$	$x = 2\frac{1}{2}$	$x = 3\frac{1}{2}$	$x = 4\frac{1}{2}$	$x = 5\frac{1}{2}$	$x = 6\frac{1}{2}$	$x = 7\frac{1}{2}$
				Values of $(1 + r)^x - 1$ for ages of $x = 0$ to $x = 19\frac{1}{2}$								
				0.000000	0.029563	0.091337	0.156817	0.226226	0.299800	0.377788	0.460456	0.548083
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
0 – ½	0.4397	0.025952	0.0000	100.0000	0.0000							
½– 1½	1.3220	0.080076	0.00045	100.0000	63.0813	0.0000						
1½– 2½	2.2147	0.137745	0.01797	100.0000	78.5378	33.6914	0.0000					
2½– 3½	3.1462	0.201206	0.16722	100.0000	88.6154	54.6052	22.0613	0.0000				
3½– 4½	4.1036	0.270122	0.75366	100.0000	89.0556	66.1867	41.9457	16.2502	0.0000			
4½– 5½	5.0749	0.344079	2.1916	100.0000	91.4081	73.4547	54.4241	34.2517	12.8689	0.0000		
5½– 6½	6.0540	0.422990	4.7260	100.0000	93.0109	78.4068	62.9265	46.5173	29.1236	10.6863	0.0000	
6½– 7½	7.0375	0.506920	8.1505	100.0000	94.1681	81.9820	69.0647	55.3724	40.8585	25.4739	9.1661	0.0000
7½– 8½	8.0238	0.596061	11.7419	100.0000	95.0403	84.6766	73.6911	62.0464	49.7031	36.6192	22.7503	8.0492
8½– 9½	9.0115	0.690612	14.4902	100.0000	95.7193	86.7745	77.2930	67.2426	56.5892	45.2967	33.3250	20.6381
9½–10½	10.0000	0.790849	15.5210	100.0000	96.2618	88.4508	80.1710	71.3945	62.0914	52.2301	41.7771	30.6969
10½–11½	10.9885	0.897029	14.4902	100.0000	96.7043	89.8178	82.5182	74.7805	66.5785	57.8845	48.6688	38.9002
11½–12½	11.9762	1.009410	11.7419	100.0000	97.0712	90.9514	84.4645	77.5882	70.2995	62.5734	54.3837	45.7026
12½–13½	12.9625	1.128275	8.1505	100.0000	97.3798	91.9047	86.1012	79.9494	73.4284	66.5163	59.1894	51.4229
13½–14½	13.9460	1.253804	4.7260	100.0000	97.6421	92.7152	87.4927	81.9568	76.0887	69.8686	63.2753	56.2864
14½–15½	14.9251	1.386125	2.1916	100.0000	97.8672	93.4106	88.6866	83.6792	78.3713	72.7450	66.7811	60.4593
15½–16½	15.8964	1.525066	0.75366	100.0000	98.0615	94.0110	89.7173	85.1661	80.3418	75.2281	69.8075	64.0617
16½–17½	16.8538	1.669934	0.16722	100.0000	98.2297	94.5305	90.6094	86.4530	82.0472	77.3771	72.4267	67.1794
17½–18½	17.7835	1.818857	0.01797	100.0000	98.3746	94.9783	91.3782	87.5622	83.5171	79.2294	74.6843	69.8666
18½–19½	18.6780	1.969365	0.00045	100.0000	98.4988	95.3621	92.0372	88.5127	84.7768	80.8168	76.6191	72.1696
19½–20	19.5603	2.126014	0.0000 100.0000	100.0000	98.6095	95.7038	92.6239	89.3591	85.8985	82.2302	78.3418	74.2202
A = Σ product of column (4) and respective condition-percent:												
A				100.000000	9590.661591	8733.625315	7826.033212	6866.702359	5862.003771	4831.481249	3810.855542	2848.753654
B = percent surviving at age x:												
B				100.00000	100.000000	99.99955	99.98158	99.81436	99.06070	96.8691	92.1431	83.9926
C = condition-percent of survivors at age x:												
C				100.000000	95.906616	87.336646	78.274750	68.794734	59.175877	49.876392	41.358013	33.916722
D = condition-percent or original 100 units at age x:												
D				100.000000	95.906616	87.336253	78.260332	68.667024	58.620038	48.314812	38.108555	28.487537

Condition-percent of each frequency group of units remaining in service at indicated ages x											
$x = 8\frac{1}{2}$	$x = 9\frac{1}{2}$	$x = 10\frac{1}{2}$	$x = 11\frac{1}{2}$	$x = 12\frac{1}{2}$	$x = 13\frac{1}{2}$	$x = 14\frac{1}{2}$	$x = 15\frac{1}{2}$	$x = 16\frac{1}{2}$	$x = 17\frac{1}{2}$	$x = 18\frac{1}{2}$	$x = 19\frac{1}{2}$
Values of $(1 + r)^x - 1$ for ages of $x = 0$ to $x = 19\frac{1}{2}$											
0.640968	0.739426	0.843792	0.954420	1.071685	1.195987	1.327746	1.467411	1.615456	1.772383	1.938726	2.115050
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)
0.0000											
7.1885	0.0000										
18.9519	6.5022	0.0000									
28.5454	17.5694	5.9348	0.0000								
36.5007	26.7467	16.4074	5.4478	0.0000							
43.1904	34.4640	25.2140	15.4090	5.0156	0.0000						
48.8781	41.0253	32.7014	23.8780	14.5253	4.6113	0.0000					
53.7583	46.6551	39.1258	31.1447	22.6848	13.7172	4.2117	0.0000				
57.9711	51.5151	44.6718	37.4178	29.7286	21.5780	12.9384	3.7805	0.0000			
61.6172	55.7212	49.4715	42.8469	35.8247	28.3812	20.4911	12.1276	3.2623	0.0000		
64.7598	59.3466	53.6086	47.5264	41.0792	34.2451	27.0011	19.3224	11.1829	2.5551	0.0000	
67.4530	62.4536	57.1541	51.5367	45.5822	39.2704	32.5800	25.4881	17.9707	10.0023	1.5558	0.0000
69.8512	65.2201	60.3111	55.1076	49.5918	43.7451	37.5477	30.9783	24.0148	16.6335	8.8094	0.5157
1996.557528	1295.832692	767.381155	406.905129	188.393819	73.497055	22.907909	5.235882	0.754565	0.050416	0.000700	0.0000
72.2507	57.7605	42.2395	27.7493	16.0074	7.8569	3.1309	0.9393	0.18564	0.01842	0.00045	0.0000
27.633746	22.434582	18.167382	14.663618	11.769170	9.354460	7.316717	5.574238	4.064670	2.737035	1.5558	0.0000
19.965575	12.958327	7.673812	4.069051	1.883938	0.734971	0.229079	0.052359	0.007546	0.000504	0.000007	0.0000

TABLE 3.8.—RENEWALS CALCULATIONS FOR MORTALITY TYPE CURVE S_3
For 10 years average life

Age interval, years	Total renewals during *n*th age interval	Age interval (0–1, 1–2, 2–3, etc.) during which renewals are made. First line of age interval (0–20) corresponds to renewals above first diagonal staggered heavy line, etc.																				
		0–1 21–22 42–43 63–64	1–2 22–23 43–44 64–65	2–3 23–24 44–45 65–66	3–4 24–25 45–46 66–67	4–5 25–26 46–47 67–68	5–6 26–27 47–48 68–69	6–7 27–28 48–49 69–70	7–8 28–29 49–50 70–71	8–9 29–30 50–51 71–72	9–10 30–31 51–52 72–73	10–11 31–32 52–53 73–74	11–12 32–33 53–54 74–75	12–13 33–34 54–55 75–76	13–14 34–35 55–56 76–77	14–15 35–36 56–57 77–78	15–16 36–37 57–58 78–79	16–17 37–38 58–59 79–80	17–18 38–39 59–60 80–81	18–19 39–40 60–61 81–82	19–20 40–41 61–62 82–83	20–21 41–42 62–63 83–84
Multipliers	....	0.0000	0.0019	0.0327	0.2209	0.8614	2.3324	4.8415	8.1736	11.6357	14.2713	15.2572	14.2713	11.6357	8.1736	4.8415	2.3324	0.8614	0.2209	0.0327	0.0019	0.0000
Original units	100.0000	0.0000	0.0037	0.0618	0.3799	1.3429	3.3219	6.3611	9.9862	13.2852	15.2573	15.2573	13.2852	9.9862	6.3611	3.3219	1.3429	0.3799	0.0618	0.0037	0.0000	
0–1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1–2	0.0037	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0003	0.0004	0.0005	0.0005	0.0005	0.0004	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
2–3	0.0618	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0005	0.0015	0.0030	0.0051	0.0072	0.0088	0.0094	0.0088	0.0072	0.0051	0.0030	0.0015	0.0005	0.0001	0.0000
3–4	0.3799	0.0001	0.0000	0.0000	0.0000	0.0000	0.0001	0.0008	0.0033	0.0089	0.0184	0.0310	0.0442	0.0542	0.0580	0.0542	0.0442	0.0311	0.0184	0.0089	0.0033	0.0008
4–5	1.3429	0.0030	0.0004	0.0000	0.0000	0.0000	0.0000	0.0004	0.0030	0.0116	0.0313	0.0650	0.1098	0.1563	0.1916	0.2049	0.1916	0.1563	0.1098	0.0650	0.0313	0.0116
5–6	3.3222	0.0286	0.0073	0.0011	0.0001	0.0000	0.0000	0.0001	0.0011	0.0074	0.0286	0.0775	0.1608	0.2715	0.3866	0.4741	0.5069	0.4741	0.3866	0.2715	0.1608	0.0775
6–7	6.3630	0.1484	0.0548	0.0140	0.0021	0.0001	0.0000	0.0000	0.0001	0.0021	0.0140	0.0548	0.1484	0.3081	0.5201	0.7404	0.9081	0.9708	0.9081	0.7404	0.5201	0.3081
7–8	9.9954	0.4839	0.2331	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0861	0.2331	0.4839	0.8170	1.1630	1.4265	1.5250	1.4265	1.1630	0.8170
8–9	13.3187	1.0886	0.6448	0.3106	0.1147	0.0294	0.0044	0.0003	0.0000	0.0000	0.0003	0.0044	0.0294	0.1147	0.3106	0.6448	1.0886	1.5497	1.9008	2.0321	1.9008	1.5497
9–10	15.3587	1.7871	1.2554	0.7436	0.3582	0.1323	0.0339	0.0050	0.0003	0.0000	0.0000	0.0003	0.0050	0.0339	0.1323	0.3582	0.7436	1.2554	1.7871	2.1919	2.3433	2.1919
10–11	15.5201	2.2149	1.8059	1.2685	0.7514	0.3620	0.1337	0.0343	0.0051	0.0003	0.0000	0.0000	0.0003	0.0051	0.0343	0.1337	0.3620	0.7514	1.2685	1.8059	2.2149	2.3679
11–12	13.8785	2.1175	1.9806	1.6149	1.1344	0.6719	0.3237	0.1195	0.0307	0.0045	0.0003	0.0000	0.0000	0.0003	0.0045	0.0307	0.1195	0.3237	0.6719	1.1344	1.6149	1.9806
12–13	11.1733	1.5946	1.7047	1.5946	1.3001	0.9133	0.5410	0.2606	0.0962	0.0247	0.0036	0.0002	0.0000	0.0000	0.0002	0.0036	0.0247	0.0962	0.2606	0.5410	0.9133	1.3001
13–14	8.4924	0.9881	1.2120	1.2957	1.2120	0.9881	0.6941	0.4112	0.1981	0.0731	0.0188	0.0028	0.0002	0.0000	0.0000	0.0002	0.0028	0.0187	0.0731	0.1981	0.4112	0.6941
14–15	6.7912	0.5551	0.7902	0.9692	1.0362	0.9692	0.7902	0.5551	0.3288	0.1584	0.0585	0.0150	0.0022	0.0001	0.0000	0.0000	0.0001	0.0022	0.0150	0.0585	0.1584	0.3288
15–16	6.5033	0.3149	0.5315	0.7567	0.9281	0.9922	0.9281	0.7567	0.5316	0.3149	0.1517	0.0560	0.0144	0.0021	0.0001	0.0000	0.0000	0.0001	0.0021	0.0144	0.0560	0.1517
16–17	7.4392	0.1735	0.3602	0.6081	0.8656	1.0617	1.1350	1.0617	0.8656	0.6081	0.3602	0.1735	0.0641	0.0164	0.0024	0.0001	0.0000	0.0000	0.0001	0.0024	0.0164	0.0641
17–18	8.9905	0.0774	0.2097	0.4353	0.7348	1.0461	1.2831	1.3717	1.2831	1.0461	0.7348	0.4353	0.2097	0.0774	0.0199	0.0029	0.0002	0.0000	0.0000	0.0002	0.0029	0.0199
18–19	10.4954	0.0232	0.0904	0.2448	0.5082	0.8579	1.2212	1.4978	1.6013	1.4978	1.2212	0.8579	0.5081	0.2448	0.0904	0.0232	0.0034	0.0002	0.0000	0.0000	0.0002	0.0034
19–20	11.5109	0.0038	0.0254	0.0992	0.2685	0.5573	0.9408	1.3394	1.6427	1.7562	1.6427	1.3394	0.9408	0.5573	0.2685	0.0992	0.0254	0.0038	0.0002	0.0000	0.0000	0.0002
20–21	11.8674	0.0002	0.0039	0.0262	0.1022	0.2768	0.5746	0.9700	1.3809	1.6936	1.8106	1.6936	1.3809	0.9700	0.5746	0.2768	0.1022	0.0262	0.0039	0.0002	0.0000	0.0000
21–22	11.6029	0.0000	0.0002	0.0038	0.0256	0.0999	0.2706	0.5618	0.9484	1.3501	1.6559	1.7703	1.6559	1.3501	0.9484	0.5618	0.2706	0.0999	0.0256	0.0038	0.0002	0.0000
22–23	10.9105	0.0000	0.0000	0.0002	0.0035	0.0241	0.0940	0.2545	0.5282	0.8918	1.2695	1.5571	1.6646	1.5571	1.2695	0.8918	0.5282	0.2545	0.0940	0.0241	0.0036	0.0002
23–24	10.0726	0.0002	0.0000	0.0000	0.0002	0.0033	0.0222	0.0868	0.2349	0.4877	0.8233	1.1720	1.4375	1.5368	1.4375	1.1720	0.8233	0.4877	0.2349	0.0868	0.0222	0.0033
24–25	9.3680	0.0031	0.0002	0.0000	0.0000	0.0002	0.0031	0.0207	0.0807	0.2185	0.4535	0.7657	1.0900	1.3369	1.4293	1.3369	1.0900	0.7657	0.4536	0.2185	0.0807	0.0207
25–26	9.9891	0.0199	0.0029	0.0002	0.0000	0.0000	0.0002	0.0029	0.0199	0.0774	0.2097	0.4352	0.7347	1.0459	1.2829	1.3715	1.2829	1.0459	0.7347	0.4352	0.2097	0.0774
26–27	8.9941	0.0775	0.0199	0.0029	0.0002	0.0000	0.0000	0.0002	0.0029	0.0199	0.0775	0.2098	0.4354	0.7351	1.0465	1.2836	1.3722	1.2836	1.0465	0.7351	0.4355	0.2098
27–28	9.3102	0.2171	0.0802	0.0206	0.0030	0.0002	0.0000	0.0000	0.0002	0.0030	0.0206	0.0802	0.2171	0.4507	0.7610	1.0833	1.3287	1.4205	1.3287	1.0833	0.7610	0.4508
28–29	9.7796	0.4735	0.2281	0.0842	0.0216	0.0032	0.0002	0.0000	0.0000	0.0002	0.0032	0.0216	0.0842	0.2281	0.4735	0.7994	1.1379	1.3957	1.4921	1.3957	1.1379	0.7993
29–30	10.2263	0.8359	0.4951	0.2385	0.0881	0.0226	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0226	0.0881	0.2385	0.4951	0.8359	1.1899	1.4594	1.5603	1.4594	1.1899

30–31	10.5158	1.2236	0.8595	0.5091	0.2453	0.0906	0.0232	0.0034	0.0002	0.0000	0.0000	0.0002	0.0034	0.0232	0.0906	0.2453	0.5091	0.8595	1.2236	1.5008	1.6044	1.5008
31–32	10.5891	1.5112	1.2321	0.8655	0.5127	0.2470	0.0912	0.0234	0.0035	0.0002	0.0000	0.0000	0.0002	0.0034	0.0234	0.0912	0.2470	0.5127	0.8655	1.2321	1.5112	1.6156
32–33	10.4660	1.5968	1.4936	1.2178	0.8555	0.5067	0.2441	0.0902	0.0231	0.0034	0.0002	0.0000	0.0000	0.0002	0.0034	0.0231	0.0902	0.2441	0.5067	0.8555	1.2178	1.4936
33–34	10.2237	1.4590	1.5599	1.4590	1.1896	0.8356	0.4950	0.2385	0.0881	0.0226	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0226	0.0881	0.2385	0.4950	0.8356	1.1896
34–35	9.9606	1.1590	1.4215	1.5197	1.4215	1.1590	0.8141	0.4822	0.2323	0.0858	0.0220	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0220	0.0858	0.2323	0.4823	0.8141
35–36	9.7607	0.7978	1.1357	1.3930	1.4892	1.3930	1.1357	0.7978	0.4726	0.2276	0.0841	0.0216	0.0032	0.0002	0.0000	0.0000	0.0002	0.0032	0.0215	0.0841	0.2276	0.4726
36–37	9.6733	0.4683	0.7907	1.1255	1.3805	1.4759	1.3805	1.1255	0.7907	0.4683	0.2256	0.0833	0.0214	0.0032	0.0002	0.0000	0.0000	0.0002	0.0032	0.0214	0.0833	0.2256
37–38	9.7034	0.2263	0.4698	0.7931	1.1290	1.3848	1.4805	1.3848	1.1291	0.7931	0.4698	0.2263	0.0836	0.0214	0.0032	0.0002	0.0000	0.0000	0.0002	0.0032	0.0214	0.0836
38–39	9.8186	0.0846	0.2290	0.4754	0.8025	1.1425	1.4012	1.4980	1.4012	1.1425	0.8025	0.4754	0.2290	0.0846	0.0217	0.0032	0.0002	0.0000	0.0000	0.0002	0.0032	0.0217
39–40	9.9676	0.0220	0.0859	0.2325	0.4826	0.8147	1.1598	1.4225	1.5208	1.4225	1.1598	0.8147	0.4826	0.2325	0.0859	0.0220	0.0032	0.0002	0.0000	0.0000	0.0002	0.0032
40–41	10.0972	0.0033	0.0223	0.0870	0.2355	0.4888	0.8253	1.1749	1.4410	1.5405	1.4410	1.1749	0.8253	0.4889	0.2355	0.0870	0.0223	0.0033	0.0002	0.0000	0.0000	0.0002
41–42	10.1720	0.0002	0.0033	0.0225	0.0876	0.2372	0.4925	0.8314	1.1836	1.4517	1.5520	1.4517	1.1836	0.8314	0.4925	0.2372	0.0876	0.0225	0.0033	0.0002	0.0000	0.0000
42–43	10.1793	0.0000	0.0002	0.0033	0.0225	0.0877	0.2374	0.4928	0.8320	1.1845	1.4527	1.5531	1.4527	1.1845	0.8320	0.4928	0.2374	0.0877	0.0225	0.0033	0.0002	0.0000
43–44	10.1299	0.0000	0.0000	0.0002	0.0033	0.0224	0.0872	0.2363	0.4904	0.8280	1.1787	1.4457	1.5455	1.4457	1.1787	0.8280	0.4904	0.2363	0.0872	0.0224	0.0033	0.0002
44–45	10.0500	0.0002	0.0000	0.0000	0.0002	0.0033	0.0222	0.0866	0.2344	0.4866	0.8214	1.1694	1.4342	1.5333	1.4343	1.1694	0.8214	0.4866	0.2344	0.0866	0.0222	0.0033
45–46	9.9704	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0220	0.0859	0.2326	0.4827	0.8149	1.1601	1.4229	1.5212	1.4229	1.1601	0.8149	0.4827	0.2326	0.0859	0.0220
46–47	9.9154	0.0219	0.0032	0.0002	0.0000	0.0000	0.0002	0.0032	0.0219	0.0854	0.2313	0.4801	0.8104	1.1537	1.4151	1.5128	1.4151	1.1537	0.8104	0.4801	0.2313	0.0854
47–48	9.8969	0.0853	0.0219	0.0032	0.0002	0.0000	0.0000	0.0002	0.0032	0.0219	0.0852	0.2308	0.4792	0.8089	1.1516	1.4124	1.5100	1.4124	1.1516	0.8089	0.4792	0.2308
48–49	9.9139	0.2312	0.0854	0.0219	0.0032	0.0002	0.0000	0.0000	0.0002	0.0032	0.0219	0.0854	0.2312	0.4800	0.8103	1.1536	1.4149	1.5126	1.4148	1.1536	0.8103	0.4800
49–50	9.9542	0.4819	0.2322	0.0857	0.0220	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0220	0.0858	0.2322	0.4819	0.8136	1.1582	1.4206	1.5187	1.4206	1.1582	0.8136
50–51	10.0006	0.8174	0.4842	0.2333	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0861	0.2333	0.4842	0.8174	1.1636	1.4272	1.5258	1.4272	1.1636
51–52	10.0377	1.1679	0.8204	0.4860	0.2341	0.0865	0.0222	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0222	0.0865	0.2341	0.4860	0.8204	1.1679	1.4325	1.5315	1.4325
52–53	10.0563	1.4352	1.1701	0.8220	0.4869	0.2345	0.0866	0.0222	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0222	0.0866	0.2345	0.4869	0.8220	1.1701	1.4352	1.5343
53–54	10.0536	1.5339	1.4348	1.1698	0.8217	0.4867	0.2345	0.0866	0.0222	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0222	0.0866	0.2345	0.4868	0.8217	1.1698	1.4348
54–55	10.0352	1.4321	1.5311	1.4321	1.1677	0.8202	0.4858	0.2341	0.0864	0.0222	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0222	0.0864	0.2341	0.4859	0.8202	1.1677
55–56	10.0096	1.1647	1.4285	1.5272	1.4285	1.1647	0.8181	0.4846	0.2335	0.0862	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0862	0.2335	0.4846	0.8181
56–57	9.9857	0.8162	1.1619	1.4251	1.5235	1.4251	1.1619	0.8162	0.4835	0.2329	0.0860	0.0220	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0220	0.0860	0.2329	0.4835
57–58	9.9710	0.4827	0.8150	1.1602	1.4230	1.5213	1.4230	1.1602	0.8150	0.4827	0.2326	0.0859	0.0220	0.0032	0.0002	0.0000	0.0000	0.0002	0.0033	0.0220	0.0859	0.2326
58–59	9.9682	0.2325	0.4826	0.8147	1.1599	1.4226	1.5209	1.4226	1.1599	0.8148	0.4826	0.2325	0.0859	0.0220	0.0032	0.0002	0.0000	0.0000	0.0002	0.0032	0.0220	0.0859
59–60	9.9755	0.0859	0.2327	0.4830	0.8153	1.1607	1.4236	1.5220	1.4236	1.1607	0.8154	0.4830	0.2327	0.0859	0.0220	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0220
60–61	9.9892	0.0220	0.0860	0.2330	0.4836	0.8165	1.1623	1.4256	1.5241	1.4256	1.1623	0.8165	0.4836	0.2330	0.0860	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033
61–62	10.0034	0.0033	0.0221	0.0862	0.2333	0.4843	0.8176	1.1640	1.4276	1.5262	1.4276	1.1640	0.8176	0.4843	0.2333	0.0862	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002
62–63	10.0138	0.0002	0.0033	0.0221	0.0862	0.2336	0.4848	0.8185	1.1652	1.4291	1.5278	1.4291	1.1652	0.8185	0.4848	0.2336	0.0862	0.0221	0.0033	0.0002	0.0000	0.0000
63–64	10.0178	0.0000	0.0002	0.0033	0.0221	0.0863	0.2337	0.4850	0.8188	1.1656	1.4297	1.5284	1.4297	1.1656	0.8188	0.4850	0.2337	0.0863	0.0221	0.0033	0.0002	0.0000
64–65	10.0158	0.0000	0.0000	0.0002	0.0033	0.0221	0.0863	0.2336	0.4849	0.8186	1.1654	1.4294	1.5281	1.4294	1.1654	0.8187	0.4849	0.2336	0.0863	0.0221	0.0033	0.0002
65–66	10.0092	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0862	0.2335	0.4846	0.8181	1.1646	1.4285	1.5271	1.4284	1.1646	0.8181	0.4846	0.2335	0.0862	0.0221	0.0033
66–67	10.0008	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0861	0.2333	0.4842	0.8174	1.1637	1.4272	1.5258	1.4272	1.1637	0.8174	0.4842	0.2333	0.0861	0.0221
67–68	9.9942	0.0221	0.0032	0.0002	0.0000	0.0000	0.0002	0.0032	0.0221	0.0861	0.2331	0.4839	0.8169	1.1629	1.4263	1.5248	1.4263	1.1629	0.8169	0.4839	0.2331	0.0861
68–69	9.9904	0.0860	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0860	0.2330	0.4837	0.8166	1.1624	1.4258	1.5242	1.4258	1.1624	0.8166	0.4837	0.2330
69–70	9.9904	0.2330	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0861	0.2330	0.4837	0.8166	1.1624	1.4257	1.5242	1.4257	1.1624	0.8166	0.4837

TABLE 3.8.—RENEWALS CALCULATIONS FOR MORTALITY TYPE CURVE S_3.—(*Concluded*)

Age interval, years	Total renewals during *n*th age interval	Age interval (0–1, 1–2, 2–3, etc.) during which renewals are made First line of age interval (0–20) corresponds to renewals above first diagonal staggered heavy line, etc.																				
		0 21 2 42 4 63 6	1 2 2 3 4 4 6	2 3 2 4 4 5 6	3 4 2 5 4 6 6	4 5 2 6 4 7 6	5 6 2 7 4 8 6	6 7 2 8 4 9 7	7 8 2 9 5 0 7	8 9 3 0 5 1 7	9 1 0 3 1 5 2 7	0 1 1 3 2 5 3 7	1 1 2 3 3 5 4 7	2 1 3 3 4 5 5 7	3 1 4 3 5 5 6 7	4 1 5 3 6 5 7 7	5 1 6 3 7 5 8 7	6 1 7 3 8 5 9 8	7 1 8 3 9 6 0 8	8 1 9 4 0 6 1 8	9 2 0 4 1 6 2 8	0 21 1 42 2 63 3 84
Multipliers	....	0.0000	0.0019	0.0327	0.2209	0.8614	2.3324	4.8415	8.1736	11.6357	14.2713	15.2572	14.2713	11.6357	8.1736	4.8415	2.3324	0.8614	0.2209	0.0327	0.0019	0.0000
Original units	100.0000	0.0000	0.0037	0.0618	0.3799	1.3429	3.3219	6.3611	9.9862	13.2852	15.2573	15.2573	13.2852	9.9862	6.3611	3.3219	1.3429	0.3799	0.0618	0.0037	0.0000	
70–71	9.9934	0.4838	0.2331	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0032	0.0021	0.0861	0.2331	0.4838	0.8168	1.1628	1.4262	1.5247	1.4262	1.1628	0.8168
71–72	9.9977	0.8172	0.4840	0.2332	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0861	0.2332	0.4840	0.8172	1.1633	1.4268	1.5253	1.4268	1.1633
72–73	10.0019	1.1638	0.8175	0.4842	0.2333	0.0862	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0861	0.2333	0.4842	0.8175	1.1638	1.4274	1.5260	1.4274
73–74	10.0049	1.4278	1.1641	0.8178	0.4844	0.2333	0.0862	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0862	0.2333	0.4844	0.8178	1.1641	1.4278	1.5265
74–75	10.0058	1.5266	1.4280	1.1642	0.8178	0.4844	0.2334	0.0862	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0862	0.2334	0.4844	0.8178	1.1642	1.4280
75–76	10.0044	1.4277	1.5264	1.4277	1.1641	0.8177	0.4844	0.2333	0.0862	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0862	0.2333	0.4844	0.8177	1.1641
76–77	10.0019	1.1638	1.4274	1.5260	1.4274	1.1638	0.8175	0.4842	0.2333	0.0862	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0861	0.2333	0.4842	0.8175
77–78	9.9998	0.8174	1.1636	1.4271	1.5257	1.4271	1.1636	0.8173	0.4841	0.2332	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0861	0.2332	0.4841
78–79	9.9977	0.4840	0.8172	1.1633	1.4268	1.5253	1.4268	1.1633	0.8172	0.4840	0.2332	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0861	0.2332
79–80	9.9970	0.2332	0.4840	0.8171	1.1632	1.4267	1.5252	1.4267	1.1632	0.8171	0.4840	0.2332	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221	0.0861
80–81	9.9971	0.0861	0.2332	0.4840	0.8171	1.1632	1.4267	1.5253	1.4267	1.1632	0.8171	0.4840	0.2332	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033	0.0221
81–82	9.9982	0.0221	0.0861	0.2332	0.4841	0.8172	1.1633	1.4269	1.5254	1.4269	1.1633	0.8172	0.4841	0.2332	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002	0.0033
82–83	9.9995	0.0033	0.0221	0.0861	0.2332	0.4841	0.8173	1.1635	1.4271	1.5257	1.4271	1.1635	0.8173	0.4841	0.2332	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000	0.0002
83–84	10.0010	0.0002	0.0033	0.0221	0.0861	0.2332	0.4842	0.8174	1.1637	1.4273	1.5259	1.4273	1.1637	0.8174	0.4842	0.2333	0.0861	0.0221	0.0033	0.0002	0.0000	0.0000

are somewhat lengthy and tedious. They are illustrated in Table 3.8 for mortality type curve S_3, with 10 years average life, and will be explained below.

The renewals of the original units are the retirements, during the respective age intervals in the first column of Table 3.8, indicated by the frequency-curve data. In Table 3.8, the renewals shown in the horizontal line designated "original" are taken directly from the frequency-curve data of mortality type curve S_3 for 10 years average life.

The renewals of renewals during each service year are the sum of the renewals this year of renewals made in previous years (as indicated by the mortality-frequency data of the original group) plus the renewals during the year of renewals made during the year.

To get the renewals during any service year of the renewals made during any previous year, it is necessary to use multipliers, each the average of the original-group-frequency data for two succeeding years,[1] as shown in Table 3.8 in the horizontal line, designated *multipliers*, for the service ages listed in the first horizontal line at the top of the table. This is because the average age of each year's renewals is only one-half year at the end of the year in which they are made; the first year of their service extends to the middle of the year after the one in which they are made.

The results obtained by applying the proper multipliers to the renewals made in previous years are shown in each service-age column of Table 3.8, below the "original units" horizontal line; in addition, the results immediately above the zigzag heavy lines indicate the renewals during the year of renewals made during the year.

To get the renewals during the year of renewals made during the year, it is necessary to multiply the sum of the renewals during the year of original units, and of renewals during the year of renewals made during previous years, by the numerical value for this mortality group of the algebraic expression[2]

$$x = \left(\frac{r}{2 - r}\right)s,$$

in which x equals renewals of renewals during the year in which they are placed in service, r equals rate of renewals the first year of service (as shown by the mortality-frequency data of the original group), and s is the sum (referred to just above) of all renewals during the year, except the renewals of renewals made during the year.

In the case of mortality type curve S_3, $r = 0$ and hence $x = 0$. This is shown in Table 3.8 by the fact that in each service-age column the entry immediately above each zigzag heavy line is 0.000.

[1] In later calculations (see *Bull.* 125) the Iowa Engineering Experiment Station is using more exact multipliers, equal to the sum of the frequencies of retirements in the *last half of the preceding* and the *first half of the succeeding* age intervals.

[2] Since the renewals $s + x$ made during the year have an average service age of one-half year at the end of the year, their average rate of retirement is $\frac{r}{2}$; whence $x = (s + x)\frac{r}{2}$, and $x = \left(\frac{r}{2 - r}\right)s$. In later calculations (see *Bull.* 125) the Iowa Engineering Experiment Station is using the more exact formula $x = \frac{1}{1 - r_1}s$, in which r_1 is the frequency of retirements during age 0 to age ½ the age interval.

In Table 3.8 the total renewals are shown in the second column for the service years shown in the first column.

Each of the entries in the second column is the sum of all the entries in the horizontal line in which it stands; and is also equal to the sum of the entries in the corresponding service-age column, between *the corresponding heavy zigzag lines.*

The renewals indicated in Table 3.8 are plotted in Fig. 3.7 to service age 19 to 20.

Although Table 3.8 has been calculated for renewals per year (with service ages stated in years), it is manifest that the calculations would be of the same character for renewals per age interval equal to 10 per cent of average life (with service ages stated in percents of average life). Such computations have been made for the mortality type renewals curves (see Table B.3 and Fig. B.8, Appendix B). Study of these renewals curves shows that they are characterized by gradually diminishing sine waves, oscillating about the "normal-renewals" horizontal line; the curves eventually straighten out at the percent of normal renewals; this is always equal to $\frac{100 \text{ percent}}{\text{average life}}$ (average life in years, for annual renewals, or in age-interval percent for renewals per age interval stated in percent of average life).

Winfrey[1] has found that the renewals curves shown in Fig. B.8, Appendix B, are fitted closely by damped sine-wave empirical curves; their general equation is

$$y = 10 + de^{-ax} \sin (bx + \theta),$$

in which y equals ordinate, x age, e the base of natural logarithms equals 2.30258, and a, b, d and θ are the empirical constants given in Table 3.9 for 13 curves.

TABLE 3.9.—SUMMARY OF RENEWAL-EQUATION CONSTANTS FOR 13 TYPE CURVES*
General equation: $y = 10 + de^{-ax} \sin (bx + \theta)$

Type No.	No. of oscillations considered	Equation constants			
		d	a	b	θ
1L	1.5	0.320310	0.005626	0.033939	1.790550
2L	2.5	8.234646	0.021513	0.061576	2.568117
3L	4	21.495494	0.019916	0.070757	1.446579
4L	5	31.221335	0.014011	0.064562	1.674037
1S	3	12.311136	0.015060	0.046657	2.024705
2S	4	13.912270	0.013980	0.052911	2.140251
3S	6	17.825392	0.010835	0.058457	1.900912
4S	10	21.624785	0.006790	0.061546	1.684121
5S	10	22.148228	0.003879	0.062788	1.452255
1R	4	12.510099	0.012506	0.044186	1.754166
2R	4.5	15.393899	0.012399	0.051064	1.713436
3R	6	17.898690	0.009567	0.055975	1.655997
4R	10	21.544835	0.006687	0.059500	1.548696

* See *Bull.* 103, *Iowa Eng. Exp. Sta.*, pp. 42–44.

3.31. The Average Condition-percents, at Given Ages, of the Survivors of All Service Ages, in Mortality Groups of Industrial-property

[1] WINFREY and KURTZ, *op. cit.*, pp. 42–44.

Units Kept at 100 Percent Numbers by Continued Renewals.—From the mortality-frequency data, survivor data, average condition-percent of the survivors data, and renewals data (developed as described in Secs. 3.24, 3.26, 3.29, 3.30), it is practicable to compute the average condition-percents, at given ages, of survivors of all ages, in mortality groups of industrial-property units kept at 100 percent numbers by continued renewals. For mortality type curve S_3 these data have been developed in Secs. 3.29 and 3.30 for 10 years average life and 6 percent net return. The yearly renewals may be found in Table 3.8. The other mortality data needed will be found in Table 3.10.

TABLE 3.10.—MORTALITY DATA FOR TYPE CURVE S_3
10 years average life, 6% net return

Age, years	Survivors, %	Average condition-percent of survivors, %	Age, years	Survivors, %	Average condition-percent of survivors, %
0	100.00000	100.0000	0	100.0000	100.00
½	100.00000	95.9066	1	100.0000	91.67
1½	99.99955	87.3366	2	99.9963	83.00
2½	99.98158	78.2748	3	99.9345	73.71
3½	99.81436	68.7947	4	99.5547	63.98
4½	99.06070	59.1759	5	98.2117	54.46
5½	96.8691	49.8764	6	94.8899	45.48
6½	92.1431	41.3580	7	88.5285	37.59
7½	83.9926	33.9167	8	78.5422	30.64
8½	72.2507	27.6337	9	65.2576	24.92
9½	57.7605	22.4346	10	50.0000	20.18
10½	42.2395	18.1674	11	34.7424	16.33
11½	27.7493	14.6636	12	21.4578	13.15
12½	16.0074	11.7692	13	11.4715	10.53
13½	7.8569	9.3545	14	5.1101	8.32
14½	3.1309	7.3167	15	1.7883	6.44
15½	0.9393	5.5742	16	0.4453	4.83
16½	0.18564	4.0647	17	0.0655	3.37
17½	0.01842	2.7370	18	0.0037	2.13
18½	0.00045	1.5558	19	0.0000	1.01
19½	0.00000	0.0000	19½	0.000000	

From the above data and the yearly renewals given in Table 3.8, the computations of the average condition-percents of the whole 100 percent group at different ages may be made, as indicated in Table 3.11.

Table 3.11.—Illustrating Computations of Average Condition-percents, at Given Ages, of Survivors of All Ages, in Age-groups of Like Units Kept at 100% Numbers by Continued Renewals

Mortality data for type curve S_3, 10 years average life, 6% net return

Age of original units when renewals made	Total renewals, %	Columns showing distribution as to ages, at given ages of the original group, of all survivors, including renewals								
		Age, 1 year,	Age, 2 years,	Age, 3 years,	Age, 4 years,	Age, 5 years,	Age, 6 years,	Age, 7 years,	Age, 8 years,	Continue
		%	%	%	%	%	%	%	%	
Original units		100.0000	99.9963	99.9345	99.5547	98.2117	94.8899	88.5285	78.5422	Continue
Renewals, years										
0.5	0.0000									
1.5	0.0037	....	0.0037	0.0037	0.0037	0.0037	0.0036	0.0036	0.0034	Continue
2.5	0.0618	....	100.0000	0.0618	0.0618	0.0618	0.0617	0.0612	0.0597	
3.5	0.3799	....	....	100.0000	0.3799	0.3799	0.3798	0.3792	0.3763	
4.5	1.3429	....	....	....	100.0001	1.3429	1.3429	1.3422	1.3404	to age 40
5.5	3.3222	....	....	....	....	100.0000	3.3222	3.3222	3.3216	
6.5	6.3630	....	....	....	....	....	100.0001	6.3630	6.3630	
7.5	9.9954	....	....	....	....	....	....	99.9999	9.9954	
8.5	13.3187	....	....	....	....	....	....	....	100.0020	Continue
Average condition—percent of all survivors............		91.670	83.000	73.724	64.116	55.157	47.857	43.631	42.984	Continue

Continued, years	Continued, %	Survivors at age 40 years, percent	Age of survivors, years	Condition-percents of survivors, %	Products survivors × condition-percents,
22.5	10.9105	0.0020	17.5	2.7370	0.0055
23.5	10.0726	0.0187	16.5	4.0647	0.0760
24.5	9.3680	0.0880	15.5	5.5742	0.4905
25.5	8.9891	0.2814	14.5	7.3167	2.0589
26.5	8.9941	0.7066	13.5	9.3545	6.6099
27.5	9.3102	1.4903	12.5	11.7692	17.5396
28.5	9.7796	2.7138	11.5	14.6636	39.7941
29.5	10.2263	4.3195	10.5	18.1674	78.4741
30.5	10.5158	6.0740	9.5	22.4346	136.2678
31.5	10.5891	7.6507	8.5	27.6337	211.4171
32.5	10.4660	8.7907	7.5	33.9167	298.1515
33.5	10.2237	9.4204	6.5	41.3580	389.6089
34.5	9.9606	9.6487	5.5	49.8764	481.2424
35.5	9.7607	9.6690	4.5	59.1759	572.1718
36.5	9.6733	9.6553	3.5	68.7947	664.2335
37.5	9.7034	9.7016	2.5	78.2747	759.3898
38.5	9.8186	9.8186	1.5	87.3366	857.5231
39.5	9.9676	9.9676	0.5	95.9066	955.9586
		100.0169	Sum of products...........................		5471.0131
			Average condition-percent................		54.701%

The upper part of the Table 3.11 illustrates the standardized form for making the computations and gives the computations in full for ages one to eight years, inclusive (of the original group).

The second vertical column shows the total renewals (taken from Table 3.8) each age-year (0 to 1, 1 to 2, 2 to 3, etc.) from the time the original group was put in service. Since the renewals are distributed throughout each year, their average age is one-half year at the end of the year in which they were made.

The horizontal line designated "original units" gives the survivors of the original group at the respective service ages of the original group. Below this, the horizontal lines show the survivors (at the given ages of the original group) of the respective annual renewals, the service age of which for each column is equal to the age given at the head of the column minus the age indicated in the first column for the horizontal line.

Thus the upper part of the Table 3.11 (continued to the right in like manner) shows the percentage distribution by ages of all survivors at the respective service ages of the original group indicated at the heads of the respective vertical columns.

TABLE 3.12.—MORTALITY-TYPE-S_3-GROUP RENEWALS AND AVERAGE CONDITION-PERCENTS

10 years average life, 6% net return

Group kept at 100% numbers by continued renewals

Age interval, years	Total renewals, %	Age interval, years	Total renewals, %	Age, years	Average condition-percent, %	Age, years	Average condition-percent, %
				0	100.000		
0–1	0.0000	20–21	11.8674	1	91.670	21	54.517
1–2	0.0037	21–22	11.6029	2	83.000	22	56.239
2–3	0.0618	22–23	10.9105	3	73.724	23	57.293
3–4	0.3799	23–24	10.0726	4	64.116	24	57.486
4–5	1.3429	24–25	9.3680	5	55.157	25	56.918
5–6	3.3222	25–26	8.9891	6	47.857	26	55.907
6–7	6.3630	26–27	8.9941	7	43.631	27	54.848
7–8	9.9954	27–28	9.3102	8	42.984	28	54.080
8–9	13.3187	28–29	9.7796	9	46.022	29	53.784
9–10	15.3587	29–30	10.2263	10	51.403	30	53.960
10–11	15.5201	30–31	10.5158	11	57.227	31	54.464
11–12	13.8785	31–32	10.5891	12	61.518	32	55.073
12–13	11.1733	32–33	10.4660	13	63.067	33	55.579
13–14	8.4924	33–34	10.2237	14	61.776	34	55.848
14–15	6.7912	34–35	9.9606	15	58.582	35	55.842
15–16	6.5033	35–36	9.7607	16	54.927	36	55.618
16–17	7.4392	36–37	9.6733	17	52.114	37	55.287
17–18	8.9905	37–38	9.7034	18	50.851	38	54.971
18–19	10.4954	38–39	9.8186	19	51.166	39	54.765
19–20	11.5109	39–40	9.9676	20	52.516	40	54.701

NOTE.—The minor variations from exactly 100 percent survivors in some age columns is due to the fact that it was considered unnecessary for the degree of accuracy required to make certain minor adjustments in the calculations of renewals (see footnotes, Sec. 3.30). The lower part of Table 3.11 gives the details of the calculations of the average condition-percent of the whole 100 percent of units in service at the service age of 40 years (of the original group).

NOTE.—In making these calculations on a computing machine, no record of the respective "products" is necessary, since they can be retained and added on the machine without separate record.

Normal Average Condition-percent.—The normal average condition-percent of a group of like industrial-property units, originally put in service in the same year, is its ultimate uniform average condition-percent when kept by renewals at 100 percent numbers long enough for the renewals to become constant (normal renewals).

The normal average condition-percent for mortality type-S_3 groups, for 10 years average life and 6 percent return, is 55.13 percent.

The average condition-percents of groups of like property units, installed the same year and kept at 100 percent numbers by renewals, fluctuate above and below the "normal" average condition-percent in very much the same manner as the annual renewals fluctuate with relation to normal renewals; and both would eventually become uniform, and equal to the "normal."

This is illustrated for mortality type-S_3 groups by the data given in Table 3.12.

CHAPTER IV

DEPRECIATION PRINCIPLES AND RELATIONS

It cannot be repeated too often that the fundamental basis of value is prevailing opinion as to the present worth of probable future returns. Depreciation is *negative value;* its fundamental basis, also, is prevailing opinion as to the *probable* future operation returns yet to be earned by physical-property units during their *probable* future service lives.

GENERAL DEPRECIATION DEFINITIONS AND RELATIONS

4.1. Depreciation Defined.—The depreciation of a physical-property unit at any service age is its loss of value during its service to date; this is due to the decrease, since it was first put into service new, of the present worth of its probable future-earned operation returns.

A unit of physical property loses value[1] due to age and use because it thereby suffers a decrease in the amount and character of the future service it can render before it will have to be retired. The present value of the service remaining in the unit at any date is the actual present worth, at that date, of the probable future operation returns which will be earned by its services.

The *basic measure* of the amount of the depreciation at any date is not the amount of service the unit has rendered, although it is during the rendering of that service that it has actually depreciated in value. The basic measure of depreciation, on the contrary, is the decrease in the present worth of the service remaining in the unit; the amount of the depreciation is the difference between the value new and the present worth of the future service remaining in the unit, allowing for salvage value, if any.

Since the remaining service lies in the future, it cannot be foretold with certainty; but the future service life of the property can be re-estimated from time to time, as indicated in Fig. 4.1, and as the unit approaches retirement such estimates can be made with increasing accuracy. The actual service life is never known with certainty until the property is finally retired.

In general, the physical-property unit's loss of value by depreciation is inevitable, for that loss cannot be made good until the future date

[1] The general nature of the increase of the depreciation of a physical-property unit from date to date during its service life is indicated in Fig. 4.1, for the numerical data of which see Sec. 4.2.

when the unit is retired from service and replaced by a new unit. No amount of wise expenditure for maintenance can accomplish more than to insure that the date of retirement of the unit is postponed to the end of its normal service life.

The main cause of depreciation is decrease in the number of future annual operation returns; this is due to decrease of expectancy of future service life. For example, the present worths of the probable future operation returns yet to be earned by the property unit whose depreciation and present value changes are illustrated in Fig. 4.1 compare as follows:

Service Age		Present Value
New, 0 years	=	present worth of 20 future yearly operation returns
When valued, 12 years	=	present worth of 9 future yearly operation returns
Decrease in value	=	present worth of the last 11 future operation returns of the original 20 (the tenth to the twentieth, incl.).

An additional cause of depreciation is decrease in the future annual value of operation returns; this is due to lowered efficiency of the unit, lowered output capacity, increased cost of maintenance, increased running costs, intermittent (stand-by) service, and/or operation at less than normal capacity. These possibilities are investigated; if the value of the probable future annual operation returns will be affected materially thereby, a correction factor is applied (designated herein PFORR, the "probable future operation-return ratio").

4.2. Figure 4.1 Illustrative of Depreciation.—Figure 4.1 illustrates the depreciation of a physical-property unit during its service life. The numerical data of the unit are as follows:

Value new	= cost new	= 100.00%	Salvage value[1]	= 10.00%
Overhead cost	= 15% of direct cost	= 13.04	Depreciable value new	= 90.00
Direct cost	=	86.96	Rate of net return	= 6.00

Service age, years	Probable life, years	Service age, years	Probable life, years
0–8	20	15–21	22
8–15	21	21–23	23
Valued at age 12 years		Retired at age 23 years	

When the unit was new, its probable life was judged to be 20 years, and it was not necessary to change this estimate during the first 8 years of service. The estimate of probable life was raised to 21 years at age 8, 22 years at age 15, and 23 years at age 21. Retirement was at age 23.

[1] Salvage value is usually much less than 10 percent and often is 0.

The estimates of total accrued depreciation were readjusted during the ninth, sixteenth, and twenty-second years, as shown in Fig. 4.1, to agree with the re-estimated probable lives. Thus the depreciation conformed throughout the entire service life to the actual life experience of the unit and checked out correctly at the actual date of its retirement.

4.3. Secondhand or Forced-sale Values.—Often some units of property (automobiles, for example) are not continued in service, with adequate yearly maintenance, until wisely retired for sound economic reasons, but are sold for what they will bring at the expiration of only a

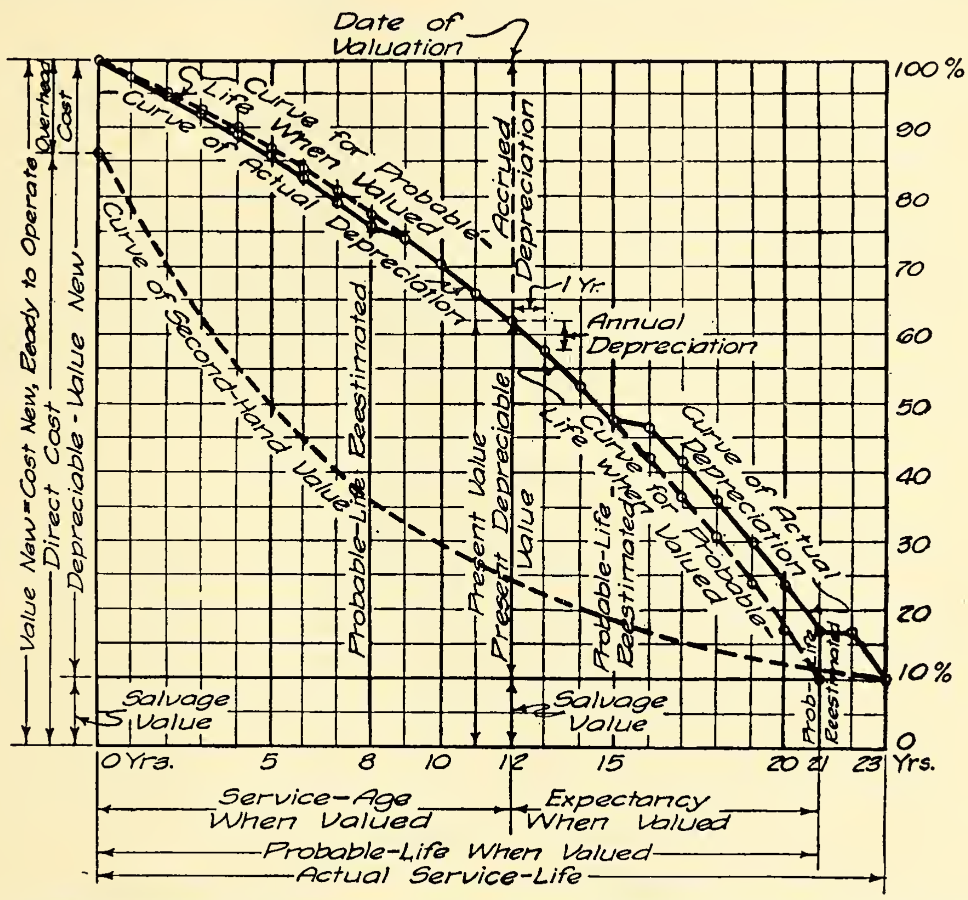


FIG. 4.1.—Diagram showing depreciation and present-value relations for the property unit for which data are given in Sec. 4.1.

fraction of their attainable service lives, because out of fashion or because sale is forced.

In such cases, there are material sacrifices of the values which might be realized by continued use by the original owner. What he receives is of the nature of salvage value; the law of depreciation is no longer that explained in Sec. 4.1 and illustrated by the upper curve of Fig. 4.1.

The prices received in secondhand or forced sales follow some such law as is illustrated by the lower curve of Fig. 4.1; this has been constructed to show approximately the loss of value at different percentages of average life which would be indicated by the 1934 automobile code allowances for used cars, if their ultimate salvage value was 10 percent (instead of 0).

4.4. Depreciation a Real Current Loss of Value and a Real Current Cost of Production.—Depreciation is a real and inescapable current loss of value, and current element of the cost of production, which must be correctly accounted for each year and adequately provided for in industrial undertakings, the same as all other operation costs and losses of capital. In a pamphlet the United States Chamber of Commerce says:[1]

> Every thousand of brick produced, every case of shoes manufactured, every fashioning of metal and turning of wood, each twist of the handle and blow of the hammer helps to wear out your plant and equipment, and the particular job or process that causes this loss should bear the cost.
>
> . . . .
>
> The business man who does not charge depreciation at all is fooling himself. He is making no provision for the inevitable day when his property must be scrapped. His supposed profits may be in fact a distribution of his capital.
>
> The business man who does wait till the end of the year to determine his depreciation according to the size of his profits may make the discovery that he has no profits, since he has inconsistently sold his product upon a cost that was incorrect—a cost that did not include such a necessary expense of manufacture as depreciation.

The fact that depreciation is a real current loss of value and element of the cost of production is often illustrated by such instances as that of the steam boiler which cost $15,000, installed new, and was retired and replaced at the end of 15 years of service, with no net salvage.

During its service, a large amount of coal was burned, the cost of which and all other expenses on the boiler were charged each year to the cost of current steam production. The boiler was consumed in the 15 years just as completely as the coal, suffering a depreciation loss of value of 100 percent, or $15,000. It manifestly would be grossly incorrect to fail to charge this depreciation to the steam-production costs, year by year. If it was not charged off at all, the accounts were in error $15,000 at the end of the 15 years of service, and the steam-production costs recorded were too small every year. If it was all charged off in the fifteenth year (retirement accountancy), the steam-production costs that year were too great by the major part of $15,000 and were too small every other year the boiler was in service.

The only correct way to handle the accounting for the costs of steam production by this boiler is to determine its actual depreciation each year of the 15 and charge the yearly actual depreciations to the steam-production costs for each year.

4.5. Actual Depreciation.—Knowledge and clear understanding of depreciation have been developed mainly since 1900. At first, estimates

[1] "Depreciation Treatment in Production Costs," Department of Manufacture, U.S. Chamber of Commerce, Washington, D.C., reprint, 1929, pp. 5 and 6.

of depreciation were based wholly on theoretical mathematical office calculations. Since 1920 the courts have ruled more and more frequently and definitely that true depreciation is *actual depreciation*, determined by the aid of actual examinations of the respective property units, rather than *theoretical depreciation*, determined mainly or wholly by office computations.

Actual depreciation is the true loss of value of property units during service, as determined by

1. Competent valuation experts.
2. Studying data obtained by careful examinations of the units.
3. The application of correct depreciation principles to the observed facts.

The actual depreciation of a property unit is not, like its color or its shape, a physical fact which can be observed. From the data which can be observed, or otherwise ascertained, the probable life of each unit and its probable future operation-return ratio (PFORR) are first determined; from these the actual depreciation can be determined by methods which are explained in Chap. V. By re-estimating the probable life and the PFORR from time to time, and readjusting the total depreciation accordingly, the depreciation is spread over the actual service life of each unit in accordance with the actual facts of its life history.

Quotations from three court decisions on depreciation are presented below to show that the courts uphold actual depreciation in preference to theoretical depreciation; at least whenever the actual depreciation is proved in court by competent and convincing testimony.

In the *Indianapolis Water Case* (Sec. 8.6), decided in 1926, Mr. Justice Butler said, in writing the decision of the United States Supreme Court:

The testimony of competent valuation engineers who examined the property and made estimates in respect to its condition is to be preferred to mere calculations based on averages and assumed probabilities.

In the *Otis Steel Co. Case*, as quoted in the decision of the *Cumberland Glass Co. Case* (Sec. 8.6), the U.S. Board of Tax Appeals said:

The depreciation written off on the books of petitioner . . . results from an actual physical examination and inspection of its plant, machinery, and equipment.

A straight-line basis for depreciation has been adopted by the Commissioner [of U.S. Internal Revenue], writing off a special percentage each year, quite apart from the actual conditions affecting depreciation. . . . We are of the opinion that the method used by petitioner, of establishing the depreciation actually sustained during the years in controversy, is more accurate and reasonable than the method used by the Commissioner.

In the *Cumberland Glass Co. Case* (Sec. 8.6), decided 1930, the United States Court of Claims said:

> In the instant case the depreciation charged off each year . . . was determined by officials of the company who had been connected with the management for many years and who possessed accurate and technical knowledge of the plant, knew the age, cost, and probable useful life of its various items of equipment and the current cost of repairs and replacements.
>
> That they acted in the utmost good faith and exercised their best judgment in estimating the depreciation charged off each year is not questioned.
>
> While the straight-line or fixed percentage method adopted by the Commissioner [of U.S. Internal Revenue] in determining plaintiff's allowable depreciation . . . is one most generally used in determining depreciation for tax purposes, and is quite generally accepted as the simplest and most accurate of the various methods used, computation made on that basis cannot stand where the facts in a particular case, as here, show that the results reached by such a method would not be a reasonable allowance within the meaning of the statute.

4.6. Theoretical Depreciation.—Theoretical depreciation is depreciation calculated by some *assumed* mathematical formula for its distribution throughout the average service lives of similar property units, serving under similar conditions. The calculation of theoretical depreciation

1. Can be made by any accountant or engineer without examination of the property units.
2. Is based on the average service lives of similar units, instead of on the actual service lives of the actual units.
3. Is based on assumed mathematical formulas for distribution of depreciation throughout the assumed average life.

4.7. Depreciation Estimated Separately for Different Units.—Industrial properties are made up of great numbers of individual units, which vary as to service age, service condition, physical condition, and expectancy of future years of service life. It is not possible to estimate correctly in a lump sum the total depreciation of an industrial property as a whole. The indiscriminate use of an average age and an average service life for groups of property items in making depreciation estimates leads to erroneous results. On the contrary, it is necessary to estimate the depreciation of each unit of property, or of each age-group of like units, separately, in the manner illustrated in Fig. 4.1. In so doing, each unit, or group of like units, must actually be examined and the probable life and the PFORR estimated.

Because of the widespread and eventually comparatively uniform distribution of retirements of individual property units, the law of depreciation of an industrial property as a whole is widely different from the law of depreciation of an individual unit of physical property. This will be clear if Fig. 4.3 is compared with Fig. 4.1.

4.8. Annual Estimates of Actual Depreciation.—The accounts of industrial enterprise will not show the real status of the property unless the yearly accounts show the actual annual depreciation, and the book-value accounts show each year the true accrued actual depreciation. The correct estimation of the actual depreciation of the various property units involves sufficiently frequent re-examinations (Sec. 3.20) to check yearly their estimated probable lives and PFORR (Sec. 4.28), separately or by groups of like units.

Each annual estimate[1] of the depreciation of each property unit or age-group of like units must be of the total depreciation to date, in accordance with its probable life and its PFORR, both being re-estimated whenever found necessary. The annual depreciation deductions from value and the corresponding annual depreciation appropriations from income must be adjusted during the following year or two so as to conform with the facts, as illustrated in Fig. 4.1. By this plan, the estimated depreciation losses of value always conform to the actual condition of the property units from year to year throughout their entire service lives.

RETIREMENTS AND THEIR CAUSES

As already stated, the main cause of the depreciation of a property unit is the inevitable approach of the date of its retirement.

4.9. Retirements.—Retirements are withdrawals from service of industrial-property units at the ends of their service lives. Retirements are due mainly to seven causes:

A. Physical Causes

- *a. Sudden physical damage*, due to
  1. Miscellaneous accidents
  2. Disasters
- *b. Physical decrepitude*, due to
  3. Physical deterioration
  4. Wear and tear

B. Functional Causes

- *c. Functional inefficiency*, due to
  5. Inadequacy
  6. Obsolescence
  7. Supersession

Except for retirements due to sudden physical destruction, the exact date when any particular property unit is retired is decided by judgment; this should be guided by a correct wise-retirement principle.

[1] In the accounts, one-twelfth the estimated actual depreciation for the year is usually charged off each month.

4.10. Replacements.—Property units retired may be renewed (Sec. 3.2) or replaced. Replacements are substitutions for property units, at the end of their service lives (usually because they are damaged, worn out, or obsolescent), of new units which may or may not substantially duplicate the former. Replacements include renewals (which duplicate the old units) but are not limited to renewals. Improvements in machinery, for example, may make it advisable to replace a former machine by one very different in character, which may cost more or cost less than the original machine.

4.11. Retirements Due to Sudden Physical Damage.—Sudden physical damage may be caused by

1. *Miscellaneous accidents*, such as explosions, collisions, falls, failures of buildings and other structures, or the breaking of machinery by extraneous agencies.
2. *Disasters*, such as fires, storms, floods, or earthquakes.

When the sudden damage amounts to destruction, the unit is retired thereby. When the damage is partial, the cost of repairs or reconstruction must be weighed against cost of renewal in deciding whether or not to retire the property.

Losses due to accidents and/or disasters should be insured against so far as wisely feasible; insurance costs should be charged to current operation expenses instead of to depreciation. Receipts from insurance realized increase income. Emergency losses which cannot wisely be insured against should be charged to current operation expenses unless too large; in such case, they should be amortized over a sufficient term of years, out of income before operation return.

4.12. Retirements Due to Physical Decrepitude.—Physical decrepitude is that physical disablement of industrial-property units which develops and increases during service in spite of wise expenditures for repairs and other maintenance. Physical decrepitude may be due to

3. *Physical deterioration* from rusting and other chemical processes, the mechanical effects of freezing and thawing and other temperature variations, the gradual decay of timber and other materials of organic origin, etc. Examples: A wooden bridge unsafe because of decay, or a steel bridge dangerously weakened by corrosion. Physical deterioration increases with age and extent of exposure to destructive elements rather than with use.
4. *Wear and tear*, due to friction, impact, vibration, stress, fatigue of materials, etc., caused by use. Examples: An engine, a lathe, a concrete mixer worn out by use. Wear and tear are proportional to use rather than to age.

4.13. Retirements Due to Functional Inefficiency.—Industrial-property units are functionally inefficient whenever their services could be rendered more efficiently by larger, better, or different units. Functional inefficiency may be due to

5. *Inadequacy;* or insufficient capacity for the service required of the unit. Example: 150-horsepower boilers and engines when 200-horsepower is required.
6. *Obsolescence;* usually due to the invention and development of later, improved devices of the same general character. Example: Boilers and engines for 300 pounds

per square inch steam pressure in large power plants where new 1,000 pounds per square inch boilers and engines would effect great savings.

7. *Supersession;* in cases where the same service can be rendered with greater efficiency by quite different kinds of structures and/or equipment. Example: In such power plants as they would be more economical, the substitution of electric power or internal-combustion engines for steam boilers and engines.

4.14. Physical and Functional Depreciation.—Physical depreciation is depreciation due to retirements for physical causes, including (1) accidents, (2) disasters, (3) physical deterioration, (4) wear and tear.

Since the mortality data for industrial-property mortality curves include all retirements, depreciation estimates correctly based on properly estimated probable lives of industrial-property units contain advance allowances for retirements due to accidents and disasters. Nevertheless, accidents and disasters should be insured against, as stated in Sec. 4.11.

Functional depreciation is depreciation due to retirements for functional inefficiency, including (5) inadequacy, (6) obsolescence, (7) supersession.

As in the case of accidents and disasters, depreciation estimates correctly based on properly estimated probable lives of industrial-property units contain advance allowances for retirements due to functional inefficiency; every reasonable effort must be made in estimating depreciation to forecast retirements for inadequacy, obsolescence, and/or supersession.

Losses due to functional depreciation which could not reasonably have been forecast should be paid out of current income *before* operation return; if too large to be paid from *current* income, they should be amortized over a limited, reasonable period of years, *before* operation return.

4.15. The Wise-retirement Principle.—It must always to an important extent be a question to be decided by good judgment just when to retire from service any industrial-property unit not destroyed by accident or disaster. In most cases, the proper retirement date is selected without theoretical calculations, although there is in each case a definite theoretical date of wise retirement.

The wise-retirement principle is that retirement is due when the actual unit cost of service by the old property unit becomes greater than the unit cost of service by a new unit.

The total cost of the services rendered by any property unit in any year is its operation costs (including overheads), plus its fair operation return (annual depreciation plus fair net return on its present value); the unit cost of service is the total annual cost divided by the total number of service units rendered.

For the old unit, the unit service costs which must be used in applying the wise-retirement principle are those estimated for the next few years. The actual costs during the last few years should be given due weight in estimating future costs for the old unit, but repairs and other maintenance work are often neglected as a property unit approaches retirement. The fair operation return for the old unit is merely

salvage value times rate of fair return; at retirement it can have no further depreciation, and its total value is merely its salvage value.

For a new unit to replace the old unit, the unit service costs should be those estimated with the help of recent experience with new units; the fair operation return should be that calculated by the formula (5–13):

$$R_u = rV_{nd}\left[\frac{(1+r)^n}{(1+r)^n - 1}\right] + rV_s.$$

There are three main factors which tend to increase the unit service costs of used industrial property units to such an extent as to make retirements advisable:

1. The annual maintenance costs often become excessive after a certain age, especially as compared with more modern equipment. Wise maintenance must not exceed the true economic limit, although the service lives of property units can be prolonged, and hence their yearly depreciations diminished, by comparatively large repair expenditures. Neglect of repairs decreases service life and therefore increases yearly depreciation.

2. The annual running costs of used units often, after a certain age, will be found to be greater than for newer or more modern equipment; owing to decreased efficiency, to the development of units inherently more efficient, to increased cost of attendance, or to a combination of these factors.

Typical running costs for mechanical equipment include those for attendance, fuel, pounds of steam, kilowatt-hours of electrical energy; for office buildings they include heat, light, and janitor service.

3. The service output capacity per unit of time may decrease materially after a certain age. Service output capacity may be measured in either value or quantity units, such as the millions-gallons-daily service capacity of a pumping engine or the annual-rent-dollars-possible service of an office building.

A Wise-retirement Formula.—The wise-retirement principle, stated above, can be expressed in the following mathematical wise-retirement formula. At the date of wise retirement,

$$[(C_m + C_r)(1+O) + rV_s]\frac{1}{S_r} = \text{or} > (C'_m + C'_r)(1+O) + rV'_{nd}\frac{(1+r)^{n'}}{(1+r)^{n'} - 1} + rV'_s, \quad (4\text{–}1)$$

in which C_m = probable future average annual direct maintenance costs.

C_r = probable future average annual direct running costs of the old unit during the next few years.

C'_m and C'_r = the corresponding costs for a new replacement unit.

O = percent of overhead operation expenses properly chargeable to these units.

S_r = service ratio to the annual amount of service which can be rendered by the new replacement unit.

n' = probable life of the new replacement unit in service.

r = fair rate of net return for this enterprise.

V_s = salvage value of the old unit.

V'_s = estimated salvage value of the new replacement unit.

V'_{nd} = depreciable value new of the replacement unit (= its cost new $- V'_s$).

Numerical Example: The retirement data in the case of a certain steam engine, which has been in service 25 years, are as follows:

Present salvage value of old engine, $200 net.

It is estimated that the future maintenance costs and running costs of the old engine will average, respectively, \$500 and \$450 per year for the next 2 years (including quite large repair costs the first year to get the engine in shape to use at all), and that it could do only 75 percent the work of a new engine.

The cost of a satisfactory new replacement engine, ready to run, would be \$8,256. Its salvage value is estimated to be \$200, its probable life to be 20 years and its average annual maintenance and running costs the next 2 years to be \$100 and \$300, respectively.

The percentage of overhead operation costs is 10 percent.

The fair rate of net return is 6 percent.

Substituting the above numerical data in the wise-retirement formula we have

$$[(\$950 \times 1.10) + \$12]\frac{1}{0.75} = \$1{,}409 > (\$400 \times 1.10) + \left(\$483.36 \times \frac{3.2071}{2.2071}\right) + \$12 = \$1{,}154.$$

Answer: The steam engine should be retired at once.

THE RELATION BETWEEN DEPRECIATION AND MAINTENANCE

Maintenance and depreciation are so closely allied that they are often confused, quite incorrectly.

4.16. Maintenance and Repairs.—The term *maintenance* conveys the idea of constantly keeping up the good condition of the property, and the maintenance costs thereby incurred constitute one important class of operation costs. Some examples of maintenance are the inspection, painting, and repair of structures, making good damages from wind, flood, or weather, and repairing machinery.

The term *repairs* conveys the idea of mending or replacing broken or worn parts by expert overhauling to restore run-down units of property to as good condition as is economical or practicable. Repair costs are a part of the maintenance costs and include both labor and materials.

Deferred maintenance is maintenance work which has been postponed beyond the date when it should have been performed, and which still remains to be done and paid for. The ultimate cost of deferred maintenance sometimes becomes a large amount, as in the case of failure of the railways to keep roadbeds, ties, rails, structures, and operating equipment in first-class condition in times of wars or severe business depressions.

4.17. Maintenance Contrasted with Depreciation.—Maintenance costs and depreciation are really quite different in character and treatment: maintenance costs are operation expenses, paid from current income as they are incurred; depreciation is the inevitable loss of value which accrues in spite of wise maintenance expenditures, and which can be made good only by replacing depreciated units by new units at the ends of their service lives.

By systematic maintenance, the depreciation of property is held down to a normal rate; nevertheless, depreciation will inevitably continue to accrue as an actual consumption of the fixed-capital property in the production process. Current depreciation should be offset by current depreciation appropriations from each year's income, of amounts sufficient to make good the depreciation during the year.

Annual maintenance expenditures are charged in accounting to the annual operation expense accounts and credited to the cash accounts; they do not involve any changes in the capital accounts.

Sometimes an arbitrary distinction is made between maintenance and depreciation by charging all renewals of parts costing less than say $100 or $200 to repairs instead of to replacements.

Annual depreciation costs should be credited to the depreciation reserve accounts and charged to the annual revenue accounts. At the same time there should be deductions from the book values of the physical-property units in the fixed-capital accounts equal in total amount to the total depreciation appropriation for the year, the units retired disappearing from the assets.

4.18. Deferred Maintenance Compared with Depreciation.—The eventual cost of deferred maintenance is an unpaid operation cost, which can be paid at any time by performing the deferred maintenance work. It is in no sense a depreciation cost. Depreciation, on the other hand, cannot be made good in any particular year, except insofar as there are actual replacements of physical items that are wisely retired in that year.

In making an engineering valuation of an industrial property, the estimated cost at the date of valuation of making good deferred maintenance must be deducted from the total value, for it is a liability which goes with the property. The deduction for deferred maintenance is stated separately in valuation reports; its amount is added to the value of the property when the deferred maintenance work is performed.

Some court special masters have erroneously assumed that certain industrial properties can be made "as good as new" by specified, limited, maintenance expenditures on designated physical-property units. They have confused deferred maintenance with depreciation.

THE RELATIONS OF DEPRECIATION TO ENGINEERING VALUATION

The decision of the United States Supreme Court in 1909 in the *Knoxville Water Co. Case* established the legal valuation principle, since upheld by numerous later decisions, that depreciation losses of value must always be deducted in determining the fair values of utility properties. The determination of the true actual depreciation is always an important part of the work of making an engineering valuation. The present fair value is always equal to the present depreciated value—the value new less depreciation to the date of valuation.

4.19. Estimates of Depreciation for Valuations.—In engineering valuation, the fair cost-values of the different property units are determined by giving "such weight as is just and right in each case" to their original costs and to their reproduction costs; the original-cost value new (Secs. 11.1, 11.2, 11.3) of each unit is ascertained, and its reproduction-cost value new (Sec. 11.2) is estimated; by giving these due weights, its fair cost-value new (Sec. 11.3) is determined.

Hence, in order to determine present values, three estimates of the depreciation of each unit must be made, based respectively on its original cost new, its reproduction cost new, and its fair cost-value new.

The same condition-percent must be applied to the three values new to get the three present values.

4.20. The Depreciation of Direct-cost Values and of Overhead-cost Values.—Part of the cost-value new of each property unit is based on its *direct costs* (Sec. 11.10), incurred for it alone, and part on its percentage share of the *overhead costs* (Sec. 11.11), incurred for all units jointly.

Both direct costs and overhead costs are subject to depreciation. This is illustrated in Fig. 2.3. The same condition-percent applies to both. Care must be taken in accounting to charge all overhead costs of replacement units to construction costs, instead of to operation costs; and to eliminate all duplication in the accounting.

4.21. Salvage Value.—The salvage value of a unit of industrial property is the net sum, over and above the cost of removal and sale, which can be realized for it when it is discarded at the end of its service life. Sometimes the highest salvage value of a unit of industrial property may be realized when the unit can be sold for further use, as when an electric motor can be sold for further use elsewhere as a motor. Salvage value often is zero, and sometimes may be negative; it is not necessarily the same as scrap value or junk value.

Scrap, or junk, value is the net value realized when the unit of industrial property is scrapped or broken up in order to use it as manufacturing material. It may be sold intact, or it may be cut or broken into pieces convenient for removal.

4.22. Depreciable Value.—This term is employed instead of the older term *wearing value* because it is more descriptive of the quality it is desired to designate. It is applied to new property and to property that has been in service.

1. *Depreciable Value New.*—The depreciable value new of a unit of industrial property is that part of the value which gradually disappears during its service life, owing to the progressive approach of its date of retirement. Depreciable value new equals the difference between value new and salvage value. Hence, depreciable value often equals the entire value new, since very often no net return at all can be secured in salvaging a unit of industrial property.

The depreciable value new of an entire industrial property is the sum of the depreciable values new of all its present existing units. Retired units are thus excluded.

There are three kinds (or estimates) of depreciable value new; that which is based on original cost, that which is based on reproduction cost, and that which pertains to fair cost-value.

2. *Present Depreciable Value.*—The present depreciable value of a unit of industrial property is that part of the total depreciable value which remains at the date of valuation. It is determined by deducting the total depreciation from the depreciable value new.

The present depreciable value of an entire industrial property is the sum of the present depreciable values of all its present existing units at the date of valuation. Retired units are thus excluded. In the case of a very large property, the present depreciable value eventually becomes nearly constant, except as increased by enlargements and improvements and as affected by changing price levels.

There are the same three kinds of present depreciable value as there are of depreciable value new; based respectively on original cost, reproduction cost, and fair cost-value.

4.23. Accrued Depreciation to Date of Valuation.—The accrued depreciation of any physical unit of industrial property to the date of valuation is its total actual loss of value between the date when it was first put into use new and the date of valuation of the property.

The accrued depreciation of an entire industrial property is the sum of the accrued depreciations of all its present existing units. Retired units are thus excluded. The accrued depreciation of a large property eventually becomes nearly constant, except for the effects of enlargements, improvements, and changing price levels.

There are three kinds of accrued depreciation, based respectively on original cost, on reproduction cost, and on fair cost-value.

4.24. Present Value.—The present value of a physical unit of industrial property is its value in its condition at the date of valuation, correct deductions from its value new having been made for accrued actual depreciation.

The present value of an entire industrial property is the sum of the present values of all its present existing units on the date of valuation. Retired units are thus excluded. The present value of a large property eventually becomes nearly constant, except for the effects of enlargements, improvements, and changes in price levels.

There are three kinds of present value, based respectively on the original cost, on reproduction cost, and on fair cost-value.

4.25. Annual Actual Depreciation.—The annual actual depreciation of a unit of industrial property is its actual depreciation during a year of its service life.

The annual depreciation of a single property unit varies from year to year of service, usually increasing with its age. *In this treatise, the annual depreciation as of the date of valuation will be taken to be the depreciation during the year following the date of valuation*, as illustrated in Fig. 4.1.

The annual depreciation of the entire property is the sum of the annual depreciations of all its units. This total annual depreciation does not change rapidly from year to year; in the case of a large property it eventually becomes nearly constant, except for the effects of enlargements, improvements, and changing price levels.

There are three kinds of annual depreciation, based respectively on the original cost new, on reproduction cost new, and on fair cost-value new.

4.26. Operation Return in Relation to Depreciation.—The annual operation returns earned by a physical-property unit are those shares, during its probable life, of the annual operation returns of the entire property whose combined present worth, at the date of installation of the unit new, is equal to its value new, ready to operate.

The operation return for an industrial property as a whole was defined in Sec. 2.21. In estimating depreciation it is necessary to deal in turn with each unit of the property, since there is no method whereby the depreciation of the property as a whole can be estimated correctly, except by summing up the separate estimates for the several units that comprise the whole. Hence, for depreciation purposes, it is necessary to recognize the fact that each unit of the physical property in reality earns a definite share of the total operation returns.

In the case of public utilities, the annual operation return for a unit should be just sufficient to repay all the yearly actual depreciation losses of value of the unit of the year incurred and, in addition, pay each year a fair rate of net return on the depreciated value of the unit at the beginning of the year. In the case of private industrial properties, the annual operation return must be sufficient to pay the yearly actual depreciation (or depletion) losses of value and in addition pay a satisfactory rate of net return on the depreciated value of the unit at the beginning of the year.

In all cases except those in which its annual unit service costs increase materially during the service life of the property unit, it is reasonable and convenient to estimate the future operation returns as a uniform annual sum.

4.27. Equivalent Uniform Annual Operation Returns.—It is desirable to introduce at this point a special conception of operation returns.

1. *The equivalent uniform annual operation returns* earned by a physical-property unit are uniform annual credits which if continued throughout the entire probable life of the unit would have a present worth at the date of installation of the unit new equal to its value new ready to operate.

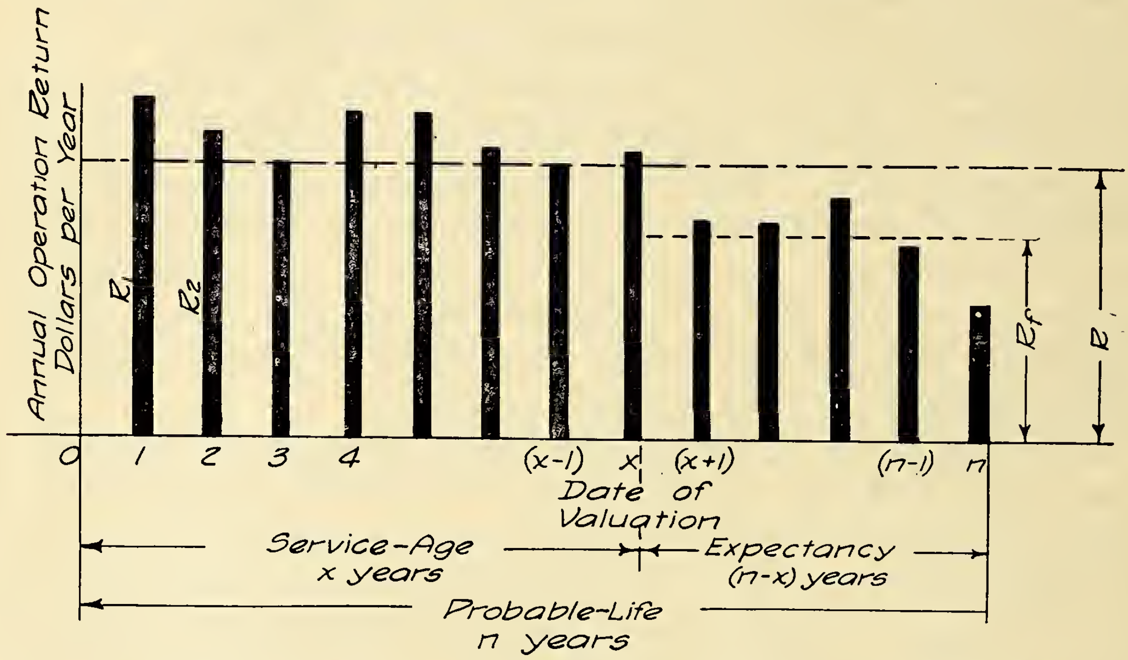
Evidently the equivalent uniform annual operation returns must have the same present worth at the date of installation of the unit new as do the operation returns probably to be earned by the unit. Because of the incidence of compound interest in the computations, the equivalent uniform returns are not the arithmetical average returns.

In determining depreciation it is never necessary actually to calculate the amount of the equivalent uniform annual operation return of the unit, for the operation return

cancels out of the equations for depreciation and for present value. Moreover, the true actual depreciation can most often be estimated with all practicable accuracy by the aid of depreciation tables (Sec. 5.18) based on uniform operation returns.

2. *Probable future equivalent uniform annual operation returns* are those estimated at any date to have the same present worth at that date as the probable future operation returns yet to be earned.

4.28. The probable future operation-return ratio (or PFORR) of any physical-property unit is the ratio of its probable future equivalent uniform annual operation returns (Sec. 4.27), during its probable future



R_1, R_2, etc. = probable operation return to be received at the end of the first, second, etc. years.

R = equivalent uniform operation return to be received throughout the probable service life.

R_f = probable equivalent uniform operation return after date of valuation.

r = fair rate of net return.

$$\text{PFORR} = \frac{R_f}{R_u} = \text{probable future operation return ratio.}$$

FIG. 4.2.—Diagram illustrating the actual, equivalent uniform, and probable future equivalent uniform annual operation returns and the PFORR.

service life, divided by its equivalent uniform annual operation returns throughout its entire service life, past and future. PFORR is the factor in estimating the depreciation of a physical-property unit by which correct allowance can be made for any decrease, as its service age increases, in the annual value of the operation return it earns.

Such decrease in the annual value of the operation return earned by the unit may be due to lowered efficiency, lessened output capacity, increased maintenance costs, increased running costs, intermittent (stand-by) service, and/or operation at less than normal capacity. This factor should be estimated directly (not calculated) by a qualified engineering-valuation expert, at the time he makes the examination of

the unit for the purpose of forecasting its expectancy (Sec. 3.20) and estimating its actual depreciation. A numerical estimate of the PFORR should be noted along with the record (Secs. 3.20, 5.17) of the present physical condition of the item and of its past and probable future service condition.

4.29. Diagram on Operation Return.—Figure 4.2 illustrates graphically the actual annual operation returns (Sec. 4.26) earned by a physical-property unit throughout its service life, its equivalent uniform annual operation returns (Sec. 4 27), its probable future equivalent uniform annual operation returns (Sec. 4.27), and its PFORR (Sec. 4.28).

In addition to the nomenclature shown on Fig. 4.2, let

V_n = the value new of the physical-property unit = the present worth of its actual operation returns, R_1, R_2, R_3, etc., at age 0.

V_{nd} = the unit's depreciable value new.

V_p = the present value, at age x, for the equivalent uniform annual operation returns R_u = their present worth at age x.

V_{pd} = the unit's present depreciable value.

V_p' = the present value, at age x, for the probable future equivalent uniform annual operation returns R_f = their present worth at age x.

V_s = the salvage value.

Then, using the principle that the value of the physical unit at any age is equal to the present worth of its probable future annual operation returns, we have, for the case when $V_s = 0$

At age 0

$$V_n = \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_x}{(1+r)^x} + \cdots + \frac{R_n}{(1+r)^n} = R_u\frac{(1+r)^n - 1}{r(1+r)^n} \quad (4\text{–}2)^1$$

At age x

If the equivalent uniform return R_u is used,

$$V_p = \frac{R_u}{(1+r)} + \frac{R_u}{(1+r)^2} + \cdots + \frac{R_u}{(1+r)^{n-x}} = R_u\frac{(1+r)^{n-x} - 1}{r(1+r)^{n-x}} \quad (4\text{–}3)$$

If the probable future equivalent operation R_f is different from R_u,

$$V_p' = \frac{R_f}{(1+r)} + \frac{R_f}{(1+r)^2} + \cdots + \frac{R_f}{(1+r)^{n-x}} = R_f\frac{(1+r)^{n-x} - 1}{r(1+r)^{n-x}} \quad (4\text{–}4)$$

$$V_p' = \left[\frac{R_f}{R_u}\right]V_p = (\text{PFORR})(V_p) \quad (4\text{–}5)$$

V_p is first estimated (usually by the aid of depreciation condition-percent tables) upon the basis of uniform equivalent operation returns (R_u). The above mathematical discussion shows the present values obtained this way must be multiplied by PFORR to get the correct present values and/or depreciations.

Formula (4–5) applies when $V_s = 0$. The general formula[2] for V_p is

[1] See Eq. (5–3).

[2] See Eq. (5–14).

$$V_p = (V_{nd})\left(\frac{\text{condition-percent}}{100}\right)(\text{PFORR}) + V_s \qquad (4\text{–}6)$$

Equation (4–6) is mathematically exact for all values of PFORR when $V_s = 0$, and for all values of V_s when PFORR = 1.00; and is more nearly exact than PFORR can be determined in all other cases.

4.30. Net Return in Relation to Depreciation.—The annual net return for any one physical unit of the property for any one year is the balance left after subtracting the actual depreciation of the unit for the year from the actual operation return which might properly be allocated to the unit for that year. The annual net return for an entire industrial property was defined in Sec. 2.23. As with operation return,

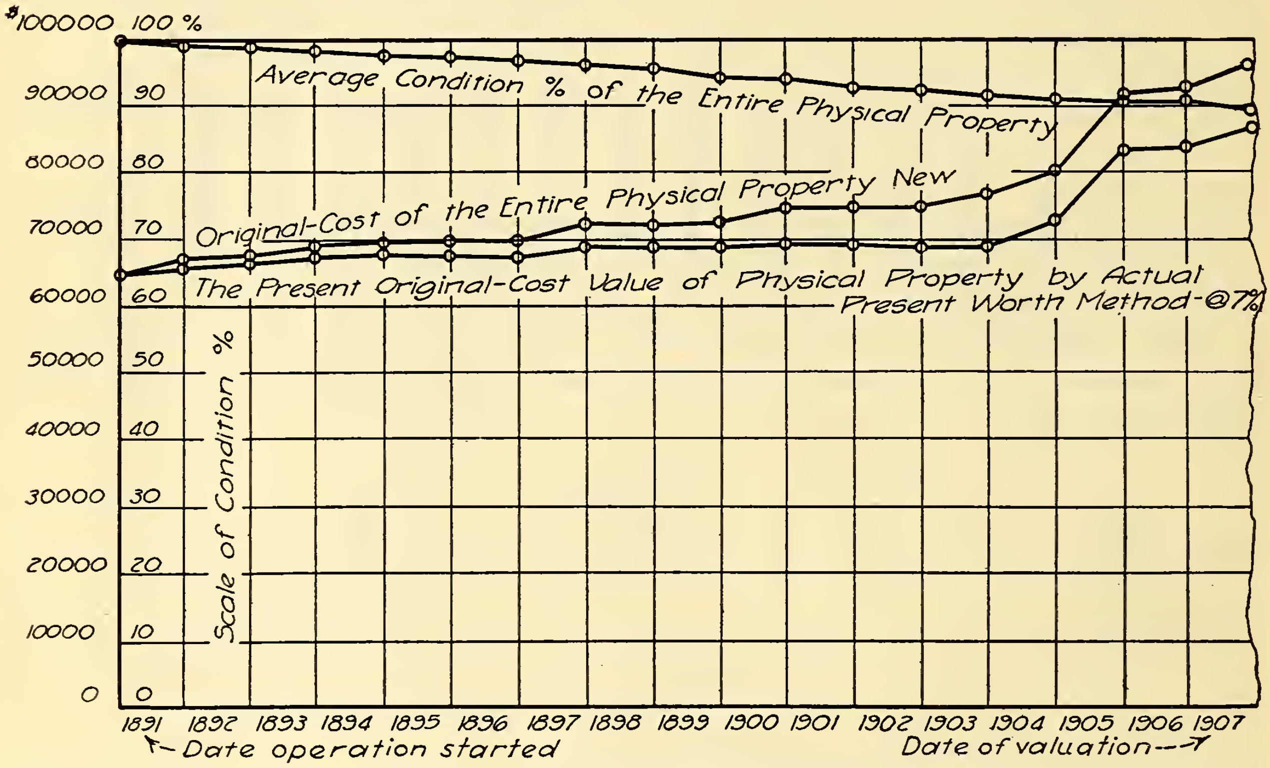


FIG. 4.3.—Original cost, present value, and condition-percent of physical property of the Caseone Waterworks Company.

the total net return is really earned by, and conceivably might be allocated to, the individual units of the property.

4.31. Rates of Net Return.—Rates of net return are computed as follows:

1. The percent rate of net return for the entire industrial property is 100 times the quotient obtained by dividing the total annual net return for the entire property by the fair value of the entire property at the beginning of the corresponding year.

2. The percent rate of net return for any one physical unit of an industrial property is equal to 100 times the quotient obtained by dividing the annual net return for the unit by the depreciated value of the unit at the beginning of the corresponding year.

4.32. The Depreciation of an Entire Industrial Property.—The depreciation discussions in this chapter to this point have referred to the

depreciations of individual units of physical property, not to the depreciation of an industrial property as a whole. Figure 4.1 illustrates the changes of the depreciation of a single unit during its service life. The variations from time to time of the depreciation of an industrial property considered in its entirety are quite different, as shown in Table 4.1 and in Fig. 4.3.

The difference between the depreciated value curves of Figs. 4.1 and 4.3 is due to the fact that each entire industrial property includes a great many physical units of different service ages and different service lives, so that new units are constantly taking the place of depreciated units retired. In fact, after a sufficiently long period a large stabilized industrial property would reach a condition, in which the normal annual depreciation would become nearly constant.

TABLE 4.1.—DEPRECIATION OF THE PHYSICAL PROPERTY OF THE CASEONE WATERWORKS COMPANY

Year	Original cost of physical property, Jan. 1	Cost of additions during year	Depreciation during year (7% basis)	Accrued depreciation, Jan. 1	Present value of physical property, Jan. 1	Condition-percent* of entire property, Jan. 1, %
1891	$64,727	$ 2,209	$324	$ 0	$64,727	100.00
1892	66,936	528	347	324	66,612	99.52
1893	67,464	942	372	671	66,793	99.01
1894	68,406	1,356	398	1,043	67,363	98.48
1895	69,762	0	426	1,441	68,321	97.93
1896	69,762	46	456	1,867	67,895	97.32
1897	69,808	2,326	488	2,323	67,485	96.67
1898	72,134	0	523	2,811	69,323	96.10
1899	72,134	183	559	3,334	68,800	95.38
1900	72,317	1,932	599	3,893	68,424	94.62
1901	72,249	200	641	4,492	69,757	93.95
1902	74,449	235	686	5,133	69,316	93.11
1903	74,684	1,404	734	5,819	68,865	92.21
1904	76,088	3,628	786	6,553	69,535	91.39
1905	79,716	11,950	842	7,339	72,377	90.79
1906	91,666	695	905	8,181	83,485	91.08
1907	92,361	3,682	968	9,086	83,275	90.16
Jan. 1, 1908	96,043	....	....	10,054	85,989	89.53

* No account of salvage value is taken in this table.

CHAPTER V

THE METHODS OF ESTIMATING DEPRECIATION

Knowledge of the true nature of depreciation is of recent development. Until 1902 even our highest courts still confused depreciation and maintenance, and it was not until 1909 that the United States Supreme Court finally ruled[1] definitely that depreciation is a true loss of value, which cannot be made good by maintenance but must be deducted in determining present value, and which the owner must make good each year out of current income before he really has any true net return at all. Most of the present knowledge of depreciation has accumulated since 1900.

After its reality began to be understood, the first attempts to estimate depreciation were by accountants without technical knowledge of industrial property, who made calculations by the simple but incorrect theoretical "fixed-percentage-of-value-new" straight-line assumption, without having even seen the property units. Only since 1920 have the courts clearly established the legal principle that true depreciation is *actual* depreciation, which must be determined by engineers or other technically qualified persons and which must be based correctly on actually observed mortality data of the property units. Even yet, the majority of engineers do not understand the mortality-curve principles and methods described in Chap. III.

THE DIFFERENT DEPRECIATION METHODS AND THEIR FUNDAMENTAL BASES

Failure to understand the basic principle that the fundamental basis of all values, including the present values of depreciated property units, is the present worth of probable future services is the explanation of the incorrectness of the methods first used (and still most widely used) in estimating depreciation.

5.1. Historical Résumé of the Development of Depreciation Methods.—The early efforts to calculate depreciation naturally developed the apparently simple fixed-percentage-of-value-new-less-estimated-salvage-value depreciation method, based on the arbitrary straight-line assumption, as applied to the average life of similar units instead of to the actual life of the particular unit. This method is more widely used than any other at the present time.

[1] See *Knoxville v. Knoxville Water Co.*, Sec. 8.4.

When knowledge of depreciation increased sufficiently to make clear some of the fallacies resulting from the entire neglect in the straight-line assumption to take any account whatsoever of interest, it was quite natural that engineers and accountants should develop the original sinking-fund depreciation method, based on the sinking-fund assumption; this, however, like the straight-line assumption, is purely arbitrary. It soon became apparent that the original method of applying the sinking-fund assumption also involved inadmissible fallacies, which engineers then sought to eliminate by a compound-interest modification of the original sinking-fund method. Sinking-fund methods are second only to straight-line methods in extent of use in depreciation accountancy.

The fixed-percentage-of-depreciated-value and the annuity depreciation assumptions and methods will be described briefly below; and the entire incorrectness of the "good-as-new" and the "intuitive-dictum" assumptions will be explained.

In their valuation study and research since 1920, the authors have discovered that the sinking-fund methods can be developed into a "present-worth" depreciation method, based upon the correct principle that the depreciated value of any property unit is always equal to the present worth of its probable future operation returns.

The authors have also developed a readily usable actual depreciation method, whereby the present-worth principle, the sinking-fund assumption, and the straight-line assumption may be applied each year to the actually observed depreciation data of particular property units and/or age-groups of like units. By this method the yearly depreciation estimates are constantly based on actual mortality facts, thus insuring that the total depreciations check out exactly correct at the dates of the actual retirements from service of the respective property units.

5.2. The Good-as-new Depreciation Assumption.—Although it is in absolute conflict with numerous decisions of the United States Supreme Court, there are still some persons who argue for the erroneous good-as-new assumption; this may be stated as follows:

Some persons argue that it ought to be assumed in valuation that property units so well maintained that their service efficiency and daily output capacity are practically undiminished are as good as new and have no depreciation.

That this assumption is entirely erroneous is made clearly apparent by the fact that the difference between the future services which *old* units are yet to render and the future services which the same units *would render if still new* is equal to the entire services already rendered by the old units.

The good-as-new assumption is not only erroneous but absurd when applied (as its advocates insist) to units evidently due for very early retirement.

Moreover, adherence in industrial-property accountancy to a practice of carrying all property units at their values new until their respective retirement dates, without maintaining a depreciation reserve equal to the accrued actual depreciation of all units in service, reprehensively deceives the stockholders and the public in two exceedingly important particulars:

1. The value of the property is much overstated.
2. The yearly net returns (profits) are also much overstated (except in the cases of very old properties which have stopped growing long since).

For further discussion of these matters, see Sec. 6.9 on retirement accountancy.

5.3. The Intuitive-dictum Actual Depreciation Assumption.—This unwarranted but formerly common actual depreciation assumption may be stated as follows:

It is sometimes assumed that a competent valuator, by merely inspecting a property unit, can decide arbitrarily how much it is actually depreciated, by expert intuition, without applying any particular depreciation principle in a process of reasoning.

It is doubtless true in many simple cases of short-lived property units, that a competent valuator who actually inspects them can determine their approximate actual depreciation percentages without formal calculations, by a reasoning process which in extremely simple cases may be so short and so informal as to be almost unconscious. In the great majority of cases, however, attempts to determine actual depreciation by intuitive dictum are sure to lead to mere arbitrary estimates, often not entirely free from unconscious influence by some bias of the estimator, or his employer.

One well-known writer on engineering valuation expresses himself on this subject very emphatically thus:

> Some experts take a look at a utility plant, nod their heads wisely, and ascribe to it a certain percent-condition, but what that percent shall be depends on whether the expert is a pessimist or an optimist, and on whose money is burning holes in his breast pocket and searing his conscience.[1]

5.4. The Straight-line Depreciation Assumption.—Although the good-as-new and the intuitive-dictum depreciation assumptions must be entirely rejected, there are two other assumptions which are used extensively, the straight line and the sinking fund, the former more widely than any other.

The straight-line depreciation assumption is that the loss of value due to the depreciation of any physical-property unit (its value new, less its net salvage value) is distributed uniformly throughout its service life.

In estimating actual depreciation, the straight-line assumption is applied to the probable life of the particular unit; this is re-estimated from time to time, as the observed facts of its service life make advisable,

[1] Wilcox, Delos F., "Depreciation in Public Utilities," 1925, p. 4.

so that the final estimate is the unit's actual life; corresponding depreciation adjustments are made, including changes in the annual depreciation allowance; by this procedure, the total depreciation always checks out correct (equal to value new, less salvage value) at the actual date of retirement.

In estimating theoretical depreciation, the straight-line assumption is applied to the average life of similar units instead of to the actual life of the particular unit. This gives a fixed-percentage-of-value-new-less-estimated-salvage-value annual depreciation, regardless of the unit's actual service life, and makes the total depreciation at the actual date of retirement nearly always too large or too small. Nevertheless, the fixed-percentage-of-value-new-less-net-salvage depreciation assumption is more widely used than any other.

When analyzed, the straight-line assumption is found to be based on the erroneous idea that the present value of a future service is the same as that of a present service. This will be clear upon studying the explanation of the straight-line assumption by the Interstate Commerce Commission in its decision in the *Texas Midland Railroad Case*, in 1918.

"As treated by the Commission, depreciation may be defined as the lessening in cost value due to the smaller number of service units in the property as found, than in the same property new. An article when new contains, so to speak, a certain number of units of service; as those units are exhausted the article depreciates, when they are all used up the service life ends."

But the present value of the future "service units in the property as found" is not the sum of their future values but only the present worth of their future values. Applied to a property unit whose services earn \$1,000 per year, and whose service life is 16 years, the Commission's reasoning implies, incorrectly, that when the unit is new its future services are worth \$16,000, instead of merely the present worth of 16 annual \$1,000 sums, received 1 to 16 years in the future; and that at age x the future services are worth $(16 - x)(\$1,000)$, instead of merely the present worth of $(16 - x)$ annual \$1,000 sums, received 1 to $(16 - x)$ years in the future.

5.5. The Fixed-percentage-of-depreciated-value Assumption.—This is a variation of the fixed-percentage depreciation idea, applying the percentage to depreciated value instead of to value new.

The fixed-percentage-of-depreciated-value theoretical depreciation assumption is that the annual depreciation of a property unit is always a fixed percentage of the unit's depreciated value at the beginning of the year.

The formula for the present value of a property unit at any age by the fixed-percentage-of-depreciated-value assumption is

$$V_x = V_n\left(1 - \frac{r_d}{100}\right)^x, \tag{5–1}$$

in which V_n = value new.

V_x = present value at age x.

r_d = the fixed-percentage-of-depreciated-value rate of depreciation.

Although the expression $\left(1 - \frac{r_d}{100}\right)^x$ is always less than unity and constantly decreases in value as the age x increases, it does not equal 0 till x equals infinity. Hence the depreciation of a property unit by the fixed-percentage-of-depreciated-value assumption would never be 100 percent at the date of its retirement.

The curve of $V_x = V_n\left(1 - \frac{r_d}{100}\right)^x$ is concave upwards, like the curve of secondhand values in Fig. 4.1, to which it can be fitted closely by trying different values of r_d. Although useful in representing the law of secondhand values, the fixed-percentage-of-depreciated-value assumption is all wrong when applied to industrial-property units continued in service till wisely retired for sound economic reasons. The assumption is little used in the United States and may be dismissed from further consideration.

It would appear from a statement by Saliers[1] that the fixed-percentage-of-depreciated-value assumption has been used to a considerable extent by some officials of the English government, who, he states, have "determined depreciation rates for many trades," for the most of these rates cited by Saliers seem to apply to "written-down" values instead of to values new.

5.6. The Fictitious Depreciation Sinking Fund of the Sinking-fund Depreciation Assumption.—To understand the sinking-fund depreciation assumption (which is more widely used than any other except the straight-line), it is necessary beforehand to understand the particular kind of sinking fund to which the assumption applies.

First, the depreciation sinking fund of the assumption is purely *fictitious;* merely *a mathematical concept,* used to supply formulas by which to compute depreciation by the sinking-fund assumption.

It is not practicable, nor if practicable would it be wise, to invest all or even a considerable part of the annual depreciation-appropriations balances of an industrial enterprise in any kind of a sinking fund.

Second, the particular kind of sinking fund to which the sinking-fund depreciation assumption applies is an *equal-annual-year-end-payment* sinking fund.

Third, the interest rates to use in the fictitious depreciation sinking funds of the sinking-fund depreciation assumption are fixed by arbitrary custom, at 3 to 5 percent, usually 4 percent.

[1] SALIERS, "Depreciation Principles and Practice," The Ronald Press, New York, 1922, pp. 310–312.

THE MATHEMATICAL THEORY OF THE EQUAL-ANNUAL-YEAR-END-PAYMENT SINKING FUND

The mathematical theory of the equal-annual-year-end-payment fictitious-depreciation sinking fund is as follows:

Let V_n = the property unit's value new.
V_s = the property unit's salvage value.
V_{nd} = the unit's depreciable value new.
V_p = the unit's present value.
V_{pd} = the unit's present depreciable value.
x = the unit's service age, in years (= the age of the sinking fund).
n = the unit's total service life (= the total life of the sinking fund).
i = the sinking-fund interest rate (3 to 5 percent, usually 4 percent).

And, for an equal annual year-end payment equal to $1.00, let
f_n = the accumulation in the fund at age n years.
pf_n = the present worth of the accumulation in the fund at age n years.
A_n = the equal annual year-end payment to give an accumulation equal to $1.00 at the end of n years.

At any life n, the successive year-end payments of $1.00 each, accumulated at compound interest i, constitute a geometrical series, from which, multiplying by $(1 + i)$, we have

$$\begin{aligned}(1+i)f_n &= (1+i)^n + (1+i)^{n-1} + \cdots + (1+i)^2 + (1+i)\\ f_n &= \phantom{(1+i)^n +{}} (1+i)^{n-1} + \cdots + (1+i)^2 + (1+i) + 1.\end{aligned}$$

Then, by subtracting and then dividing by i, we have

$$f_n = \frac{(1+i)^n - 1}{i} \tag{5-2}$$

$$pf_n = \frac{(1+i)^n - 1}{i(1+i)^n} \tag{5-3}$$

The accumulation for any other year-end payment than $1.00 is equal to f_n multiplied by the payment. Hence

$$A_n = \frac{i}{(1+i)^n - 1} \tag{5-4}$$

The fictitious depreciation sinking fund used in estimating the depreciation of any particular property unit must have an accumulation at the end of the service life of the unit (n years) equal to its depreciable value new V_{nd} (equals value new minus salvage value). From this fact, and Eqs. (5-2) and (5-4), we readily derive the following depreciation sinking-fund formulas:

$$\text{The equal annual year-end payment} = V_{nd}\frac{i}{(1+i)^n - 1} \tag{5-5}$$

$$\text{The sinking-fund accumulation, at age } x = V_{nd}\frac{(1+i)^x - 1}{(1+i)^n - 1} \tag{5-6}$$

$$\text{The sinking-fund interest, age } x \text{ to age } x+1, = iV_{nd}\frac{(1+i)^x - 1}{(1+i)^n - 1} \tag{5-7}$$

$$\text{The annual depreciation, age } x \text{ to age } x+1, = iV_{nd}\frac{(1+i)^x}{(1+i)^n - 1} \tag{5-8}$$

$$\text{The present depreciable value } V_{pd} = (V_{nd})\left(\frac{\text{condition-percent}}{100}\right) \tag{5-9}$$

where

$$\text{Condition-percent} = \left(\frac{(1+i)^n - (1+i)^x}{(1+i)^n - 1}\right)(100) \tag{5-10}$$

Introducing salvage value and PFORR (Secs. 4.28, 4.29) into the equation for V_p, it becomes, closer than PFORR can be estimated (Sec. 4.29),

$$V_p = (V_{nd})\left(\frac{\text{condition-per cent}}{100}\right)(\text{PFORR}) + V_s \tag{5-11}$$

The fictitious depreciation sinking fund used in estimating depreciation upon the sinking-fund depreciation assumption is illustrated in Figs. 5.1 and 5.2 for an industrial-property unit whose depreciable value new is $10,000, salvage value 0, and service life n is 25 years. The customary 4 percent sinking-fund interest rate is used.

The computations for the first 5 years of the Fig. 5.1 sinking fund give

	1st year	2d year	3d year	4th year	5th year
Sinking fund, Jan. 1	$ 0	$240.12	$489.84	$ 749.56	$1,019.66
Interest during year	0	9.60	19.60	29.98	40.78
Year-end payment	240.12	240.12	240.12	240.12	240.12
Increment for year	240.12	249.72	259.72	270.10	280.90
Sinking fund, Dec. 31	240.12	489.84	749.56	1,019.66	1,300.56
Total payments into fund	240.12	480.24	720.36	960.48	1,200.60
Total interest in fund	0	9.60	29.20	59.18	99.96

For 25 years' service life, Fig. 5.2 shows the relative depreciations (and present depreciable values) which sinking-fund interest rates of 0, 2, 4, 6, and 8 percent would give. It appears that the straight-line depreciation assumption is merely a particular case (for interest rate equals zero) of the sinking-fund assumption.

It should be said, further, that the shorter the service life the flatter the sinking-fund curves; for service lives not exceeding 5 years there is little difference between straight-line and sinking-fund depreciations.

5.7. The Sinking-fund Depreciation Assumption.—This assumption is more widely used in depreciation accountancy than any other except the straight line.

The sinking-fund depreciation assumption is that the total accrued depreciation of any property unit at any date is equal to the corresponding accumulation (of payments into the fund and compound interest thereon) in a fictitious equal-annual-year-end-payment depreciation sinking fund, in which the total accumulation at the end of the service life would be just sufficient to repay the unit's value new, less salvage value.

In estimating actual depreciation, the sinking-fund assumption is applied to the probable-life of the particular unit; this is re-estimated from time to time, as the observed facts of its service life make advisable, so that the final estimate is the unit's actual life; corresponding depreciation adjustments are made, including changes in the annual depreciation allowances; by this procedure, the total depreciation always checks

out correct (equal to value new, less net salvage) at the actual date of retirement.

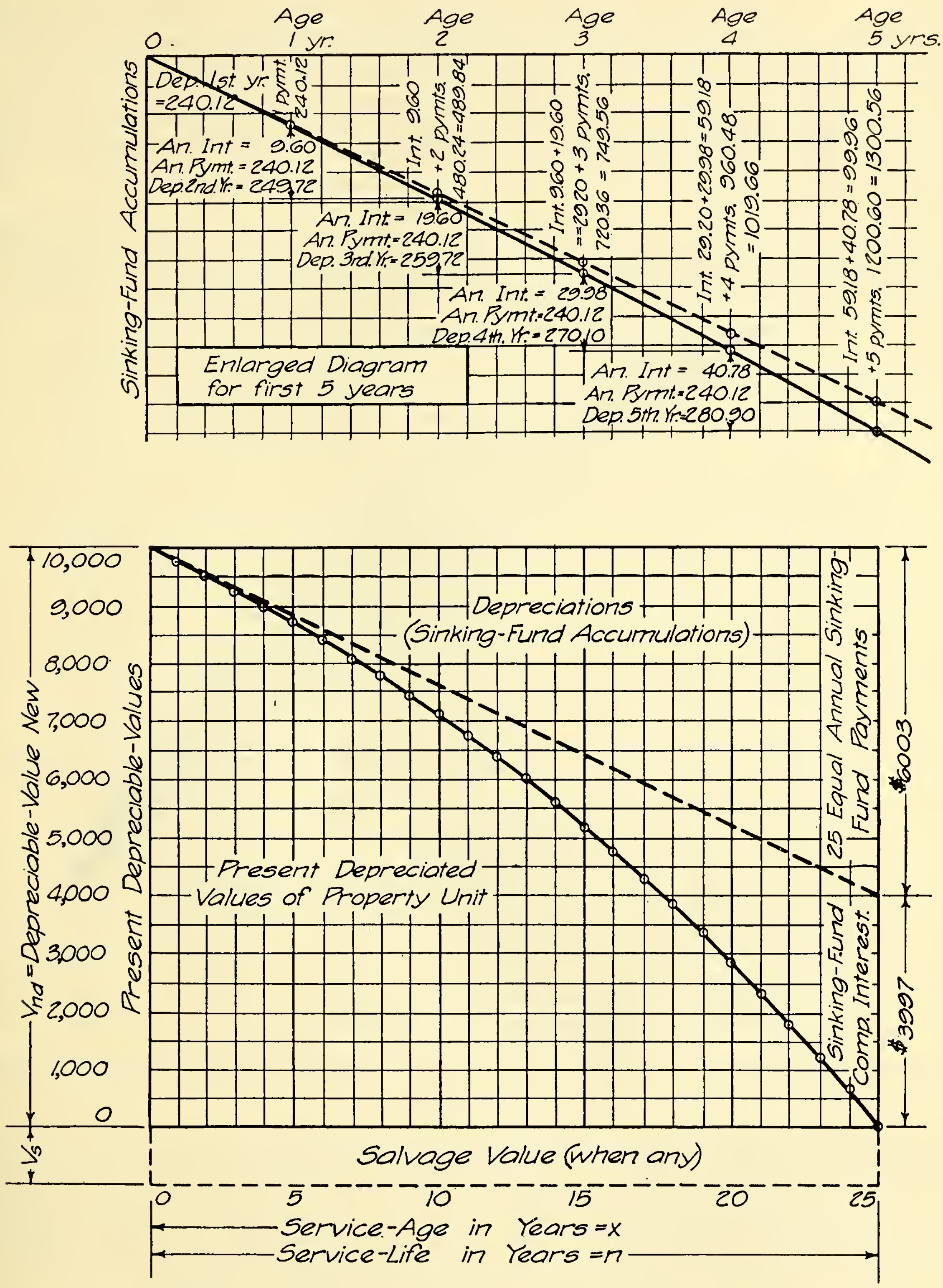


FIG. 5.1.—Illustrating fictitious depreciation sinking fund (used in sinking-fund depreciation assumption). For 25 years' service life, 4 percent interest rate, $10,000 depreciable value.

In estimating theoretical depreciation, the sinking-fund assumption is applied to the average life of similar units instead of to the actual life of the particular unit. This makes the total depreciation at the actual date of retirement nearly always too large or too small.

The sinking-fund assumption is purely arbitrary, but its supporters claim in its favor that

[It] . . . is the result of observation by engineers and operators of the relatively slow rate of depreciation of water works [and other] structures during the early years of their use, and the accelerated rate toward the end of their useful lives. . . . These requirements were not met by the straight-line method, but they were met, in fair degree, by the sinking-fund method of computing depreciation.[1]

Mathematical analysis of the sinking-fund assumption proves that (when using the interest rates 3 to 5 percent prescribed by custom) it

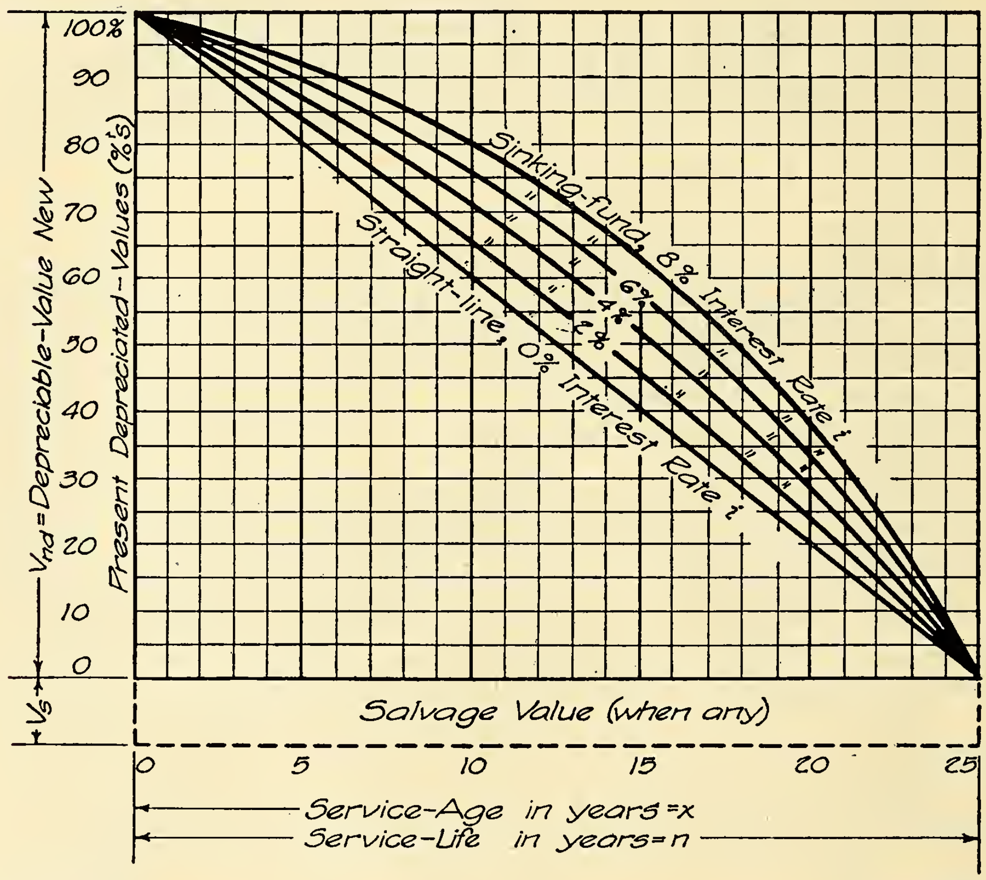


FIG. 5.2.—Illustrating effect of interest rates on depreciation sinking funds for 25 years' service life.

gives results which, while nearer than those by the straight-line assumption, are not equal to those required by the correct depreciation principle (that the present depreciated value of a property unit is always equal to the present worth of its probable future operation returns).[2]

Note that if the sinking-fund interest rate used were equal to the fair rate of return (7 percent in Figs. 5.5 and 5.6), and if in addition the life of the sinking fund were made equal to the probable life of the particular

[1] METCALF, LEONARD, "Manual of American Water Works Practice," American Water Works Association, 1925, p. 527.

[2] See Figs. 5.5 and 5.6 for illustrations of the relative magnitudes when 4 percent sinking-fund interest and a 7 percent rate of return are used.

unit (instead of the average life of similar units), the sinking-fund method would give the same depreciations as the (correct) present-worth method (Secs. 5.10, 5.11).

5.8. The Annuity Depreciation Assumption.—This incorrect and little used assumption may be stated as follows:

First, it is assumed that the total annual depreciation charge for each physical-property unit should include sinking-fund interest on its depreciated value at the beginning of the year in addition to its loss of value during the year.

Second, it is further assumed that the unit's depreciation charges, including interest, are best paid by an annuity extending over a period equal to the average life of similar units.

The first assumption is incorrect. The interest on the depreciated value of the unit is a part of its fair net return, not a part of depreciation cost. The second assumption is just an assumption.

From Eqs. (5–8), (5–9), and (5–10), it may readily be proved [Eq. (5–13)] that the proper formula by which to compute the annuity would be

$$\text{Annuity} = iV_{nd}\frac{(1+i)^n}{(1+i)^n - 1} + iV_s \tag{5–12}$$

in which V_{nd} = depreciable value new.
V_s = salvage value.
n = average life of similar units.
i = sinking-fund interest rate.

It is not considered that the annuity assumption is correct for use in depreciation accountancy or that it warrants further consideration.

5.9. The Present-worth Actual Depreciation Principle.—Sections 5.2 to 5.8 all deal with depreciation *assumptions*. It seems plainly evident that the correct *principle* by which to determine the loss of value during service which we call depreciation is that (Sec. 1.13) the fundamental basis of a property unit's value is the present worth of the probable future values of its future services. This applies to the values of all property units at all times; including the actual depreciated values of industrial-property units throughout their service lives. This present-worth actual depreciation principle may be stated as follows:

The present-worth actual depreciation principle is that the depreciated value of an industrial-property unit, at any date during its service life, is the present worth at that date of the probable future operation returns yet to be earned by its probable future services.

The authors consider this to be the correct depreciation principle, not a mere depreciation assumption.

The present-worth actual depreciation principle takes into account correctly the fact that the present value of a future service is less than that of a present service; much less, if the future service is remote; a well-known and plainly evident fact which is entirely inconsistent with the straight-line depreciation assumption.

5.10. Summary of Depreciation Assumptions and Principles.—What has been said in Secs. 5.2 to 5.9 about the various depreciation assumptions which have been used or suggested, and the one depreciation principle which the authors consider to be correct, may be summarized as follows:

The good-as-new and the intuitive-dictum assumptions are entirely inadmissible; the first because it is contrary to law and to fact, and the second because it assumes an impossibility.

The fixed-percentage-of-depreciated-value assumption is little used and is inapplicable to property units continued in service until wisely retired for sound economic reasons.

The annuity assumption is erroneous and should not be used.

The straight-line and the sinking-fund depreciation assumptions are in wide use; especially the straight line which is, in reality, just a special case (for interest rate equals zero) of the sinking-fund assumption. The interest rates prescribed by custom for use with the sinking-fund assumption are 3 to 5 percent, usually 4 percent.

The present-worth depreciation principle is the only correct principle by which to determine depreciation. It will be found later that its use gives the same depreciations which would be obtained by the sinking-fund assumption if the interest rate used were the fair rate of net return, usually 5 to 8 percent, instead of the customary 3 to 5 percent sinking-fund interest rates.

When the straight-line and the sinking-fund assumptions and the present-worth principle are applied (as they should be in estimating actual depreciation) to the probable lives (re-estimated from time to time) of particular industrial-property units, they all three give correct total depreciations at the actual dates of retirement; but the straight-line and the sinking-fund assumptions give too great total depreciations at all dates between those of installation and of retirement. The sinking-fund assumption gives results closer to the true depreciations than does the straight line.

THE HISTORY, DERIVATION, AND VERIFICATION OF THE PRESENT-WORTH DEPRECIATION PRINCIPLE

It is such a short time since depreciation came really to be understood that it is not surprising that most depreciation calculations are still based on the arbitrary straight-line and sinking-fund assumptions, although neither gives the true distribution of actual depreciations throughout the service lives of industrial-property units.

5.11. History of Present-worth Depreciation Principle.—One of the authors of this treatise, who with his colleagues has been teaching engineering valuation continuously since 1920, to a total, up to 1935, of some 3,000 engineering college senior students, came gradually to realize in his

study and researches that the only correct principle by which true actual depreciations can be determined is the present-worth principle, stated in Sec. 5.9. About 1924, he worked out the mathematical present-worth solution[1] presented in Sec. 5.12; he found that fortunately the resulting formulas, *though based entirely on present-worth principles*, gave the same results which the sinking-fund assumption would call for if an interest rate equal to the fair rate of net return, say 5 to 8 percent, were used instead of the 3 to 5 percent, usually 4 percent, prescribed by general custom for sinking-fund depreciation computations.

This, and the additional fact that the straight-line assumption is really only a special case (for interest rate equals zero) of the sinking-fund assumption, made possible the use of the single condition-percent table in Appendix C of this treatise, for depreciation computations based on all three ideas, the present-worth principle, the sinking-fund assumption, and the straight-line assumption.[2]

The present-worth mathematical solution,[1] presented in Sec. 5.12 was published in the second (mimeographed) edition of this treatise in 1926.

The remaining step in the completion of the present-worth depreciation method was to develop the idea and the method of use of the factor PFORR, to provide correctly for any decrease in the present worth of the probable future operation returns due to probable decrease in their future annual values. The idea of this factor and the method for its use in making depreciation computations were worked out in 1931, by the same author who had worked out the original present-worth solution.[3]

5.12. The Derivation of the Present-worth Depreciation-principle Formulas.—In the process of estimating the actual depreciations of a property unit at different dates during its service life, it is necessary to estimate and re-estimate its probable life repeatedly, first at the date of installation new and thereafter from time to time (Secs. 3.19, 3.20, 3.21) as may be required to make the estimate agree with the actual mortality facts observed. The present-worth depreciation principle (Sec. 5.9) and the resulting present-worth formulas must be applied to each separate estimate (n) of the probable life; such corresponding adjustments of present value, total depreciation, and annual depreciation must be made as may be required by changes in the estimated probable life.

[1] The first solution did not include the PFORR.

[2] The condition-percent table, calculated to four decimal places under the direction of Dr. Marston, was published in 1924, as *Bull.* 70, *Iowa Eng. Exp. Sta.*, Robley Winfrey, editor. The table has been recalculated to six decimal places for the present edition of this treatise.

[3] The first publication of the probable-future-operation-return-ratio concept and of the method for its use was in 1932, in the fourth (mimeographed) edition of this treatise.

Numerical Example: A certain waterworks company installed two duplicate pumping engines at the same date; for each of these the direct costs were \$8,971, the overhead costs 10 percent, and the estimated salvage value \$500.

For Engine 1, the actual mortality data observed during its service life proved to justify the following estimates: Probable life, age 0 to 15, 25 years; 15 to 20, 26 years; 20 to 25, 27 years; 25 to 28, 28 years; PFORR, 1.00, age 0 to 20, and 0.90, age 20 to 28. Retired at age 28. Net salvage realized, \$450.

For Engine 2, the corresponding estimates justified by its observed actual mortality data proved to be: *Probable life,* age 0 to 15, 25 years; 15 to 18, 24 years; 18 to 20, 23 years; 20 to 22, 22 years; PFORR, 1.00, age 0 to 18, and 0.85, age 18 to 22. Salvage value estimated at \$500, age 0 to 18, and \$400, age 18 to 21. Retired at age 22. Net salvage realized, \$350.

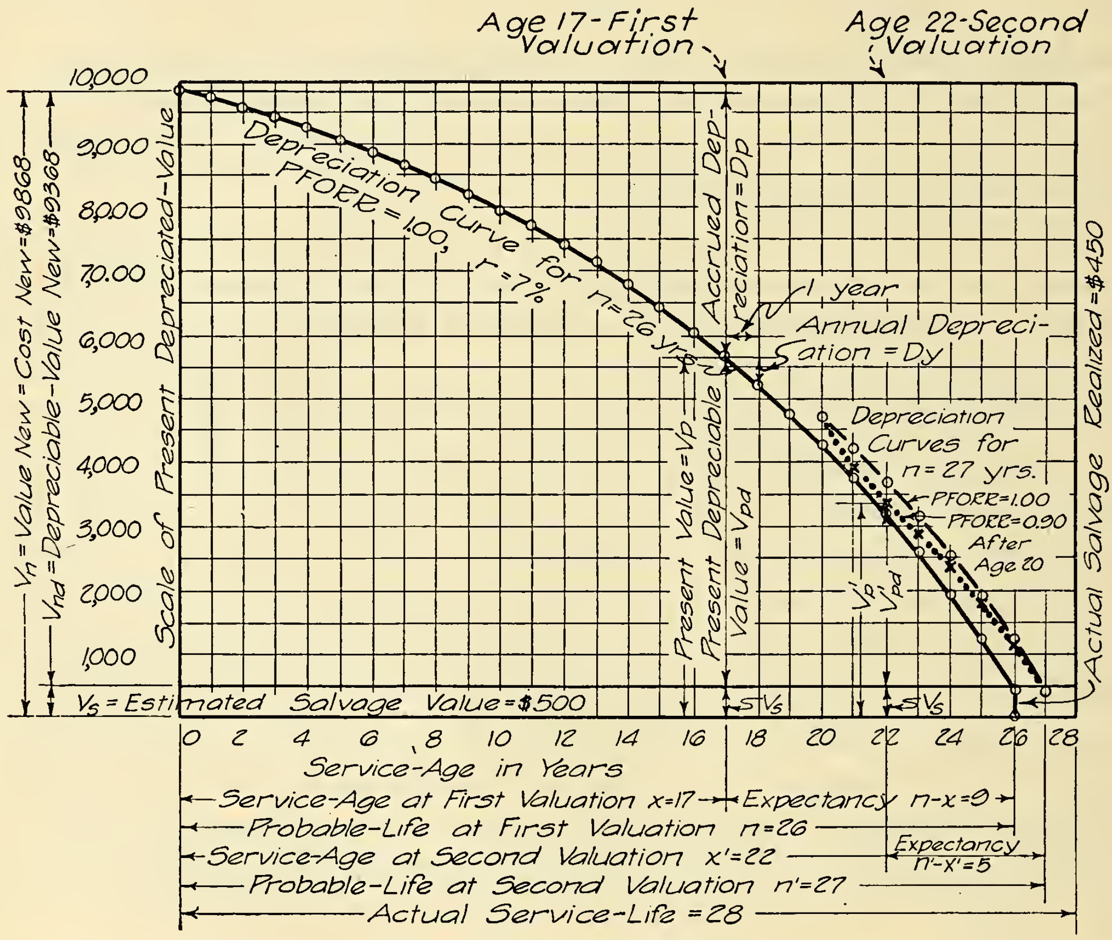


FIG. 5.3.—Illustrating derivations of present-worth depreciation-principle formulas.

Engine 1 was valued at ages 17 and 22 in formal valuations of the entire property, and Fig. 5.3 has been constructed to illustrate the corresponding present-worth formulas and computations.

NOMENCLATURE

Let V_n = the property unit's value new.
V_{nd} = the unit's depreciable value new.
V_s = the unit's salvage value (estimated when valued).
V_p = the unit's present value (depreciated) at age x.
V_{pd} = the unit's present depreciable value at age x.
D_p = the unit's present total accrued depreciation at age x.
D_y = the unit's annual depreciation, age x to age $x + 1$.
x = the unit's service age in years.

n = the unit's probable life in years (estimated when valued).
r = *the fair rate of net return on the entire property.*
R_u = the unit's equivalent uniform annual operation return (Sec. 4.27) for the probable life n.
R_f = the unit's probable future equivalent uniform annual operation return (Sec. 4.28) for the probable future life $n - x$.
PFORR = the unit's probable future operation-return ratio (Sec. 4.29) for the probable future life $n - x$.

Derivations of the Present-worth Depreciation-principle Formulas

In order to estimate the depreciations at different ages of a property unit by the present-worth depreciation principle, three formulas based on the principle are required: (1) for the unit's present value V_p at age x; (2) for the unit's total accrued depreciation D_p at age x; (3) for the unit's annual depreciation D_y at age x to age $x + 1$.

Each of these three formulas applies to the particular probable life n, estimated for the particular age x.

1. *The present-worth depreciation-principle formula for V_p* (the present value of a property unit at age x).

Since, when PFORR = 1.00, the unit's present value at age x is equal to the present worth of $n - x$ future equivalent uniform annual operation returns R_u, plus the present worth of the estimated salvage value to be realized $n - x$ years in the future, we have

$$V_p = R_u\frac{(1 + r)^{n-x} - 1}{r(1 + r)^{n-x}} + \frac{V_s}{(1 + r)^{n-x}}, \text{ for PFORR} = 1.00.$$

Note.—Equation (5–2), shows that the accumulation at compound interest rate r of a \$1.00 year-end annuity for n years would be $\frac{(1 + r)^n - 1}{r}$, whence its present worth would be $\frac{(1 + r)^n - 1}{r(1 + r)^n}$.

Also, since at age 0 the unit's value new was equal to the present worth of n future equivalent uniform annual operation returns R_u, plus the then present worth of the estimated salvage value V_s, to be realized n years in the future, we have

$$V_n = V_{nd} + V_s = R_u\frac{(1 + r)^n - 1}{r(1 + r)^n} + \frac{V_s}{(1 + r)^n},$$

whence

$$R_u = rV_{nd}\frac{(1 + r)^n}{(1 + r)^n - 1} + rV_s, \tag{5–13}$$

and by substituting this value for R_u in the equation for V_p, above, we get

$$V_p = V_{nd}\frac{(1 + r)^n - (1 + r)^x}{(1 + r)^n - 1} + V_s, \text{ for PFORR} = 1.00.$$

More exactly than PFORR can be estimated (Sec. 4.29), this becomes

$$V_p = V_{nd}\frac{(1 + r)^n - (1 + r)^x}{(1 + r)^n - 1} \times \text{PFORR} + V_s, \tag{5–14a}$$

or

$$V_p = (V_{nd})\left(\frac{\text{condition-percent}}{100}\right) \times (\text{PFORR}) + V_s, \tag{5–14}$$

where

$$\text{Condition-percent} = \left(\frac{(1+r)^n - (1+r)^x}{(1+r)^n - 1}\right)(100) \tag{5–14b}$$

Note.—Comparing Eqs. (5–14), (5–14*a*), and (5–14*b*), above, with Eqs. (5–10) and (5–11), the fortunate fact is apparent that, *although the present-worth depreciation-principle formulas are based entirely on present-worth principles, without any consideration of sinking funds*, yet the sinking-fund formulas would give the correct present-worth depreciations and present values if the fair rate of net return r were substituted in them for the sinking-fund interest rate i. It is this fortunate fact which permits the use of a single condition-percent table (Appendix C) for the present-worth and the sinking-fund depreciation methods; and even for the straight-line method, which is only a special case (for interest rate $i = 0$ percent) of the sinking-fund method.

2. *The present-worth depreciation-principle formula for* D_p (the total accrued depreciation of a property unit at age x).

From Eqs. (5–14), (5–14*a*), and (5–14*b*), we readily find that

$$D_p = (V_{nd})\left(1 - \frac{\text{condition-percent}}{100}\right)(\text{PFORR}) \tag{5–15}$$

3. The present-worth depreciation principle formula for D_y (the annual depreciation of a property unit age x to age $x + 1$).

The correct formula for annual depreciation is manifestly

$$D_y = (V_p \text{ for age } x) - (V_p \text{ for age } x + 1) \tag{5–16}$$

This formula is correct for all depreciation methods. For the present-worth depreciation-principle method, D_y during any year not affected by depreciation adjustments required by change in the estimated probable life n may be calculated by the formula

$$D_y = rV_{nd}\frac{(1+r)^x}{(1+r)^n - 1}. \tag{5–16a}$$

5.13. Rate of Interest Applicable to Present-worth Method.—In deriving the present-worth formulas of Sec. 5.12, the correct rate of interest to use in determining the present worth of the probable future annual operation returns is the fair rate of net return for the entire property of which the depreciated property unit under consideration is a part.

This is because the annual depreciation allowance, out of current income, by which the depreciation of the property of every industrial enterprise must be made good each year before any true net return is really earned, should always be reinvested, if possible, in the property itself. The depreciation allowance, therefore, is entitled to earn at the fair rate of net return on the property (Sec. 6.22).

Any part of the annual depreciation appropriations which it is not possible to reinvest in the property itself goes to the stockholders, as return in excess of the true net return, and they are free to invest it in other enterprises equally profitable.

NOTE.—In the case of properties which the experience of years has demonstrated cannot earn the fair rate of net return, the average rate of actual net return may be used in applying the present-worth depreciation principle.

5.14. A Numerical Verification of the Present-worth Depreciation Principle.—The theoretical derivation of the present-worth depreciation-principle formula (Eq. 5–14), for the present value V_p, may be verified by the following numerical example, based on an actual case:

Numerical Example: A waterworks company hires certain deep-well pumping done by a power and light company, which owns the pump. The numerical data are as follows:

Price, \$0.02 per 1,000 gallons. Cost, \$0.014, exclusive of depreciation and net return. Average annual pumpage 166⅔ million gallons. Fair rate of net return 7 percent, PFORR 1.00. Service age of pump 8 years. Probable life 16 years. The annual operation return = (0.02 − 0.014) × 166,666⅔ = \$1,000.

The real value of the pump to the power and light company would be the present worth of 16 future \$1,000 probable future annual operation returns, if the pump were new; but the present value of the 8-year-old pump is only the present worth of 8 such \$1,000 probable future annual returns.

1. The condition-percent of the pump by the present-worth depreciation principle can be read directly in the condition-percent table in Appendix C, for age 8, probable life 16, and interest rate 7 percent. It is found to be 63.21 percent.

2. The above condition-percent can be verified by direct computation of 100 times the ratio of the present worth of the 8 probable future \$1,000 operation returns, at age 8, to the present worth of the 16 probable future \$1,000 operation returns at age 0 (pump new), using the formula $\frac{(1+r)^n - 1}{r(1+r)^n}$ for n annual \$1.00 annuities (see Eq. 5–3).

At age 8,

$$\$1{,}000\frac{1.07^8 - 1}{0.07 \times 1.07^8} = \$1{,}000\frac{0.718186}{0.07 \times 1.718186} = \$5{,}971.30.$$

At age 0,

$$\$1{,}000\frac{1.07^{16} - 1}{0.07 \times 1.07^{16}} = \$1{,}000\frac{1.952164}{0.07 \times 2.952164} = \$9{,}446.65.$$

Hence the real value of the pump to its owner, the power and light company, would be \$9,446.65 if it were new, but now, at age 8, is only \$5,971.30. Therefore, its condition-percent at age 8 = 100 × \$5,971.30/\$9,446.65 = 63.21 percent. This verifies the present-worth depreciation principle in the case of this property unit.

THE ESTIMATION OF ACTUAL DEPRECIATION BY THE PRESENT-WORTH, THE SINKING-FUND, AND THE STRAIGHT-LINE METHODS

The present-worth depreciation principle is intended to be applied only in estimating actual depreciation; the sinking-fund and the straight-line assumptions are used in estimating actual and/or theoretical depreciation.

As explained in Sec. 4.5, actual depreciation can be estimated correctly only by competent experts, who make actual examinations of the prop-

erty units and who apply correct depreciation principles to the observed mortality facts. The authors have developed what they consider to be a correct and feasible method for estimating actual depreciation by the correct present-worth depreciation principle, and/or by the sinking-fund and/or the straight-line depreciation assumptions. This method is presented in Secs. 5.15 to 5.20.

5.15. Making Actual Depreciation Observations and Determinations. It must be observed that, in general, actual depreciation cannot be reliably determined by accountants, or other persons, who are without special technical knowledge of engineering equipment and structures, including their mortality characteristics.

1. *Very large industrial properties* more and more generally maintain regular valuation departments, which in the case of holding-company owners frequently serve numbers of individual properties. Competent valuation engineers are required to estimate correctly the actual depreciations of large industrial properties, and consulting valuation-engineering advice may occasionally be of great service.
2. *Moderately large industrial properties* which do not maintain regular valuation departments can obtâin the regular, part-time service of engineering appraisal companies to make and keep up to date correct determinations of their actual depreciations.
3. *Comparatively small industrial properties* often utilize the services of salaried officials who have technical training, to estimate their actual depreciations.

It is increasingly apparent that the cooperation of competent engineers and competent accountants is essential in all industrial-property accountancy. In every line of modern industry, whether public utility or private enterprise, correct perpetual property inventories, in which the true actual depreciation of every property unit (and/or age-group of like units) is determined correctly and accounted for currently, are more and more evidently essential to the development of any adequate, feasible industrial plan whereby owners, employees, and consumers can be assured of justice and security.

5.16. The Frequency of Actual Depreciation Examinations and Re-examinations.—In general, the actual depreciation for each year ought to be estimated at the beginning of the year and entered in the books of the enterprise in equal monthly installments during the year.

This does not necessarily mean that every physical-property unit, or group of like units, has to be examined in minute detail every year by a special depreciation expert. The valuation engineers, or technically trained officials, who determine the actual depreciation do require, however, access to the property ledger record of each unit; and, in addition, sufficient direct knowledge of the units to insure that their forecasts of probable service lives and their estimates of the probable uture operation return ratios are substantially correct and up to date. Detailed

re-examinations must be made as often as may be found necessary to accomplish this result.

5.17. The Process of Estimating Actual Depreciation.—As already repeatedly intimated, it is necessary, in estimating at different dates during service the actual depreciations of any particular property unit (and/or age-group of like units), to estimate and re-estimate its probable life from time to time; corresponding adjustments must be made in the estimates of total depreciations and of annual depreciations, so that they

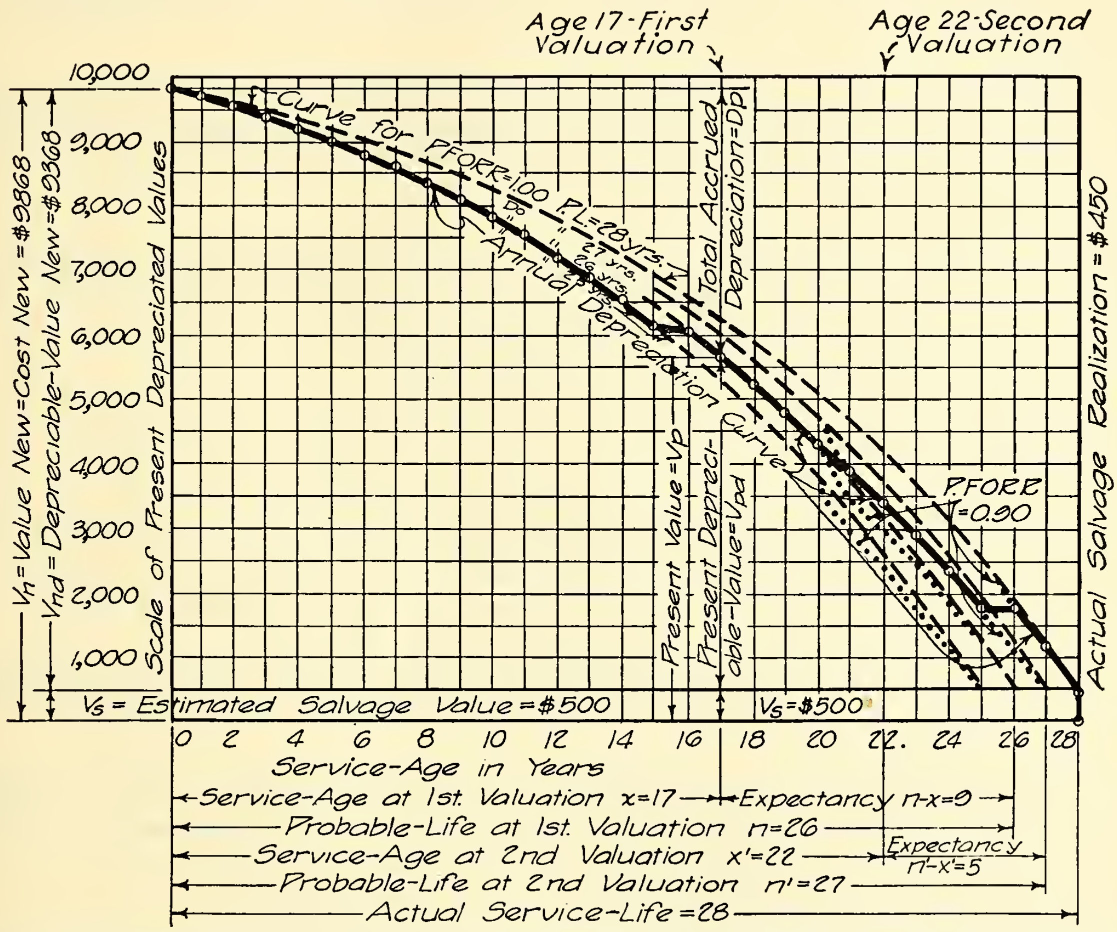


Fig. 5.4.—Illustrating the actual depreciations of pumping engine 1, as determined by the present-worth method.

conform at all times to the observed mortality facts, and so that the total depreciation checks out at retirement exactly equal to the value new, less the net salvage actually realized.

This process is illustrated in Fig. 5.4, as applied to pumping engine 1, of which the observed actual mortality numerical data are given in Sec. 5.12.

At installation new, the engine's probable life was estimated at 25 years, and the depreciations were determined on this basis till age 15, when the observed mortality facts caused a change of estimated probable life to 26 years. The total accrued depreciation was then estimated on the 26-years' probable-life basis at ages 16 to 20, and the annual depreciation during the year, age 15 to age 16, was adjusted accordingly. Sim-

ilar changes in estimated probable life and corresponding adjustments of the depreciation estimates were made at ages 20 and 25; at age 20 the estimated PFORR was changed from 1.00 to 0.90, with corresponding adjustments of the estimated depreciations.

Although the depreciation determinations illustrated in Fig. 5.4 were based on the present-worth depreciation principle, the same general process of making estimates and re-estimates of the unit's probable life and PFORR, with corresponding adjustments of the depreciation estimates, should be used with either the sinking-fund or the straight-line assumption.

This general method, described above and illustrated in Fig. 5.4, has been developed by the authors into an orderly *five-step process for determining actual depreciation,* as follows:

Step 1.—Make actual inspections of the various property units, observing all pertinent data of each unit bearing on its actual depreciation, and making a book record for each unit, and/or age-group of like units, of

Its book value new, and estimated salvage value.
Its service age.
Its past service conditions—mild, favorable, average, unfavorable, severe (M, F, A, U, S).
Its probable future service conditions—mild, favorable, average, unfavorable, severe (M, F, A, U, S).
Its physical condition—excellent, good, average, poor, bad, FOR ITS AGE (E, G, A, P, B).
Its probable future operation return ratio (PFORR) (a numerical estimate).

Step 2.—Estimate and make a book record of the probable life of each unit, and/or age-group of like units.

In estimating the probable lives, use the methods described in detail in Secs. 3.20 and 3.21, utilizing mortality curves to the fullest extent practicable. A type mortality curve should be selected (and made a matter of book record) for each property unit, and/or age-group of like units, if at all practicable.

Step 3.—Determine and make a book record of the condition-percent of each unit, and/or age-group of like units.

In the case of individual units, the tabular condition-percents (those found in the condition-percent table in Appendix C, for the proper interest rate and the unit's service age and corresponding probable life) must be multiplied by the unit's PFORR.

In the case of age-groups of like units, the tabular condition-percent must be taken for the service age and the probable life indicated by the group's type mortality curve for its *average survivor.* A correction[1] must be applied to this tabular condition-percent, and the corrected condition-percent must be multiplied by PFORR.

Step 4.—Calculate the numerical present value of each unit, and/or age-group of like units, and its total depreciation accrued to date.

The formula for present value is

$$V_p = (V_{nd})\left(\frac{\text{tabular condition-percent}}{100}\right)(\text{PFORR}) + V_s, \qquad (5\text{-}17)$$

[1] Correction tables for this purpose have not yet been published but are being prepared by the Iowa Engineering Experiment Station.

in which V_p is the present value, V_{nd} the depreciable value new, and V_s the estimated net salvage value.

The formula for total depreciation is

$$D_p = V_n - V_p, \tag{5-18}$$

in which D_p is the total depreciation and V_n the total value new.

Step 5.—Calculate the numerical annual depreciation during the following year. The formula for annual depreciation is

$$D_y = (V_p \text{ at age } x) - (V_p \text{ at age } x + 1), \tag{5-19}$$

in which D_y is the annual depreciation and x the service age.

The annual depreciation should be estimated at the beginning of the year; one-twelfth the total should be charged to the production-expense accounts each month during the year.

The annual depreciation estimated at the beginning of the year should not be changed in making the depreciation adjustments required by later changes in the estimates of probable life and/or PFORR; all of these adjustments are to be made during the following year (or years).

Numerical Example: Calculate the present values and total and annual depreciations, for ages 16, 17, 18, and 19 years, of a property unit of which the data are

Total value new, $1,123.26; estimated salvage value, $50.

Probable life, 25 years, age 16 to 17, and 26 years, age 17 to 19.

PFORR, 0.95. Present-worth method. Fair net return 7%.

Solution: Find the tabular condition-percents in the table in Appendix C.

Age, years	Probable life, years	Tabular condition-percent	Condition-percent ×PFORR	Present value	Total depreciation	Age interval, years	Annual depreciation
16	25	55.91	53.1145	$620.06	$503.20		
						16–17	$47.62
17	25	51.24	48.6780	572.44	550.82		
	26	55.09	52.3355	611.70	511.56		
						17–18	7.65
18	26	50.49	47.9655	564.79	558.47		
						18–19	50.16
19	26	45.57	43.2915	514.63	608.63		

5.18. Condition-percent Tables.—An extensive condition-percent table is provided in Appendix C, in which the tabular condition-percents required for Step 3 in the actual depreciation process, described in Sec. 5.17, can be obtained without computation, for service ages 0 to 100 years, for all probable lives 0 to 100 years, and for interest rates of 0, 2, 3, 4, 5, 6, 7, and 8 percent.

In using the condition-percent table, the service age and the probable life used should each be stated to the nearest year only. This is closer than the possible accuracy of probable-life predictions.

5.19. The Correct Rates of Interest to Use in Estimating Actual Depreciation.—As already explained in Secs. 5.6 and 5.13, the correct interest rates to use in estimating actual depreciation are

Present-worth principle, the fair rate of net return (usually 5 to 8 percent).

Sinking-fund assumption, 3 to 5 percent (usually 4 percent).

Straight-line assumption, 0 percent.

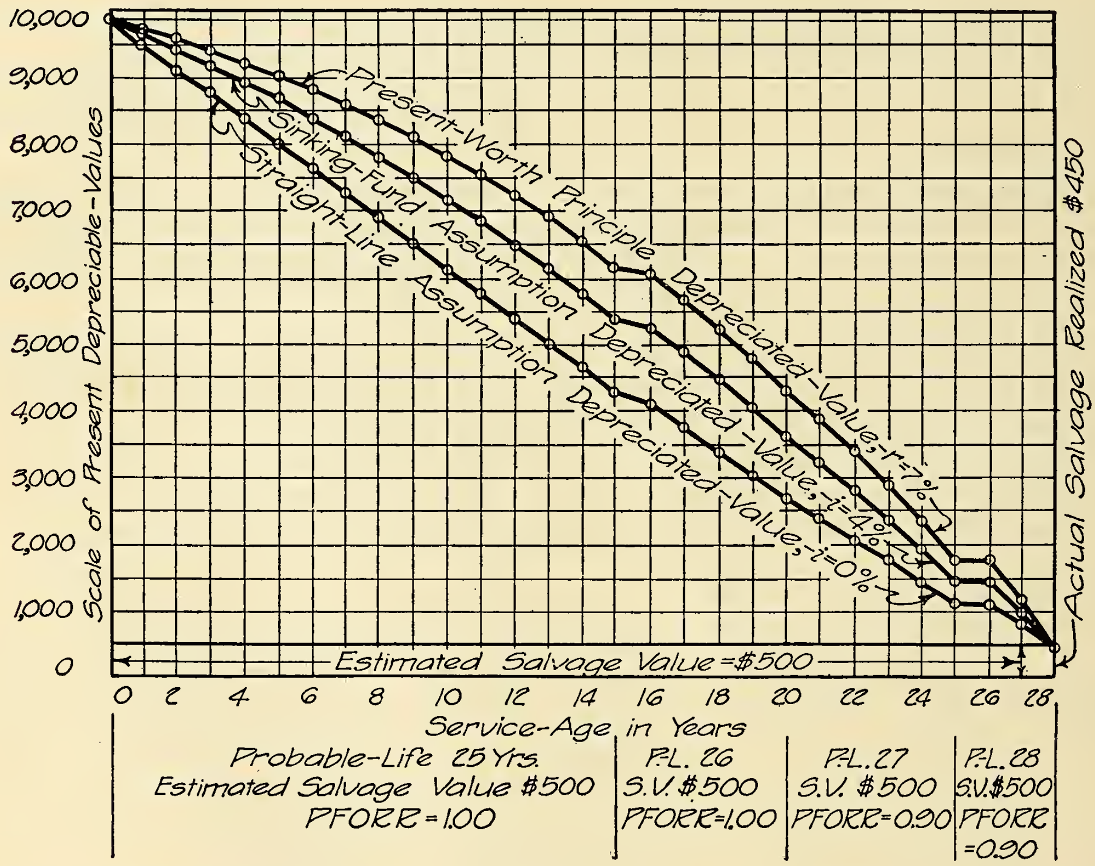


FIG. 5.5.—The actual depreciations of pumping engine 1, by the present-worth, the sinking-fund, and the straight-line methods.

In the judgment of the authors, the possible preciseness of depreciation determinations does not warrant using fractional rates of interest, intermediate between those used in the condition-percent table in Appendix C.

5.20. Determination of Actual Depreciation of Two Pumping Engines. The calculations of the actual depreciations of two duplicate pumping engines, whose numerical mortality data are given in Sec. 5.12, will now be presented, for the present-worth principle, the sinking-fund assumption, and the straight-line assumption, using the process described in Sec. 5.17.

The results for Engine 1 are given in Table 5.1 and Fig. 5.5.

The results for Engine 2 are given in Table 5.2 and Fig. 5.6.

TABLE 5.1.—COMPUTATIONS OF ACTUAL DEPRECIATION, PUMPING ENGINE 1

Direct costs new, $8,971. Overheads, 10%. Salvage value estimated at $500. Average service life of similar engines, 25 years. This engine retired at age 28. $450 net salvage realized.

Age, years	Probable life, years	PFORR ratio	Condition-percent			Present value			Age interval, years	Annual depreciation			Net return†			Operation return		
			0% S.L.,* %	4% S.F.,* %	7% P.W.,* %	0% S.L.	4% S.F.	7% P.W.		0% S.L.	4% S.F.	7% P.W.	0% S.L.	4% S.F.	7% P.W.	0% S.L.	4% S.F.	7% P.W.
0	25	1.00	100.00	100.00	100.00	$9,868	$9,868	$9,868	0–1	$375	$225	$148	$691	$691	$691	$1,066	$916	$839
1	25	1.00	96.00	97.60	98.42	9,493	9,643	9,720	1–2	374	234	158	665	675	680	1,039	909	838
2	25	1.00	92.00	95.10	96.73	9,119	9,409	9,562	2–3	375	244	170	638	659	669	1,013	903	839
3	25	1.00	88.00	92.50	94.92	8,744	9,165	9,392	3–4	375	253	182	612	642	657	987	895	839
4	25	1.00	84.00	89.80	92.98	8,369	8,912	9,210	4–5	375	263	194	586	624	645	961	887	839
5	25	1.00	80.00	86.99	90.91	7,994	8,649	9,016	5–6	374	273	208	560	605	631	934	878	839
6	25	1.00	76.00	84.07	88.69	7,620	8,376	8,808	6–7	375	285	222	533	586	617	908	871	839
7	25	1.00	72.00	81.03	86.32	7,245	8,091	8,586	7–8	375	296	237	507	566	601	882	862	838
8	25	1.00	68.00	77.87	83.78	6,870	7,795	8,349	8–9	374	307	255	481	546	584	855	853	839
9	25	1.00	64.00	74.59	81.06	6,496	7,488	8,094	9–10	375	321	272	455	524	567	830	845	839
10	25	1.00	60.00	71.17	78.16	6,121	7,167	7,822	10–11	375	332	291	428	502	548	803	834	839
11	25	1.00	56.00	67.62	75.05	5,746	6,835	7,531	11–12	375	347	312	402	478	527	777	825	839
12	25	1.00	52.00	63.92	71.72	5,371	6,488	7,219	12–13	374	360	334	376	454	505	750	814	839
13	25	1.00	48.00	60.08	68.16	4,997	6,128	6,885	13–14	375	374	357	350	429	482	725	803	839
14	25	1.00	44.00	56.08	64.35	4,622	5,754	6,528	14–15	375	390	382	324	403	457	699	793	839
15	25	1.00	40.00	51.92	60.27	4,247	5,364	6,146	15–16	144	110	82	297	375	430	441	485	512
	26	1.00	42.31	54.81	63.41	4,464	5,635	6,440										
16	26	1.00	38.46	50.75	59.39	4,103	5,254	6,064	16–17	360	396	403	287	368	424	647	764	827
17	26	1.00	34.62	46.52	55.09	3,743	4,858	5,661	17–18	360	411	431	262	340	396	622	751	827
18	26	1.00	30.77	42.13	50.49	3,383	4,447	5,230	18–19	361	429	461	237	311	366	598	740	827
19	26	1.00	26.92	37.55	45.57	3,022	4,018	4,769	19–20	360	445	493	212	281	334	572	726	827
20	26	1.00	23.08	32.80	40.31	2,662	3,573	4,276	20–21	288	367	423	186	250	299	474	617	722
	27	0.90	23.34	33.08	40.46	2,686	3,599	4,290										
21	27	0.90	20.00	28.89	35.79	2,374	3,206	3,853	21–22	312	408	469	166	224	270	478	632	739
22	27	0.90	16.67	24.53	30.79	2,062	2,798	3,384	22–23	313	423	502	144	196	237	457	619	739
23	27	0.90	13.33	20.01	25.43	1,749	2,375	2,882	23–24	312	443	537	122	166	202	434	609	739
24	27	0.90	10.00	15.29	19.70	1,437	1,932	2,345	24–25	312	458	573	101	135	164	413	593	737
25	27	0.90	6.67	10.40	13.57	1,125	1,474	1,772	25–26	23	19	16	79	103	124	102	122	140
	28	0.90	9.64	14.99	19.46	1,403	1,904	2,323										
26	28	0.90	6.43	10.19	13.41	1,102	1,455	1,756	26–27	301	469	607	77	102	123	378	571	730
27	28	0.90	3.21	5.19	6.93	801	986	1,149	27–28	351	536	699	56	69	80	407	605	779
28	28	0.90	0.00	0.00	0.00	450	450	450										

* S.L. = straight line; S.F. = sinking fund; P.W. = present worth.
† Fair rate of net return = 7%.

Table 5.2.—Computations of Actual Depreciation, Pumping Engine 2

Direct costs, new, $8,971. Overheads, 10%. Salvage value estimated at $500 till age 18, then at $400. Average service life of similar engines, 25 years. This engine retired at 22 years. $350 net salvage realized.

Age, years	Probable life, years	PFORR ratio	Condition-percent 0% S.L.,* %	Condition-percent 4% S.F.,* %	Condition-percent 7% P.W.,* %	Present value 0% S.L.	Present value 4% S.F.	Present value 7% P.W.	Age interval, years	Annual depreciation 0% S.L.	Annual depreciation 4% S.F.	Annual depreciation 7% P.W.	Net return† 0% S.L.	Net return† 4% S.F.	Net return† 7% P.W.	Operation return 0% S.L.	Operation return 4% S.F.	Operation return 7% P.W.
0	25	1.00	100.00	100.00	100.00	$9,868	$9,868	$9,868	0–1	$ 375	$ 225	$ 148	$691	$691	$691	$1,066	$ 916	$ 839
1	25	1.00	96.00	97.60	98.42	9,493	9,643	9,720	1–2	374	234	158	665	675	680	1,039	909	838
2	25	1.00	92.00	95.10	96.73	9,119	9,409	9,562	2–3	375	244	170	638	659	669	1,013	903	839
3	25	1.00	88.00	92.50	94.92	8,744	9,165	9,392	3–4	375	253	182	612	642	657	987	895	839
4	25	1.00	84.00	89.80	92.98	8,369	8,912	9,210	4–5	375	263	194	586	624	645	961	887	839
5	25	1.00	80.00	86.99	90.91	7,994	8,649	9,016	5–6	374	273	208	560	605	631	934	878	839
6	25	1.00	76.00	84.07	88.69	7,620	8,376	8,808	6–7	375	285	222	533	586	617	908	871	839
7	25	1.00	72.00	81.03	86.32	7,245	8,091	8,586	7–8	375	296	237	507	566	601	882	862	838
8	25	1.00	68.00	77.87	83.78	6,870	7,795	8,349	8–9	374	307	255	481	546	584	855	853	839
9	25	1.00	64.00	74.59	81.06	6,496	7,488	8,094	9–10	375	321	272	455	524	567	830	845	839
10	25	1.00	60.00	71.17	78.16	6,121	7,167	7,822	10–11	375	332	291	428	502	548	803	834	839
11	25	1.00	56.00	67.62	75.05	5,746	6,825	7,531	11–12	375	347	312	402	478	527	777	825	839
12	25	1.00	52.00	63.92	71.72	5,371	6,488	7,219	12–13	374	360	334	376	454	505	750	814	839
13	25	1.00	48.00	60.08	68.16	4,997	6,128	6,885	13–14	375	374	357	350	429	482	725	803	839
14	25	1.00	44.00	56.08	64.35	4,622	5,754	6,528	14–15	375	390	382	324	403	457	699	793	839
15	25	1.00	40.00	51.92	60.27	4,247	5,364	6,146	15–16	625	727	769	297	375	430	922	1,102	1,199
	24	1.00	37.50	48.77	56.81	4,013	5,069	5,822										
16	24	1.00	33.33	44.16	52.06	3,622	4,637	5,377	16–17	389	449	475	254	325	376	643	774	851
17	24	1.00	29.17	39.37	46.99	3,233	4,188	4,902	17–18	391	467	509	226	293	343	617	760	852
18	24	1.00	25.00	34.38	41.56	2,842	3,721	4,393	18–19	1,043	1,354	1,575	199	260	308	1,242	1,614	1,883
	23	0.85	18.48	25.47	30.91	2,150	2,811	3,327										
19	23	0.85	14.78	20.77	25.54	1,799	2,367	2,818	19–20	350	463	544	126	166	197	476	629	741
20	23	0.85	11.08	15.88	19.79	1,449	1,904	2,274	20–21	683	969	1,194	101	133	159	784	1,102	1,353
	22	0.85	7.73	11.09	13.90	1,132	1,450	1,716										
21	22	0.85	3.87	5.65	7.18	766	935	1,080	21–22	416	585	730	54	65	76	470	650	806
22	22	0.85	0.00	0.00	0.00	350	350	350										

* S.L. = straight line; S.F. = sinking fund; P.W. = present worth.
† Fair rate of net return = 7 %.

THE ESTIMATION OF THEORETICAL DEPRECIATION BY THE FIXED-PERCENTAGE-OF-VALUE-NEW AND BY THE SINKING-FUND DEPRECIATION METHODS

As stated in Secs. 5.4 and 5.7, in using the straight-line and the sinking-fund depreciation assumptions to estimate theoretical depreciation they are applied to the average lives of similar property units, *not*, as in estimating actual depreciation, to the probable lives (re-estimated from time to time) of the particular units. This use of average instead of particular service lives is what makes it possible for accountants who have never

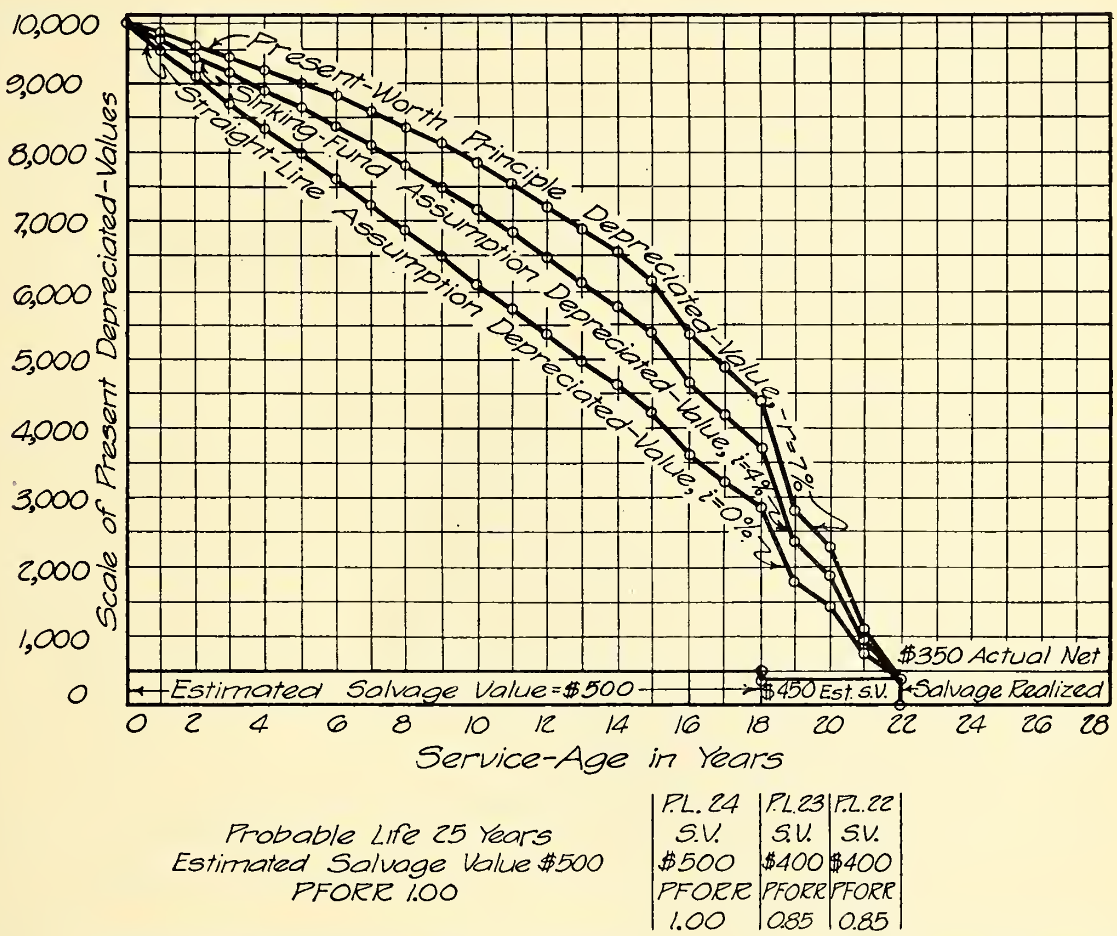


Fig. 5.6.—The actual depreciations of pumping engine 2, by the present-worth, the sinking-fund, and the straight-line methods.

seen the units, and who are not competent to judge their probable lives if they did see them, to make computations of theoretical depreciation.

5.21. The Estimation of Average Service Lives for Computing Theoretical Depreciation.—It is highly important that the estimated average service lives used in computing the theoretical depreciations of particular properties agree with the actual average service lives of the actual units, without appreciable errors.

As already explained in Secs. 3.13 and 3.14, average lives ought to be estimated with the aid of actual mortality curves, made up for each property from the actual records of the physical-property units, including

those still in use and those retired. When it is impossible to secure the aid of actual mortality curves in estimating average lives, the common practice is to estimate them by reference to published average-life tables[1] (Sec. 3.14 and Appendix A), aided by the estimator's own personal experience. Average lives estimated in this manner are likely to be materially in error for any particular property, causing corresponding errors in computations of theoretical depreciation.

5.22. Failure of Theoretical Depreciation Estimates to Agree with Actual Retirements.—The estimated average service lives of similar units usually differ materially from the actual service life of any particular unit; hence, the theoretical depreciations computed for a particular unit by different methods do not agree with each other or check with the true depreciation of the unit (its value new, minus its net salvage value, if any) at the date of its actual retirement (as is the case with all actual depreciation methods). Various arbitrary accounting devices become necessary to provide for the differences between the theoretical and actual accrued depreciations of particular units at the dates of their actual retirements.

Theoretical depreciation estimates are not automatically continually checked and corrected by the actual retirement experience of the properties. Nevertheless, whenever a theoretical depreciation method is used, much care should be taken to compare the results with yearly retirement experience, using every reasonable effort to secure as close an average agreement between actual and theoretical depreciation as is practicable.

If the estimated average service lives are materially too small or too large, not even an average agreement of the theoretical depreciation with the retirement experience is possible.

5.23. Adjustment of Theoretical Depreciation Accounts.—Since about one-half of any large group of similar property units will have actual service lives shorter, and the other half longer, than the average for the group, it is evident that theoretical depreciations computed for the average life, even when it is estimated correctly, will be too small for about one-half, and too large for the other half, of the individual units at the dates of their actual retirements. These discrepancies may be adjusted at the actual dates of retirement, to agree with the actual depreciations of the several units, by the following accounting devices:

1. For those units whose actual lives prove to be shorter than the estimated average lives of similar units, charge off in a single amount during the last year of actual service life all deficiencies in the total previous theoretical depreciation allowances. When these deficiency charges

[1] The U.S. Bureau of Internal Revenue has published a good average-life table for property items whose depreciations are legal deductions in calculating income taxes. Depreciation Studies, *Prelim. Rept., Bur. Internal Rev.*, Treasury Dept., January, 1931, Washington, D.C.

prove to be so large as materially to disarrange finances, they may sometimes be charged to a suspense account, to be amortized later (see Tables 5.3, 5.4, 5.5, and Fig. 5.7).

2. For those units whose actual lives prove to be longer than the estimated average lives of similar units, stop charging off depreciation at the end of the estimated average life. Such units must be carried thereafter at merely their net salvage values (see Tables 5.3, 5.4, 5.5, and Fig. 5.7).

5.24. The Fixed-percentage-of-value-new-less-net-salvage Theoretical Depreciation Method.—This widely used theoretical depreciation method is based on the straight-line assumption (Sec. 5.4), applied to the average lives of similar units, instead of the actual lives of particular units.

In the fixed-percentage-of-value-new-less-net-salvage depreciation method, the annual theoretical depreciation percentage of the value new less estimated salvage value of each property unit is assumed to be equal to 100 percent, divided by the average life of similar units, in years.[1]

As stated in Sec. 5.4, it is the view of the authors that straight-line distribution, of either theoretical or actual depreciation, is merely a simple and convenient but erroneous assumption.

Nevertheless, theoretical fixed-percentage depreciation is more widely used in depreciation accountancy than any other. It "is the one [method] most generally used for determining depreciation for tax purposes" (Sec. 4.5). It is the method prescribed by the Interstate Commerce Commission for steam railways, electric railways, express properties, and telephone properties (Sec. 2.10). It is widely used in depreciation accountancy for all sorts of industrial properties.

5.25. The Original Sinking-fund Theoretical Depreciation Method.—Both this often-used theoretical depreciation method and its compound-interest modification are based on the sinking-fund assumption, applied to the average lives of similar units, instead of the actual lives of particular units.

In the original sinking-fund theoretical depreciation method

1. The total accrued theoretical depreciation of each property unit at each service age is assumed to be equal to the corresponding accumulation in a fictitious depreciation sinking fund, whose total life is the estimated average life of similar units.

2. The annual depreciation annuity, set aside out of current annual income, is not the full annual theoretical depreciation but merely the equal annual year-end payment into the fictitious depreciation sinking fund.

[1] In cases where the amount of service rendered by particular units varies in different years, the assumption is sometimes changed to make the annual fixed percentage of depreciation vary from year to year in proportion to the extent of current service rendered.

3. To compensate for the shortages between the depreciation annuity and the total annual depreciations, a nominal net return is allowed on the unit's full value new, although the United States Supreme Court has repeatedly decided that the correct rate base is the present depreciated value.

The meaning of the above three assumptions is illustrated in columns (1) to (9), Table 5.3. These show the numerical results of the application of the original sinking-fund method to the computations of the depreciations of Pumping engine 2, whose mortality data are given at the top of the table.

Column (4) shows the total annual depreciations, ranging from $225 the first year to $493 the twenty-first (and $513 the twenty-second). But the depreciation annuity [see column (5)] is only $224.94, which falls short of the total annual depreciations by the amounts shown in column (6) (the sinking-fund interest in the different years).

Because of these annual shortages between the depreciation annuity and the total annual depreciation, the *real* net returns, shown in column (8), are smaller than the *nominal* net returns in column (7) (which are calculated on the full value new instead of on the correct rate bases equal to the present depreciated values (column 2).

5.26. The Compound-interest Modification of the Original Sinking-fund Theoretical Depreciation Method.—This plan was devised to correct the errors in the original sinking-fund method, due to its false assumption that the total annual and accrued depreciations could be adequately provided for by a depreciation annuity, equal to the equal year-end payment into the fictitious depreciation sinking fund of the sinking-fund depreciation assumption. In the compound-interest modification of the original sinking-fund theoretical depreciation method

1. The total accrued theoretical depreciation of each property unit at each service age is assumed to be equal to the corresponding accumulation in a fictitious depreciation sinking fund, whose total life is the estimated average life of similar units.

This assumption is the same as in the original sinking-fund depreciation method.

2. The annual depreciation allowance is the total annual depreciation called for by the sinking-fund assumption (1, just above); and, hence, is the total annual increment in the fictitious depreciation sinking fund.

In the original sinking-fund method the annual depreciation allowance is merely the depreciation annuity instead of the total annual increment of the fictitious sinking fund.

3. The net returns are calculated on the unit's present depreciated values, Jan. 1, each year.

Table 5.3.—Comparison of Original Sinking-fund Method and Its Compound-interest Modification Theoretical Depreciation Computations: 4% interest, 7% return.

Pumping engine 2. Cost new, $9,868. Salvage, estimated $500, realized $350. Life, estimated 25, actual 22 years.

Age, years	Both methods, present depreciated values	Age interval, years	Both methods, annual depreciations	Original sinking-fund method					Compound-interest modification	
				Depreciation annuity	Sinking-fund interest	Nominal net return at 7 %	Real net return (7) − (6)	Operation return (4) + (8)	Net return at 7 %	Operation return (4) + (10)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	$9,868	0–1	$ 225	$224.94	0	$690.76	$691	$916	$691	$916
1	9,643	1–2	234	224.94	9	690.76	682	916	675	909
2	9,409	2–3	244	224.94	19	690.76	672	916	659	903
3	9,165	3–4	253	224.94	28	690.76	663	916	642	895
4	8,912	4–5	263	224.94	38	690.76	653	916	624	887
5	8,649	5–6	273	224.94	48	690.76	643	916	605	878
6	8,376	6–7	285	224.94	60	690.76	631	916	586	871
7	8,091	7–8	296	224.94	71	690.76	620	916	566	862
8	7,795	8–9	307	224.94	82	690.76	609	916	546	853
9	7,488	9–10	321	224.94	96	690.76	595	916	524	845
10	7,167	10–11	332	224.94	107	690.76	584	916	502	834
11	6,835	11–12	347	224.94	122	690.76	569	916	478	825
12	6,488	12–13	360	224.94	135	690.76	556	916	454	814
13	6,128	13–14	374	224.94	149	690.76	542	916	429	803
14	5,754	14–15	390	224.94	165	690.76	526	916	403	793
15	5,364	15–16	405	224.94	180	690.76	511	916	375	780
16	4,959	16–17	421	224.94	196	690.76	495	916	347	768
17	4,538	17–18	439	224.94	214	690.76	477	916	318	757
18	4,099	18–19	455	224.94	230	690.76	461	916	287	742
19	3,644	19–20	474	224.94	249	690.76	442	916	255	729
20	3,170	20–21	493	224.94	268	690.76	423	916	222	715
21	2,677	21–22	2,327*	224.94	288	690.76	403	916	187	700
22	350									

* Depreciation $1,814 in excess of sinking-fund allowance. When such depreciation excess is so large as materially to affect the yearly finances, it sometimes is charged to a suspense account and amortized over a period longer than one year.

In the original sinking-fund method, the net returns are calculated on the unit's value new.

Columns (1) to (4) and (10) and (11), Table 5.3, show the numerical results obtained by the application of the compound-interest modification to the same pumping engine 2 to which the original sinking-fund method was applied, as explained in Sec. 5.25. The differences between the two methods are illustrated clearly in Table 5.3.

The authors consider the compound-interest modification to be much more nearly correct than the original sinking-fund method.

5.27. Application of Fixed-percentage and Sinking-fund Methods.—Tables 5.4, 5.5, and Fig. 5.7 show the numerical results of the application

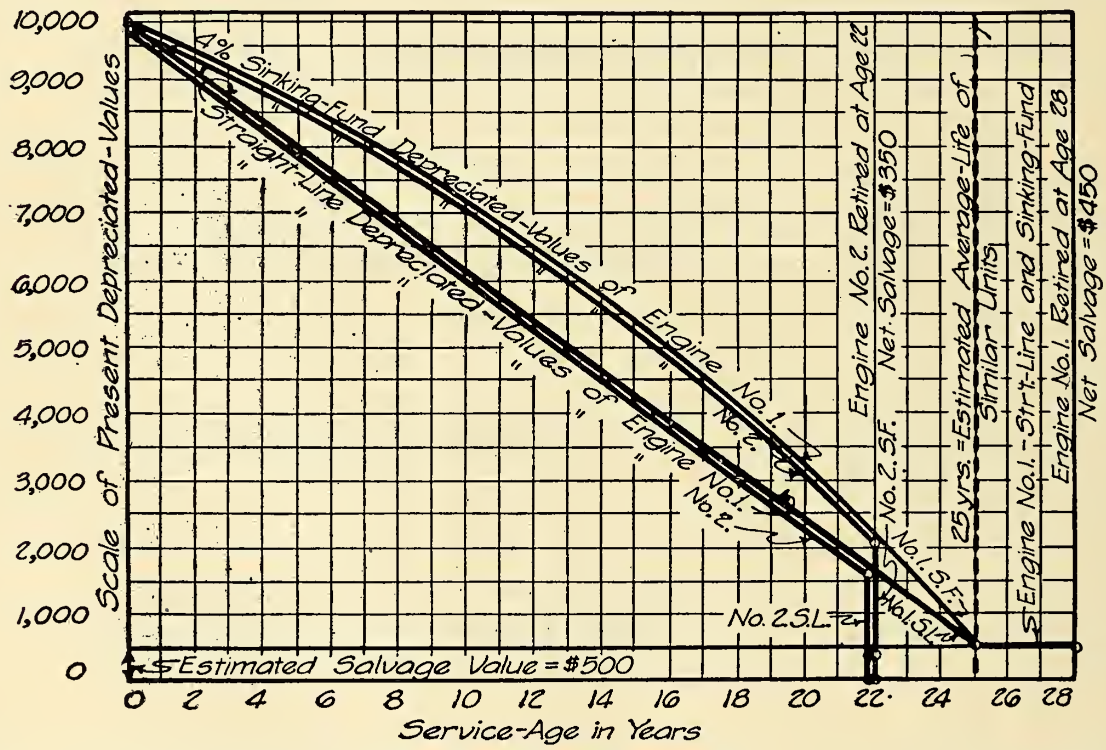


FIG. 5.7.—The theoretical depreciations of pumping engines 1 and 2, by the sinking-fund and the straight-line methods.

of the fixed-percentage and the sinking-fund theoretical depreciation methods in calculating the depreciations of two duplicate pumping engines throughout their service lives.

These duplicate pumping engines 1 and 2 are the same engines for which the numerical results of the application of the present-worth, the sinking-fund, and the straight-line actual depreciation methods were shown in Tables 5.1, 5.2, and Figs. 5.5 and 5.6.

Table 5.4.—Computations of Theoretical Depreciation, Pumping Engine 1
Direct costs new, $8,971. Overhead costs, 10%. Estimated salvage value, $500.
Average service life, similar engines, 25 years. Retired at age 28.
Net salvage value, $450.

Age, years	Condition-percent		Present value		Age interval, years	Annual depreciation		7% net return		Operation return	
	S.L., %	4% S.F., %	S.L.	4% S.F.		S.L.	4% S.F.	S.L.	4% S.F.	S.L.	4% S.F.
0	100.00	100.00	$9,868	$9,868	0–1	$375	$225	$691	$691	$1,066	$916
1	96.00	97.60	9,493	9,643	1–2	374	234	665	675	1,039	909
2	92.00	95.10	9,119	9,409	2–3	375	244	638	659	1,013	903
3	88.00	92.50	8,744	9,165	3–4	375	253	612	642	987	895
4	84.00	89.80	8,369	8,912	4–5	375	263	586	624	961	887
5	80.00	86.99	7,994	8,649	5–6	374	273	560	605	934	878
6	76.00	84.07	7,620	8,376	6–7	375	285	533	586	908	871
7	72.00	81.03	7,245	8,091	7–8	375	296	507	566	882	862
8	68.00	77.87	6,870	7,795	8–9	374	307	481	546	855	853
9	64.00	74.59	6,496	7,488	9–10	375	321	455	524	830	845
10	60.00	71.17	6,121	7,167	10–11	375	332	428	502	803	834
11	56.00	67.62	5,746	6,835	11–12	375	347	402	478	777	825
12	52.00	63.92	5,371	6,488	12–13	374	360	376	454	750	814
13	48.00	60.08	4,997	6,128	13–14	375	374	350	429	725	803
14	44.00	56.08	4,622	5,754	14–15	375	390	324	403	699	793
15	40.00	51.92	4,247	5,364	15–16	375	405	297	375	672	780
16	36.00	47.60	3,872	4,959	16–17	374	421	271	347	645	768
17	32.00	43.10	3,498	4,538	17–18	375	439	245	318	620	757
18	28.00	38.42	3,123	4,099	18–19	375	455	219	287	594	742
19	24.00	33.56	2,748	3,644	19–20	374	474	192	255	566	729
20	20.00	28.50	2,374	3,170	20–21	375	493	166	222	541	715
21	16.00	23.24	1,999	2,677	21–22	375	513	140	187	515	700
22	12.00	17.76	1,624	2,164	22–23	375	533	114	151	489	684
23	8.00	12.07	1,249	1,631	23–24	374	554	87	114	461	668
24	4.00	6.16	875	1,077	24–25	375	577	61	75	436	652
25	0.00	0.00	*500	*500	25–26	*0	*0	35	35	35	35
26	0.00	0.00	500	500	26–27	0	0	35	35	35	35
27	0.00	0.00	500	500	27–28	50	50	35	35	85	85
28	0.00	0.00	450	450							

* Note that after the expiration of the estimated average life (25 years in this case) the unit's book value is merely salvage value. No further depreciation expense can be charged; the production-expense accounts are correspondingly incorrect and only an insignificant net return can be claimed fairly.

Table 5.5.—Computations of Theoretical Depreciations, Pumping Engine 2
Direct costs new, $8,971. Overhead costs, 10%. Estimated salvage value, $500. Average service life, similar engines, 25 years. Retired at age 22. Net salvage value, $350.

Age, years	Condition-percent		Present value		Age interval, years	Annual depreciation		7% net return		Operation return	
	S.L. %	4% S.F., %	S.L.	4% S.F.		S.L.	4% S.F.	S.L.	4% S.F.	S.L.	4% S.F.
0	100.00	100.00	$9,868	$9,868	0–1	$ 375	$ 225	$691	$691	$1,066	$916
1	96.00	97.60	9,493	9,643	1–2	374	234	665	675	1,039	909
2	92.00	95.10	9,119	9,409	2–3	375	244	638	659	1,013	903
3	88.00	92.50	8,744	9,165	3–4	375	253	612	642	987	895
4	84.00	89.80	8,369	8,912	4–5	375	263	586	624	961	887
5	80.00	86.99	7,994	8,649	5–6	374	273	560	605	934	878
6	76.00	84.07	7,620	8,376	6–7	375	285	533	586	908	871
7	72.00	81.03	7,245	8,091	7–8	375	296	507	566	882	862
8	68.00	77.87	6,870	7,795	8–9	374	307	481	546	855	853
9	64.00	74.59	6,496	7,488	9–10	375	321	455	524	830	845
10	60.00	71.17	6,121	7,167	10–11	375	332	428	502	803	834
11	56.00	67.62	5,746	6,835	11–12	375	347	402	478	777	825
12	52.00	63.92	5,371	6,488	12–13	374	360	376	454	750	814
13	48.00	60.08	4,997	6,128	13–14	375	374	350	429	725	803
14	44.00	56.08	4,622	5,754	14–15	375	390	324	403	699	793
15	40.00	51.92	4,247	5,364	15–16	375	405	297	375	672	780
16	36.00	47.60	3,872	4,959	16–17	374	421	271	347	645	768
17	32.00	43.10	3,498	4,538	17–18	375	439	245	318	620	757
18	28.00	38.42	3,123	4,099	18–19	375	455	219	287	594	742
19	24.00	33.56	2,748	3,644	19–20	374	474	192	255	566	729
20	20.00	28.50	2,374	3,170	20–21	375	493	166	222	541	715
21	16.00	23.24	1,999	2,677	21–22	*1,649	*2,327	140	187	515	700
22	0.00	0.00	350	350							

* Depreciations $1,274 in excess of straight-line allowance and $1,814 in excess of sinking-fund allowance. When such depreciation excesses are so large as materially to affect yearly finances, they sometimes are charged to a suspense account and amortized over a period longer than one year.

CHAPTER VI

DEPRECIATION ACCOUNTANCY

It is now generally agreed by good authorities that depreciation is a highly important, real production expense; correct current accountancy for it is essential to the wise management of industrial enterprises, and to true accounts of their operations and financial affairs. Nevertheless, a considerable diversity of opinion still exists upon a number of important depreciation questions; also, the depreciation-accountancy methods still most widely used give only roughly correct, average approximations to the true depreciations.

THE HISTORY, OBJECTIVES, VALUE, AND FEASIBILITY OF CORRECT DEPRECIATION ACCOUNTANCY

6.1. An Outline History of the Legal Status of Depreciation Accountancy.—Prior to 1909, the courts generally confused depreciation with repairs, and replacements with construction. It seemed impossible for them to visualize a kind of expense which *accrued* as a loss of value of units still in service; an expense which could not be paid in money until the future dates, often far remote, of the actual retirements of the units. When, in litigated cases, property owners sought tax allowances, and/or higher utility rate schedules, on account of depreciation, the courts at first held that there was no such thing as accrued depreciation.

The United States Supreme Court so held in the *Kansas Pacific Case* (99 U.S. 459), in ____; in *Reagen v. Farmers Loan and Trust Company* (154 U.S. 562), in 1894; and in *San Diego Land and Town Co. v. Jasper* (189 U.S. 439), in 1903.

In the first-named case, the Court said unequivocally of depreciation, "We are of the opinion that it is not a fair charge"; in the second case, it was held proper to charge the costs of retirements to operating expenditures; and in the third case, Mr. Justice Holmes, who did not retire from the bench until 1932, wrote the opinion of the Court holding against "the contention of the appellant, that there should have been allowance for depreciation over and above the allowance for repairs."

The California Supreme Court followed the lead of the United States Supreme Court by ruling against depreciation in the *San Diego Water Company Case*, in 1897, and the *Redlands Water Company Case*, in 1898; as also did the Iowa Supreme Court in *Cedar Rapids Water Company v. Cedar Rapids*, in 1902, saying: " . . . to hold otherwise is to say that the public must not only pay the reasonable and fair value of the services rendered, but must in addition pay the company the full value of its works every forty years. . . . "

In 1909, however, the United States Supreme Court, in its celebrated opinion in the *Knoxville Water Case* (Sec. 8.4), abandoned its earlier

position and announced unequivocally that accrued depreciation is a real loss of value, which must be deducted in determining fair present value; and is therefore a real operating expense, which the owners of utilities are entitled to recoup by current collections from customers. The *Knoxville Case* decision was vigorously reaffirmed in 1913, in the *Minnesota Rate Case* decision (Sec. 8.4); and has been supported by numerous other decisions since 1909.

The position announced in the *Knoxville Case* has been maintained in the face of opposition by many utility owners, who have urged strongly, and continue to urge, that their properties are as good as new, so long as their operating efficiency is maintained at a high standard. Although the United States Supreme Court has ruled in favor of utilities in several cases where practically no accrued depreciation was deducted (in one such case at the same date as the *Knoxville Case*), such ruling has been made only in cases where the amount of depreciation was not the determining factor; no approval of the good-as-new doctrine has been involved in the decisions.

Since 1920, the various courts, including the United States Supreme Court, have been ruling ever more strongly in favor of "actual" in preference to "theoretical" depreciation (Sec. 4.5).

6.2. Brief Outline of the History of Depreciation Accountancy.— Accountants, like the courts, until less than 30 years ago quite generally confused depreciation with repairs. Mr. Justice Brandeis of the United States Supreme Court, says[1] of depreciation accountancy that "among street railways, the Milwaukee Electric Railway and Light Company became the pioneer by adopting it in 1897. Others followed in 1905."

Early accounting dealt mainly with historical records of actual money payments and receipts. It was hard for the accountant to visualize and understand a kind of expense which could not be paid for currently, but must be left to *accrue*, as a loss of value, until some future date before it could actually be made good physically; and still harder to devise correct depreciation-accountancy methods, involving the aid of technical knowledge and experience, for charging the depreciation expenses currently to production costs and for providing *currently* from income for making *future* retirement payments.

When depreciation began to be understood, it was natural for the clerks who made the book records of money payments and receipts to want some simple mathematical rule for calculating depreciation expense; hence the early and still continued widespread vogue of the straight-line fixed-percentage depreciation method.

Many accountants still desire some mathematical rule, rather than a valuation principle, for estimating depreciation; but valuation accounts should never be based merely on clerical records and computations. The judgment of a qualified valuator is always necessary; depreciation accountancy deals with losses of value, instead of with money payments.

[1] See the *Baltimore Street Railway Case* decision, Sec. 8.6.

The correct estimation of depreciation involves special technical knowledge, experience, and judgment, which cannot be expected of clerks; correct depreciation accountancy requires the periodic aid of qualified technical valuators.

Depreciation accountancy is constantly developing, and its value and necessity are constantly becoming more widely known. Doubtless many of its present shortcomings are due to lack of knowledge of the mortality characteristics of physical property, discussed in Chap. III; this is a handicap similar to that which the actuaries of insurance companies would experience if they were not familiar with human mortality curves, tables, and characteristics.

6.3. Depreciation-accountancy Objectives.—The *ultimate objectives* of depreciation accountancy are

A. *To Account Correctly and Currently for All Losses of Value by Depreciation.*
   1. To determine each year the true present values of all physical-property units.
   2. To provide correctly and currently for maintaining the original investment in the property unwasted by keeping a correct depreciation reserve account and by making corresponding annual depreciation appropriations from current income; this provides in advance for the inevitable costs of the ultimate retirements of units.

B. *To Account Correctly and Currently for All Depreciation Production Expenses.*
   3. To keep true accounts of the current annual depreciation costs of the various plant operations and units of equipment.
   4. To make correct current depreciation allowances in determining fair rates and prices for the various services rendered, and/or commodities produced by the plant.

C. *To Furnish at Call and Correctly All Current Depreciation Data Needed in the Wise Management of the Enterprise.*
   5. To determine the true net return each year, after making the correct annual depreciation appropriation from current income.
   6. To determine what rate on the true present value of the property each year's true net return gives.
   7. To furnish all correct current depreciation data needed in determining the efficiency and any possible increase in efficiency of each plant operation and machine.

6.4. Depreciation-accountancy Errors and Inaccuracies.—Correct current depreciation accountancy is one of the major essentials in attaining the depreciation-accountancy objectives enumerated in Sec. 6.3; depreciation is just as real a cost of production as money paid for

labor, fuel, materials, and supplies. Moreover, depreciation is one of the major costs of production.

This is illustrated by the following data of the actual costs of the electricity produced and sold by the Ames, Ia., municipal electric utility in 1932.

	Money expenditures*	Depreciation*	Total*
Production costs	$ 77,144.33	$33,688.31	$110,832.64
Distribution costs	13,605.92	20,149.69	33,755.61
Commercial costs	10,896.37	....	10,896.37
Total costs	$101,646.62 65.37%	$53,838.00 34.63%	$155,484.62 100.00%

* Taken from a complete valuation made by the authors. Large current retirements for obsolescence made the current depreciation somewhat greater than normal.

Without correct current depreciation accountancy, the managers of this utility could only guess at 34.63 percent of their total costs (59.69 percent of their distribution costs).

Errors and inaccuracies in estimating and recording depreciation expense and loss of value are exactly as serious and objectionable as errors and inaccuracies of like amounts in pay rolls and bills of all kinds: for labor, fuel, materials, supplies, equipment, buildings, and plant.

A large number of quotations could readily be offered in support of the view that depreciation is a real loss of value and a real production expense, and that correct depreciation accountancy is one of the major essentials to true accounts.

The Federal Trade Commission[1] says

> Depreciation is one of the most important of all the overhead expenses, because it is generally the largest. . . . It is universally admitted, however, that depreciation does exist, that it is an element of cost just as much as labor or material, and that any system that does not provide for including it is faulty and one that will not give true costs.

In his recent book,[2] A. Hamilton Church says of depreciation, "This third class is a most important one, embracing, as it does, nearly all capital investment—building, machinery, equipment, tools."

While errors and inaccuracies of every kind in estimating and accounting for depreciation are serious and objectionable, the not unknown

[1] Fundamentals of a Cost System for Manufacturers, Government Printing Office, Washington, D.C., 1916, p. 12.

[2] CHURCH, A. HAMILTON, "Manufacturing Costs and Accounts," 2d ed., McGraw-Hill Book Company, Inc., New York, 1929, p. 4.

practice of manipulating depreciation accounts to conceal either profits or losses is reprehensible. Depreciation, including accrued depreciation of units not yet retired, must be correctly accounted for currently; otherwise, the accounts will be false accounts. It is not permissible to charge off less than the true depreciation in years of depression, or more than the true depreciation in years of unusual prosperity; the so-called "business judgment" on which such practices are based is just a deceptive name for deception of the investing public, including the enterprise's own stockholders.

6.5. The Feasibility of Correct Current Depreciation Accountancy.—Objections are sometimes made to the maintenance of correct current depreciation-accountancy systems because of the work and expense required. The same objections might be raised to all kinds of correct accounts; accountancy of every kind is expensive. The real question is whether the expense is necessary; if so, it will be profitable if the enterprise is sound; it is the practically unanimous opinion of those best qualified to judge that depreciation accountancy is not only necessary but feasible, and not only feasible but profitable.

As to the necessity for depreciation accountancy, the United States Chamber of Commerce says:[1] "It is our feeling that depreciation, more often than any other item of cost, blurs the line dividing profits from losses."

As to feasibility, the Interstate Commerce Commission states[2] that the witnesses who appeared before it for various industries testified " . . . that private industries generally recognize depreciation as a factor affecting their operations which should be accounted for currently."

The Interstate Commerce Commission has said further,[3] referring to the contention by steam-railway representatives that complete depreciation accountancy will be so expensive as to require an increase in railway rates:

> This contention, after all, begs the question. If depreciation accounting is a method by which the facts with regard to the cost of operating railroad and telephone companies are recorded with approximate accuracy, then that method should be adopted regardless of its effect on rates. Nothing is to be gained by refusal to face facts or by deferring to some future date burdens which ought to be borne now. As we have seen, however, there are substantial reasons for believing that no additional burdens on patrons will result.

[1] "Depreciation Treatment in Production Costs," Department of Manufactures, U.S. Chamber of Commerce, Washington, D.C., 1929 reprint, p. 20.

[2] "Depreciation Charges of Telephone and Steam Railroad Companies," *Proposed Rept.*, Nos. 14700 and 15100, Interstate Commerce Commission, Washington, D.C., 1929, pp. 4, 5.

[3] 118 I.C.C., 1926, pp. 317, 318.

The Federal Trade Commission has said[1] of correct cost systems, of which correct depreciation-accountancy records are an important essential feature:

A system will not run itself; neither will it in itself reduce costs or increase efficiency. This is strictly up to the manufacturer himself. A system will give him the information, and if this system is properly used, he will unquestionably find that his system is not an item of expense, but a very valuable asset.

DEPRECIATION-ACCOUNTANCY GENERAL PRINCIPLES AND METHODS

There are a number of depreciation-accountancy general principles and methods which still are subjects of difference of opinion and of active discussion. These include

The depreciation base.
Unit vs. group depreciation accountancy.
Current depreciation accountancy.
Retirement accountancy.
Actual vs. theoretical depreciation accountancy.
The net return base in depreciation accountancy.

6.6. The Depreciation Base.—The depreciation base of a property unit is its value new, less estimated salvage value (that is, its depreciable value new).

As stated in Secs. 5.4, 5.6, and 5.12, the accrued depreciation of the unit is calculated by multiplying its value new, less estimated salvage value by $\frac{1}{100}$ (100 percent − condition-percent).

There are three possible depreciation bases of property units which might be used in depreciation accountancy, namely,

Original-cost new, less estimated salvage value.
Reproduction-cost new, less estimated salvage value.
Fair cost-value[2] new, less estimated salvage value.

The United States Supreme Court has recently (Jan. 6, 1930) decided[3] that the legal depreciation base is "present fair value" (by which is meant what the authors term "fair cost-value new") less estimated salvage value, saying:

It is the settled rule of this Court that the rate base [for utility net returns] is present value, and it would be wholly illogical to adopt a different rule for depreciation.

In spite of this ruling by the highest court of our land, it is hardly too much to say that the original cost new less salvage is used almost universally as the depreciation base in actual accountancy for depreciation.

[1] "Cost System for Manufacturers," Federal Trade Commission, Washington, D.C., 1916, p. 31.

[2] Fair cost-value is determined by giving "such weight as may be just and right in each case" to original cost and to reproduction cost (Chap. XI).

[3] See the *Baltimore Street Railway Case*, Sec. 8.6.

Mr. Justice Brandeis, in his dissenting opinion in the *Baltimore Street Railway Case,* gives long lists of accountancy authorities, and of associations of business men, mine owners, manufacturers, state commissions, and federal agencies, that declare unequivocally for original cost new, less estimated salvage value, as the depreciation base. Among the federal agencies are the Interstate Commerce Commission, the U.S. Commissioner of Internal Revenue, the U.S. Bureau of the Census, the U.S. Department of Agriculture, the U.S. Bureau of Mines, the Federal Trade Commission, and the Federal Power Commission. The U.S. Chamber of Commerce strongly favors original cost new less salvage.

It is not correct, however, to argue, as some do, that the adoption of the fair present cost-value new, less estimated salvage value, for the depreciation base, in compliance with the United States Supreme Court's *Baltimore Street Railway Case* decision, would throw all depreciation estimates into the realm of mere conjecture; they need not lead to depreciation estimates which sometimes would be greater and sometimes smaller, when property units are retired, than their replacement costs less salvage. By depreciation adjustments for changes in fair values new, similar to the adjustments required by changes in estimated probable life, as described in Secs. 5.17 and 5.20 (illustrated in Tables 5.1, 5.2, and Figs. 5.4, 5.5, 5.6), it is entirely possible to make the annual depreciation allowances and the total depreciation reserve comply at all times with the Supreme Court's ruling.

By this plan, the depreciation reserve for each property unit would always be just sufficient at its retirement to cover its replacement costs, less salvage actually realized; thus always keeping the investment unwasted without returning old, or requiring new, capital.

6.7. Unit and Group Depreciation-accountancy Systems.—Two depreciation-accountancy systems are in use, the unit system and the group system.

1. In the *unit depreciation-accountancy system,* the depreciation is determined and recorded for each unit separately. In general, this is the most desirable system, because (*a*) it furnishes the depreciation data in complete detail, permitting classifications and summations to meet all the needs of valuation and cost accounting; (*b*) by the unit system, actual (not theoretical) depreciations can be definitely checked, precisely, for each unit at the date of its actual retirement. However, the unit system is not practicable for very numerous, comparatively small units (or items), such as ties, rails, and poles.

2. In the *group depreciation-accountancy system,* the depreciation is accounted for and determined by groups of units of the same general class.

In the case of large units, the group system is best adapted to the fixed-percentage theoretical depreciation method. If, for example, a railway has 100 locomotives of the same general character in service, by the group system it would estimate the yearly theoretical depreciation

of the whole group, by applying the fixed percentage of depreciation which it uses for that class of locomotive to the sum of the depreciable costs of the whole 100.

There are two main objections to the group system: (*a*) it does not furnish the depreciation data in detail for any particular unit; (*b*) group depreciation is not automatically checked from time to time by the retirement data of the individual units.

However, the Interstate Commerce Commission[1] prefers and proposes to require the group system, for use with the fixed-percentage-of-value-new-less-salvage theoretical depreciation which it prescribes. In this fixed-percentage method, only the average life of the group is estimated, and the actual service lives of most of the individual units of the group are sure to be shorter or longer than the estimated average life of similar units. When the fixed-percentage method is applied to individual units, its inconsistencies are exposed by the disagreement of the actual service lives with the estimated average life, as explained and illustrated in Secs. 5.23 and 5.27. The argument of the Interstate Commerce Commission against the unit system seems to be based very largely upon the assumption that only the average life of similar units is to be used in estimating depreciation; the actual lives of the actual units should be used, checked, and re-estimated from time to time till actual retirement.

It should be observed that the actual depreciation of age-groups of many like units (ties, rails, poles) can be determined and accounted for reliably by the use of the mortality curves and principles described in Chap. III; the group system is the only one practicable in such cases.

In general, the authors favor the unit system, supplemented by the use of the group system for age-groups of many like units, and for other groups of small units.

6.8. Current Depreciation Accountancy.—There are two general methods which are advocated and used at the present time for accounting, the one wholly and the other only in part, for depreciation expense:

1. Current depreciation accountancy.
2. Retirement accountancy.

In *current depreciation accountancy,* effort is made to estimate the correct total annual depreciation each year; all the current estimated annual depreciation is charged to the current production expenses and credited to the depreciation reserve. The reserve account is charged with the book value new, less actual net salvage, of each property unit at the date of its retirement; thus the balance in the reserve shows at all times the total depreciation which has been estimated as accrued on the property units still in service. If the depreciation has been estimated correctly, the expense accounts will then show the correct production costs; also, the depreciation reserve accounts will always show the property's true loss of value by depreciation.

[1] 118 I.C.C. 1926, pp. 367–372. *Proposed Rept.* I.C.C. 1929, pp. 42–46.

If, however, the depreciation is not correctly estimated, both the expense accounts and the depreciation reserve accounts will be correspondingly incorrect. They will be only approximately correct, if the estimated depreciations merely approximate the true depreciations.

Merely keeping correct current depreciation expense and depreciation reserve accounts will not save the investment in the property from wasting by depreciation. In addition, an *appropriation of actual cash from income, reserved before net return is calculated,* must be made each year, just equal in amount to the true depreciation during the year.

If these depreciation appropriations are invested wisely, as discussed in Sec. 6.22, the investment will thereby be completely safeguarded against uncompensated wasting by depreciation. See the numerical example in Sec. 6.25.

Current depreciation accountancy is advocated by the great majority of accountants, business and manufacturing associations, valuation authorities, and governmental agencies. It is strongly favored by the Interstate Commerce Commission. The authors of this treatise are convinced, first, that it is the only method which permits of true depreciation expense and valuation accounts; and, second, that its use is to the best interests of both producers and consumers, of both owners and the public.

6.9. Retirement Accountancy.—In retirement accountancy, only such part of the current depreciation expense each year is currently provided for as is actually paid out for retirements during the year. This expense is termed retirement expense. The depreciation accruing on property units still in service remains unprovided and unaccounted for.

The production expense accounts in retirement accountancy are not true accounts; they omit the accruing depreciation expenses on property units still in service, an important part of the total; they charge the retirement expenses of units wholly to the year in which retirements happen to be made, instead of in the years in which they have actually been accrued. Nor are the valuation accounts in retirement-accountancy systems true accounts; they show no deductions from the value new of property units on account of depreciation, even when it is known that many such units must be retired in the near future.

A *retirement reserve* is provided for in the retirement-accountancy systems recommended for intrastate gas and electric utilities by the National Association of Railway and Utility Commissioners (Sec. 2.11).

The main purpose of such a reserve is to provide in advance for extra large replacement demands for cash in any one year. The reserve may be established and maintained by appropriations from income, from surplus, or from both. The amount of the reserve is discretional, subject to regulation; it may even be omitted entirely.

Retirement accountancy does not safeguard the owners of utilities against ultimate important losses whenever actual sales of their properties occur; the highest courts have repeatedly held that the true accrued actual depreciation must be deducted in determining present fair value; such sales are sometimes forced.

One of the main arguments for retirement accountancy by its supporters is that it permits preserving a utility's credit by using business judgment in reducing depreciation appropriations in years of poor business, to which practice opponents of the system object as a deception of the stock- and bondholders and the general investing public.

The arguments for and against retirement accountancy will be illustrated by two quotations.

L. R. Nash says,[1] in favor:

> . . . its inflexibility [that of current depreciation accountancy] has a disturbing effect on utility credit. The author has studied this question in the light of the history of a large number of properties covering a long period of years, and is assured that the application of a system of unvarying provisions for depreciation would have been disastrous to many of them, and would have prevented their successful functioning and development. The alternative flexible [retirement-accountancy] system of appropriations which has been in effect upon these properties has in no case been responsible for inadequate or unreliable service, or otherwise operated to the disadvantage of either patrons or investors. This system requires conscientious and broad-minded administration, but no more than should be accorded to any important problems of complex enterprises.

On the other hand, the Interstate Commerce Commission, Mr. Eastman, chairman, says,[2] against:

> . . . Obviously the financial condition of the company will be less secure under the retirement than under the depreciation method, and it will be less well equipped to face the future and reap the benefit of improvements in the art.
>
> If the revenue resources are not adequate, it is true that the retirement method may enable a company, at least in the early years of development, to make a better apparent show of net earnings; but it does not follow that this is a blessing to the investor. It may well be argued that the best way for a company engaged in a quasi-monopolistic public service to obtain adequate revenue is to disguise its actual requirements as little as possible. The history of such enterprises is full of instances where investors have been misled to their hurt by delusive income and balance sheet statements.

6.10. Actual vs. Theoretical Depreciation Accountancy.—It apparently is the well-settled rule of our highest courts that true depreciation is actual depreciation, as defined and discussed in Sec. 4.5; hence true

[1] "Economics of Public Utilities," McGraw-Hill Book Company, Inc., New York, 1925, p. 407.

[2] 118 I.C.C. 1926, p. 312.

expense and true valuation accounts require the use of actual depreciation in depreciation accountancy. It can hardly be doubted that, as this fact and its implications come to be widely realized, the use of actual depreciation in accountancy will increase greatly.

In the meantime, the use of theoretical depreciation (Sec. 4.6) is very widespread in accountancy work; especially as calculated by the fixed-percentage-of-value-new-less-estimated-salvage-value method. When theoretical depreciation is used, both the depreciation expense and the present depreciated value accounts must be considered as only approximately correct; they are subject to rejection in legal cases where adequate proof of actual depreciation is offered in evidence.

6.11. The Net-return Rate Base in Depreciation Accountancy.—The rate base in depreciation accountancy is that value for an industrial property which is used as the base for calculating the rate of net return, when the total net return for any year is known; and, conversely, for calculating the total fair yearly net return when the fair rate of net return is known.

Although there has been a great deal of discussion as to whether or not accrued depreciation must be deducted from value new in determining the rate base of a public-utility property, it is the settled rule of the United States Supreme Court that the legal rate base of a public-utility property is the depreciated present fair value of the property which it employs in the public's service.

This rule is not entirely inconsistent with the use, in the original sinking-fund depreciation method and in retirement-accountancy

TABLE 6.1.—CORRECT REAL AND NOMINAL NET-RETURN RATE BASES

Accountancy system	Depreciation-accountancy method	Net-return rate base	
		Value new or depreciated	Real or nominal
Current-depreciation-accountancy-systems	Actual-present-worth method	Depreciated value	Real rate base
	Straight-line method	Depreciated value	Real rate base
	Compound-interest modification of sinking-fund method	Depreciated value	Real rate base
	Original sinking-fund method	Value new	Nominal rate base
Retirement-accountancy systems		Value new	Nominal rate base

systems, of a *nominal* rate base equal to the fair value new of the property.

The correct real or nominal rate bases to use with different depreciation methods and systems are as shown in Table 6.1.

DEPRECIATION-ACCOUNTANCY PROCEDURES

A satisfactory technique for depreciation accountancy is rapidly developing, though not yet sufficiently widely understood by accountants and engineers. It is based on the mortality characteristics of industrial-property units (Chap. III); on correct depreciation principles and relations (Chap. IV); and on the modern methods of estimating actual depreciation (Chap. V).

6.12. Classification of Property for Depreciation Accountancy.—For depreciation-accountancy purposes, the physical units of industrial properties must be grouped in accordance with the primary accounts adopted in the accounting systems used. Otherwise the depreciation accounts would not be conveniently arranged for furnishing the cost and valuation data needed for the separate operations and divisions of the property.

In general, the uniform classification of standard systems of accounts (like those mentioned in Secs. 2.10 and 2.11) should be followed wherever at all practicable.

The *division into accountancy units*, for the property ledger and for determining depreciation, should be carefully studied with a view to minimizing accountancy labor and cost.

Like units should be grouped so far as practicable.

Very numerous, comparatively small units (or items), such as ties, rails, and poles, must be accounted for by age-groups of like units.

Concerning division of fixed-capital property into units, the Wisconsin Railroad Commission has said:[1]

"In general, such classifications are made either upon an operating unit basis or upon an inventory basis. The operating unit classification involves grouping the items in accordance with their use as a particular operating unit. The inventory classification involves simply the grouping of like kinds of equipment together without particular reference to their operating function or identity. Both of these types of classification are useful. . . . "

K. Lee Hyder[1] illustrates the distinction between the "operating-unit" and the "inventory" bases for classifying property into units by explaining the application of each to a machine foundation, thus:

Operating-unit Basis: "The foundation would be described as a unit, with the dimensions, location, character of material, etc., and with the original cost determined and assigned thereto."

Inventory Basis: "The foundation would be broken down to show the number of barrels of cement, the number of cubic yards of sand and gravel, the number of hours of labor, the charge for plant rental, etc."

The view of the authors is that the division of fixed-capital property into units should be upon the operating-unit basis, but that the original-

[1] *Bull.* 833, The American Appraisal Co. Milwaukee, Wis.

costs entries on the property ledger sheets should be broken down and classified sufficiently to permit their ready use in preparing cost indices, sometimes called cost-time factors, as described in Sec. 9.14.

6.13. Construction Contracts, Work Orders, Invoices, Requisitions, Work-order Reports and Engineering Audits.—The first essential in depreciation accountancy for the physical property of an industrial enterprise is to ascertain correctly the actual original costs new of the various property units at the various times when they were first installed. This requires effective cooperation between the accountants and the engineers of the organization.

Part of the property units are likely to have been constructed by contract, and part direct by the owner. In the case of those constructed by the owner, part of the purchased equipment units and other construction materials are likely to have been kept in "stores" for some time before use.

Construction Contracts.—The direct costs new (contract costs) of the various contractor-built property units are to be found in the construction contracts and their corresponding vouchers.

Construction Work Orders.—Work orders should be issued in advance for the construction of all property units built direct by the owner. The orders should contain full and explicit instructions for the construction ordered, including formal detailed plans and specifications whenever needed.

Invoices.—The f.o.b. direct costs new of the purchased equipment units, and other purchased construction materials required for the execution of work orders, are to be found in the invoices of the purchases and their corresponding vouchers.

The *stores direct costs new* of purchased equipment units and other construction materials are equal to their invoice prices, plus transfer-to-storage costs, plus storage costs (see Stores-inventory Prices, Sec. 6.14).

Requisitions.—Formal requisitions for all equipment units and other construction materials taken from stores to execute work orders should be made and filed with the custodian of stores before any withdrawals are permitted.

Work-order Reports.—During the execution of construction work orders and immediately upon their completion, work-order reports of all materials used, work performed, and costs incurred should be required; they should be made out in sufficient detail to enable ready identification in the materials vouchers, construction pay rolls, and other contract-cost records of all the direct costs new, of each property unit, and/or age-group of like units, constructed.

Engineering Audits.—1. In starting property ledgers (Secs. 6.15, 6.16, 6.19, 6.20), the entries of original costs new should be carefully checked by competent engineers. Accountants cannot be expected to possess the engineering construction knowledge and experience necessary to understand, classify, and properly set up the data from the contracts, work orders, plans, specifications, and reports.

2. Throughout the service lives of the various property units, the organization's engineers should audit each year's depreciation computations; they must make personal examinations of the units and re-estimate their probable lives and PFORR's from time to time, in accordance with the observed mortality facts.

Direct Costs New of owner-built property units include the f.o.b. invoice costs of all construction materials used direct from shipments, and/or the stores-inventory prices of all materials requisitioned from stores, plus all construction-pay-roll costs of labor, superintendence, etc., plus all other construction direct-cost charges.

Owner's Overhead Costs.—These must be added to the direct costs new of the contractor-built and owner-built property units to get their total original costs new, installed ready for use.

6.14. Accountancy for Stores.—It is customary and wise for industrial enterprises to keep considerable stocks of construction and operation materials and supplies constantly on hand in stores.

One object is to secure lower prices by purchasing in quantity; another is to avoid losses of time, and consequent financial losses, in making repairs and replacements.

Materials and supplies kept in stores should be systematically accounted for, and guarded and cared for at all times, to prevent loss and waste.

Stores Inventories.—A complete card index or loose-leaf stores inventory should be made, and be kept constantly up to date. This inventory should be simplified as far as practicable by putting like property units and miscellaneous small units into inventory groups. The inventory should be checked physically every year.

Stores-inventory Prices.—These should include (1) the f.o.b. purchase invoice prices, (2) the costs of placing in stores, and (3) the cost of keeping in stores.

Stores Requisitions.—Withdrawals from stores should be permitted only upon filing of completely detailed requisitions, from which the stores inventory can be kept constantly up to date.

The stores accountancy system should be as simple and inexpensive as practicable. Experience shows that costly, complex systems are in danger of being abandoned.

In making formal valuations of the entire property of an industrial enterprise it is necessary to make an approximate division of stores materials and supplies between those kept for construction needs and those for operation requirements.

The interest lost on construction materials constitutes a part of the construction overhead costs (Chap. XI); it must not be duplicated in the stores-inventory prices.

Operation materials and supplies constitute a part of working capital (Chap. XIV).

6.15. Property Ledgers.—Adequate industrial-property accountancy requires keeping a separate property ledger account, constantly up to date, for every property unit, and/or age-group of like units. The property ledger data should include the original cost new of each unit and/or group; redeterminations of values new (if and when made); estimated and re-estimated salvage values, service ages, probable lives, and PFORR's.

The annual ledger entries, based on the above data, should include the condition-percents, annual depreciation allowances, the depreciation reserve entries, and the present values.

Such property ledgers constitute a perpetual inventory, constantly up to date, of all the physical property owned by the enterprise; they furnish many data needed for a revaluation of the physical property at any date.

Physical-property ledger forms are best of the loose-leaf type, in standard sheets, approximately 12 by 12 inches in size. Discussions

of property ledgers and suggestions for standard forms can be found in numerous accountancy publications,[1] but the authors have not succeeded in finding published forms well adapted to modern depreciation-accountancy methods. They, therefore, present their own suggestions for standard property ledger forms for industrial-property accountancy in Secs. 6.16, 6.19, and 6.20.

6.16. Property Ledger Form for Physical Units.—*Physical-property Ledger Form* I, herewith, has been selected by the authors as the best yet developed for modern current depreciation accountancy for physical-property individual units.

1. Form I provides, in column (9), for estimating and re-estimating the unit's probable life at different service ages, with the aid of its mortality type curve, and in accordance with the observed mortality facts of its actual service life, all as described in Secs. 3.20 and 5.17.
2. Form I also provides, in column (11), for estimating and re-estimating the unit's PFORR from time to time, in accordance with the observed mortality facts of its actual service life, as described in Secs. 4.28 and 5.17.
3. Form I provides, in columns (10), (12), (13), (14), and (15), for the use of the condition-percent table in Appendix C for computing the property unit's present values, total depreciations at different service ages, and annual actual depreciations; all readjusted from time to time to agree with changes in its estimated probable life and PFORR; the depreciations conform at all service ages to the unit's actual service-life mortality experience; they check out at the time of the unit's actual retirement to its value new, less net salvage actually realized.
4. Form I provides, in column (13), the data for making annual depreciation allowances from current income which will maintain at all times a real depreciation reserve, of amounts shown in column (14), just equal to the unit's corresponding total accrued actual depreciations.

Form I is not a form usable by accountants without the aid of engineering help. It requires repeated actual examinations of the industrial-property units by engineers qualified to observe and understand their actual mortality data, and to use these data scientifically and correctly in estimating the corresponding actual depreciation. Correct industrial-property current depreciation accountancy requires the active cooperation of qualified engineers and qualified accountants.

6.17. Accountancy for Simple, Composite, and Continuous Units.—Form I may be used for these three (Sec. 2.6) types of property units. Composite and continuous units may require more than one sheet; in this case, use the first sheet for the main unit (or the longest lived subunit). Often more than one short-lived subunit may find sufficient room on the same sheet; each should be treated as if it were a separate unit.

[1] Examples: SALIERS, "Depreciation Principles and Applications," The Ronald Press, New York; CHURCH, "Manufacturing Costs and Accounts," 2d ed., McGraw-Hill Book Company, Inc., New York, 1929.

Physical-property Ledger Form I
(For Indivdual Property Units)

Sac County Electric Company, Sac City, Ia.

Unit Permutit Water Softener
Manufacturer Crane Mfg. Co., Chicago, Ill.
Purchased from Mid-West Supply Co., Des M.
Mfg's Serial Number 341289-12

Location Power Plant Building
Date Installed Oct. 19, 1929
Rating or Size 12,000 gallon per day
Type, Kind, etc. E-4

Division Sac Citv Power Plant
Account 315 (Miscellaneous Power)
Plant Equipment
Assigned Number P 321

Date	Number in service	Item	Cost f.o.b. or stores	Installation costs	Total cost installed	Estimated net salvage value	Depreciation: Age, years	P.L.,* years	Tabular condition-percent†	PFORR	Depreciable value new of survivors	Annual depreciation allowance	Depreciation reserve	Present or residual value
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Oct. 19, 1929		Work Order 306—												
	1	12,000 Gal. Permutit Softener	$4,290.00	$310.00	$4,600.00									
		Piping:												
	10′	6″ C.I.B. & S.	9.65											
	1	6″ T Bell	11.00											
	1	6″ Gate Valve Bell	24.00											
	178′	4″ C.I.B. & S.	91.60											
	4	4″ 90 Degree Bell	19.70											
	1	4″ 45 Degree Bell	4.80											
	1	4″ T Bell	7.70											
	2	4″ Gate Valve Bell	24.00											
	1	4″ Float Valve Bell	60.00											
	1	4″ × 4″ × 2″ Y Bell	6.50											
	1	4″ × 4″ × 2″ T Bell	7.15											
	20′	2″ C.I.B. & S.	3.50											
	1	2″ 90 Degree Bell	4.50											
	1	2″ Gate Valve Bell	7.50											
	22′	2½″ Steel Pipe through Reservoir	8.10											
	2	2½″ 45 Degree Elbows	1.70											
		Total Piping	291.40	104.00	395.40									
Oct. 19, 1929	1	Total Water Softener	$4,581.40	$414.00	$4,995.40	$ 0					$4,995.40			$4,995.40

Sample Form I

This form sheet for Physical Property Record is for use in the accounting for individual units by the present-worth actual depreciation method. Provision is made to record accurately the original costs, depreciation reserve, and present value of the unit over its entire service life. Large maintenance and repair items or costs can be entered in column (3) so that they are available in applying the "wise retirement formula."

Dec. 31, 1929	1	Allocated General Overhead (6.1%)			304.72					1.00	5,300.12			5,300.12
Dec. 31, 1929						0	0	15	100.0	1.00	5,300.12			5,300.12
Dec. 31, 1930						0	1	15	96.0	1.00	5,300.12	$ 212.00	$ 212.00	5,088.12
Dec. 31, 1931						0	2	15	91.8	1.00	5,300.12	222.61	434.61	4,865.51
Dec. 31, 1932						0	3	12	82.0	1.00	5,300.12	519.41	954.02	4,346.10
Dec. 31, 1933		(City Water Softened)				300.00	4	12	75.2	.80	5,000.12	1,038.03	1,992.05	3,308.07
Dec. 31, 1934						300.00	5	12	67.9	.50	5,000.12	1,310.53	3,302.58	1,997.54
Dec. 31, 1935						300.00	6	12	60.0	.50	5,000.12	197.50	3,500.08	1,800.04
Dec. 31, 1936						300.00	7	13	57.0	.60	5,000.12	0.00	3,500.08	1,800.04‡
Dec. 1, 1937		Work Order 2178—												
	1	6″ Gate Valve Bell (Retired-Broken)	24.00	10.56	34.56						4,965.56		3,465.52	1,800.04
	1	6″ Gate Valve Bell (Replacement)	20.00	10.00	30.00						4,995.56		3,465.52	1,830.04
Dec. 31, 1937	1	Allocated General Overhead (8.0%)			2.40						4,997.96		3,465.52	1,832.44
Dec. 31, 1937						300.00	8	13	49.1	.60	4,997.96	60.04	3,525.56	1,772.40
Dec. 31, 1938						200.00	9	12	33.0	.60	5,097.96	563.00	4,088.56	1,209.40
Dec. 31, 1939						200.00	10	12	22.8	.60	5,097.96	312.00	4,400.56	897.40
Dec. 31, 1940						200.00	11	12	11.8	.60	5,097.96	336.46	4,737.02	560.94
Oct. 8, 1941		Work Order 3063—												
		Entire Unit Retired and Salvaged									5,197.96	460.94	5,197.96	100.00
		Sold to John Blank and Co., Chicago, Ill.				100.00								
						$100.00					$5,197.96		$5,197.96	0.00

NOTE.—This form has been prepared mainly by Wallace A. Rogers, Ames, Ia., under the direction of Dr. Marston.

* Mortality-type curve S_3, age 0–11.

† Fair return, 7%.

‡ Adjusted owing to changes in P.L. and PFORR.

6.18. Extraordinary Repairs.—Formerly the renewals of the subunits of composite and continuous property units were sometimes called "extraordinary repairs," and accounted for by special accounting devices; the best present practice is to account for each subunit as if it were a separate unit. Form I may be used as described in Sec. 6.17.

6.19. Property Ledger Form for Age-groups of Property Units.—*Physical-property Ledger Form* II, herewith, has been selected by the authors as the best yet developed for modern current depreciation accountancy for age-groups of like physical-property units (Sec. 3.2).

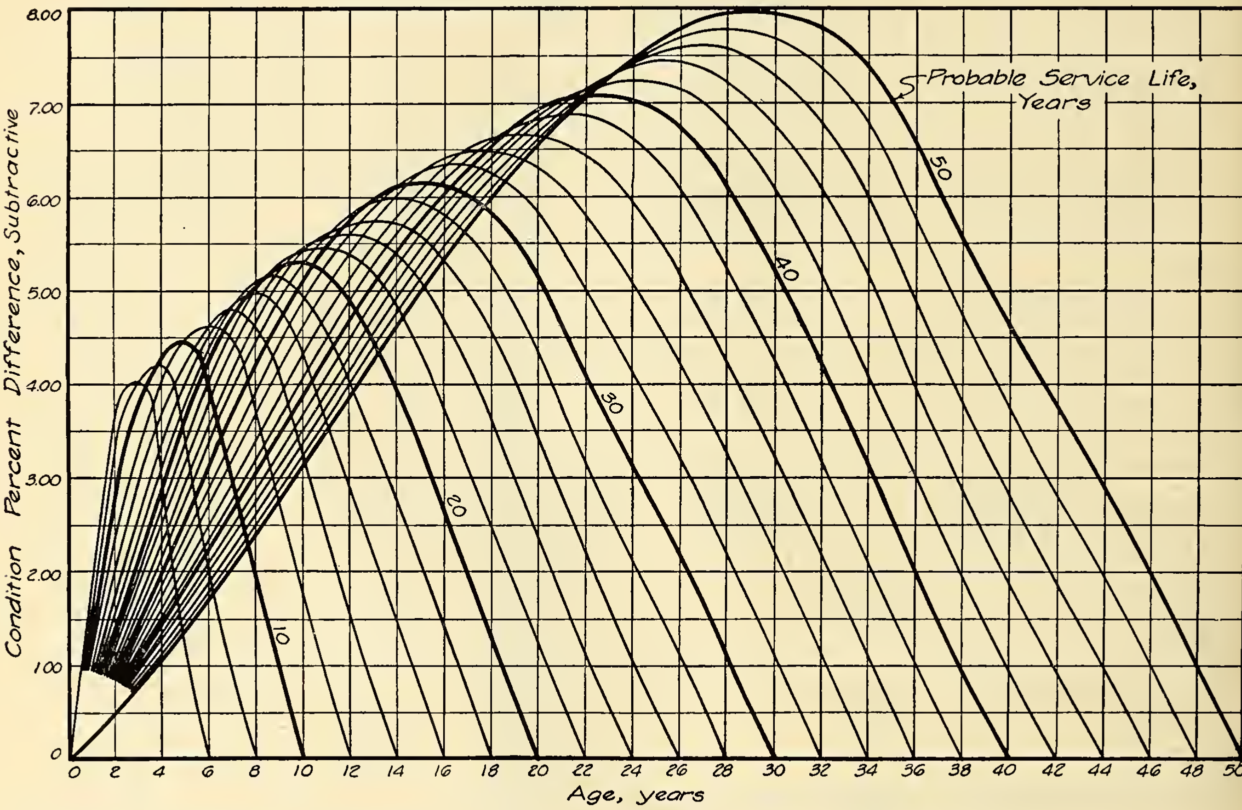


FIG. 6.1.—Correction curves for obtaining average condition-percent of survivors. Applicable only to S_2 type curve and 6 percent interest.

In the interest of reduction of work, time, and expense, it is desirable to use age-groups of like units in physical-property ledger accountancy to the fullest extent practicable.

The present difficulty is the present lack of condition-percent correction tables, by which the true average condition-percent of the survivors at any service age of any age-group can be ascertained easily and quickly, by merely deducting the proper correction from the *condition-percent of the average survivor at the age* (which can be read directly from the condition-percent tables in Appendix C). A full discussion of this subject is given in Sec. 3.29. One of the needed correction curves is shown in Fig. 6.1.

The use of Form II requires the selection of the proper mortality type curve for each group; to enable the selection of the right condition-percent correction [see column (14)] and to aid in estimating the probable

life of the age-group's average survivor, at different service ages [see column (12)].

Columns (2) and (8) supply the data for gradually compiling and plotting the actual mortality curve of the particular age-group; the mortality type curve selected can be compared with this and changed when found advisable at any service age.

Physical-property Ledger Form II provides [in columns (16), (17), (18), (19)], for determining all the data required for correct current depreciation accountancy for the entire age-group; the correct annual depreciation appropriations [column (16)], from current income, will maintain a real depreciation reserve [column (18)], just equal at all service ages to the corresponding total accrued actual depreciations of the age-group survivors.

6.20. Property Ledger Form for Miscellaneous Small Units.—*Physical-property Ledger Form* III, herewith, has been selected by the authors as the best yet developed for modern current depreciation accountancy for groups of miscellaneous, small, like, physical-property units. This form is for use in accounting for miscellaneous, small, property units which are used in quantities. Examples: Insulators, pipes, and conduits (grouped by sizes); pipe fittings, bolts, nails, and similar materials (in pounds); brackets, insulators, pins, etc.; various small devices.

Many such materials may be used repeatedly and do not have definite service ages or service lives. Their condition-percents can usually be determined by direct examination, with all practicable accuracy, using the straight-line depreciation assumption.

Under some conditions, the condition-percents of groups of this type of property can rationally be assumed to be the same as that of some age-group of larger units to which they are attached; column (11) is provided in Form III for such cases. As an example, the primary line insulators used for illustration in Form III are assumed to have the same condition-percents as the crossarms to which they are attached.

Form III provides [in columns (15), (16)] for regular annual depreciation appropriations, from current income, sufficient to maintain at every date a depreciation reserve just equal to the corresponding total accrued actual depreciation.

6.21. The Depreciation Deductions to Determine Present Values.—The *legal* and only *correct* way to determine the present values of an industrial physical-property unit at its different service ages (the entries in the last columns of Property Ledger Forms I, II, III) is to deduct the unit's true total accrued actual depreciations, at the ages, from its corresponding values new. This principle is unshakably established by repeated decisions of our highest court.

The use of condition-percent tables to compute the present values is just a convenient method of applying the principle.

Physical-property Ledger Form II
(For Age-groups of Like Property Units)

Sac County Electric Company, Sac City, Ia.

Unit Electric Poles
Manufacturer Various
Purchased from Various
Mfg's Serial Number —

Location Within City Limits
Date Installed 1916
Rating or Size 30′-Class D-6″
Type, Kind, etc. Western Red Cedar—Untreated

Division Sac City Distribution
Account 331 (Poles, Towers, and Fixtures)
Assigned Number E-19′6

Date	No. in service	Additions					Retirements		Estimated net salvage value	Depreciation								Present or residual value
		No.	Unit cost f.o.b. or stores	Unit installation cost	Total unit installation cost	Total installed cost	No.	Deduction from depreciable value and reserve		Age, years	P.L. of average survivor†	Tabular condition-percent*	Corrected condition-percent‡	PFORR	Depreciable value new of survivors	Annual depreciation allowance	Depreciation reserve	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
Mar. 2, 1916	50	50	$6.50	$2.00	$ 8.50	$ 425.00			0						$ 425.00			$ 425.00
June 8, 1916	130	80	6.50	2.00	8.50	680.00			0						1,105.00			1,105.00
July 17, 1916	150	20	7.50	2.00	9.50	190.00			0						1,295.00			1,295.00
Aug. 20, 1916	179	29	7.50	2.00	9.50	275.50			0						1,570.50			1,570.50
Dec. 11, 1916	309	130	8.50	2.00	10.50	1,365.00			0						2,935.50			2,935.50
Dec. 31, 1916	309	Allocated general overhead (5.263%)				154.50			0						3,090.00			3,090.00
Dec. 31, 1916	309								0	0	15	100.0	100.0	1.00	3,090.00			3,090.00
Dec. 31, 1917	309								0	1	15	95.7	95.0	1.00	3,090.00	$154.50	$ 154.50	2,935.50
July 6, 1918	306						3	$ 30.00	0						3,060.00		124.50	2,935.50
Dec. 31, 1918	306								0	2	15	91.2	89.0	1.00	3,060.00	212.10	336.60	2,723.40
May 22, 1919	297						9	90.00	0						2,970.00		246.60	2,723.40
Dec. 31, 1919	297								0	3	12	81.1	77.5	1.00	2,970.00	421.65	668.25	2,301.75
Feb. 6, 1920	296						1	10.00	0						2,960.00		658.25	2,301.75
Dec. 31, 1920	296								$296.00	4	12	74.1	69.7	1.00	2,664.00	148.94	807.19	2,152.81
Sept. 3, 1921	289						7	63.00	289.00						2,601.00		744.19	2,145.81
Dec. 31, 1921	289								289.00	5	12	66.6	62.5	1.00	2,601.00	231.18	975.37	1,914.63
Mar. 10, 1922	278						11	99.00	278.00						2,502.00		876.37	1,903.63
Dec. 31, 1922	278								278.00	6	12	58.7	56.4	1.00	2,502.00	214.50	1,090.87	1,689.13
June 3, 1923	260						18	162.00	260.00						2,340.00		928.87	1,671.13
Nov. 22, 1923	248						12	108.00	248.00						2,232.00		820.87	1,659.13
Dec. 31, 1923	248								248.00	7	12.4§	52.4	50.4	1.00	2,232.00	286.20	1,107.07	1,372.93
May 11, 1924	224						24	216.00	224.00						2,016.00		891.07	1,348.93
Dec. 31, 1924	224								112.00	8	12.8	46.3	43.7	1.00	2,128.00	306.99	1,198.06	1,041.94
June 11, 1925	177						47	446.50	88.50						1,681.50		751.56	1,018.44
Dec. 31, 1925	177								88.50	9	13.2	40.4	38.5	1.00	1,681.50	282.56	1,034.12	735.88

Sample Form II

This form sheet for Physical Property Record is for use in the accounting for age-groups of like items by the present-worth actual depreciation method. Provision is made to record accurately all additions and retirements of the group, depreciation, cost new, estimated net salvage value, and the depreciation reserve over the entire service lives of its many items. Accurate mortality data on the items are also made available.

Sept. 13, 1926	149						28	266.00	74.50						1,415.50		768.12	721.88
Dec. 31, 1926	149								74.50	10	13.7	35.1	33.2	1.00	1,415.50	177.43	945.55	544.45
Dec. 1, 1927	142						7	66.50	71.00						1,349.00		879.05	540.95
Dec. 31, 1927	142								0	11	14.4	31.5	29.5	1.00	1,420.00	122.05	1,001.10	418.90
Sept. 6, 1928	121						21	210.00	0						1,210.00		791.10	418.90
Dec. 31, 1928	121								0	12	14.7	25.2	25.6	1.00	1,210.00	109.14	900.24	309.76
Apr. 5, 1929	101						20	200.00	0						1,010.00		700.24	309.76
Dec. 21, 1929	93						8	80.00	0						930.00		620.24	309.76
Dec. 31, 1929	93								0	13	15.1	19.6	22.5	1.00	930.00	100.51	720.75	209.25
May 15, 1930	82						11	110.00	0						820.00		610.75	209.25
Dec. 31, 1930	82								0	14	15.8	16.5	19.0	1.00	820.00	53.45	664.20	155.80
Apr. 30, 1931	64						18	180.00	0						640.00		484.20	155.80
Dec. 31, 1931	64								0	15	16.6	13.1	17.0	1.00	640.00	47.00	531.20	108.80
June 10, 1932	49						15	150.00	0						490.00		381.20	108.80
Dec. 31, 1932	49								0	16	17.5	13.0	14.0	1.00	490.00	40.20	421.40	68.60
Nov. 3, 1933	33						16	160.00	0						330.00		261.40	68.60
Dec. 31, 1933	33								0	17	18.4	11.8	12.0	1.00	330.00	29.00	290.40	39.60
Mar. 18, 1934	27						6	60.00	0						270.00		230.40	39.60
Dec. 31, 1934	27								0	18	18.9	7.6	10.0	1.00	270.00	12.60	243.00	27.00
Oct. 30, 1935	26						1	10.00	0						260.00		233.00	27.00
Dec. 31, 1935	26								0	19	19.3	2.5	8.2	1.00	260.00	5.68	238.68	21.32
June 11, 1936	22						4	40.00	0						220.00		198.68	21.32
Dec. 31, 1936	22								0	20	20.2	1.6	6.5	1.00	220.00	7.02	205.70	14.30
Aug. 19, 1937	5						17	170.00	0						50.00		35.70	14.30
Dec. 31, 1937	5								0	21	21.2	1.5	4.5	1.00	50.00	12.05	47.75	2.25
Nov. 18, 1938	0						5	50.00	0						0.00	2.25	0.00	0.00

Note.—This form has been prepared mainly by Wallace A. Rogers, Ames, Ia., under direction of Dr. Marston.
* Fair return, 6%.
† Mortality-type curve S_2, age 0–21.
‡ Corrections by field judgment aided by correction curves similar to those of Fig. 6.1.
§ The authors would estimate probable life to nearest full year only.

Physical-property Ledger Form III
(For Groups of Miscellaneous, Small, Like Items)

Sac County Electric Company, Sac City, Ia.

Unit	Primary Line Insulators	Location	Within City Limits	Division	Sac City Distribution
Manufacturer	Locke Insulator Co.	Date Installed	1916	Account	332 (Overhead Conductors and Devices)
Purchased from	L.M. Co.	Rating or Size	8. KV. Top Groove	Assigned Number	
Mfg's Serial Number	40-B	Type, Kind, Etc.	Pin—1" screw head		

Date	No. in service	Additions					Retirements		Estimated net salvage value	Depreciation						Present or residual value
		No.	Unit cost, f.o.b. or stores	Unit installation cost	Total unit installation cost	Total installed cost	No.	Deduction from depreciable value and reserves		Condition-percent same as	Condition-percent	PFORR	Depreciable value new of survivors	Annual depreciation allowance	Depreciation reserve	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Oct. 15, 1916	10	10	$0.12	$0.07	$0.19	$1.90			$0				$ 1.90			$ 1.90
Oct. 21, 1916	61	51	0.12	0.07	0.19	9.69			0				11.59			11.59
Nov. 9, 1916	110	49	0.13	0.07	0.20	9.80			0				21.39			21.39
Dec. 31, 1916	110	Allocated General Overhead (5.15%)				1.10			0				22.49			22.49
Dec. 31, 1916	110								0	X arms	95.0	1.00	22.49	$1.12	$1.12	21.37
May 13, 1917	115	5	0.14	0.08	0.22	1.10			0				23.59		1.12	22.47
June 22, 1917	125	10	0.14	0.08	0.22	2.20			0				25.79		1.12	24.67
Oct. 6, 1917	145	20	0.15	0.09	0.24	4.80			0				30.59		1.12	29.47
Oct. 13, 1917	142						3	$0.61	0				29.98		0.51	29.47
Dec. 31, 1917	142	Allocated General Overhead (3.33%)				0.27			0				30.25		0.51	29.74
Dec. 31, 1917	142								0	X arms	89.0	1.00	30.25	2.82	3.33	26.92
June 3, 1918	135						7	1.49					28.76		1.84	26.92
June 8, 1918	145	10	0.15	0.10	0.25	2.50			0				31.26		1.84	29.42
Dec. 17, 1918	150	5	0.15	0.10	0.25	1.25			0				32.51		1.84	30.67
Dec. 31, 1918	150	Allocated General Overhead (0.27%)				0.01			0				32.52		1.84	30.68
Dec. 31, 1918	150								0	X arms	84.3	1.00	32.52	3.17	5.01	27.41

Sample Form III

This form sheet for Physical Property Record is for use in the accounting for miscellaneous, small, like items by the present-worth actual depreciation method. The items in this group are such as those which cannot easily be identified as to age and make up a very small part of the entire property. It is advised that the use of this form be limited as much as possible because more accurate mortality data are furnished by Forms I and II.

Note.—This form has been prepared mainly by Wallace A. Rogers, Ames, Ia., under the direction of Dr. Marston.

In accountancy practice, however, the depreciations unfortunately are most frequently calculated on a theoretical fixed-percentage or sinking-fund basis; the depreciation deductions are often made arbitrarily and irregularly, in accordance with what is claimed to be business judgment.

6.22. Depreciation Appropriations and Their Disposition.—Only by making each year, from current income *before net return*, a depreciation appropriation of cash equal in amount to the total *actual* depreciation during the year can the owners of an industrial property make good its depreciation losses of value, keep the investment in the property unwasted by depreciation, and provide currently for meeting the inevitable future costs of the renewals of the property's physical units at the dates of their inevitable future retirements.

The obligation to keep the investment in an industrial property unwasted by depreciation rests entirely upon its owners.

If the owners omit in any year to make any annual depreciation appropriation at all, or if they make it too small, their accounts for the year show profits greater than the true profits. Making depreciation appropriations larger than the true actual depreciations is unjust to consumers and/or customers. In both cases, the property's accounts are not true accounts; and in both cases the owners of the property must be held fully responsible for all the consequences.

On the other hand, the owners of industrial enterprises are entitled, in law, in the case of public utilities, and in justice, in the case of private enterprises, to charge fair rates and/or fair prices,[1] which, in addition to paying all operation and depreciation expenses, will yield fair net returns on the fair depreciated values of the properties "used and useful" in the production of their services and/or commodities.

THE DISPOSITION OF ANNUAL DEPRECIATION APPROPRIATIONS

All depreciation appropriations should be treated as obligated to the purpose of paying back to the property owners the total accrued actual depreciation liabilities on the various property units at the dates of their actual retirements (that is, their values new, less the net salvages actually realized then).

The right way to devote depreciation appropriations to the above purpose is to reinvest them in the fixed-capital physical property of the enterprise, as follows:

1. *Current Replacement Costs.*—Pay all these each year out of the current annual depreciation appropriation, so far as it is sufficient.
2. *Actual Depreciation Sinking Fund* (Sec. 6.24).—Only when, and then only to the extent such a fund is really needed, limited fractions of the depreciation appropriations may be kept invested in an actual depreciation sinking fund. Such funds are for the purpose of guarding against occasional excessive demands for cash to make replacements.

[1] But fair rates and/or fair prices must never exceed the reasonable worths of the services and/or commodities; that is, the rates and/or prices at which they can be supplied from other sources.

3. *Improvements and Enlargements of the Fixed-capital Physical Property of the Enterprise.*—All remaining annual depreciation-appropriation balances should be invested in improvements and enlargements if possible; this is usually possible with successful enterprises.

Such depreciation-appropriation investments are in lieu of new capital which it otherwise would be necessary to procure.

4. *Buying Back from Stockholders Part of Their Equity in the Property of the Enterprise.*—Excess depreciation-appropriation balances, left after wise investments of the characters described in 1, 2, and 3, above, must go to the stockholders of the enterprise as excess dividends, and/or other returns, larger than the true net returns.

Whether so admitted at the time or not, the distribution of such excess dividends and/or other returns, has the effect of decreasing the stockholders' true equity in the concern's property, to the extent of the excess payment; this should be made clear at the time, both to the stockholders and to the public, in the regular statements of the enterprise.

6.23. The Depreciation Reserve Account and the Depreciation Reserve.—The *depreciation reserve account*[1] is a valuation book account, to which all depreciation losses of value charged to production-expense accounts are credited; and to which the book values new, less net salvages actually realized, of all physical-property units are charged at the dates of their actual retirements. Thus the depreciation reserve account constantly shows the amount of the total accrued depreciation of the present existing property units. This total accrued depreciation, shown in the depreciation reserve account, has been charged currently to past production expenses; but direct repayment therefor has not yet been made, for the reason that the property units upon which this depreciation has accrued will not be renewed (and thus restored to new condition) until their respective future retirement dates.

The depreciation reserve account is merely a book account, not a reserve at all, and might be kept without ever creating a real reserve. A real depreciation reserve, corresponding to the depreciation reserve account, can be created only by, first, making each year an appropriation, from current income of actual cash equal to the depreciation during the year, shown by the depreciation reserve account; and, second, by investing such appropriations in the property, as described in Sec. 6.22.

The *depreciation reserve* is an undivided amount of the fixed-capital property assets of the enterprise, equal, in present value, to the total accrued present actual depreciation. When the owners fail to make an adequate annual depreciation appropriation, their nominal net returns exceed their true net returns; the stockholders' equity is reduced accordingly. A depreciation reserve equal to the total accrued actual depreciation is always a preferred liability on the property, prior even to bondholders' liens.

[1] See the next to the last columns of Physical-property Ledger Forms I, II, III, Secs. 6.16, 6.19, 6.20.

The depreciation reserve must not be confused with either a fictitious or an actual depreciation sinking fund (Sec. 6.24). When an actual depreciation sinking fund is maintained it is a part, but usually a relatively small part, of the depreciation reserve.

6.24. Actual and Fictitious Depreciation Sinking Funds.—1. An *actual depreciation sinking fund* is a really existing sinking fund, created and maintained by actual transfers thereto of cash (or liquid securities purchased by cash) from the yearly depreciation appropriations from income. As stated in Sec. 6.22, an actual depreciation sinking fund should be created and maintained only when it is really needed and should then be restricted to the smallest safe size. Very often, no such fund is needed at all.

Sinking-fund investments can earn only at the comparatively low prevailing sinking-fund interest rates, usually 3 to 5 percent; whereas investments in the property itself earn at the rate of the net actual return on the entire property, usually 5 to 8 percent, sometimes even higher.

The sinking-fund interest may or may not be kept in the sinking fund, accumulating at compound interest. In either case, the interest should be counted as part of the gross income of the entire property.

The total amount in the (wisely maintained) actual depreciation sinking fund should be counted as part of the whole property, entitled to earn at the same rate of net return as the rest of the property. The difference between the sinking-fund interest rate and the rate of net return on the entire property is thus made up out of the gross income.

2. A *fictitious depreciation sinking fund* is a purely imaginary sinking fund, used merely as a mathematical concept, to facilitate the estimation of depreciation by the sinking-fund method (Sec. 5.6).

Such sinking funds do not really exist. No cash and securities are transferred thereto, or kept therein. Yet fictitious are often confused with actual depreciation sinking funds by students of depreciation.

6.25. Numerical Example Illustrating Depreciation-accountancy General Procedure.—The depreciation-accountancy general procedure in handling depreciation deductions from value, depreciation reserves, depreciation appropriations, and actual depreciation sinking funds, may be illustrated by its very general numerical application to a simple case.

Assume:

1. That the value of a certain Industrial Property X, on Jan. 1, the first year of its operation, was $1,000,000.

2. That no additions or improvements were made with new capital during the first 5 years of operation.

3. That the yearly losses of value due to actual depreciation, the yearly depreciation appropriations from income, the yearly investments of depreciation appropriations, and the yearly additions to the depreciation reserve, were as shown in Table 6.2 during the first 5 years of operation.

It should be noted, in connection with Table 6.2, that all differences between the replacement and the retirement costs of property units retired are added algebraically (they may be plus or minus) to the investments in improvements.

In the case of Industrial Property X, it was found to be wise to allocate $3,000 per year during the first 5 years to establish a small actual depreciation sinking fund. In this case, sinking-fund interest earned was turned into the general income, instead of being kept in the fund accumulating at compound interest.

All balances of the yearly depreciation appropriations, not required for paying the yearly retirement costs of units retired (and for increasing the depreciation sinking fund), were invested in additions to the property and in improvement of existing units. By this method, the depreciation-reserve liability for accrued depreciation is fully balanced at all times by actual property assets (including the actual depreciation sinking fund); and the investment in the property is constantly maintained unwasted

Table 6.2.—Yearly Depreciation-accountancy Data of Industrial Property X

Year	Value lost by actual depreciation during year	Accruals of actual depreciation during year	Yearly depreciation appropriations				Additions during year to depreciation reserve
			Total for year	Paid for retirement costs	Additions and improvements	Actual depreciation sinking fund	
First	$15,000	$14,000	$15,000	$ 1,000	$11,000	$ 3,000	$14,000
Second	16,000	14,000	16,000	2,000	11,000	3,000	14,000
Third	18,000	15,000	18,000	3,000	12,000	3,000	15,000
Fourth	21,000	17,000	21,000	4,000	14,000	3,000	17,000
Fifth	25,000	20,000	25,000	5,000	17,000	3,000	20,000
Total	$95,000	$80,000	$95,000	$15,000	$65,000	$15,000	$80,000

Table 6.3.—Depreciation Book-value Capital Accountancy Data of Industrial Property X

Date, Jan. 1: year	Total accruals of actual depreciation	Depreciation reserve			Value new, including additions and improvements	Depreciated value, including additions and improvements	Total present value, including depreciation sinking fund
		Total depreciation reserve	Depreciation-reserve investments				
			Additions and improvements	Actual depreciation sinking-fund			
First	$ 0	$ 0	$ 0	$ 0	$1,000,000	$1,000,000	$1,000,000
Second	14,000	14,000	11,000	3,000	1,011,000	997,000	1,000,000
Third	28,000	28,000	22,000	6,000	1,022,000	994,000	1,000,000
Fourth	43,000	43,000	34,000	9,000	1,034,000	991,000	1,000,000
Fifth	60,000	60,000	48,000	12,000	1,048,000	988,000	1,000,000
Sixth	80,000	80,000	65,000	15,000	1,065,000	985,000	1,000,000

by depreciation. From the yearly depreciation-accountancy data in Table 6.2, starting with a value new of the entire property of $1,000,000, Table 6.3 has been prepared, showing the resulting book-value capital accounts of the property Jan. 1, each year, to the beginning of the sixth year of operation.

The value new of the property increases each year owing to the additions and improvements; in spite of which the present depreciated value decreases, in the case of this property, owing to the increase of accrued depreciation. This decrease in present value is just equal to the actual depreciation sinking fund maintained. Hence, without procuring any new capital, the investment in the entire property is fully maintained, unwasted by depreciation, out of the excess of the annual incomes over the sum of the actual net returns realized, plus amortization payments (if any), and plus operation costs.

THE RELATION OF INDUSTRIAL-PROPERTY LEDGER ACCOUNTANCY TO ENGINEERING VALUATION

An important present (1935) development in engineering valuation and industrial-property accountancy is the vigorous movement now under way to develop, and bring into general use by all industrial enterprises, continuous property-ledger inventory-accountancy methods, similar to those described in Sec. 6.12 to 6.20. The authors earnestly believe that the general lack of such accountancy in the past has been a major cause of the delays and the large expense not uncommon in engineering valuations; and of the increasingly criticised long delays and general ineffectiveness of public regulation of public utilities. Lack of such accounts is now one of the major obstacles in the way of a successful square-deal reorganization of modern industries; to insure real industrial security for all; and to assure real industrial justice and fairness, to consumers, labor, and capital, alike.

The successful development of feasible continuous property-ledger inventory accountancy for industrial properties requires that engineers take a prominent part in accountancy organizations. They must be called upon to cooperate with the accountants; supplying the technical knowledge and experience necessary for the understanding and the correct accountancy treatment of engineering constructions and devices.

6.26. Valuation and Property Accounts.—Not even continuous, complete property ledger accounts can do away with the need for engineering appraisals of the true fair values of industrial properties, at reasonable intervals, by competent and impartial engineers. In making such appraisals, the valuation engineer must determine the true fair values of the different parts of the properties for himself, independently of the book values shown in the accounts.

Reliable continuous property ledger inventories save much of the time and expense of engineering valuations. They also make better estimates of the true fair values possible; values of a comparatively

limited number of sizable property units, readily identifiable for field examination from the continuous inventory, constitute about 80 percent of the total value of an industrial property. Besides the saving of time and expense in preparing inventories, continuous property-ledger inventory accounts furnish reliable data of original costs, and data for cost-time factors for estimating reproduction costs; to obtain either of these without such ledgers requires an almost endless amount of work to get data certain to be less reliable.

Although complete and reliable accountancy records are almost indispensable in making an engineering valuation of a particular property, the valuator using them should keep in mind the following principles:

1. *The present fair value* of the property is a matter of fact, which the engineer must determine for himself, not a mere matter of book value which he can look up in the accountancy records. The book values carried in the various accounts show merely the values claimed by the owners.

2. *The present actual depreciation* of the property is also a matter of fact, which the valuator must determine for himself, not merely look up in the accounts.

The depreciation reserve account shows merely the depreciations claimed by the owners, which often are much (sometimes 100 percent) too low or too high. However, the depreciation reserve account should contain many data indispensable to the engineering valuator in making his own determinations of the true present actual depreciations. Retirement reserve accounts have no particular relation to true present actual depreciations.

3. *Property paid for from depreciation* (*or retirement*) *reserve appropriations* must be included in the valuation the same as if paid for from any other sources.

4. *Any actual depreciation* (*or retirement*) *sinking funds and/or other liquid reserve funds* maintained should be included in the valuation to the extent (and only to the extent) really needed. The securities and/or cash shown by the accounts to be in such liquid reserve funds must be checked by actual examination and must be valued at their actual present values (Secs. 14.6, 14.7).

5. *Working Capital.*—The average working capital (Secs. 2.17, 14.1 to 14.4) should be included in the valuation to the extent (and only to the extent) really needed.

Working-capital materials and supplies should be shown in the stores inventories (Sec. 6.14), but in any case must be checked in the field as of one or more specific dates. Working-capital operation cash resources and temporary investments in paid-up operation expenses must be ascertained mainly from the accountancy records.

6.27. Valuation without Property-ledger Inventory Accounts.—In making engineering valuations of industrial properties which do not keep

continuous property-ledger inventory accounts, their lack can be supplied only imperfectly by the expenditure of much time and effort.

1. Complete detailed inventories must be prepared by comprehensive field work, aided by laborious examinations of the plans, specifications, and other book records (Chap. X).

2. Laborious studies of the property's book records must be made, to find all obtainable data of the actual original costs new of the various property units, and of cost-time factors for estimating their present reproduction costs new; the many unavoidable gaps must be filled by outside data and by judgment (Chap. XI).

The very best inventory and price data obtainable by these means will unavoidably be much less complete and reliable than those supplied by continuous property-ledger inventory accounts.

6.28. Use of Property-ledger Inventory Accounts in Valuation.—It already, for years, has been "the standard policy of many of the larger industrial corporations" owning private industrial enterprises to maintain "a continuous property record and control";[1] the public service commissions of various states, particularly New York and Wisconsin, are taking part in a general movement to develop and require the adoption of similar records and controls by public utilities.

The authors' ideas at the present time about continuous-inventory property-ledger forms and their use have been set forth in Secs. 6.12 to 6.25. These ideas require perfecting by years of experience with their actual applications to different types of property.

The successful installation and maintenance of continuous property-ledger inventory accounts require a clear understanding of several phases of the work.

An initial inventory is the first necessity. Except for a new enterprise, it must be carefully prepared by much arduous field work and study of book records, supplying many gaps in data by the best means practicable (Chap. XI).

The continuous inventory requires competent work each year, keeping each property-ledger account up to date, adding new ledgers for all new property units, withdrawing the old ledgers of all units retired.

Inventory Prices.—Several inventory-price possibilities should be understood.

1. *Original Costs.*—Each property-ledger account should be started at a value new, equal to the total original costs to the first owner, at the date of installation new. The authors agree with various public service commissions in adopting this view, rejecting arguments of later purchasers to use the purchase price paid by the present owners.
2. *Piecemeal Construction.*—Justice would seem to require that in pricing inventories no deductions should be made for estimated higher costs of piecemeal construction, except in cases where wholesale construction was both feasible and so advantageous as to be necessary for prudent investment reasons. Under usual circumstances, much of the existing property of well-established industrial

[1] *Bull.* 833, The American Appraisal Co., Milwaukee, Wis.

enterprises has necessarily been installed piecemeal, in making replacements, improvements, and extensions.

3. *Cost-time Indexes.*—As stated in Sec. 6.12, the actual original costs of the inventory units should be broken down in their property-ledger entries in detail sufficient to enable the ready construction of cost-time indexes (Sec. 9.12), which will show the percentage relation between original costs incurred in any particular year to the corresponding reproduction costs current in any subsequent year.
4. *Reproduction Costs.*—At such times as desired, current reproduction-cost prices of inventory units may be estimated from the current costs of similar units, in the particular property or others similar, aided by the cost-time indexes.
5. *Book Values New and Their Revaluations.*—The book values new, carried on the inventory property ledgers, should start with the original costs new to the original owner. No blanket revaluations should be made; the policy as to revaluations of particular units should be conservative, changing their book values new only at comparatively infrequent intervals when materially different new price levels have become well established and are practically certain to continue for at least several years.

   In general, changes in values lag materially behind changes in price levels (Secs. 7.15, 7.16).

The Determination of Depreciation Deductions.—The continuous-inventory property-ledger accounts should be kept strictly up to date each year by deducting the total accrued actual depreciations, determined by competent engineers who actually examine the various property units.

Engineering Audits.—The initial inventory should be prepared with the aid of engineers, who should check all additions thereto of new units and removals therefrom of units retired. Engineers should determine the true actual accrued depreciations each year and the present values of the units, including any revaluations. The entire property-ledger accounts should be established and maintained by active cooperation between competent accountants and competent engineers.

SUPERVISION OF INVENTORIES BY REGULATORY AUTHORITIES

It is quite obvious that complete and reliable continuous property-ledger accounts will greatly simplify, speed up, and lessen the costs of engineering valuations of all industrial properties. The greatest benefits can be realized in the case of public utilities regulated by public service commissions, of private industrial enterprises under code regulation, and of all industrial enterprises whose securities must be sold under regulation by securities departments and commissions. For all these, regular joint examinations of the property-ledger accounts by the regulatory authorities' engineers and accountants would be sufficient to enable prompt decisions to be made at all times on fair-value questions affecting fair rates, fair prices, and securities-sales licenses.

PART II

ENGINEERING-VALUATION PRINCIPLES AND METHODS

Part I was devoted to studies of value, of industrial-property accountancy, of physical-property mortality characteristics, and of the various principles and aspects of depreciation.

Part II is devoted, first, to studies of the general fundamental principles of engineering valuation; second, to studies of the various specialized methods which have been developed for its practice.

CHAPTER VII

THE FUNDAMENTAL GENERAL PRINCIPLES OF ENGINEERING VALUATION

Various specialized processes and a general method (Sec. 1.15) for making engineering valuations have been gradually developed over a long period of years. These processes and methods are all based on certain fundamental general principles; Chap. VII will be devoted to their enumeration and discussion.

THE ORIGIN, BASIS, AND PRESENT STATUS OF THE FUNDAMENTAL GENERAL PRINCIPLES OF ENGINEERING VALUATION

7.1. Present Status of General Principles of Valuation.—The origin of these principles has been the publication and discussion of increasing numbers of valuation decisions by federal and state courts and by utility commissions, as well as of the results of valuation and rate studies and researches by experts; all of which has been the inevitable accompaniment of the development of engineering valuation, especially since 1890 (Secs. 1.2, 7.4).

The basis of the fundamental general principles of engineering valuation is the law of the land, both common and statute, as interpreted and thereby established by our courts of final resort, in the large body of decisions which they have made on litigated valuation questions.

The present status of the development of the fundamental general principles of engineering valuation is that quite a large number of such principles have gradually attained widespread acceptance and authority. Several quite important principles are still involved in controversy; among these are (1) depreciation; (2) depreciation accountancy; (3) the controversy over the *Smyth v. Ames* rule (that all factors affecting value must be given "such weight as may be just and right in each case") vs. the reproduction-cost-new-less-depreciation and the prudent-investment valuation formulas.[1]

Engineering valuators must be well informed on all sides of such controverted questions and must be careful to be open-minded and fair.

Court Procedure in Valuation Litigation

A very brief, general explanation of court procedure in valuation litigation cases is presented here, to afford some assistance in studying,

[1] For summaries of some 68 important legal valuation decisions, see Chap. VIII.

interpreting, and giving proper weight to such court decisions on valuation questions as are cited hereinafter.

7.2. State and Federal Courts—Jurisdiction and Procedure.—Valuation questions are sometimes litigated in state and sometimes in federal courts.

State courts deal primarily with intrastate cases, involving questions of state common and statute law. Appeals may be taken to the state supreme court, but in some states must go first to an appellate division. Appeal may be taken to the United States Supreme Court only in those cases in which competent showing is made that some question of federal law or federal constitutional right is sufficiently at issue.

Federal courts deal primarily with *interstate* cases, and with other cases involving questions of *federal* common and statute law, including federal constitutional rights. Most cases started in federal courts go first to the United States District Courts. United States Circuit Courts of Appeal are provided to decide most appeals; only certain cases can reach the United States Supreme Court. The United States Court of Claims and the United States Board of Tax Appeals are examples of special tribunals; provided to try, in the first instance, certain special types of litigations.

7.3. Classes of Law Cases Involving Valuation Principles.—Law cases involving valuation principles may be roughly classified as follows:

1. Valuation litigations between private individuals and/or corporations. The value at issue may be that of either a private or a public-utility property. The litigation is initiated by filing a suit in a court of appropriate jurisdiction; sometimes in a state and sometimes in a federal court. Examples: Litigations over property sales or leases; divisions of estates; divisions of jointly owned properties; between corporation stockholders.

2. Valuation litigations between private individuals, and/or corporations, and the public, or legally qualified representatives of the public.

The value at issue may be that of either a private property or a public-utility property. The litigation is sometimes in a state and sometimes in a federal court; it must be in a court of appropriate jurisdiction. In most cases the litigation is not started in court until after some owner of a public utility, or some governmental officer, jury, board, commission, or legislative agency has first made some definite official decision involving the value at issue; from this decision a qualified party at interest appeals to the courts. Examples: Litigations over property tax valuations; income taxes; condemnations of private property (private or utility) for public use; public-utility rate cases.

Except in interstate cases, and in other cases where questions of federal law or federal constitutional rights are at issue, litigations over

private-property rights are most often carried on in state courts. In *public-utility litigations,* however, the right of appeal to the federal courts exists so often, and is exercised so often, that there is a strong tendency to start such litigations in the federal courts in the first instance, whenever practicable.

A large proportion of the court decisions on which the fundamental principles of engineering valuation are based have been rendered in public-utility litigations; they have been closely related to the rapid development of public utilities in the United States since 1830.

The Historic Development of American Valuation Litigation

7.4. Important American Valuation Decisions.—A list is presented below of 68 of what the authors consider the most important American valuation decisions. They are arranged by chronological periods; the progress of the development of valuation principles in each period is stated with utmost conciseness. With each decision, one or two of its most important rulings are named. Every decision listed will be found briefed in Chap. VIII (some of the "briefs" are very brief).

7.4–1. *The Period Prior to* 1830.—Engineering valuation can hardly be said to have existed. The general legal principles of ordinary property rights, established in statute and common law by centuries of legislation and court decisions, were still made to answer in such valuations of industrial properties as became necessary.

There were no great utilities. By common and statute law, the public had acquired the right to prescribe maximum rates of charges for grist and some other mills, toll roads, toll bridges, ferries, inns, and some other utilities; and to regulate them in some other particulars. Such regulation was mainly by ordinances, by franchise provisions, and by license provisions.

7.4–2. *The Period from* 1831 *to* 1865.—An extensive railway net was constructed in the United States to the Mississippi River. A few lines extended somewhat farther west. There was a considerable development of city waterworks, sewer systems, gas lighting systems, and horse-drawn street cars. Many telegraph lines were built.

At first, public utilities were regulated mainly by franchise provisions and general statutes. Railway abuses led to the development of more extensive and systematic public regulation. Seven state railway commissions were established between 1844 and 1859, in New England and New York.

1. *The Louisiana Bread Case.* La. Sup. Ct. 1857. Enterprise held subject to public regulation.
2. *Mellerish v. Keen.* 1860. English good-will decision.

7.4–3. *The Period from* 1866 *to* 1900.—During this period, the rapid development of modern utilities, and the many resulting abuses and dissensions, brought out a considerable number of important court valuation decisions and rulings; by these, many fundamental legal principles of utility regulation and valuation were gradually established.

The "Granger Laws," to control railways, were passed in 1871 to 1874, in Illinois, Iowa, and Wisconsin. The Interstate Commerce Commission was established in 1887. Some 27 state railroad commissions had been established by 1885.

Local utilities multiplied; electric utilities gained a strong foothold. Telephone systems were added to telegraphs.

3. *Munn v. Illinois.* U.S. Sup. Ct. 1876. Pioneer decision upholding public regulation of public warehouses.
4. *Stone et al. v. Farmers Loan & Trust Co.* U.S. Sup. Ct. 1886. One of the "Railroad Commission Cases." The Mississippi Railroad Commission law was upheld.
5. *Chicago, Milwaukee & St. Paul Railway v. Minnesota.* U.S. Sup. Ct. 1890. Decision upheld railway regulation.
6. *Budd v. New York.* U.S. Sup. Ct. 1892. Railway rate decision, upholding public regulation.
7. *Monongahela Navigation Co. v. United States.* U.S. Sup. Ct. 1893. Condemnation case decision, analyzing tangible and intangible elements of value.
8. *Brass v. North Dakota.* U.S. Sup. Ct. 1894. Railroad case.
9. *Reagan v. Farmers Loan & Trust Co. et al.* U.S. Sup. Ct. 1894. Noted decision upholding Texas Railroad Commission law.
10. *National Water Works v. Kansas City.* U.S.C.C.A. 1894. Pioneer decision, discussing and upholding going value.
11. *Page v. Ratliffe.* 1896. English good-will decision.
12. *Smyth v. Ames.* U.S. Sup. Ct. 1898. The most famous and most cited valuation decision. In holding a Nebraska railway rate law unconstitutional, because the rates would be confiscatory, the United States Supreme Court established the "*Smyth v. Ames* rule"; that all factors affecting value must be given "such weight as is just and right in each case."
13. *People ex rel A. J. Johnson Co. v. Roberts,* Comptroller. N.Y. Ct. of App. 1899. Tax-assessment case involving the values of good will and of copyrights.

7.4–4. *The Period from* 1901 *to* 1915.—During this period, construction-cost prices maintained a nearly constant level. Adequate, reliable book records of the actual original costs of existing property units were generally lacking; partly in consequence, many engineering valuations were based mainly on reproduction costs new less depreciation. In 1909, the United States Supreme Court made it the "law of the land" that depreciation must be deducted in ascertaining fair values.

14. *Cedar Rapids Water Co. v. Cedar Rapids.* Ia. Sup. Ct. 1902. It was held that depreciation was not an allowable expense of producing service.
15. *San Diego Land & Town Co. v. Jasper.* U.S. Sup. Ct. 1903. The United States Supreme Court ruled against a depreciation operating-cost allowance.
16. *Brunswick & Topsham Water District v. Maine.* Me. Sup. Ct. 1904. Valuation instructions important in connection with service-worth value.
17. *Von Au v. Magenheimer et al.* N.Y. Sup. Ct. 1906. Ruling decision on good-will value.
18. *In re Keahon's Estate.* N.Y. Sup. Court. 1908. Good-will value.
19. *Knoxville v. Knoxville Water Co.* U.S. Sup. Ct. 1909. The United States Supreme Court first established the present rule that depreciation must be deducted to get fair value, and must be allowed as real production cost.
20. *Willcox v. Consolidated Gas Co.* U.S. Sup. Ct. 1909. New York 80-cent gas rate upheld. Appreciation of lands approved.
21. *Omaha v. Omaha Water Co.* U.S. Sup. Ct. 1910. Purchase appraisal case in which going value was allowed.
22. *The Minnesota Rate Cases.* U.S. Sup. Ct. 1913. The United States Supreme Court again upheld depreciation deductions from value and established the present rule that lands must be valued at current market values.
23. *The Missouri Rate Cases.* U.S. Sup. Ct. 1913. Railway rates.
24. *In re Ball's Estate.* N.Y. Sup. Ct. 1914. Good-will value case.

25. *German Alliance Insurance Co. v. Lewis.* U.S. Sup. Ct. 1914. Utility border-line decision, upholding Kansas statute regulating insurance rates.
26. *San Joaquin & Kings River Canal & Irrigation Co. v. County of Stanislaus.* U.S. Sup. Ct. 1914. Valuation of water rights.
27. *In re Demarest's Estate.* 1914.
28. *Yellow Taxicab Case.* 1915.
29. *In re McMullin's Estate.* 1915. These were three New York good-will value cases. New York courts.
30. *Des Moines Gas Co. v. Des Moines.* U.S. Sup. Ct. 1915. Rate case. Decision established the present rule against including a value for costs not actually incurred for cutting paving over mains.

7.4–5. *The Period from* 1916 *to* 1925.—The great rise in prices due to the World War aroused violent controversies over the "*Smyth v. Ames* rule" that all factors affecting value must be given due weight; it was attacked by the supporters of the prudent-investment formula, which would give dominant weight to original cost, and by those who favored the reproduction-cost-new-less-depreciation formula, which would give dominant weight to present prices. The United States Supreme Court continued to uphold the *Smyth v. Ames* rule.

31. *The City and County of Denver et al. v. Denver Union Water Company.* U.S. Sup. Ct. 1918. Rate case in which water rights were found to have a high value.
32. *Newton v. Consolidated Gas Co.* U.S. Sup. Ct. 1922. Although approved in 1909, 80-cent gas rate enjoined by United States Supreme Court because of increase in operation-cost prices. See case 20, above.
33. *Galveston Electric Company v. Galveston.* U.S. Sup. Ct. 1922. Rate case. Original cost and "period" reproduction cost given equal weight. Famous United States Supreme Court decision.
34. *State of Missouri ex rel. Southwestern Bell Telephone Co. v. Public Service Commission of Missouri et al.* U.S. Sup. Ct. 1923. Commission's rate order enjoined because no weight given to reproduction cost. Famous United States Supreme Court decision.
35. *Bluefield Waterworks & Improvement Co. v. Public Service Commission of West Virginia et al.* U.S. Sup. Ct. 1923. Rate order enjoined because no weight given reproduction cost.
36. *Georgia Railway & Power Co. v. Railroad Commission of Georgia.* U.S. Sup. Ct. 1923. Property revalued at reproduction cost new less depreciation.
37. *Superior Water Light & Power Co. v. Superior.* U.S. Sup. Ct. 1923. Wisconsin legislature could not change franchise contract.
38. *Colorado Power Co. v. Grant E. Halderman et al.* U.S. Dist. Ct. 1924. Rate-order case. Interesting valuation rulings.
39. *Pacific Gas & Electric Co. v. City and County of San Francisco.* U.S. Sup. Ct. 1924. Property made obsolescent by new process should be amortized. Rate order enjoined.
40. *New York & Queens Gas Company v. William A. Prendergast et al.* U.S. Dist. Ct. 1924. The New York \$1.00 gas rate was enjoined by the district court. See also cases 46, 47, 48, below.
41. *Southern Bell Telephone and Telegraph Company v. Railroad Commission of South Carolina et al.* U.S. Dist. Ct. 1925. Several interesting valuation rulings.

7.4–6. *The Period from* 1926 *to* 1930.—A "plateau" of construction-cost levels about double pre-war prices was maintained from 1924 to 1929. The United States Supreme Court continued to uphold the *Smyth v. Ames* rule, that all factors affecting

value must be given due weight; despite the arguments of the supporters of the reproduction-cost-new-less-depreciation and the prudent-investment valuation formulas. Depreciation questions were prominent in valuation litigation; actual depreciation was upheld in preference to theoretical; our highest court called attention, in 1930, to the anomaly involved in permitting public utilities to charge for annual theoretical depreciation allowances greater than experience shows are necessary to maintain depreciation reserves just equal to total accrued actual depreciation; in 1930, also, our highest court ruled that the depreciation base for estimating actual depreciation should be fair present value new, instead of the customary original cost new. In numerous valuation litigations, the courts ruled on numerous other valuation questions.

42. *Fort Smith v. Southwestern Bell Telephone Company.* U.S. Sup. Ct. 1926. The "cost of establishing the business" approved by United States District Court as fair measure of going value (but soon rejected elsewhere).
43. *Monroe Gas, Light & Fuel Co. v. Michigan Public Utilities.* U.S. Dist. Ct. 1926. Dominant weight given by a United States District Court to reproduction cost in a rate case.
44. *Public Utilities Commission v. New York Telephone Co.* U.S. Sup. Ct. 1926. The "New Jersey Telephone Case." The United States Supreme Court held that excessively high past accrued depreciation allowance charges do not justify future confiscatory rates.
45. *McCardle et al. v. Indianapolis Water Co.* U.S. Sup. Ct. 1926. Famous rate case decision in which the United States Supreme Court held that due weights must be given to both original cost and reproduction cost.
46. *Ottinger v. Consolidated Gas Co.* U.S. Sup. Ct. 1926.
47. *Ottinger v. Brooklyn Union Gas Co.* U.S. Sup. Ct. 1926.
48. *Ottinger v. Kings County Lighting Co.* U.S. Sup. Ct. 1926. In these three "New York Gas Cases," decided the same day, the United States Supreme Court affirmed federal district court injunction decrees against New York City $1.00 gas rates. See also case 40, above.
49. *United States v. Los Angeles and Salt Lake Railroad Co.* U.S. Sup. Ct. 1927. Rulings on valuation of water rights.
50. *Idaho Power Co. v. Thompson et al.* U.S. Dist. Ct. 1927. Interesting United States District Court rate case valuation rulings.
51. *United Fuel Gas Co. et al. v. Railroad Commission of Kentucky.* U.S. Sup. Ct. 1929. Rate case decision involving value of gas rights and allocation of property in different states to Kentucky service.
52. *Gilchrist et al. v. Interborough Rapid Transit Co. et al.* U.S. Sup. Ct. 1929. Interlocutory United States District Court injunction against New York City subway contracts' 5-cent fare clauses stayed, pending litigation in state courts. See case 56.
53. *St. Louis & O'Fallon Railway Co. et al. v. United States et al.* U.S. Sup. Ct. 1929. The United States Supreme Court rejected an Interstate Commerce Commission "recapture" order, because the Commission gave no weight to reproduction cost in valuing the railway property (except land).
54. *New York Telephone Company v. William A. Prendergast, et al.* U.S. Dist. Ct. 1929. Important telephone-rate legislation, begun in 1924, the record of which included over 100,000 pages of testimony and exhibits. An appeal to the United States Supreme Court was dismissed in 1934.
55. *United Railways & Electric Co. of Baltimore v. West, chairman, et al.* U.S. Sup. Ct. 1930. The United States Supreme Court made the important

ruling that the legal depreciation base for each physical-property unit is its present fair value new, instead of the customary original cost new.

56. *City of New York v. Interborough Rapid Transit Co.* N.Y. Sup. Ct. 1930. Decision upheld the validity of the New York City subway contracts' 5-cent fare clauses. See case 52.
57. *Cumberland Glass Mfg. Co. v. United States.* U.S. Ct. of Claims. 1930. Actual depreciation upheld in preference to theoretical.
58. *Michigan Bell Telephone Co. v. Odell et al.* U.S. Dist. Ct. 1930. Rate order enjoined. Reproduction cost given dominant weight. Theoretical annual depreciation allowances upheld, although only actual depreciation deducted to get fair value. Decision soon withdrawn on account of Illinois Bell Telephone Co. United States Supreme Court decision, below.
59. *Smith et al. v. Illinois Bell Telephone Co.* U.S. Sup. Ct. 1930. The United States Supreme Court set aside a United States District Court injunction decree against a 1923 Illinois Commerce Commission rate order; the case was remanded for further hearing and findings; with an admonition that the company's annual depreciation allowances ought to be made to check with its actual experience in retiring property. See case 67.

7.4–7. *The Period from* 1931 *to* 1934.—The valuation decisions during this period have been of two classes:

a. Cases originated prior to 1931 and others involving ordinary valuation questions. The United States Supreme Court has consistently upheld its former decisions on the *Smyth v. Ames* rule, on going value, on depreciation deductions, and on actual vs. theoretical depreciation; in addition, it has ruled decisively that annual depreciation allowances, as well as depreciation deductions from value, must be limited to actual depreciation, as shown by experience.

b. Cases originating in questions raised by the great depression which began late in 1929. Most of these remain yet to be adjudicated. One decision gave some light on the border line in 1932 between public utilities and private industrial enterprises.

60. *Excess Income of Richmond, Fredericksburg and Potomac Railroad Co.* I.C.C. 1931. This was an attempt by the Commission to comply with the United States Supreme Court rulings in the *O'Fallon Case* (54, above). This time, the Commission gave weight to both original cost and reproduction cost.
61. *Georgia Public Service Commission et al. v. United States et al.* U.S. Sup. Ct. June 1, 1931.
62. *State of Alabama et al., Appellants, v. United States et al.* U.S. Sup. Ct. June 1, 1931. In these two cases the court ruled that states must not establish intrastate railway rates which are not in harmony with proper Interstate Commerce Commission interstate rate orders.
63. *City of Logansport v. Public Service Commission et al.* Indiana Sup. Ct. 1931. The Court ruled that municipally owned public utilities are subject to state regulation.
64. *New State Ice Co. v. Liebmann.* U.S. Sup. Ct. 1932. In this decision, the United States Supreme Court discussed the border-line characteristics of public utilities and private industrial enterprises; reviewing former decisions, it held that the business of manufacturing, selling, or distributing ice was not a public utility, subject to license under an Oklahoma statute. In a previous decision, it had held that a public cotton gin was subject to the statute in question.
65. *Wabash Valley Electric Co. v. Young.* U.S. Sup. Ct. 1933. The United States Supreme Court affirmed a United States District Court's adverse

decree, denying an injunction against enforcement of a rate order by the Indiana Public Service Commission, applying to rates in Martinsville, Ind. The company served a large territory. It was held that the fair value of property and the cost of service must be segregated to determine Martinsville fair rates. The segregation was made on the basis of the percentage of kilowatt-hours sold in Martinsville.

66. *Los Angeles Gas and Electric Corp. v. Railroad Commission of California et al.* U.S. Sup. Ct. 1933. In delivering its opinion, the United States Supreme Court made a comprehensive, authoritative statement of present valuation law; this statement completely upheld its former rulings. The Court found that the California commission had made a number of errors in finding the 1930 value on which it based an order reducing rates—but that, by a coincidence, these errors neutralized each other; for this reason, the rate order was affirmed. The Court found that dominant weight was due, *in this case*, to "historical cost" (original cost).
67. *Lindenheimer et al. v. Illinois Bell Telephone Co.* U.S. Sup. Ct. 1934. See Case 59. After the 1930 United States Supreme Court decision, the United States District Court upon further hearing, adopted many technical findings and conclusions. The United States Supreme Court swept them all away because the district court had failed to limit the company's annual depreciation allowances to amounts just sufficient to maintain depreciation reserves just equal to the true, total accrued depreciation. The district court's injunction decree was ordered dissolved. Refunds ordered to subscribers are said to have amounted, with interest, to more than $20,000,000.
68. *Harold E. West et al. v. Chesapeake and Potomac Telephone Co. of Baltimore.* U.S. Sup. Ct. 1935. This decision is noteworthy because it is the first authoritative ruling on the proper use of cost indexes (Chap. IX) in making valuations. The court set aside the commission's valuation because it was based merely on an empirically weighted average of some sixteen construction-cost indexes, without making an appraisal of the physical property, but said: "This is not to suggest that price trends are to be disregarded; quite the contrary is true." The court upheld the rule of *Smyth v. Ames.*

A SUMMARY ENUMERATION OF THE FUNDAMENTAL GENERAL PRINCIPLES OF ENGINEERING VALUATION

In Secs. 7.5 and 7.6, the authors state summarily their interpretations of the well-established, fundamental, general economic and legal principles of engineering valuation; with the caution that it is not claimed that all authorities on valuation will agree with the authors in all cases.

7.5. General Principles of Valuation.

Part I. Value in General

Principle 1. The Fundamental Basis of Value.—The fundamental basis of the value of a property at any time is the prevailing judgment, at that time, as to the *present worth* of its *probable future net returns.*

Note.—Value is *not* based on the *actual* future returns; these can never be known with certainty until the future dates when they are actually realized.

Principle 2. Factors Affecting Value.—*All* factors which affect the value of any particular property must be taken into account in determining the property's present fair value, giving each factor "such weight as may be just and right in each case."

Note 1.—The factors affecting value are as follows:

1. The original-cost value, indicating the actual investment in the property at the date of valuation.
2. The reproduction-cost value, indicating value on the basis of present prices.
3. The earning value, indicating value on the basis of past and present net earnings.
4. (Needed only in some cases.) The service-worth value, indicating what the earning value would be if the prices charged for the commodities or services produced were just equal to what they are "reasonably worth" to the customers.
5. The stock-and-bond value, or market value, indicating the public's present guess at the value of the entire property.
6. All other pertinent factors affecting value.

Note 2.—Each factor which affects the value of any particular property does so by affecting the *probable* amounts of its future returns.

Principle 3. Legal Weights of Factors Affecting Value.—The "just and right weights" which must be given the different factors affecting the values of particular properties must be determined by fair, correct judgment in each case; *not* by any valuation formula.

Note 1.—The just and right weights which must be given the different factors affecting value vary widely in different cases.

Note 2.—All factors affecting value must be given thorough study, and judgment thereon must be carefully considered. Judgment is not guess work.

Note 3.—All factors affecting value must be given fair consideration. Judgment must not be biased.

Note 4.—The reproduction-cost-new-less-depreciation and the prudent-investment valuation formulas are the ones most frequently urged.

Principle 4. The Value of a Particular Property.—The value of any particular property is the sum total of its

1. *Physical value;* the total present value of all its present existing physical units.

Note 1.—Discarded units no longer have any value.

Note 2.—The existing units include those paid for from income, surplus, and/or any other reserves, as well as those paid for from new capital obtained by selling corporation securities.

2. *Intangible value;* the total value of all its present existing intangible elements.

Note 1.—The intangible values are

1. The preliminary-expense value.
2. The going value.
3. The good-will value.
4. Other intangible values such as the values of contracts, patent rights, and water rights.

Note 2.—Only such amounts must be allowed for intangible values as can be proven by the actual facts of the particular property.

3. *Working capital and other liquid funds wisely tied up in the business;* the total value of
   a. The short-time capital investment required to meet the day-to-day working needs of the enterprise.
   b. The permanent, and/or comparatively permanent, investments in liquid funds to meet emergency needs arising in connection with the operation and management of the property.

Part II. Physical Value

Principle 5. Overhead and Direct Costs of Physical-property Units.—The original costs and the reproduction costs of physical-property units include

1. The *direct* construction costs (sometimes termed contract costs) which are directly assignable to different specific units.

2. Their fair shares of the necessary general construction costs, termed *overhead* costs.

Note 1.—The percentages of overhead construction costs necessarily incurred are usually smaller for property units installed after operation begins than they were for the units of the original property; they will vary from year to year.

Note 2.—The overhead construction costs of the property units installed after operation begins must be added to the total costs of the units; they must not be charged to operation expenses.

Principle 6. The Deduction of Depreciation in Determining the Present Values of Physical-property Units.—The total accrued actual depreciations of physical-property units at the dates of their valuations must be deducted from their fair values new to determine their present fair values.

Note 1.—The correct determination of actual depreciation requires personal examination and study of the physical-property units by experts qualified to determine depreciation.

Note 2.—Theoretical depreciation, based on the average service lives of similar units, and computable without personal examination of the existing units, is rejected by the courts in favor of actual depreciation.

Note 3.—In addition to physical depreciation, such functional depreciation must be deducted in determining present fair value as can be forecasted with reasonable certainty by qualified experts. See Principle 19.

Principle 7. Annual Depreciation Expense.—The annual actual depreciation of all the existing physical units of an industrial property is a necessary annual expense; in addition to operation costs, it must be deducted from income to determine the annual net return from the property; it is the right and the duty of the owners to recoup it each year from income, before net return, in order to maintain properly the investment unwasted.

Note 1.—Neither undercharges nor overcharges by the owners for annual depreciation expense affect the present fair value of the property.

Note 2.—Past errors in making annual depreciation expense charges must not be perpetuated in future charges.

Principle 8. The Depreciation Base.—By decision of the United States Supreme Court, the legal "depreciation base," for estimating both the accrued and the annual actual depreciation of a physical-property unit, is its present fair value new, revised from time to time, whenever necessary.

Note.—Despite Principle 8, the most usual practice in depreciation accountancy is to use the actual original cost of each physical-property unit as its depreciation base.

Principle 9. Property Paid For from the Depreciation Reserve.—Property paid for from the depreciation reserve must be included in the property to be valued, the same as if paid for from other sources.

Part III. Intangible Value

Principle 10. Preliminary-expense Value.—The necessary preliminary expense of originating, organizing, and promoting an industrial enterprise, in preparation for

actual construction, may be capitalized properly as part of the intangible value of the property.

NOTE.—Necessary, customary, and fair brokerage and other legitimate expenses of selling securities are a part of the legitimate preliminary expenses; discounts and premiums must be excluded. See Principle 17.

Principle 11. Going Value.—An industrial property already in successful operation, regularly earning a valuable income, has a going value, over and above the sum of the values of the other elements of the property. The going value is the excess of the present worth of an already established income above the present worth of an income yet to be developed.

NOTE 1.—The going value is *not* necessarily equal either (1) to the capitalized deficits of the property during the "development period" (during which the property is developing a paying business), or (2) to the "cost of development" of such business, although both of these may be helpful evidence bearing on the true going value. The courts rule that past losses cannot be capitalized.

NOTE 2.—Going values allowed in actual cases frequently equal about 10 percent of the sum of the present values of the fixed-capital physical elements of public-utility properties; usually, they are not less than 5, or more than 15 percent.

Principle 12. Good-will Value.—The good will of the customers who habitually patronize a particular industrial enterprise may have a real money value, the carefully determined actual amount of which is a part of the intangible value of the property.

NOTE 1.—Good-will value must not be confused with going value.

NOTE 2.—In the courts, the good-will value of ordinary business enterprises is customarily measured by the number (usually 1 to 6) "year's purchase" of the excess of the actual net returns over a fair rate of "interest" (usually 6 percent) on the fair value of the other property elements. More exact methods may be used in engineering valuations of industrial properties.

NOTE 3.—Good-will value is not usually allowed public-utility properties.

Principle 13. "Other Intangibles" Values.—Particular industrial properties may be entitled to "other intangibles" values, such as those of patent rights, water rights, and contracts.

PART IV. WORKING CAPITAL AND OTHER LIQUID FUNDS TIED UP IN THE BUSINESS

Principle 14. Working Capital.—The working capital required to meet the day-to-day working needs of an industrial enterprise, for materials, supplies, cash, and temporary investments in paid-up operation expenses, must be valued as a part of the property.

NOTE.—The amount of working capital required varies from day to day.

1. For price and rate-making purposes, the value of the working capital is its *average* value.
2. For transfers of title purposes, the value of the working capital is the value, on the day of the transfer, of such of its items as are transferred to the new owners.

Principle 15. Other Liquid Funds Wisely Tied Up in the Business.—Permanent and/or comparatively permanent investments wisely tied up by the enterprise, in liquid funds, to meet emergency needs arising in connection with the operation and management of the property, must be valued at their actual present values.

NOTE 1.—For price and rate-making purposes, only such funds (usually investments in high-grade liquid securities) as are wisely kept tied up to meet really probable emergency operation requirements of the enterprise must be included in making its valuation.

1. The present yearly average values of such funds should be included in the rate base, on which the enterprise is entitled to earn a fair rate of net return.

2. The incomes earned by such funds should be included in the gross income of the enterprise.

NOTE 2.—Owing to the apparent, present (1935) fact that the market value of high-grade liquid securities at any particular date may be greatly below or above their real worths (unless they are sold immediately), a move has developed to value them at the actual present worths of future interest payments at the dates due, and the payment of principal at maturity.

PART V. INTEREST AND AMORTIZATION

Principle 16. Interest.—Interest on bonds and/or other securities representing funded indebtedness, usually incurred to obtain funds for investment in the property, is a part of the net return on the property; *not* a part of production expense.

Principle 17. Discounts on Securities and Their Amortization.—1. Discounts and/or premiums on industrial securities are not property, and must not be valued as a part of the property of the enterprise.

2. In effect, discounts and/or premiums on funded-indebtedness securities, which must be paid at par when due, are increases or decreases in the interest on such securities, above or below their nominal interest rates. Necessary and reasonable discounts on funded-indebtedness securities may be recouped properly by amortization payments, extending throughout the lives of the securities; such amortization payments, like interest, must be treated as a part of the net return on the property.

NOTE 1.—Discount amortization payments falling due during the construction period, before operation earnings begin, if on account of securities sold to obtain funds for construction, are sometimes included in the valuation; in effect, they are "interest lost during construction."

NOTE 2.—Discounts and/or premiums on shares of stock sold have to do with the cost of ownership shares in the property; they must not be included in its valuation, or in any amortization payments except such as are paid out of net return.

NOTE 3.—Legitimate expenses of selling securities are a part of "preliminary expense." See Principle 10.

Principle 18. The Amortization of Unforeseeable Special Losses.—Unforeseeable special losses, if of such magnitude that they cannot wisely be recouped from current income, and if of such character that they cannot reasonably be forecasted, are properly recouped by amortization payments which extend over a reasonable period of years, and which are treated as a part of necessary expense outside of net return.

NOTE.—Such losses as can reasonably be forecasted (fire and tornado loss, for example) are properly recouped, so far as practicable, by insurance; the premiums for which are treated as a part of production expense.

Principle 19. The Amortization of Functional Depreciation.—When functional depreciation losses[1] are of such character that it is not feasible for qualified experts to forecast them sufficiently in advance, they may properly be recouped by amortization payments, extending over a reasonable period of years, treated as a part of necessary expense before net return.

NOTE 1.—By making the amortization period shorter or longer, the yearly amortization payment may be made (1) greater than, (2) equal to, or (3) less than the yearly saving in operation expenses due to the improvement.

NOTE 2.—Such functional depreciation as it is feasible for qualified experts to forecast sufficiently in advance must be charged off to depreciation in advance of the retirement dates. See Note 3, Principle 6.

[1] Those due to obsolescence, inadequacy, or supersession.

Part VI. The Fair Rate of Net Return

Principle 20. The Fair Rate of Net Return.—The fair rate of net return for any particular industrial enterprise is that rate which is just large enough to secure what capital is really needed for investment in enterprises of the particular character.

Note 1.—The fair rate of net return of an industrial enterprise must be calculated on the basis of the present depreciated fair value of the property, not on the investment therein or the value new.

Note 2.—The fair rate of net return for any particular industrial enterprise will be greater than the prevailing rate of interest (less taxes, etc.) on high-grade liquid securities, by an amount just sufficient to serve as an adequate insurance against the special risks of investments in enterprises of this particular character.

Note 3.—The actual fair rates of net return on private industrial properties (excluding public utilities, for which see Sec. 7.6, Special Public-utility Valuation Principle 5) may readily vary between 5 and 20 percent for different properties, depending upon the business risks of investments in the different kinds of enterprises.

7.6. Special Public-utility Valuation Principles.—In this section, the authors state summarily their interpretation of certain well-established fundamental special public-utility valuation principles; these are based upon the special public character with which such utilities are endued, and upon their resulting subservience to public regulation. The 20 fundamental general principles of engineering valuation enumerated in Sec. 7.5 apply to public-utility valuation,[1] in addition to the five special public-utility valuation principles about to be enumerated in Sec. 7.6.

Special Public-utility Valuation Principle 1. The Subservience of Public Utilities to Public Regulation.—Public-utility rates, operation, and valuation are subservient to just and fair public regulation by

1. Legislative bodies, such as Congress, state legislatures, and city councils.
2. Utility commissions, such as the Interstate Commerce Commission, the railroad and other commissions of the several states, and certain city commissions; *when duly authorized by constitutional statute laws.*

Special Public-utility Valuation Principle 2. The Authority of the Courts in Public-utility Rate and Valuation Litigation.—The several state and United States courts have jurisdiction and authority to protect both the public and public-utility owners against unfair rates and/or regulation, no matter how imposed.

Note 1.—Both utility owners and qualified representatives of the public have the right of appeal to the courts.

Note 2.—State courts have primary jurisdiction in intrastate cases, and in the enforcement of state laws; their decisions are subject to the right of appeal to the federal courts as to infringements of federal laws and the federal constitution; the federal courts have primary jurisdiction in interstate cases, and in the enforcement of federal laws.

Note 3.—The authority of the courts is conferred by their duty and power to enforce the laws.

Note 4.—The constitutional authority of the federal courts to protect public-utility owners against confiscatory rates is conferred by

[1] Many of the 20 principles stated in Section 7.5 have been enunciated most clearly, and supported most completely, in court decisions in public-utility litigations.

1. That portion of the Fifth Amendment to the Constitution of the United States which provides that no person shall be "deprived of life, liberty, or property, without due process of law; nor shall private property be taken for public use without just compensation."
2. That part of the Fourteenth Amendment to the Constitution of the United States which provides: "Nor shall any state deprive any person of life, liberty, or property without due process of law, nor deny to any person the equal protection of the laws."

Special Public-utility Valuation Principle 3. The Function and the Practice of the Courts in Public-utility Rate and Valuation Cases.—It is the function of the courts to decide whether particular public-utility rate or valuation laws or utility commission decisions, and/or orders, are constitutional and/or legal; it is not their function to establish definite utility rates, or make definite utility valuations, on their own initiative.

Note 1.—The courts decide whether particular rates established by law and/or by utility commission orders are, or are not, so low as to be confiscatory of property, and therefore in violation of the constitutional rights of the owners.

Note 2.—In practice, the courts give great weight to the authority of legislative bodies and to the decisions and orders of utility commissions. In general, the rule is that positive and convincing proof of injustice, amounting to the violation of legal rights, is necessary to justify setting aside rates and valuations established by laws, or by utility commission orders.

Note 3.—This principle of practice may lead to the approval, in different cases, from time to time, or even at the same time, of rates and valuations *apparently* based on conflicting theories.

Special Public-utility Valuation Principle 4. The Legal Principle for Determining Fair Public-utility Rates.—The legal principle which most often applies in determining fair public-utility rates is that the utility owners are entitled to a fair rate of return upon the present fair value of the property used and useful in rendering the public service; but this principle is subservient to the ruling principle that the public is entitled to demand that no more shall be exacted from it for the services of a public utility than the services rendered are reasonably worth.

Note 1.—The above principles were first stated clearly by the United States Supreme Court in 1898 in the celebrated case of *Smyth v. Ames* (Sec. 8.3); they have been confirmed by the same final authority in an unbroken line of later decisions.

Note 2.—Note particularly that the rate base is the fair value of the property used and useful; *not* the investment therein.

Special Public-utility Valuation Principle 5. The Fair Rates of Return on Public-utility Properties.—The fair rates of return on public-utility properties must be determined by the same principles as for other industrial properties (see Principle 20, Sec. 7.5); since public-utility rates are subject to revision, at reasonable intervals, down or up to make them "fair," the range of variation in fair public-utility rates of return is likely to vary only between 5 and 10 percent; it usually falls between 5 and 8 percent.

THE LAW OF THE LAND AS TO THE FACTORS AFFECTING VALUE

The law of the land is determined and announced by the United States Supreme Court in its decisions in litigated cases, and is the subject of constant development. In rare instances [as in the case of deprecia-

tion (Sec. 6.1)], the law of the land, even when not changed by new statute law, may be modified or even completely reversed in later decisions; usually because of advances in knowledge and in civilization. Engineering valuation is of such recent origin that the development of the law of the land on valuation questions is still progressing. Some of its fundamental principles are still the subject of controversy.

7.7. The Rule of Smyth v. Ames.—The law of the land as to the factors affecting value, as stated in Principles 2 and 3 (Sec. 7.5), is often known as the "*Smyth v. Ames* rule," because it was first enunciated clearly by the United States Supreme Court in that famous case.

The *Smyth v. Ames* rule is that *all* factors affecting value must be taken into account and given such weight as may be just and right in each case, as determined by sound judgment (not by formula). By enunciating the rule, the United States Supreme Court, *in effect*, has upheld the view that experts and courts must determine the values of properties in formal valuations upon the same basis as prevails in ordinary business affairs; the values of property in ordinary affairs are not computed mathematically but are fixed in each transaction by the judgments of the parties thereto.

7.8. The Controversy over the Rule of Smyth v. Ames.—Although the *Smyth v. Ames* rule has been firmly established and consistently upheld ever since it was announced by the United States Supreme Court in 1898, there nevertheless has been continuing clamor for some valuation formula to substitute for the sound judgment which the law of the land makes the final criterion of value. The formulas urged are

1. The reproduction-cost-new-less-depreciation valuation formula.
2. The prudent-investment valuation formula.

The clamor for these valuation formulas has come, and continues to come, mainly from three classes of people:

1. Some accountants, economists, valuation writers, and other people who, although the values of properties in all ordinary sales and purchases are determined by the *judgments* of sellers and buyers, nevertheless wish to substitute in valuations of a formal character some hard and fast rule in place of the technical training, experience, careful study, straight thinking, and sound judgments of qualified experts.

2. Some people, mainly owners, whose interests or beliefs lead them always to desire the highest possible valuations; they generally favor the reproduction-cost-new-less-depreciation formula in periods of such high prices as prevailed in 1918 to 1929; they are prone to shift allegiance to the prudent-investment formula during periods of such depressed prices as prevailed during 1930 to 1933.

3. Some people, including some representatives of the public in utility controversies, whose interests or beliefs lead them always to desire the

lowest possible valuations; they are prone to shift allegiance also, but in their case from the prudent-investment formula during high price periods to the reproduction-cost-new-less-depreciation formula during low price periods.

7.9. The Reproduction-cost-new-less-depreciation Valuation Formula.—The reproduction-cost-new-less-depreciation valuation formula may be stated as follows:

$$\left.\begin{array}{l}\text{The physical}\\ \text{value of an}\\ \text{industrial}\\ \text{property}\end{array}\right\} = \left\{\begin{array}{l}\text{(1) the sum of the reproduction costs new of all its existing}\\ \quad\text{physical units, less}\\ \text{(2) the sum of their total reproduction-cost actual depreciations}\\ \quad\text{at the date of valuation of the property.}\end{array}\right.$$

The above formula gives the present reproduction-cost "physical value" of the property; to this must be added its "intangible values" and the values of the "working capital" and "other liquid funds" necessary to tie up in the business; the sum gives the "reproduction-cost value" of the entire property.

The reproduction-cost-new-less-depreciation valuation formula was developed and used extensively in the period 1900 to 1915; during those years, construction-cost price levels were nearly constant, so that original-cost values and reproduction-cost values did not differ widely; also, the book records of many industrial properties were so imperfect as not to supply reliable data for determining original-cost values.

During the period 1915 to 1930, owing to the tremendous rise in construction-cost price levels due to the World War and to the slowness of their fall after its close (Figs. 7.1, 9.8), reproduction-cost values, in general, were much higher than original-cost values, and the reproduction-cost-new-less-depreciation valuation formula was ardently supported by those who desired high valuations. The supporters of the formula have claimed that it has been upheld by the United States Supreme Court, which, since the World War, has repeatedly overruled valuations in which *little or no* weight was given to reproduction cost. Some lower courts even adopted the erroneous view that the highest court of the land had held that reproduction cost must be given *dominant* weight; but the United States Supreme Court has ruled merely that reproduction cost shall be given "such weight as is just and right in each case," never that it shall be predominant. In its decision in the famous *O'Fallon Railway Case,* 1929 (case 53, Sec. 8.6), for example, in which the valuations made by the Interstate Commerce Commission were overruled because no weight was given to reproduction costs (except for land), the Court was careful to say:

> . . . The weight to be accorded thereto [to reproduction costs] is not the matter before us. No doubt there are some, perhaps many, railroads the ultimate value of which should be placed far below the sum necessary for reproduction.

7.10. The Prudent-investment Valuation Formula.—The prudent-investment valuation formula may be stated as follows:

The total fair value of an industrial property = The total actual prudent investment (Sec. 1.18) in the property at the date of valuation.

As already explained in Secs. 1.16 and 1.17, the "prudent investment" in an industrial property at the date of valuation is equal to its "original-cost value," determined as described in Chap. XI, less all "imprudent" investments; imprudent because not made in a careful, business-like, and competent manner.

Ordinary, nonspeculative business investments are made for the purpose of earning profits on the sums invested. Hence, in spite of the fact that ordinary investments are not immune to losses from changes of value, and in spite of the further fact that it is not investment but value which the Constitution of the United States and the other laws of the land protect against unjust confiscation, it is not surprising that there has been much support from an early date for a valuation formula which would make prudent investment the *dominant* factor in determining the rate base, for calculating fair rates or prices.

The "Massachusetts rule," that "stated tersely, it is the money honestly and prudently invested and devoted to the service of the public that is entitled to a fair return,"[1] was established as early as 1914. During the period of high prices following 1915, the public-utility commissions of several states favored the prudent-investment valuation formula; doubtless in order to protect the public from being required to pay returns upon great increases in value gained by utility properties without corresponding increases in investment. A number of valuators and economists have supported the formula. In the *Southwestern Bell Telephone Case* (Sec. 8.5), in 1923, a respectable minority of the United States Supreme Court, led by Mr. Justice Brandeis, strongly supported the prudent-investment formula and attacked the *Smyth v. Ames* rule in an able dissenting opinion.

Nevertheless, not only in the *Southwestern Bell Telephone Co. Case* but repeatedly in later cases, the United States Supreme Court has consistently continued to uphold the *Smyth v. Ames* rule, by which prudent investment is only *one* of the factors affecting value which must be taken into account and given just and right weight in ascertaining fair value.

[1] The Massachusetts Department of Public Utilities, *in re New England Telephone and Telegraph Co.*, July 30, 1925. See Whitten and Wilcox, "Valuation of Public Service Corporations," 2d ed., 1928, p. 318. Note that the issuance of securities in Massachusetts has long been safeguarded by strict state law.

Moreover, it may be remarked that the great depression beginning in 1929 has demonstrated again that prudent investment is not the predominant factor in determining value in ordinary business transactions; otherwise, billions of dollars of losses would have been saved on investments whose prudence was considered unquestioned when made.

Note on Split Inventories.—Split inventories, in which the physical-property units, except land, are segregated by price periods for convenience in supplying omissions in book records of original cost new, have been used in certain cases since 1915 by certain utility commissions (including the Interstate Commerce Commission) in estimating the prudent investments in certain properties (Sec. 8.6). Although the valuations adopted were overruled in some such cases, the adverse decisions were not rendered because of the use of split inventories but because no, or at least insufficient, weight was given to reproduction costs.

THE JUST AND RIGHT WEIGHTS DUE THE DIFFERENT FACTORS AFFECTING THE VALUES OF INDUSTRIAL PROPERTIES

The law of the land that *all* factors affecting value must be given just and right weights in valuation is so completely in accord with the methods by which all ordinary exchange values are fixed that it is hard to explain the continued controversy on the subject; it may be due in part to lack, heretofore, of sufficient analysis and discussion of the general principles by which sound judgment should be guided in deciding what weights are just and right in particular cases. The authors present below their own views on this too little discussed subject.

7.11. Weights Due Original Cost and Reproduction Cost in Determining Fair Cost-value.—After making a detailed analysis of nine valuation decisions (Secs. 7.15, 7.16) and carefully studying many others, the authors conclude that the general principles governing the selection of the just and right relative weights due original cost and reproduction cost under different circumstances are about as follows:

Case 1. *When physical-property units are new,* their fair cost-values are equal to their total original costs new, installed ready for service; present original-cost value will continue to be the correct fair present cost-value of each unit until there is a change in its price level.

This principle has been stated by the United States Supreme Court [in its decision in the *Indianapolis Water Case* (Sec. 8.6)] in the following words:

" . . . Undoubtedly the reasonable cost of a system of water works, well-planned and efficient for the public service, is good evidence of value at the time of construction. And such actual cost will continue fairly well to measure the amount to be attributed to the physical elements of the property so long as there is no change in the level of applicable prices."

Case 2. *During periods of rising or falling construction-cost price levels,* the present fair cost-values of physical-property units are no longer equal to their present original-cost values. The changes in fair cost-values lag behind (Secs. 7.15, 7.16) the changes in "spot" (Sec. 11.18) reproduction-cost prices, but shift gradually closer to reproduc-

tion-cost levels; the relative weights given original cost and reproduction cost should shift accordingly.

Case 3. *During periods of uniform construction-cost price levels,* distinction must be made between property units installed *before* and those installed *after* the beginning of the uniform price-level period.

1. The shift of the cost-values of previously installed units toward reproduction-cost levels continues for some years, and cost-values should eventually be based on prevailing prices. For such units, the relative weights given original cost and reproduction cost should shift accordingly.
   This principle has been stated by the United States Supreme Court [in its decision in the *Indianapolis Water Case* (Sec. 8.6)] in the following words: "And, as indicated in the report of the commission, it is true that, if the trend of prices is not definitely upward or downward and it does not appear probable that there will be a substantial change of prices, then the present cost of constructing the plant less depreciation, if any, is a fair measure of the value of the physical elements of the property."
2. The fair present cost-values of units installed after the beginning of the uniform price-level period are equal to their present depreciated original costs.
3. The combined total effect, if the uniform price level continues a sufficient number of years, is that eventually there usually is little difference between the present original-cost and the present reproduction-cost values of the entire physical property (Sec. 7.16), both depreciated.

7.12. Weights Due Cost-values, Earning Value, and Service-worth Value.—In the process of determining the present fair value of the entire property, the valuator must compare its cost-value, determined as discussed in Sec. 7.11, with its earning value, and sometimes with its service-worth value (Secs. 1.25, 1.26, 15.1 to 15.5); he must decide by considered judgment what relative weights will be just and right for the comparison under the circumstances of the particular case.

Earning value and service-worth value are based on the probable average future net earnings of the property; forecasted on the basis of its *actual* past and present and its *estimated* comparatively near future earnings and expenses in the case of earning value; forecasted, in the case of service-worth value, on the basis of what such earnings and expenses would be if the prices charged customers were the true reasonable worths to them of the services and/or commodities sold.

The Just and Right Relative Weights Due Cost-values.—The relations of cost to value, and of investment to cost and to value, are analyzed in Secs. 1.16 and 1.17. In comparing cost-values and earning values it must be remembered that, although probable future net earnings are the true fundamental basis of all values, the net earnings on which earning value, as herein defined, is based, are not necessarily the most probable future net earnings; they are merely those which have been forecasted on the basis of actual past and present and estimated comparatively near future earnings and expenses.

In comparing cost-values and earning values, distinction must be made between private properties and public-utility properties.

1. *Private Industrial Properties.*—The just and right weights due the cost-values of private industrial properties are usually greater than are due their earning values.
   This is because, *in the long run*, owing to the effects of competition and/or the principle of fair prices, the cost-values of industrial properties are usually better indicators of their future net earnings than are averages of their *past* earnings and expenses.[1]
2. *Public-utility Properties.*—The just and right weights due the cost-values of public-utility properties must usually be great enough to exclude giving any weight whatsoever to their earning values.[1]
   This is because the service rates charged by public utilities are subject to regulation, up or down, to make them just sufficient to yield a fair return on fair value. Hence, to give earning value a material weight would be to enter upon a vicious circle of reasoning.

The Just and Right Relative Weights Due Earning Values.—In discussing the just and right weights due earning values, three cases may be considered; in each of these, distinction must be made between private industrial properties and public-utility properties.

Case 1. When the actual past and present and/or the estimated comparatively near future net earnings are *approximately equal* to a fair return on the fair present cost-value of the property.

1. *Private Industrial Properties.*—The earning value simply confirms the cost-value. Cost-value should be given dominant weight.
2. *Public-utility Properties.*—Earning value should be given no weight in determining fair value, but its equality with fair present cost-value indicates that present rates are fair rates.

Case 2. When the actual past and present and/or the estimated comparatively near future net earnings are materially *less* than a fair return on the cost-value.

1. *Private Industrial Properties.*—The cost-value should be given the greater weight if there is a reasonable prospect of increasing the earnings materially in the future; in the absence of reasonable prospect of such increase, earning value should be given the greater weight; if the deficiency of earnings is practically certain to continue a very long time, earning value should be given dominant weight.
2. *Public-utility Properties.*—In valuations for rate making, deficient earnings should be given no weight except when the rates necessary to earn a fair return are higher than the reasonable worths of the services rendered. For this case, see service-worth values, below, in this section.

Note.—For *immediate* sales purposes, deficient earnings may be given some minor weight.

Case 3. When the actual past and present and/or the estimated comparatively near future net earnings are materially *greater* than a fair return on the cost-value.

1. *Private Industrial Properties.*—Cost-value should be given the greater weight unless there is practical certainty that the excess earnings will continue a long time. If there is practical certainty of such long-time continuance, earning value should be given some material weight. If the excess of earnings is practically certain to be permanent, earning value should be given dominant weight.
2. *Public-utility Properties.*—Excess earnings should be given no weight in valuations for rate-making purposes; the rates which give earnings in excess of fair return are subject to compulsory revision downward.

[1] For exceptions to the above rules, see below, in this section.

Note.—For *immediate* sales purpose, excess earnings may be given some minor weight when action for their reduction by regulatory authority is not likely to be started soon.

The Just and Right Relative Weights Due Service-worth Values.—As already stated, the service-worth value (Secs. 15.2 to 15.5) of an industrial property is what its earning value would be if the charges for its services rendered, and/or its commodities sold, were just equal to their respective reasonable worths to the customers.

It is the law of the land that " . . . What the public has a right to demand is that no more be expected from it for the services of a highway" [or other public utility] "than the services rendered by it are reasonably worth"[1]; the corresponding principle for private industrial enterprises is gaining ground.

In considering the just and right weights due service-worth values, distinction must be made between private industrial properties and public-utility properties.

1. *Private Industrial Properties.*—Until recently, it usually has been unnecessary to estimate the service-worth values of private industrial properties, because the reasonable worths of the products have been assumed to be the prices fixed by competition in the open market. Now, however, more and more attention is being given to fair prices, just sufficient to pay the costs of production by reasonably efficient plants plus fair net returns on their fair present cost-values. The service-worth values of industrial plants, estimated on a fair-price basis, would therefore seem entitled to material weight in determining fair values.

Moreover, determination of the estimated net earnings of industrial properties on the basis of fair prices for their products will often afford some aid in determining whether abnormally high or low net returns are likely to continue in the future, and how long.

2. *Public-utility Properties.*—When the service-worth value of a public-utility property, determined upon an adequate, just basis, supported by convincing evidence, is materially *less* than its cost-value, the service-worth value is entitled to relatively high, in extreme cases predominant, weight.

Note 1.—In the cases of the majority of public-utility properties, a comparatively brief general investigation readily shows that the reasonable worths of the services rendered are at least equal to charges which will yield a fair return on fair value; in such cases, it is unnecessary to estimate the service-worth values.

Note 2.—A striking illustration of extremely low service-worth values is afforded by the 4,049 miles of railways which were abandoned in the United States during 1920 to 1928, because their total earnings were insufficient to pay even operating expenses.

Their cost-values may have been greater than $200,000,000.

Their service-worth values were merely the comparatively insignificant salvage realized on their physical items.

7.13. The Just and Right Weights Due Stock-and-bond Values.—For a definition and a description of the methods of determining stock-and-bond value, see Secs. 1.28, 15.6, and 15.7. It is a part of the law of the land[1] on valuation that stock-and-bond values must be given such weights as are just and right in each case. However, a study of court decisions in many valuation litigations shows that

1. In general, stock-and-bond value is entitled to little weight, relative to cost-value and earning value, in determining the values of industrial property.

[1] See *Smyth v. Ames*, Sec. 8.3.

a. The fair values sought in valuations are comparatively stable values, which do not fluctuate in close accord with stock tickers. Stock-and-bond values show merely the public's guesses at values and are subject to speculative gambling.

b. Stocks and bonds are often based in part on property owned but not used, and therefore not to be included in fair values for rate making and/or the determination of fair prices.

c. It is often impossible to ascertain the stock-and-bond values of properties because their securities are not listed on the stock exchanges.

2. In some abnormal cases, stock-and-bond value may be entitled to material or even predominant weight.

For example, in the final decision, 1934, in the *Illinois Bell Telephone Co. Case* (Sec. 8.7), the United States Supreme Court used the high and greatly increasing values of the stocks and bonds of the company during 1923 to 1934 to help sweep away the mass of technical rulings on annual depreciations, net returns, and fair values by which the lower court decided that the company, in spite of its prosperity, had not been earning fair returns on the fair values of its property devoted to public service.

7.14. Weights Due Other Pertinent Factors.—All other pertinent factors, found to affect values in particular cases, must be thoroughly investigated; they must be given such weights, of a general character, as are found to be just and right in each case. No definite general instructions as to the magnitudes of such weights can be given. Other pertinent factors affecting value include: Available materials supply; available labor supply; transportation facilities and costs; the size, character, and prospects of growth of the local and of neighboring communities; present and probable future local and general markets; present and probable future competition; present and probable future local and general business conditions; present and probable future price trends.

THE APPROXIMATE RELATIVE WEIGHTS GIVEN ORIGINAL COST AND REPRODUCTION COST IN ACTUAL VALUATION DECISIONS

Owing largely to the great World War of 1914 to 1918, and its aftereffects, the construction-cost levels of industrial properties fluctuated violently up and down between 1913 and 1934 in the manner shown in the upper part of Fig. 7.1. These fluctuations brought to the front many puzzling and highly controverted questions as to what relative weights should be given to original cost and reproduction cost in determining the true fair present cost-values of industrial properties; especially of those owned and operated by public utilities.

7.15. Table and Diagram of Valuation Decisions.—In Table 7.1 and Fig. 7.1, herewith, the authors present the results of a study and analysis of nine important valuation decisions made between 1918 and 1931, inclusive; six by the United States Supreme Court; two by United States District Courts; one, for the same railroad in four successive years, by the Interstate Commerce Commission.

The valuation data in Table 7.1 and Fig. 7.1 are restricted to those of the present (depreciated) original-cost and reproduction-cost values, and the fair present (depreciated) cost-values of the fixed-capital physical property; excluding land, wherever the information available permitted, because, by law since 1913, the lands had to be valued at the present market values of similar adjacent lands.

Intangible values are excluded, because they are most often estimated by direct methods without formal consideration of the weights due original and reproduction costs.

Working capital is excluded, because its value is not affected by the weights due original costs and reproduction costs.

The authors believe that the data in Table 7.1 and Fig. 7.1, though only approximate, are sufficiently exact, although in some of the cases the information available was not entirely complete.

Although the nine valuation decisions studied were promulgated at dates between 1918 and 1931, inclusive, the law's delays were such that none of the valuations were of a date later than Jan. 1, 1924. However, valuation studies of Iowa public-utility properties have shown that, in the cases of some properties, both their "spot" (Sec. 11.18) and their "period" (Sec. 11.19) reproduction-cost physical values were actually somewhat lower in 1932 than the corresponding original-cost values; a condition which previously had been found to exist during the last former great depression (1893 to 1897). This was true, for example, in the cases of the Kansas City Water Works and of the Nebraska railways. See *National Waterworks v. Kansas City*, and *Smyth v. Ames*, both in Sec. 8.3.

7.16. Discussion of Valuation Decisions.—The following quotations from decisions by the United States Supreme Court state authoritative general rules about the relative weights due original cost and reproduction cost in determining fair cost-values:

Indianapolis Water Case Decision (1926).—" . . . to ascertain value 'the present as compared with the original cost of construction' are, among other things, matters for consideration. But this does not mean that the original cost or the present cost or some figure arbitrarily chosen between the two is to be taken as the measure. The weight to be given to such cost figures and other items or classes of evidence is to be determined in the light of the facts of the case in hand."

Southwestern Bell Telephone Case Decision (1923).—" . . . An honest and intelligent forecast of future values, made upon a view of all the relevant circumstances, is essential. If the highly important element of present costs is wholly disregarded, such a forecast becomes impossible. Estimates for tomorrow cannot ignore prices of today."

Construction-cost price levels, after recovering from their fall during the severe depression of 1893 to 1897, were nearly uniform (Fig. 9.9) from 1900 to 1914; since that date they have fluctuated as illustrated in Fig. 7.1. The fluctuations of the

Table 7.1.—Original- and Reproduction-cost Relative Weights in Nine Valuation Decisions, 1918 to 1932

Case no.	Dates of		Approximate depreciated physical values				Value ratios				Value weights			
	Valuation	Decision	Original cost	Reproduction cost		Fair value	Original cost	Period reproduction	Spot reproduction	Fair value	Period prices		Spot prices	
				Period prices	Spot prices						Original cost	Reproduction cost	Original cost	Reproduction cost
1	1914	March 4, 1918	....	....	....	....	1.00	1.00	1.00	1.00	0.50	0.50		
2	June 2, 1919	April 10, 1922	$ 1,326,000	$ 1,926,000	$ 2,317,000	$ 1,626,000	1.00	1.45	1.75	1.23	0.50	0.50	0.70	0.30
3	December 1, 1919	May 21, 1923	18,963,000	24,709,000	....	(a)21,787,000	1.00	1.30	....	(a)1.15	0.50	(a)0.50		
4	1920	June 11, 1923	(b)451,000	....	908,000	611,000	1.00	....	2.01	1.36	....	....	0.64	0.36
			409,000	....	....	....	....	....	....	1.49				
5_1	1920	April 7, 1931	17,157,000	26,980,000	32,720,000	24,773,000	1.00	1.57	1.91	1.44	0.23	0.77	0.52	0.48
5_2	1921	April 7, 1931	17,600,000	26,435,000	26,530,000	24,452,000	1.00	1.50	1.51	1.39	0.22	0.78	0.23	0.77
5_3	1922	April 7, 1931	17,762,000	26,340,000	24,410,000	24,378,000	1.00	1.48	1.37	1.37	0.23	0.77	0.00	1.00
5_4	1923	April 7, 1931	18,314,000	26,870,000	27,630,000	25,084,000	1.00	1.47	1.51	1.37	0.21	0.79	0.27	0.73
6	January 1, 1923	February 27, 1926	237,000	359,000	....	359,000	1.00	1.52	....	1.52	0.00	1.00		
7	August 16, 1923	December 1, 1930	91,463,000	130,500,000	....	(a)117,803,000	1.00	1.43	....	(a)1.29	0.33	(a)0.67		
8	December 31, 1923	April 30, 1925	5,120,000	6,254,000	....	6,095,000	1.00	1.22	....	1.19	0.14	0.86		
9	January 1, 1924	November 22, 1926	13,200,000	20,378,000	21,323,000	(a) 17,059,000	1.00	1.54	1.62	(a)1.29	0.46	(a)0.54	0.53	(a)0.47

(a) Not less than these values.
(b) Upper line, company's figures; lower line, Commission's.

Case	Decided by
1. Denver v. Denver Union	United States Supreme Court
2. Galveston Electric Co. v. Galveston	United States Supreme Court
3. Missouri ex rel. v. Southwestern Bell Telephone Co.	United States Supreme Court
4. Bluefield Waterworks & Improvement Co. v. Public Service Commission (W. Va.)	United States Supreme Court
5_1–5_4. Excess Income of Richmond, Fredericksburg & Potomac R. R. Co.	Interstate Commerce Commission
6. Monroe Gas, Light & Fuel Co. v. Michigan Public Service Commission	United States District Court
7. Smith et al v. Illinois Bell Telephone Co.	United States Supreme Court
8. Southern Bell Telephone & Telegraph Co. v. Public Service Commission (S. C.)	United States District Court
9. McCardle et al. v. Indianapolis Water Co.	United States Supreme Court

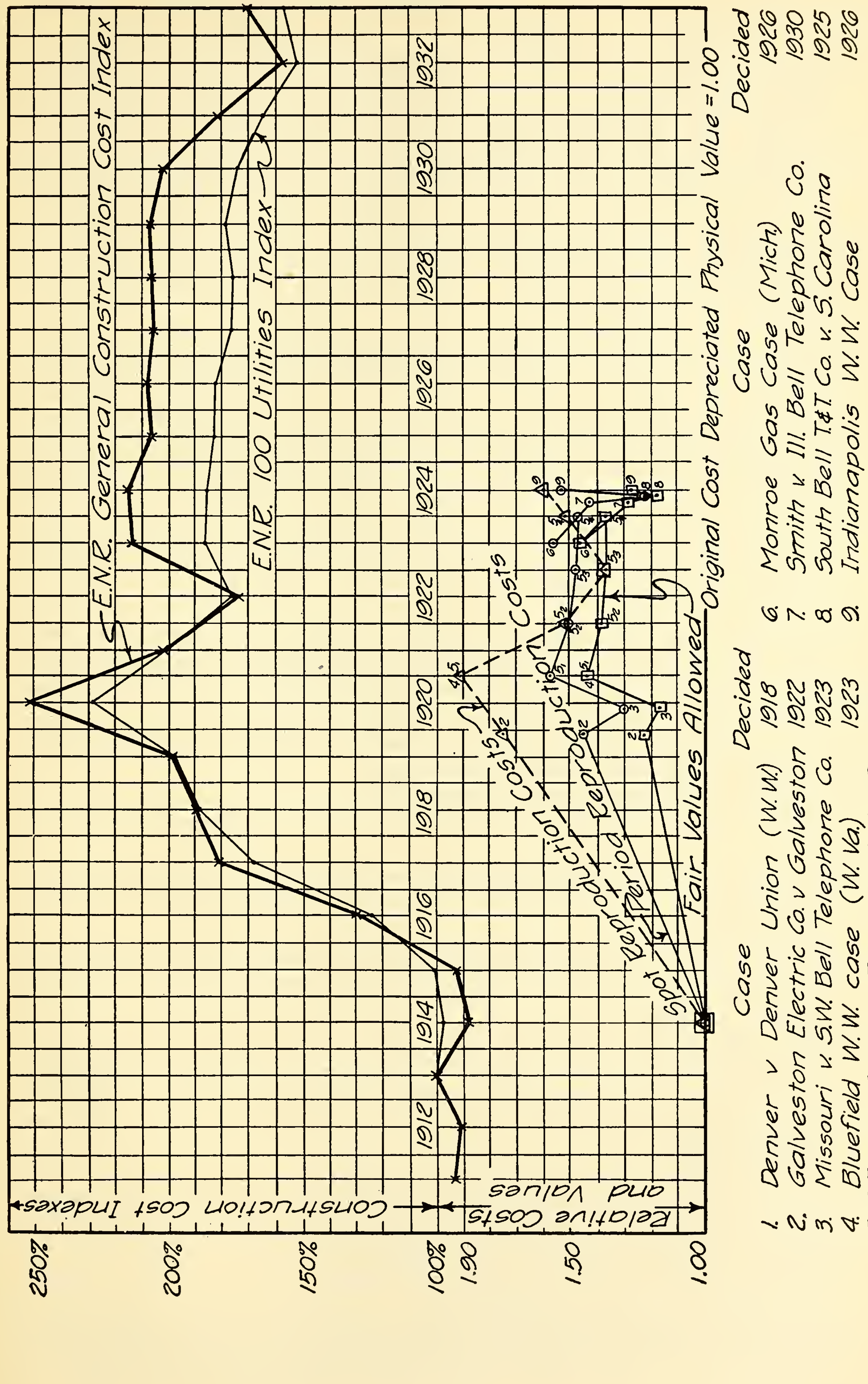


FIG. 7.1.—Price-level changes and valuation decisions since 1914.

relative weights given original costs and reproduction costs during these years will be discussed by periods.

1. *During the Early Years of Uniform Construction Cost-level Periods. Examples,* 1900 *to* 1907 *and* 1923 *to* 1929.—During the early years of uniform construction-cost price levels, the original costs and reproduction costs of new property units are equal, and the relative weights due the reproduction costs of old units gradually increase.[1] After a sufficient number of years, reproduction costs would be the true measure of value if, like original costs, they were matters of reliable book records instead of mere estimates.

   During 1900 to 1907, reliable book records of original costs were lacking in the case of most properties; largely for this reason, great weight was often given to estimated reproduction costs.

2. *During the Later Years of Long Uniform Construction Cost-level Periods. Example,* 1907 *to* 1914.—For many properties valued during 1907 to 1914, reproduction costs did not differ materially from the actual original costs; for most other properties valued, the differences were not greater than 15 percent. During 1907 to 1914, reliable book records of actual original costs were still comparatively rare; it was partly for this reason that such great weight continued to be given to reproduction costs (usually estimated on the five-year average past-prices basis). In the *Denver v. Denver Union* (Sec. 8.5) *Waterworks Case,* for example, the United States Supreme Court approved giving dominant weight to reproduction-cost estimates of this kind in a valuation dated 1914.

Because the long previous maintenance of the prevailing average price levels made their future continuation very probable, giving great weight to reproduction costs during 1907 to 1914 was in accord with the general principle stated by the United States Supreme Court in its *Indianapolis Water Case* decision.[1]

3. *During Periods of Rapidly Rising or Falling Construction-cost Price Levels. Examples,* 1915 *to* 1920, 1921 *to* 1923, 1929 *to* 1934.

During 1915 to 1920 (see Table 7.1 and Fig. 7.1).

a. Construction-cost levels rose rapidly to about 225 to 250 percent of pre-war levels.

b. Original-cost values increased gradually, owing to the continuing installation of new property units, at the higher prices, for replacements, improvements, and enlargements.

c. Spot (Sec. 11.18) reproduction-cost values rose with rapidity corresponding to that of construction costs, but only to about 190 per cent of the increased original-cost values.

d. Period (Sec. 11.19) reproduction-cost values, which should forecast prices for a few years in the future, lagged far behind spot cost levels; they reached only to about 160 percent of the increased original-cost values. This was logical in view of the uncertainty as to how far and how long prices would rise and remain high.

e. The fair values allowed lay between the original-cost values and the period reproduction-cost values; period reproduction cost was given a relative weight varying between 0.50 and 0.77 in the valuations shown in Table 7.1 and Fig. 7.1.

During 1921 to 1923 (see Table 7.1 and Fig. 7.1).

a. Construction-cost levels dropped violently in 1921, 1922, but recovered rapidly in 1922, 1923, though not to 1920 levels.

b. The gradual increase of original-cost values above pre-war levels continued, due to replacements, improvements, and enlargements at the new cost levels.

[1] See Case 3, Sec. 7.11.

c. Spot reproduction-cost levels dropped rapidly to below the period reproduction-cost-value level in 1922, but rose again above it in 1923.

d. Period reproduction-cost levels dropped somewhat in 1921, 1922, and then partially recovered in 1922, 1923; they did not fluctuate either down or up nearly as much as either construction costs or spot reproduction costs.

e. The fair values allowed lay between the original-cost values and the period reproduction-cost values; period reproduction cost was given a relative weight of 0.67 to 0.86 in the valuations shown in Table 7.1 and Fig. 7.1.

During 1929 to 1934 (see Fig. 7.1).

a. The violent drop in construction-cost levels during 1929 to 1933 caused both the spot and the period reproduction-cost values of many industrial properties to fall below their original-cost values.

b. The rise in construction-cost prices since March, 1933, has been too much affected by emergency depression relief measures to enable any reliable determination of future costs and their probable effects on valuation practice to be made as yet (1935).

The final extent and the permanency of the devaluation of the dollar and of the emergency relief measures which have increased labor costs and materials costs are as yet unknown; reliable data of their effects on values remain yet to be collected.

CHAPTER VIII

BRIEF SUMMARIES OF SIXTY-EIGHT IMPORTANT COURT VALUATION DECISIONS

Chapter VIII will be devoted to the presentation of brief summaries by the authors of the court decisions in the 68 valuation cases listed in Sec. 7.4. It is not claimed that all authorities will agree with all the authors' interpretations; the summaries have been prepared by engineers, not by lawyers.

Note.—The * which appears in a citation of a case indicates that the full text of the decision should be available to graduate students majoring in engineering valuation.

8.1. The Period before 1830 (Sec. 7.4–1).—There was no extensive development of modern utilities prior to 1830. Valuation questions were settled by the principles of the common and statute laws of property.

8.2. The Period 1831 to 1865 (Sec. 7.4–2).—Although extensive development of railway and some other modern utilities occurred, the general common and statute laws of property were made to answer, though with increasing difficulty.

1. *The Louisiana Bread Case. Guillote v. New Orleans.* 12 La. Ann. 432. La. Sup. Ct., June, 1857. Bakers can be regulated.

This was an early case, in which it was held that public utilities are subject to public regulation.

2. *Mellerish v. Keen.* 28 Beav. 453. English courts, July 6, 1860.

This was an English case, which is often referred to in the courts in the United States as furnishing an important precedent in good-will valuation. The good-will value was decided to be "worth one year's purchase of the net annual profits of the banking business, calculated on an average of the three years ending on the 31st day of December, 1858."

8.3. The Period from 1866 to 1900 (Sec. 7.4–3).—During this period the rapid development of modern utilities and the many resulting abuses and dissensions brought out a considerable number of important, ruling court valuation decisions, which did much to establish the present "law of the land" on formerly disputed valuation questions.

3. *Munn v. Illinois.* 94 U.S.* 123, 24 L. ed. 77. U.S. Sup. Ct., October, 1876.

In writing the decision in a later case, Mr. Justice Lamar has said: "The *Munn Case* is a landmark in the law. It is accepted as an authoritative and accurate statement of the principle on which the right to fix rates is based."

The *Munn Case* decision, written by Mr. Chief Justice Waite, upheld the right of the general assembly of Illinois "to fix by law the maximum of charges for the storage

of grain in warehouses at Chicago and other places in the state having not less than one hundred thousand inhabitants, 'in which grain is stored in bulk.' . . . "

Quoting in support a statement "by Lord Chief Justice Hale [England] more than two hundred years ago," the *Munn Case* decision held that, by common law: "Property does become clothed with a public interest when used in a manner to make it of public consequence, and affect the community at large. When, therefore, one devotes his property to a use in which the public has an interest, he, in effect, grants to the public an interest in that use, and must be controlled by the public for the common good, to the extent of the interest he has thus created."

Supporting cases cited included *Bolt v. Sennett*, England, beginning of nineteenth century; *Aldnutt v. Inglis*, England, 1810; *Mobile v. Yuille*, Ala. Sup. Ct., 1841.

The *Munn Case* decision further held that the Illinois statute questioned was not repugnant to section 8 or 9 of Art. 1, of the Constitution of the United States (establishing and limiting the right of Congress to regulate commerce), or to that part of the Fourteenth Amendment which ordains that no state shall "deprive any person of life, liberty, or property, without due process of law, nor deny to any person within its jurisdiction the equal protection of the laws."

4. *Stone et al. v. Farmers Loan & Trust Co.* 116 U.S. 307, 29 L. ed. 636. 6 Sup. Ct. Rep.* 334. U.S. Sup. Ct., Jan. 4, 1886.

This is one of the "railroad commission cases," in which the right of the several states to regulate their intrastate railways, by statute, through supervisory state railroad commissions, was upheld by the United States Supreme Court. Stone et al. constituted the Mississippi State Railroad Commission. The Farmers Loan and Trust Co. was a New York corporation which brought suit to enjoin the commission from enforcing, against the Mobile and Ohio Railway, the provisions of a Mississippi railway statute, passed March 11 and 15, 1884. The United States Circuit Court allowed an injunction by decree; from which appeal was taken by the Mississippi Railroad Commission.

The United States Supreme Court upheld the commission and reversed the circuit court decree for injunction. The opinion was written by Mr. Chief Justice Waite. It cites the decision, in *Munn v. Illinois*, "that, as to natural persons and corporations subject to legislative control, the state could, in cases like this, fix a maximum beyond which any charge would be unreasonable; . . . " The Court rejected the plea that, in this Mississippi case, the state had surrendered this right of control by a charter contract. The Court also rejected other claims against the validity of the Mississippi Railway Act.

On the other hand, Mr. Chief Justice Waite remarked, significantly, that: "it is not to be inferred that this power of limitation or regulation is itself without limit. This power to regulate is not a power to destroy, and limitation is not the equivalent of confiscation."

5. *Chicago, Milwaukee & St. Paul Ry. v. Minnesota.* 134 U.S. 418. 33 L. ed. 970. 10 Sup. Ct. Rep. 462. U.S. Sup. Ct., March 24, 1890.

It was held in this case that the courts necessarily have the power to declare a rate fixed by a commission to be unreasonable and illegal.

6. *Budd v. New York.* 143 U.S. 517. 36 L. ed. 247. 12 Sup. Ct. Rep. 468. U.S. Sup. Ct., Feb. 29, 1892.

This and *Brass v. North Dakota* (see below) are two of the pioneer decisions, upholding the power of rate regulation. "The underlying principle is that business of certain kinds holds such a peculiar relation to the public interests that there is super-induced upon it the right of public regulation." It is often said that such businesses are "endued with a public interest."

7. *Monongahela Navigation Co. v. United States.* 148 U.S. 312. 37 L. ed. 463. 13 Sup. Ct. Rep.* 622. U.S. Sup. Ct., March 27, 1893.

This is an often quoted decision in the case of a condemnation of a lock and dam (one of seven on the same river owned by a navigation company), which, by proceedings in the United States Circuit Court, had been valued at $209,000, "not considering or estimating in this decree the franchise of the company to collect tolls."

In its decision, written by Mr. Justice Brewer, the United States Supreme Court held "that the navigation company rightfully placed this lock and dam in the Monongahela River; that with the ownership of the tangible property, legally held in that place, it has a vested franchise to receive tolls for its use; that such franchise was as much a vested right of property as the ownership of the tangible property; . . . that just compensation requires payment for the franchise to take tolls as well as for the value of the tangible property; and that the assertion by Congress of its purpose to take the property does not destroy the state franchise." Accordingly, the judgment of the circuit court was reversed, and the case was remanded for new trial.

In the decision, the Court ruled that all the various elements of its value must be duly considered in determining the value of a property. "The value of property, generally speaking, is determined by its productiveness—the profits which its use brings to the owner. Various elements enter into this matter of value [mentioning fertility, situation, demand, and economy of cost of utilization as instances]. . . . For each separate use of one's property by others the owner is entitled to a reasonable compensation. . . . Whatever be the true value of that which it [Congress] takes from the individual owner must be paid to him. . . . and the question of just compensation is not determined by the value to the government which takes, but the value to the individual from whom the property is taken."

8. *Brass v. North Dakota.* 153 U.S. 391. 38 L. ed. 757. 14 Sup. Ct. Rep. 857. U.S. Sup. Ct., May 14, 1894. See case 6, above.

9. *Reagan v. Farmers Loan & Trust Co. et al.* 154 U.S. 362. 38 L. ed. 1014. 14 Sup. Ct. Rep.* 1047. U.S. Sup. Ct., May 26, 1894.

This decision was rendered in the case of an appeal by the Railroad Commission of Texas from an injunction decree, by the United States Circuit Court, restraining the enforcement of certain rates and regulations prescribed by the commission for the International and Great Northern Railroad Company; also restraining the commission "from making, issuing, or delivering [any further] tariff or tariffs, circular or circulars, order or orders," applying to the said company.

The United States Supreme Court decided "that the decree, as entered, must be reversed, in so far as it restrains the railroad commission from discharging the duties imposed by this [state railway] act, and from proceeding to establish reasonable rates and regulations, but must be affirmed so far only as it restrains the defendants from enforcing the rates already established."

The decision was written by Mr. Justice Brewer. That portion overruling the state's objection to including the annual "cost of road, equipment, and permanent improvements" as part of the annual operating expenses has already been cited (Sec. 6.1) to show that the United States Supreme Court had not yet come to recognize the true, essential character of depreciation, afterwards so completely stated and upheld by it in many decisions, beginning with *Knoxville v. Knoxville Water Co.*, 1909.

The Supreme Court overruled the state's contention that this was, "in effect, a suit against the State of Texas, brought by a citizen of another state, and therefore, under the eleventh amendment to the Constitution, beyond the jurisdiction of the federal court."

The right of the state to regulate railways through the commission was upheld; " . . . there can be no doubt of the general power of a state to regulate the fares and

freights which may be charged and received by railroad or other carriers, and that this regulation can be carried on by means of a commission." Mr. Justice Brewer said further, however, that " . . . while it is not the province of the courts to enter upon the merely administrative duty of framing a tariff of rates for carriage it is within the scope of judicial power and a part of judicial duty to restrain anything which, in the form of regulation of rates, operates to deny to the owners of property invested in the business of transportation that equal protection which is the constitutional right of all owners of other property."

10. *National Waterworks v. Kansas City.* 62 Fed. 853. U.S. Circ. Ct. of App., July 2, 1894.

This was an appraisal case, to determine the fair value of the waterworks property for purchase by the city, in accordance with a franchise contract. The decision was written by Circuit Justice Brewer, whose statement therein on "going value" is often cited as a pioneer ruling. He said:

"We are not satisfied that either method [capitalization of earnings, or the mere value of the naked physical property] by itself, will show that which, under all the circumstances, can be adjudged 'the fair and equitable value.' Capitalization of the earnings will not, because that implies a continuance of earnings, and a continuance of earnings rests upon a franchise to operate the water works. The original cost of the construction cannot control, for 'original cost' and 'present value' are not equivalent terms. Nor would the mere cost of reproducing the water works plant be a fair test, because that does not take into account the value which flows from the established connections between the pipes and the buildings of the city. . . . It [the city] steps into the possession of a property which not only has the pledge to earn, but is in fact earning. It should pay therefore not merely the value of the system which might be made to earn, but that of a system which does earn. . . . That valuation, after much discussion, comparison of figures, and readjustments, we have all agreed, is three millions of dollars; . . . "

In the lower court, the valuation was $2,714,000, without any allowance for the going value; proper addition for which this decision first definitely made a part of the law of the land. Evidently the going value allowed was about 10.54 percent of the present physical value.

11. *Page v. Ratliffe.* 75 Law Times Reports* 371. English High Court of Justice, Chancery Division, Oct. 29, Nov. 18, 1896.

This litigation was over the valuation of a partnership business (brewers, maltsters, and hop factors) at Northampton, for the purchase of a deceased partner's share. Concerning the good-will value of certain "tied" public houses (to which the beer was sold), the decision said: "It is agreed that good-will is ascertained by finding three years' net income."

12. *Smyth v. Ames.* 169 U.S. 466. 18 Sup. Ct. Rep.* 418. 42 L. ed. 819. U.S. Sup. Ct., March 7, 1898.

In this decision, written by Mr. Justice Harlan, the United States Supreme Court upheld a United States Circuit Court decree enjoining the enforcement of an 1893 Nebraska Railway Rate Act. There were three suits.

The Smyth v. Ames decision is the most important and the most often cited ever rendered in a valuation litigation. It established the fundamental principles of the engineering valuation of public-utility properties.

Because of the importance, selected verbatim quotations are given herewith, at considerable length, as follows:

"The first question to be considered is one common to all the cases. While it was not objected at the argument that there had been any departure from the ninety-fourth equity rule, it was contended that the plaintiffs had an adequate remedy

at law, and that the circuit court of the United States, sitting in equity, was therefore without jurisdiction. This objection is based upon the fifth section of the Nebraska statute, authorizing any railroad company to show, in a proper action brought in the supreme court of the state, that the rates therein prescribed are unreasonable and unjust, and, if that court found such to be the fact, to obtain an order upon the board of transportation permitting the rates to be raised. . . . We cannot accept this view of the equity jurisdiction of the circuit courts of the United States. The adequacy or inadequacy of a remedy at law for the protection of the rights of one entitled upon any ground to invoke the powers of a federal court is not to be conclusively determined by the statutes of the particular state in which suit may be brought. One who is entitled to sue in the federal circuit court may invoke its jurisdiction in equity whenever the established principles and rules of equity permit such a suit in that court; and he cannot be deprived of that right by reason of his being allowed to sue at law in a state court on the same cause of action.

. . . .

" . . . These principles must be regarded as settled:

"1. A railroad corporation is a person within the meaning of the fourteenth amendment declaring that no state shall deprive any person of property without due process of law, nor deny to any person within its jurisdiction the equal protection of the laws.

"2. A state enactment, or regulations made under the authority of a state enactment, establishing rates for the transportation of persons or property by railroad that will not admit of the carrier earning such compensation as, under all the circumstances, is just to it and to the public, would deprive such carrier of its property without due process of law, and deny to it the equal protection of the laws, and would, therefore, be repugnant to the fourteenth amendment of the constitution of the United States.

"3. While rates for the transportation of persons and property within the limits of a state are primarily for its determination, the question whether they are so unreasonably low as to deprive the carrier of its property without such compensation as the constitution secures, and therefore without due process of law, cannot be so conclusively determined by the legislature of the state, or by regulations adopted under its authority, that the matter may not become the subject of judicial inquiry.

. . . .

"If a railroad corporation has bonded its property for an amount that exceeds its fair value, or if its capitalization is largely fictitious, it may not impose upon the public the burden of such increased rates as may be required for the purpose of realizing profits upon such excessive valuation or fictitious capitalization; and the apparent value of the property and franchises used by the corporation, as represented by its stocks, bonds, and obligations, is not alone to be considered when determining the rates that may be reasonably charged. . . .

. . . .

"A corporation maintaining a public highway, although it owns the property it employs for accomplishing public objects, must be held to have accepted its rights, privileges, and franchises subject to the condition that the government creating it, or the government within whose limits it conducts its business, may, by legislation, protect the people against unreasonable charges for the services rendered by it. It cannot be assumed that any railroad corporation, accepting franchises, rights, and privileges at the hands of the public, ever supposed that it acquired, or that it was intended to grant to it, the power to construct and maintain a public highway simply for its benefit, without regard to the rights of the public. But it is equally true that the corporation performing such public services, and the people financially interested

in its business and affairs, have rights that may not be invaded by legislative enactment in disregard of the fundamental guaranties for the protection of property. The corporation may not be required to use its property for the benefit of the public without receiving just compensation for the services rendered by it. How such compensation may be ascertained, and what are the necessary elements in such an inquiry, will always be an embarrassing question. As said in the case last cited: 'Each case must depend upon its special facts; and when a court, without assuming itself to prescribe rates, is required to determine whether the rates prescribed by the legislature for a corporation controlling a public highway are, as an entirety, so unjust as to destroy the value of its property for all the purposes for which it was acquired, its duty is to take into consideration the interests both of the public and of the owner of the property, together with all other circumstances that are fairly to be considered in determining whether the legislature has, under the guise of regulating rates, exceeded its constitutional authority, and practically deprived the owner of property without due process of law. . . . The utmost that any corporation operating a public highway can rightfully demand at the hands of the legislature, when exerting its general powers, is that it receive what, under all the circumstances, is such compensation for the use of its property as will be just both to it and to the public.'

"We hold, however, that the basis of all calculations as to the reasonableness of rates to be charged by a corporation maintaining a highway under legislative sanction must be the fair value of the property being used by it for the convenience of the public. And, in order to ascertain that value, the original cost of construction, the amount expended in permanent improvements, the amount and market value of its bonds and stock, the present as compared with the original cost of construction, the probable earning capacity of the property under particular rates prescribed by statute, and the sum required to meet operating expenses, are all matters for consideration, and are to be given such weight as may be just and right in each case. We do not say that there may not be other matters to be regarded in estimating the value of the property. What the company is entitled to ask is a fair return upon the value of that which it employs for the public convenience. On the other hand, what the public is entitled to demand is that no more be exacted from it for the use of a public highway than the services rendered by it are reasonably worth. But even upon this basis, and determining the probable effect of the act of 1893 by ascertaining what could have been its effect if it had been in operation during the three years immediately preceding its passage, we perceive no ground on the record for reversing the decree of the circuit court. On the contrary, we are of opinion that as to most of the companies in question there would have been, under such rates as were established by the act of 1893, an actual loss in each of the years ending June 30, 1891, 1892, and 1893; and that, in the exceptional cases above stated, when two of the companies would have earned something above operating expenses in particular years, the receipts or gains, above operating expenses, would have been too small to affect the general conclusion that the act, if enforced, would have deprived each of the railroad companies involved in these suits of the just compensation secured to them by the constitution. Under the evidence, there is no ground for saying that the operating expenses of any of the companies were greater than necessary."

Some of the high points of the famous *Smyth v. Ames* decision may be summarized as follows:

a. The Right to Appeal from a State Rate Law to the Federal Courts.—The fact that the state law provided that the companies might appeal to the Nebraska Supreme Court did not take away the right of appeal to the federal courts by any one entitled to sue therein.

b. Suit against the officers of the state to enjoin the enforcement of the rate law was not a suit against the state, and therefore was not forbidden by the Eleventh Amendment to the federal Constitution.

c. Intrastate Railway Rates Primarily under State Jurisdiction.—The making of railway rates between points wholly within a state "is a subject primarily within the control of the state."

d. Railway Corporations Entitled to Protection under Fourteenth Amendment to the Constitution.—A railway corporation is a person within the meaning of the Fourteenth Amendment to the federal Constitution, and hence is entitled to protection against state laws establishing intrastate railway rates so low as to violate the provision in the Fourteenth Amendment that "no state shall deprive any person of property without due process of law, nor deny to any person within its jurisdiction the equal protection of the laws."

e. Stock-and-bond Value.—" . . . the apparent value of the property and franchises used by the corporation, as represented by its stocks, bonds, and obligations, is not alone to be considered. . . . "

f. Utility Corporations Subject to Regulation by Public.—A utility corporation " . . . must be held to have accepted its rights, privileges, and franchises subject to the condition that the government . . . may by legislation protect the people against unreasonable charges for the services rendered by it."

g. Utility Corporations Entitled to Protection against Unjust Regulation.—"But it is equally true that the corporation performing such public services and the people financially interested in its business and affairs, have rights that may not be invaded by legislative enactment in disregard of the fundamental guaranties for the protection of property."

h. How just compensation for the services rendered by a utility " . . . may be ascertained, and what are the necessary elements in such an inquiry, will always be an embarrassing question. . . . "

i. "We hold, however, that the basis of all calculations as to the reasonableness of rates to be charged . . . must be the fair value of the property being used . . . for the convenience of the public."

j. In determining fair value all factors affecting value must be given the weight due in each case. "In order to ascertain the fair value [the factors affecting the fair value] . . . are all matters for consideration, and are to be given such weight as may be just and right in each case."

While "we do not say that there may not be other matters to be regarded . . . ," the court enumerated: (1) "original cost," (2) "the present as compared with the original cost," (3) "the probable earning capacity . . . under particular rates prescribed by statute, and the sum required to meet operating expenses," (4) "the amount and market value of its bonds and stock."

k. "What the company is entitled to ask is a fair return upon the value of that which it employs for the public convenience."

l. "On the other hand, what the public is entitled to demand is that no more be exacted from it . . . than the services rendered . . . are reasonably worth."

Note on Depreciation in Court Decisions Prior to 1909.—No mention of depreciation was made in the *Smyth v. Ames* decision. Neither the federal nor the state courts came to understand and announce clearly the true character of depreciation in relation to fair value and to production expense until the United States Supreme Court decision in *Knoxville v. Knoxville Water Co.*, 1909.

13. *People ex rel. A. J. Johnson Co. v. Roberts, Comptroller.* 159 N.Y. 70. 53 N.E. Rep.* 685. N.Y. Ct. of App., April 25, 1899.

This was a privilege tax case, appealed from the appellate division of the New York Supreme Court, which affirmed the comptroller's assessment, based on property values including intangible values of copyrights and of good will. Although incorporated in West Virginia, the company conducted its business of printing and selling Johnson's Encyclopedia almost entirely in New York.

a. Copyrights and Patent Rights Not Taxable.—"Copyrights stand on the same basis as patent rights, with reference to taxation by the state, and, as we have held that the former are exempt, the latter should be held exempt also."

b. Good-will Value. Its Nature and Its Taxability.—"The remaining question presented for decision, therefore, is whether the good will of the relator was 'capital employed by it within this state.' 'Good will' is a modern but important growth of the law, not mentioned by some of the early writers, but given great prominence at the present time."

The Court cites some seven English and four American decisions on good will, and quotes explanations of good will by various legal authorities.

"In *Barber v. Insurance Co.*, 15 Fed. 312, it was said that 'the good will of an established business, which is a common subject of contract, is nothing but the chance of being able to keep the business which has been established.'"

In this Johnson Encyclopedia case, the New York Court of Appeals held that the good will was taxable in New York, and said: "While the order of the Appellate Division must be reversed, we base our reversal solely upon the ground that the copyrights of the relator are not subject to taxation by the state."

8.4. The Period 1901 to 1915 (Sec. 7.4–4).—During this period, construction-cost prices maintained a nearly constant level. Adequate, reliable book records of the actual original costs of existing property units were generally lacking; partly in consequence, many engineering valuations were based mainly on reproduction costs new less depreciation. In 1909, the United States Supreme Court made it the law of the land that depreciation must be deducted in determining fair values and allowed in determining annual revenues yielding fair rates of net return.

14. *Cedar Rapids Water Co. v. Cedar Rapids.* 118 Ia. 234. 91 N. W. 1081. Iowa Sup. Ct., Oct. 27, 1902.

This rate decision is cited because it shows the entire failure of the courts, at that time, to realize the real existence or the true nature of depreciation; any allowance for which, in determining the annual revenues required to yield fair returns, the Court overruled, saying: " . . . to hold otherwise is to say that the public must not only pay the reasonable and fair value of the services rendered, but must in addition pay the company the full value of its works every forty years."

15. *San Diego Land & Town Co. v. Jasper.* 189 U.S. 439. 23 Sup. Ct. Rep. 571. 47 L. ed. 892. U.S. Sup. Ct., April 6, 1903.

In this decision, written by Mr. Justice Holmes, who remained a member of our highest court till 1932, the United States Supreme Court overruled the "contention of the appellant, that there should have been an allowance for depreciation, over and above the cost of repairs," in the annual revenues required to yield a fair rate of net return.

In the Knoxville v. Knoxville Water Co. decision, the same Court reversed itself on this question only six years later.

16. *Brunswick & Topsham Water District v. Maine.* 99 Me. 371. 59 Atl. 537. Me. Sup. Ct., Dec. 14, 1904.

This waterworks valuation case is noteworthy especially for the definite instructions issued by Judge Savage to govern the appraisers in their work. Such of these instructions as have a special bearing on service-worth value are quoted in Sec. 15.5.

17. *Von Au v. Magenheimer et al.* First trial. 115 App. Div. 84. 100 N.Y. Supp.* 659, Oct. 12, 1906. Second trial, 126 App. Div. 257. 110 N.Y. Supp.* 629, May 1, 1908. N.Y. Sup. Ct., Appellate Division, Second Department.

The decision in this case is frequently cited in the United States on good-will valuation questions in connection with ordinary private business enterprises. It was a damage case, tried twice.

It was held in the decision following the first trial that our courts have not adopted the English rule that the measure of good-will value is one year's purchase of the three years average net profits of the business as in case 2, above, or three years' purchase, as in case 11, above; but that our courts, "on the contrary, incline to the more equitable rule that

"The value of good-will may be fairly arrived at by multiplying the average net profits [above fair "interest," (fair net return)] by a number of years, such number being suitable and proper, having reference to the nature and character of the particular business under consideration, and the determination of such proper number of years should be submitted to and determined by the jury as a question of fact."

As a result of the first appeal, the jury's award was found excessive and the case was remanded for a new trial. The above rule was read to the second jury, which found that the good-will value in this case was approximately six times the average annual net profits over 6 percent. Upon the second appeal, this verdict was upheld. (The original verdict awarded 13 "years' purchase.")

Note.—The number of "year's purchase" found to be proper varies in different cases. In various cases (*in re McMullin's Estate*, below, and cited therein), one and two-thirds, two, three, five, and six years' purchase have been allowed; most often three.

18. *In re Keahon's Estate.* 113 N.Y. Supp. and 147 N.Y. Rep.* 926. Surr. Ct., New York County, September, 1908.

This was another New York good-will valuation case, appealed from a tax appraisal. The decision followed the principles laid down in the *Von Au Case*, just above, but, under the circumstances proven in the *Keahon Case*, the good-will value was held to be three "years' purchase" (instead of six) of the "net profits," (over six percent net return).

19. *Knoxville v. Knoxville Water Co.* 212 U.S. 1. 53 L. ed. 271. 29 Sup. Ct. Rep.* 148. U.S. Sup. Ct., Jan. 4, 1909.

This decision is famous as the first in which the United States Supreme Court established, as the law of the land, the principles that all accrued actual depreciation must be deducted from the value of property new to determine its present "fair value"; that it is the duty of the owner of the property to set aside each year, out of current earnings before net return, sufficient annual depreciation appropriations to keep his investment unwasted by depreciation; and that it is the owner's right to charge "fair rates" and/or "fair prices" which will yield "fair net returns," in addition to correct annual depreciation allowances.

The case came to the United States Supreme Court on an appeal by the city from a United States Circuit Court decree enjoining the enforcement of a rate ordinance passed by the city council. The adverse decree was overruled, because it was only the error of the lower court in failing to deduct depreciation in determining fair value which made the proposed rates appear confiscatory. The opinion was written by Mr. Justice Moody, who said, in part:

"The first fact essential to the conclusion of the court below is the valuation of the property devoted to the public uses, upon which the company is entitled to earn a return. That valuation ($608,000) must now be considered. It was made up by adding to the appraisement, in minute detail of all the tangible property, the sum of $10,000 for 'organization, promotion, etc.,' and $60,000 for 'going concern.' The latter sum we understand to be an expression of the added value of the plant as a whole over the sum of the values of its component parts, which is attached to it because it is in active and successful operation and earning a return. We express no opinion as to the propriety of including these two items in the valuation of the plant, for the purpose for which it is valued in this case, but leave that question to be considered when it necessarily arises. We assume, without deciding, that these items were properly added in this case. The value of the tangible property found by the master is, of course, $608,000 lessened by $70,000, the value attributed to the intangible property, making $538,000. This valuation was determined by the master by ascertaining what it would cost, at the date of the ordinance, to reproduce the existing plant as a new plant. The cost of reproduction is one way of ascertaining the present value of a plant like that of a water company, but that test would lead to obviously incorrect results if the cost of reproduction is not diminished by the depreciation which has come from age and use. The company contends that the master, in fixing upon the valuation of the tangible property, did make an allowance for depreciation, but we are unable to agree to this. The master nowhere says that he made allowance for depreciation, and the language of his report is inconsistent with such a reduction. . . .

"The cost of reproduction is not always a fair measure of the present value of a plant which has been in use for many years. The items composing the plant depreciate in value from year to year in a varying degree. Some pieces of property, like real estate for instance, depreciate not at all, and sometimes, on the other hand, appreciate in value. But the reservoirs, the mains, the service pipes, structures upon real estate, stand pipes, pumps, boilers, meters, tools and appliances of every kind begin to depreciate with more or less rapidity from the moment of their first use. It is not easy to fix at any given time the amount of depreciation of a plant whose component parts are of different ages, with different expectations of life. But it is clear that some substantial allowance for depreciation ought to have been made in this case. [The Court here mentions $77,000, about 14 percent of $538,000, estimated by the company, and $118,000, about 32 percent of $363,000, estimated by the city.] In the view we take of the case it is not necessary that we should undertake the difficult task of determining exactly how much the master's valuation of the tangible property ought to have been diminished by the depreciation which that property had undergone. It is enough to say that there should have been a considerable diminution, sufficient, at least, to raise the net income found by the court above 6 per cent upon the whole valuation thus diminished. If, for instance, the master's valuation should be diminished by $50,000, allowed for depreciation, the net earnings found by him would show a return of substantially 6.5 per cent. . . .

"A water plant, with all its additions, begins to depreciate in value from the moment of its use. Before coming to the question of profit at all the company is entitled to earn a sufficient sum annually to provide not only for current repairs, but for making good the depreciation and replacing the parts of the property when they come to the end of their life. The company is not bound to see its property gradually waste, without making provision out of earnings for its replacement. It is entitled to see that from earnings the value of the property invested is kept unimpaired, so that, at the end of any given term of years, the original investment remains as it was at the beginning. It is not only the right of the company to make such a provision, but it is its duty to its bond and stockholders, and, in the case of a public service corpora-

tion, at least, its plain duty to the public. If a different course were pursued the only method of providing for replacement of property which has ceased to be useful would be the investment of new capital and the issue of new bonds or stocks. This course would lead to a constantly increasing variance between present value and bond and stock capitalization,—a tendency which would inevitably lead to disaster either to the stockholders or to the public, or both. If, however, a company fails to perform this plain duty and to exact sufficient returns to keep the investment unimpaired, whether this is the result of unwarranted dividends upon overissues of securities, or of omission to exact proper prices for the output, the fault is its own. When, therefore, a public regulation of its prices comes under question, the true value of the property then employed for the purpose of earning a return cannot be enhanced by a consideration of the errors in management which have been committed in the past."

. . . .

Some of the high points of the Knoxville decision may be summarized as follows:

a. "The courts, on constitutional grounds, may exercise the power of refusing to enforce legislation"; but "that power ought to be exercised only in the clearest cases."

b. Findings of Special Masters Not Conclusive.—The "respect due from the judicial to the legislative authority" renders it "not tolerable" that the findings of fact by the master, even when confirmed by the trial court, and though not without weight shall be conclusive unless they are "proved to the satisfaction" of the higher court.

c. Accrued depreciation must be deducted in determining the "fair value" of utility property.

d. It is both the right and the duty of the owner company to recoup annual depreciation out of earnings, before figuring any net return, so as to keep the investment in the property unimpaired. If this has not been done, "the true value of the property then employed for the purpose of earning a return cannot be enhanced by the errors in management that have been committed in the past."

e. The Court assumed, "without deciding," that the sum of $10,000 was properly allowed in the valuation for "organization, promotion, etc." This is about 2 percent of the present depreciated physical value.

f. The Court assumed, "without deciding," that the sum of $60,000 was properly allowed in the valuation for the "going concern." This is about 13 percent of the present depreciated physical value.

g. In this case, the capitalization in stocks and bonds was entitled to little weight, because "considerably in excess of valuation testified to by any witness, or which can be arrived at by any process of reasoning."

h. Evidence of actual, pertinent, subsequent history should have been considered. The ordinance was passed in 1901, while the final decision was announced Jan. 4, 1909. The Court held that evidence offered by the city that the company's gross income had largely increased after 1901 should have been considered (though it was not) by the lower court in making its decision.

20. *Willcox v. Consolidated Gas Co.* 212 U.S. 19. 29 Sup. Ct. Rep.* 192. 53 L. ed. 382. U.S. Sup. Ct., Jan. 4, 1909.

In this New York case, decided the same day as the *Knoxville Case,* the United States Supreme Court reversed a United States Circuit Court decree, voiding the 80-cent New York gas law, but did not rule definitely on the lower court's approval of the master's conclusion that substantially no depreciation should be deducted from reproduction-cost physical value new; the questioned rates were found not to be confiscatory whether or not depreciation was deducted.

Three important rulings in the decision were:

a. Appreciation.—It was held that the owners of the property were entitled to the benefit of increase in value by appreciation, with possible exceptions where the increase in value may be so enormous "as to render a rate permitting a reasonable return upon such increased value unjust to the public."

b. Franchise Value.—The Court recognized as binding, "the value of the company's franchises as fixed in 1884 under legislative authority and made the basis for stock issued at that time." The franchise value thus approved was $7,781,000.

c. The Court excluded good-will value, because "The complainant has a monopoly in fact, and a consumer . . . will resort to the 'old stand' because he cannot get gas anywhere else."

21. *Omaha v. Omaha Water Co.* 218 U.S. 180. 54 L. ed. 991. 30 Sup. Ct. Rep. 615. U.S. Sup. Ct., May 31, 1910.

This decision was rendered in an appeal from the lower court's approval of an appraisal for purchase by the city, under a franchise contract.

The Court approved the appraisal, which included an allowance of $562,712.45 for "going value," about 9.9 percent of the present physical value. The Court expressly stated that "the difference between a dead plant and a live plant [the going value] is a real value, and is independent of any franchise to go on or any mere good will as between such a plant and its customers. That kind of good will as suggested in *Willcox v. Consolidated Gas Co.*, 212 U.S. 19, is of little or no commercial value when the business is, as here, a natural monopoly, with which the customer must deal, whether he will or no."

22. *The Minnesota Rate Cases.* 230 U.S. 352 (three cases, *George T. Simpson et al.*, Appts., *v. David C. Shepard, also v. Emma B. Kennedy et al., also v. William Shillaber*). 33 Sup. Ct. Rep.* 729. 57 L. ed. 1511. U.S. Sup. Ct., June 9, 1913.

This is one of the leading valuation decisions, often cited. The cases involved the constitutionality of intrastate railway rate schedules prescribed by Minnesota railway acts, approved April 4 and 18, 1907, and commission orders dated Sept. 6, 1906, and May 3, 1907. *Simpson et al.* constituted the Minnesota Railroad and Warehouse Commission. The suits were brought by stockholders of the Northern Pacific Railway Co., the Great Northern Railway Co., and the Minneapolis & St. Louis Railroad Co. The United States Circuit Court held in favor of the railways; its adverse decrees were reversed by the United States Supreme Court in this decision, written by Mr. Justice Hughes. High points of the decision include the following:

a. The Right of the State to Regulate Intrastate Rates for Interstate Carriers.—In a long, exhaustive discussion, the right of the state to regulate intrastate rates on interstate carriers was upheld.

(1) This right of the states was repeatedly approved by the courts prior to the passage of the Interstate Commerce Act.

(2) The Interstate Commerce Act did not take away this right.

(3) "The decisions of this court since the passage of the act to regulate commerce have uniformly recognized that it was competent for the state to fix such rates, applicable throughout its territory."

b. The Proper Function of the Courts in Passing on Rate Legislation.—The Court held that "the rate making power is a legislative power, and necessarily implies a range of discretion. We do not sit as a board of revision to substitute our judgment for that of the legislature, or of the commission lawfully constituted by it, as to matters within the province of either. . . . "

c. The Governing Principles for Ascertaining Fair Value.—The Court held that, in determining whether, by these acts and orders, "the state has overstepped

the constitutional limit by making the rates so unreasonably low that the carriers are deprived of their property without due process of law and denied the equal protection of the laws":

"(1) The basis of calculation is the 'fair value of the property' used for the convenience of the public. *Smyth v. Ames*. . . ." [This excludes property owned but not used.]

"(2) The ascertainment of that value is not controlled by artificial rules. It is not a matter of formulas, but there must be a reasonable judgment having its basis in a proper consideration of all relevant facts."

d. Stock-and-bond Value.—Referring to the stocks and bonds of the Northern Pacific Railway, one of the parties to the litigation, the Court found that: "These securities and their values rest upon the entire property of the company" (including assets of considerable value not "devoted to the public service"). It was held that "the master was undoubtedly right" in his view that "it follows that value of bonds and stocks is wholly unreliable and cannot be used in these cases as an element in determining the value. . . . "

e. Valuation of Railway Lands.—The *Minnesota Rate Case* decision held that **railway lands must be valued at their present market value,** as determined by the actual present market value of similar lands adjacent, or near by; without allowing any "railway value," for severance damages and condemnation or other extra acquisition costs (which previously had often been allowed in railway cases, in accordance with a "reproduction-cost rule"). The "present-market-value rule" here established is now followed generally in railway valuations, including all those of the Interstate Commerce Commission.

f. Overhead Expenses Should Not Be Added in Valuing Railway Lands.—It was held that: "We also think it was error to add" overhead expenses for "'engineering superintendence,' 'legal expenses,' 'contingencies,' and 'interest during construction,'" to the present market value of the lands.

g. Depreciation.—The *Minnesota Rate Case* decision strongly and clearly upheld the *Knoxville Water Case* decision, that **accrued depreciation must be deducted from value new** in determining fair value.

"The property other than land, as the detailed statement shows, embraced all items of construction, including roadbed, bridges, tunnels, etc., structures of every sort, and all appliances and equipment. . . . "

"The master allowed the cost of reproduction new without deduction for depreciation. It was not denied that there was depreciation in fact. As the master said 'everything on and above the roadbed depreciates from wear and weather stress. The life of a tie is from eight to ten years only. Structures become antiquated, inadequate, and more or less dilapidated. Ballast requires renewal, tools and machinery wear out. Cars, locomotives, and equipment, as time goes by, are worn out or discarded for newer types.'" The master proposed to offset depreciation by the appreciation, from the consolidation and gradual improvement of embankments, roadbeds, etc., which he claimed more than offset the depreciation.

"We cannot approve this disposition of the question of depreciation. It appears that the master allowed, in the cost of reproduction, the sum of $1,613,612 for adaptation and solidification of roadbed, this being included in the item of grading, and being the estimate of the engineer of the state commission of the amount to be allowed. It is also to be noted that the depreciation in question is not that which has been overcome by repairs and replacements, but is the actual existing depreciation in the plant as compared with a new one. It would seem to be inevitable that in many parts of the

plant there would be such depreciation, as, for example, in old structures and equipment remaining on hand. And when an estimate of value is made on the basis of reproduction new, the extent of existing depreciation should be shown and deducted." The company, the Northern Pacific Railway, did so in its statement to the Interstate Commerce Commission as of March, 1907. In the present case, the engineer of the state commission estimated between $8,000,000 and $9,000,000 existing depreciation.

h. Intrinsic Appreciation.—The United States Supreme Court, in holding that it could not approve of the disposition made of depreciation by the master, by offsetting it against intrinsic appreciation, said: "If there are items entering into the estimate of the cost which should be credited with appreciation, this also should appear, so that instead of a broad comparison there should be specific findings showing the items which enter into the account of physical valuation on both sides."

i. Apportionment of Value between Intrastate and Interstate Business.—In apportioning the value of the railway properties between intrastate and interstate business, the United States Supreme Court rejected gross earnings as a basis and held that "there should be assigned to each business that proportion of the total value of the property which will correspond to the extent of its employment in that business."

The units of measurement of such "extent of employment" will vary in different cases. Freight ton-miles and passenger train-miles are examples of possible "comparable use units."

23. *The Missouri Rate Cases, Knott v. Chicago, Burlington & Quincy Railroad Co.* 230 U.S. 474. 33 Sup. Ct. Rep. 976. 57 L. ed. 1571. U.S. Sup. Ct., June 16, 1913.

Some nine companies were involved in the Missouri rate cases. The decision was written by Mr. Justice Hughes. It was held that:

a. The proof of fair value must be full and convincing.

b. Tax assessments are not necessarily good evidence of true value.

c. The contention raised by complainants (the railroad companies) "that these legislative acts cannot be enforced against one company unless enforced against all cannot be sustained . . . each complainant in order to succeed must show that as to it the rates are confiscatory."

24. *In re Ball's Estate.* 161 App. Div. 79. 146 N.Y. Supp.* 499. N.Y. Sup. Ct., App. Div., Second Department, March 6, 1914.

This was a case appealed from the surrogate's court, Suffolk County, involving the valuation of the good will of the business of the merchandising firm of Best & Co. The appellate division allowed three years' purchase of the annual net profits (averaged over the three years preceding) instead of the two allowed by the surrogate.

25. *German Alliance Insurance Co. v. Lewis.* 233 U.S. 389. 34 Sup. Ct. Rep. 612. 58 L. ed. 1011. U.S. Sup. Ct., April 20, 1914.

The question at issue in this case was the right, under the Constitution, of a state (Kansas in this instance) to regulate fire-insurance rates. Such right was upheld, three justices dissenting, though little physical property was owned.

The decision suggests the possibility of a wide future extension of the lists of business enterprises which may be held sufficiently "clothed with a public interest" to give the public the right to regulate them as public utilities. But see the decision in *New State Ice Co. v. Liebmann* (Sec. 8.7).

26. *San Joaquin & Kings River Canal & Irrigation Co. v. County of Stanislaus.* 233 U.S. 454. 34 Sup. Ct. Rep. 652. 58 L. ed. 1041. U.S. Sup. Ct., April 27, 1914.

This litigation was over the regulation by the county of the rates charged by the irrigation company. The United States Supreme Court, reversing a decree of the

circuit court, held that the water rights should have been valued as part of the property, and, in consequence, that the proposed rates were confiscatory and should be enjoined.

Note on Legal Bases for Valuing Water Rights.—The Court said: "We are not called upon to decide what the rate shall be, or even the principle by which it shall be measured." Three bases for valuing water rights have been suggested in various litigations:

a. The Present Market Value of Similar Water Rights.—This seems to have the greatest weight of authority in support. See case 31, below, *Denver v. Denver Union Water Co.*, report of Special Master Chinn, 1915, and *re Pacific Gas & Electric Company*, California Commission, decided Dec. 30, 1922. P.U.R. 1923C, 385.

b. The actual cost of acquiring the water rights in question.

c. The capitalized value of the saving in cost of service from the water rights in question, as compared with service from other sources (for example, water power as compared with steam power). This has to be based on estimates, subject to dispute, and giving results varying widely with different assumptions as to conditions and prices.

27. *In re Demarest's Estate.* 157 N.Y. Supp.* 653. Surrogate's Court, New York County, May 6, 1914.

28. *Yellow Taxicab Case.* 156 N.Y. 5580. N.Y. Sup. Ct., App. Div., 1915.

29. *In re McMullen's Estate.* 92 Misc. Rep. 637. 157 N.Y. Supp.* 655. Surrogate's Court, Bronx County, December, 1915.

The decisions in cases 27, 28, and 29 are cited herein because of their rulings on the good-will values of three New York state businesses. The unit of value adopted in each case was the average annual profit (called one "year's purchase") in excess of 6 percent on other values. The good-will values were held to be:

In case 27, three years' purchase.

In case 28, three years' purchase.

In case 29, one and two-third years' purchase.

The number of years' purchase adopted was based on the evidence as to the facts and conditions in each case.

30. *Des Moines Gas Co. v. Des Moines.* 238 U.S. 153. 35 Sup. Ct. Rep.* 811. 59 L. ed. 1244. P.U.R. 1915D, 577. U.S. Sup. Ct., June 14, 1915.

This was a rate ordinance case. The decision, written by Mr. Justice Day, affirmed a modified decree of the United States District Court, upholding the ordinance and dismissing without prejudice the company's bill for an injunction. The decision is often cited for its ruling against including in fair value the extra costs of reproduction due to paving built after mains were laid; also for its pronouncements on going value.

a. The Court upheld, for this case, the reproduction-cost-new-less-depreciation method, which the master found to be the only practical way, in his judgment, to value the physical property (outside of land and supplies), under the circumstances of this particular case.

b. Paving.—The Court upheld the action of the master and the decision of the lower court that the item of $140,000, claimed for extra cost of reproduction of water mains due to existing pavements constructed after the mains had been laid, should be excluded.

c. Overheads.—The Court approved the master's addition of 15 percent overhead construction costs ($296,245), for items as follows:

(1) General overhead, including the cost of administration, legal expenses, accidents, and injuries to workmen; and "time and money expended in

promotion, . . . interesting capital, . . . obtaining franchise, . . . incorporating. . . . "

(2) Engineering.

(3) A moderate allowance for omissions and contingencies.

(4) Interest lost during construction.

d. Organization Expenses.—The Court approved a separate allowance of $6,923 for organization expenses, in addition to preliminary expenses included in general overhead, above. This was 0.35 percent of the present depreciated physical value.

e. Going Value.—The Court said about going value "that there is an element of value in an assembled and established plant doing business and earning money over one not thus advanced is self-evident. This element of value is a property right and should be considered in determining the value of the property, upon which the owner has a right to make a fair return."

Nevertheless, the Court upheld the master's exclusion of his $300,000 estimated going value; on the ground that going value was already sufficiently allowed in the items for overhead costs and organization expenses.

f. Working Capital.—The Court quoted, with apparent but no stated approval, the language of the master in allowing $140,000 for working capital. This was 7.1 percent of the present depreciated physical value.

g. Depreciation.—The Court commended the master's method of valuation in which 14.7 percent accrued depreciation was deducted from the reproduction cost new, and the 15 percent overhead allowance was included in the depreciation base, in order to determine the present value of all physical property except land and supplies.

h. Upon the subject of fair return rate and confiscatory rate, the Court said: "Nor do we think there was error in refusing an injunction upon the conclusion reached that a return of 6 per cent per annum on the valuation would not be confiscatory. This is especially true in view of the fact that the ordinance was attacked before there was opportunity to test the results by actual experience. It is true the master reported that in his opinion the company ought to earn 8 percent, but he also found that in his judgment gas could be produced for 60 cents per thousand, and the actual effect of the 90-cent rate on an economically managed plant had not had the test of experience."

8.5. The Period 1916 to 1925.—The tremendous rise in construction costs between 1915 and 1920, due to the World War, forced the reproduction costs of items of industrial properties away above their original costs and brought on violent differences of opinion as to their "just and right" relative weights (Sec. 7.16, Fig. 7.1); the reproduction-cost-new-less-depreciation and the prudent-investment valuation formulas each gained ardent support. In important valuation decisions after the war, the United States Supreme Court continued to reject all valuation formulas, and to uphold the *Smyth v. Ames* rule, that all elements of value must be given such weight as sound judgment determines are "just and right" in each particular case.

31. *The City and County of Denver et al. v. Denver Union Water Co.* 246 U.S. 187. 38 Sup. Ct. Rep.* 278. 62 L. ed. 649. P.U.R. 1918C, 640. U.S. Sup. Ct., March 4, 1918.

The company's franchise expired in 1910; in 1914 the city passed a rate ordinance, reciting in the preamble that the company was a mere tenant by sufferance. Upon a report filed and testimony taken in 1915, by Special Master William J. Chinn, the United States District Court enjoined the enforcement of the ordinance; upon appeal, the United States Supreme Court upheld the injunction decree. The decision was written by Mr. Justice Pitney, and is often cited. Justices Holmes, Brandeis, and Clark dissented.

a. The Legal Effect of the Expiration of the Franchise.—As against the claim that the expiration of the franchise in 1914 impaired the fair value of the waterworks property, it was held that: "Without attributing to the initiators and to the city council a purpose to subject the inhabitants to grave danger of disease or worse, . . . the alternative, which we adopt, is to construe the ordinance as the grant of a new franchise of indefinite duration, terminable either by the city or by the company at such time and under such circumstances as may be consistent with the duty that both owe to the inhabitants of Denver."

b. The Method of Determining the Fair Value.—The special master made the valuation on the basis of reproduction cost at pre-war prices, less depreciation, the general trend of pre-war waterworks construction-cost prices being neither up nor down. This was approved, Mr. Justice Pitney saying: "What we have said establishes the propriety of estimating complainant's property on the basis of present market values as to land and reproduction cost less depreciation as to structures." The total value allowed, including overheads and intangibles was $13,415,899.

c. Paving.—The special master and the Court rejected the item of $539,558 claimed on account of pavement constructed over mains after they were laid.

d. Property to Be Included in the Valuation.—In characterizing the property to be valued, Mr. Justice Pitney used the expressions: "Adequate compensation for its property employed, and necessarily employed, in the public service"; "property used and actually useful in a public service."

e. Land Valuation.—The land was valued at its "present market value," indicated by the values of similar neighboring lands, as in the Minnesota rate cases.

f. Overhead Costs.—The special master allowed 12½ percent of the "fair contract cost" of the structural property for "engineering and contingencies," in addition to interest during construction, but allowed no other general overheads.

g. Water Rights.—An allowance of $2,947,617 for water rights (which were of special value in this arid country) was made by the special master, who said: "The measure of value of water rights used by me is the present market value of similar water rights in the same locality." The United States Supreme Court did not find it necessary to decide the correctness of the water-rights valuation, but the city proceeded to purchase the waterworks at the master's valuation (construction costs had risen greatly since the valuation date). The value of the water rights acquired by actual purchase (for $949,500) was not disputed.

h. Going Value.—On this question, the special master said: "There is no absolute standard by which the fair value of this element can be determined and I adopt $800,000, because no matter how often I have considered the evidence and the arguments, my mind always comes back to this amount as reasonable and fair to all parties." The $800,000 is 8.43 percent of the value of the physical property. The Court said that this amount "is not open to serious question from the standpoint of appellants [the city]."

i. Working Capital.—The special master allowed $174,686 for materials and supplies (which are a part of "working capital") but rejected a claim for

$100,000 cash working capital because the company secured this (and much more) from its consumers by requiring them to pay in advance at the beginning of each six months period. The $174,686 allowed is 1.84 percent of the present depreciated physical value.

j. Depreciation.—The actual depreciation was found to be 12.69 percent ($1,320,970) of the cost new of depreciable property ($10,411,438). This included depreciation on the overheads and on supplies.

k. Confiscatory Rate of Net Return.—The Court held definitely that 4.28 percent net return would be confiscatory in this case.

32. *Newton v. Consolidated Gas Co.* 258 U.S. 165. 42 Sup. Ct. Rep. 264. 66 L. ed. 538. P.U.R. 1922B, 752. U.S. Sup. Ct., March 6, 1922.

As already stated,[1] the United States Supreme Court sustained the New York 80-cent gas ordinance in 1909; in spite of which the conditions were so changed by the World War that after it the company tried for higher rates again, in *Newton v. Consolidated Gas Co.* This time the United States Supreme Court approved an injunction decree against the enforcement of the 80-cent rate. The opinion was written by Mr. Justice McReynolds.

a. The Confiscatory Characters of Specific Rate Schedules May Vary with Time.—The decision in this case upheld the principle that rates once non-confiscatory and lawful may later (in this case by increases in costs of operation) become confiscatory and unlawful.

b. Franchise Value.—This decision required no re-examination of the allowance of $7,781,000 for franchise value made in *Willcox v. Consolidated Gas Co.* because of certain special conditions, peculiar to this property.

c. Depreciation.—The decision did not carry with it approval of the conclusions of the special master or the lower court as to depreciation. The rate was proved to be confiscatory whether or not depreciation was deducted in ascertaining fair value.

33. *Galveston Electric Co. v. Galveston.* 258 U.S. 388. 42 Sup. Ct. Rep.* 351. 66 L. ed. 678. P.U.R. 1922D, 159. U.S. Sup. Ct., April 10, 1922.

In this decision, the United States Supreme Court upheld the United States District Court in refusing an injunction decree, sought by the company and recommended by a Special Master, against the enforcement of a 5-cent street car rate ordered by the Galveston Board of Commissioners in June, 1919. The decision was written by Mr. Justice Brandeis. It is noteworthy as the first by our highest court to allow a material increase (33⅓ percent) in the values of physical-property items installed before 1915, on account of their greatly increased reproduction costs caused by the World War.

a. Relative Weights Given Original-cost and Reproduction-cost Physical Values:

(1) Fixed-capital physical property, other than land, installed prior to Jan. 1, 1915, was valued 33⅓ percent higher than its actual original cost new less depreciation. This gave equal weight to original cost and "period" reproduction cost (the future "plateau of prices," 166⅔ percent of pre-war prices, claimed by the company's expert witnesses). "Spot" reproduction costs were 110 percent higher than pre-war prices.

(2) Fixed-capital physical property, other than land, installed after Jan. 1, 1915, was valued on the basis of actual cost.

(3) All land was valued at present market value, $58,836. Its actual cost was $15,000.

[1] See *Willcox v. Consolidated Gas Co.*, Sec. 8.4.

b. Allowances for Physical Value, Including Overheads:

Original Cost New.—It was agreed by the parties that the undepreciated "historical reproduction costs" new of the physical property were $1,715,825 (including land at $58,836).

Overheads.—This agreed value new includes $202,000 for overheads.

Original-cost depreciation was set at $390,000.

"From the agreed valuation of $1,715,825, the court deducted $425,117 for property not subject to this appreciation [land and items installed since Jan. 1, 1915]. The balance was appreciated one-third; and accrued depreciation, likewise appreciated one-third; the $425,117 was added again; and accrued depreciation, likewise appreciated, was subtracted. The Court thus obtained a base value of $1,626,061."

From this it may be calculated that the average rate of appreciation of the physical property except land was $2,087,225 divided by ($1,715,825 minus $58,836) equals 1.25965; and, further, that

Fair value new direct costs (except land) = "overhead base"	=	$1,832,776
Fair value new overheads (appreciated 25.965%)	=	254,449
Fair value new of physical property (except land)	=	$2,087,225
Fair value accrued depreciation (24.91%)	=	520,000
Fair value of physical property, depreciated (except land)	=	$1,567,225
Land	=	58,836
Total fair value, depreciated	=	$1,626,061
Original-cost value	=	1,325,825
"Period" reproduction-cost value (166⅔% "plateau" price)	=	1,926,297
"Spot" reproduction-cost value (210% price level)	=	2,316,604

c. Brokerage.—An item of $67,078 for "brokerage fees" allowed by the special master was disallowed by both courts. There was no direct evidence that such an amount of fees had actually been paid. It was held that "discounts" on securities sold should be amortized instead of capitalized.

d. Going Value.—An item of $520,000 allowed by the special master for "development cost," or going value, was disallowed by both courts. The master called this allowance "development expense." According to Mr. Justice Brandeis, he had calculated it by capitalizing the net deficits, on an 8 per cent basis, over the entire 15 years since the company purchased the property.

In the lower court, Circuit Judge Hutcheson declared that there should be an allowance made for the fact that the property was a going concern; he refused, however, to allow the so-called "development cost," "it being apparent that the engineers of the parties, in fixing the value which they did fix, had valued the plant as an operating, going entity." This is closely in line with the decision of the United States Supreme Court in the *Des Moines Gas Case* (Sec. 8.4).

Mr. Justice Brandeis upheld this view; in addition, he made a distinction between condemnation and rate cases in allowing going value, saying: "Going-concern and development cost, in the sense in which the master used these terms, are not to be included in the base value for the purpose of determining whether a rate is confiscatory." This statement has been quoted frequently against going-value allowances in rate cases.

In the opinion of the authors, repeated later decisions of the United States Supreme Court have determined, beyond all question, that going value is a "property right" which should be allowed in rate cases the same as in purchase cases.[1]

34. *State of Missouri ex rel. Southwestern Bell Telephone Co. v. Public Service Commission of Missouri, et al.* 262 U.S. 276. 43 Sup. Ct. Rep.* 544. 67 L. ed. 981. P.U.R. 1923C, 193. U.S. Sup. Ct., May 21, 1923.

In this case, the United States Supreme Court decision, written by Mr. Justice McReynolds, reversed a Missouri Supreme Court decision which upheld a rate order by the Missouri Public Service Commission, reducing telephone rates charged by the company in the state. The reversal was because the commission valued the property entirely on the basis of original cost new less depreciation, giving no weight at all to reproduction costs, which had been greatly increased by the World War.

Mr. Justice Brandeis, though concurring in the reversal, wrote a long opinion dissenting from the reason assigned by the majority. He argued strongly for the prudent-investment valuation formula, with Mr. Justice Holmes concurring.

The company's valuation claims and the commission's allowances compare as follows:

	Company's claims	Commission's allowances
Original cost new of fixed-capital physical property..	$ 21,837,379	$21,337,379
Original-cost depreciation† (13.16205%)............	(2,874,247)†	3,774,501
Depreciated, original-cost value of fixed-capital physical property..............................	$(18,963,132)†	$17,562,878
Working capital................................	1,051,564	1,034,050
Going value....................................	....	1,859,693
Total present original-cost value................	....	$20,456,621
Reproduction cost new of fixed-capital physical property..................................	$ 28,454,488	
Reproduction-cost depreciation† (13.16205%).......	3,745,193	
Depreciated reproduction-cost value of fixed-capital physical property..........................	$ 24,709,295	
Working capital................................	1,051,564	
Going value ("establishing business")............	5,594,816	
Total present reproduction-cost value............	$ 31,355,675	

† Calculated by the authors.

The commission's calculations given above are on the basis of the company's reported original costs of fixed-capital physical property, deducting $500,000 for property "not used or useful," and $17,514 for excess working capital. The commission made no complete valuation of its own; by comparing its own and the company's valuations of the properties in St. Louis, Springfield, and Caruthersville, it estimated that the total value for the state would be $20,350,000 to $20,400,000.

[1] See the *Los Angeles Gas Case* decision, 1933 (Sec. 12.11).

The United States Supreme Court said: " . . . we think the proof shows that, for the purposes of the present case, the valuation should be at least $25,000,000."

a. Reproduction-cost Value Must Be Given Due Weight.—Mr. Justice McReynolds said: "An honest and intelligent forecast of future values, made upon a view of all the relevant circumstances, is essential. If the highly important element of present costs is wholly disregarded, such a forecast becomes impossible. Estimates for tomorrow cannot ignore prices of today."

b. Allowances for Physical Value, Working Capital, and Going Value.—The United States Supreme Court made no direct authoritative rulings on these in the *Southwestern Bell Telephone Case*, and no indirect rulings except as may be argued from the fact that its "minimum value" of "at least $25,000,000" is not far from midway between the commission's value of not to exceed $20,456,621, and the company's claimed value of $31,555,675.

Although no exact analysis is possible, the authors will interpret the Court's ruling to imply that, approximately

Original-cost depreciated value of fixed-capital physical property	=	$18,963,000
Reproduction-cost depreciated value of fixed-capital physical property	=	24,709,000
Present fair (depreciated) value of fixed-capital physical property	=	21,787,000
Working capital (= 4.746%)	=	1,034,000
Going value (= 10%)	=	2,179,000
		$25,000,000

c. The Fair Rate of Return.—Five and one-third percent was held to be confiscatory. "Deducting this [depreciation] would leave a possible (5⅓ percent) return upon the minimum value of the property [$25,000,000], which is wholly inadequate, considering the character of the investment and interest rates then prevailing."

d. Annual Depreciation.—Six percent was allowed by the commission and used by the Court in determining the actual rate of net return earned.

35. *Bluefield Waterworks & Improvement Co. v. Public Service Commission of West Virginia et al.* 262 U.S. 679. 43 Sup. Ct. Rep.* 675. 67 L. ed. 1176. P.U.R. 1923D, 11. U.S. Sup. Ct., June 11, 1923.

In this case, the West Virginia Public Service Commission had issued an order reducing the water rates; on appeal by the company, the West Virginia Supreme Court upheld the commission's order; on further appeal, the United States Supreme Court reversed the lower court in a decision written by Mr. Justice Butler, and remanded the case for further action.

The company's engineer made the following estimates of reproduction cost new less depreciation, including working capital and going value:

(*a*) At pre-war prices (approximating original-cost value)	$ 624,548
(*b*) At 1920 prices	1,194,663
The company claimed a value about midway between original and reproduction cost	$ 900,000
The commission's valuation, based on reproduction cost new less depreciation at 1915 prices (approximating original cost), plus additions since 1915 at actual cost, was	$ 460,000

(This included 10 percent going value, and an allowance of $10,000 for working capital.)

After the case was remanded by the United States Supreme Court, the commission revalued the property at	$700,000

a. Reproduction-cost Value Must Be Given Due Weight.—"It is clear that the court also failed to give proper consideration to the higher cost of construction in 1920 . . . and failed to give weight to cost of reproduction less depreciation on the basis of 1920 prices. . . . The final figure, $460,000, was arrived at substantially on the basis of actual cost less depreciation plus ten per cent for going value and $10,000 for working capital. This resulted in a valuation considerably less than would have been reached by a fair and just consideration of all the facts. The valuation cannot be sustained."

b. Allowances for Physical Value, Working Capital, and Going Value.—Although no exact analysis is possible, the authors will interpret the available data of the case, including the final valuation, to imply that approximately

Original-cost depreciated value of fixed-capital physical property (company)[1]	$451,000
Original-cost depreciated value of fixed-capital physical property (Commission)	409,000
Reproduction-cost depreciated value of fixed-capital physical property (company)[1]	908,000
Fair depreciated value of fixed-capital physical property	611,000
Working capital	10,000
Going value, $61,000; preliminary expense, $18,000	79,000
Final valuation	$700,000

c. The Fair Rate of Return.—Six percent was held to be confiscatory in this case.

d. Water Rights.—The company claimed $50,000. This was disallowed. The commission found that water rights had no real value apart from the value of the land on which the springs forming the source of supply were located.

36. *Georgia Railway & Power Company et al. v. Railroad Commission of Georgia et al.* 262 U.S. 625. 43 Sup. Ct. Rep.* 680. 67 L. ed. 1144. P.U.R. 1923D, 1. U.S. Sup. Ct., June 11, 1923.

This litigation was over a rate order by the commission reducing the gas rate in Atlanta and vicinity 10 cents per 1,000 cubic feet (to $1.55); this was on the basis of the commission's valuation of the property at $5,250,000, as of Dec. 30, 1921; the valuation was based substantially on depreciated original-cost value (prudent investment), with land at present market value, though the commission claimed to have given reproduction costs the due consideration which it agreed they should receive. The company claimed a valuation of $9,500,000 and applied to the federal district court for an injunction. This was denied, because " . . . we think no constitutional wrong clearly appears"; although " . . . though correct rules were announced by the commission, we do not think they were exactly followed."

Upon appeal, the lower court was sustained by the United States Supreme Court, Mr. Justice Brandeis writing the decision. He said that: "This case is unlike *Missouri*

[1] Assuming $10,000 working capital, 10 percent going value, and deducting items not included by the commission as follows: preliminary organization, etc., $14,500; Bluefield Valley plant, $25,000; paving over mains, $28,500; water rights, $50,000. For reproduction cost, double these, except water rights.

ex rel. Southwestern Bell Telephone v. Public Service Commission et al. Here the commission gave careful consideration to the cost of reproduction. . . . The refusal of the commission and of the lower court to hold that, for rate-making purposes, the physical properties of a utility must be valued at the replacement cost less depreciation was clearly correct." Mr. Justice McKenna dissented from the opinion.

The case was then reheard by Standing Master Dorsey, who, on Sept. 25, 1924, reported a new valuation, saying: "The master considers replacement cost rather than investment the dominant element in this case."

Land	$ 239,000
Other fixed-capital physical property items	6,730,000
Total value of naked properties	$6,969,000
Working capital	350,000
Going value (10%)	696,000
Total fair value, as of Jan. 1, 1924	$8,015,000

Master Dorsey also reported that: "A valuation based upon historical cost or the prudent investment theory leads to about $6,500,000."

The case does not appear to have been litigated further. Master Dorsey's reasoning in reaching his conclusions, is persuasive, though not authoritative. He said:

"I believe the rules of law deducible from the Supreme Court decisions, including the *Georgia Case*, are substantially as follows: (1) The rate base should represent the present value of the property used and useful in the public service. (2) In arriving at the value of such property, original cost should be considered and given such weight as may be just and right under the circumstances of each particular case. (3) Replacement cost of property less depreciation should not be used as an arbitrary measure of value, but (4) cost of reproduction less depreciation of the utility's property representing investment by the utility, when ascertained with the proper degree of certainty and reasonably applied, may be a proper method of arriving at fair present value. This rule excludes franchise, undisturbed paving, and similar items. (5) An honest and intelligent forecast of probable future values made upon a view of all the relevant circumstances is essential."

37. *Superior Water, Light & Power Co. v. Superior.* 263 U.S. 125. 44 Sup. Ct. Rep. 82. 68 L. ed. 204. U.S. Sup. Ct., Nov. 12, 1923.

Here, the 30-year franchise, expiring in 1917, contained a clause providing for optional purchase by the city. Upon expiration of the 30 years, the company started an action to compel the city to purchase the property or to extend the franchise; the Wisconsin Supreme Court decided against the company, upholding a law passed by the Wisconsin legislature in 1911 making compulsory the transformation of existing franchises into "indeterminate permits."

Upon appeal, the United States Supreme Court, in an opinion written by Mr. Justice McReynolds, reversed the Wisconsin Supreme Court, holding that: "It was beyond the competency of the legislature to substitute an 'indeterminate permit' for rights acquired under a clear contract."

The litigation was compromised at a later date, the company accepting an indeterminate permit under the statute.

38. *Colorado Power Co. v. Grant E. Halderman et al.* 295 Fed. 178. P.U.R. 1924D,* 789. U.S. Dist. Ct., Jan. 4, 1924.

This litigation developed over an application by the company to the commission for permission to raise the existing rates for the services rendered by its "central

system"; alleging that these had become confiscatory because of the increase in operation-cost price levels. The central system included two large main plants and two small standby plants; the main plants, both hydro-electric, were respectively at Shoshone, Colorado River, west of the continental divide, and near Boulder, east of the divide; the standbys were at Georgetown, hydro-electric, and Leadville, steam. Over 60 percent of the output of the central system was wholesaled, by "special contracts," to the Denver Gas & Electric Light Co., the Denver Tramway Co., and the Western Light & Power Co.

In passing on the application, the Commission threw out the value of all property (including the entire expensive Boulder plant) which it did not consider "reasonably necessary for the service of all the customers," exclusive of the three "special contracts." The rate base thus found was $4,221,460. The application for permission to increase rates was denied.

The company then appealed to the United States District Court: first, attacking the constitutionality of the Colorado Power Act; second, claiming the existing rates to be confiscatory.

The United States District Court's opinion, by District Judge Symes, held the act to be constitutional, "insofar as it affects this case," but granted an injunction against enforcing the existing rates on the ground that they were confiscatory.

a. The Constitutionality of the Colorado Power Act.—This was attacked on the ground that section 52 thereof denies full right of appeal to an impartial court (the act provided for reviews by the Colorado Supreme Court); in that it forbids the consideration of new evidence, limits the extent of the review, and provides that "the findings and conclusions of the commission on disputed questions of fact shall be final and shall not be subject to review."

The United States District Court upheld the constitutionality of the act "in so far as it affects this case," because "in a proper case, the court might hold it could set aside an order of the commission as 'not supported' by the evidence, even if there was some conflict."

b. Property "Used and Useful."—The United States District Court disagreed with the commission as to what property should be included in this rate-base valuation as "used and useful"; the Court held that the Boulder plant should be included, but that the small Georgetown and Leadville stand-by plants should be excluded.

c. Marketing of Securities.—Claims by the company for inclusion of the costs of marketing securities were rejected.

d. The Court's Findings on Value.—These were made on the basis of detailed data of valuations made by consulting engineers employed by the company, and on other evidence; "prices and costs prevailing June 1, 1922," were used. Large deductions from the company's claimed values were made by the Court in various instances.

Value new of fixed-capital physical property (including some water rights)	$11,529,971
Accrued depreciation	2,941,645
Depreciated value of fixed-capital physical property	$ 8,588,326
Working capital	225,000
Going value	300,000
Water rights (in addition to some above)	500,000
Rate base, as of June 1, 1922	$ 9,613,326

e. Fair Rate of Net Return.—This was held to be 8 percent.

39. *Pacific Gas & Electric Co. v. City and County of San Francisco.* 265 U.S. 403. 44 Sup. Ct. Rep.* 537. 68 L. ed. 1075. P.U.R. 1924D,* 817. U.S. Sup. Ct., June 2, 1924.

This litigation arose over three gas-rate ordinances, July 1, 1913, 1914, 1915, each ordering gas rates not exceeding 75 cents per 1,000 cubic feet for the following year; by appealing each order to the United States District Court, temporary restraining orders were secured, giving bonds to secure repayments with interest if the rates ordered should be sustained. The maximum rate thereafter was 85 cents. The cases were consolidated Dec. 15, 1916; a master took much testimony and presented an elaborate report March 2, 1920, recommending dismissal of the bills and proper repayments to customers; the United States District Court affirmed the report. The company appealed to the United States Supreme Court, which reversed the lower court. The opinion was written by Mr. Justice McReynolds. Mr. Justice Brandeis filed a dissenting opinion, in which Mr. Justice Holmes concurred.

a. Weight Due Original-cost Value and Reproduction-cost Value.—In his dissenting opinion, Mr. Justice Brandeis said that, in this case: "Counsel, master and Court have throughout endeavored to apply the rule of *Smyth v. Ames*. . . . It is not shown that the rule has, in any respect, been departed from." The Justice went on to support the "prudent-investment" valuation formula (which he has favored in so many dissenting opinions) as well adapted to the retirement of the obsolescent plant whose treatment was the main question to be decided.

b. Actual Depreciation.—The depreciation had been estimated by the master on the basis of the modified sinking-fund method, using a 5 percent interest rate. In the majority opinion, Mr. Justice McReynolds said:

"Appellant objects to the application of this method and insists that depreciation should have been ascertained upon a full consideration of the definite testimony given by competent experts who examined the structural units, spoke concerning observed conditions and made estimates therefor. . . . we think the criticism is not without merit. Facts shown by reliable evidence were preferable to averages based on assumed probabilities."

c. Functional Depreciation. Value of Patent Rights.—The patent rights to use certain highly improved and valuable newly discovered production processes had been acquired by the company in 1915, and manufacturing costs had been greatly reduced thereby. The failure of the master (and the lower court) "properly to appraise" these patent rights; and "to make proper allowances for the successful use of such rights" is the main cause stated by Mr. Justice McReynolds, in the majority opinion, for reversing the decree below.

"Obsolescence of one or more stations and perhaps other property theretofore of great value [possibly $800,000] followed installation of the patents, but the remaining plant plus the patents gave better results. As an operating unit the new combination had greater value than the old; but the court below disregarded the demonstrated worth of the element that wrought this change. . . . "

"Whether, under the peculiar circumstances here presented, the rate base should be fixed by adding to the agreed inventory some fair valuation of the patent rights, or whether prompt recoupment [out of the savings due to the patents] should be allowed for the obsolescence caused by their introduction, or whether appellant should be saved from actual ultimate loss by some other feasible method, we will not undertake to determine upon the present record. To the end that the issues may be reconsidered in view of

this opinion, the decree below is reversed and the cause remanded for such further proceedings as the circumstances require, including another reference to the master if deemed advisable."

d. Franchise Value.—The company claimed $1,476,000 for franchise value. This was disallowed in the lower court; and the disallowance was not criticised by the Supreme Court.

e. Going Value.—The master allowed $1,500,000 for going value, adding "approximately twelve per cent to the value of the physical property to cover this element. . . . " The lower court approved this allowance; and it was not mentioned in the opinion of the Supreme Court.

f. Fair Rate of Return.—The master found that anything less than 7 percent would be confiscatory. For the Supreme Court, Mr. Justice McReynolds said: "We think the evidence supports the finding that a net return of seven per centum was necessary in order to avoid confiscation."

40. *New York & Queens Gas Co. v. William A. Prendergast et al.* 1 F. (2d) 351. P.U.R. 1924E,* 59. U.S. Dist. Ct., June 16, 1924.

Following the 1922 decision of the United States Supreme Court (in *Newton v. Consolidated Gas Co.*, above) that the 80-cent statutory New York gas rate was confiscatory, the New York state legislature passed, in 1923, a new statute prescribing a maximum New York city rate of $1.00 per 1000 cubic feet of 650 British thermal units gas. The companies affected appealed to the United States District Court, which held the rate to be confiscatory and issued an injunction against its enforcement. The opinion was by Winslow, District Judge.

The case was first referred to Special Master Graham, who expressed the correct opinion that the United States Supreme Court held to the *Smyth v. Ames* rule, yet proceeded to estimate the rate base for this property on the reproduction-cost basis. Concerning this procedure, Judge Winslow said:

"In the previous trial of the suit by this plaintiff against the members of the then Public Service Commission, the special master found,—and his decision was approved by both this court and the Supreme Court,—that the fair present value of plaintiff's property was at least the amount of its original investment therein. In view of the fact that the statutory rate in the instant case is insufficient to cover operating charges and distribution, the question of a fair and reasonable return on capital is not one for the court to speculate upon. I do not believe that it is now the function of the court to determine a rate base, and hence, the findings and opinion of the special master, so far as they relate to valuation on a reproduction-cost basis, will be disregarded by the court in the instant case."

The Court's decree was appealed to the United States Supreme Court, where it was affirmed in the New York gas cases, below.

41. *Southern Bell Telephone & Telegraph Co. v. Railroad Commission of South Carolina et al.* 5 F. (2d) 77. P.U.R. 1926A,* 6. U.S. Dist. Ct., No. 253, April 30, 1925.

The South Carolina legislature passed an act, dated April 3, 1922, limiting the telephone rates to those in effect Jan. 1, 1921. The company applied to the United States District Court for an injunction; this was recommended by Special Master J. Waties Waring, after due hearings. The commission filed exceptions, but the Court overruled these and granted the injunction decree. The Court's opinion by Ernest F. Cochran, district judge, stated that: "Upon a full consideration of the whole matter, and laying aside for the present any presumption in favor of the correctness of the master's report upon the facts, I have reached the same conclusions that he reached."

a. Division of the Total Property in the State between Intrastate and Interstate Business.—This division was made on the basis of use, giving, for 1923,

	Total	Intrastate
Original-cost value	$6,718,417	$6,025,091
Reproduction-cost value	7,973,696	7,158,404
Fair value	....	7,000,000

b. Allowances for Physical Value, Working Capital, and Going Value.—The master's allowances for total working capital and total going value were $193,433 and $815,000, respectively. Assigning the same proportion of these as of total value to intrastate business would give

	Fixed-capital physical value, depreciated	Working capital	Going value	Total value
Original-cost value	$5,120,256	$173,554	$731,281	$6,025,091
Reproduction-cost value	6,253,569	173,554	731,281	7,158,404
Fair value	6,095,165	173,554	731,281	7,000,000

c. Weights Given Original-cost and Reproduction-cost Physical Values.—Judge Cochran said: "The master reached his conclusions, and I have reached mine, in finding the present value, upon due consideration of [†] the original cost of construction, the amount expended in permanent improvements, the present as compared with the original cost of construction, the probable earning capacity under the rates prescribed by the statute, the sum required to meet operating expenses, [†] and other relevant matters, in aid of a reasonable judgment."

d. Unused Property.—The Court approved including in the valuation certain equipment installed in planning for the future growth of the business in certain towns and cities, saying:

"Of course, proper business prudence also requires that additional equipment for future use, which must in a certain sense be unused for a period of time, should not be laid out, where such additional equipment can as readily be placed in the plant at the time it is needed. But the evidence in this case shows that the provision for future use was not of that character, but a proper provision such as would effect a large saving in the long run."

e. Actual Depreciation.—After describing "straight-line" and "curved-line" "theoretical" depreciation, and stating that "in physical examination the depreciation rests upon estimates and opinions of witnesses," the Court said: "My own opinion is that certainly in a case of this kind the actual personal examination of the property by competent witnesses and their estimate and opinion of the actual depreciation is a better guide than any of the theoretical methods that have been suggested. The decisions practically sustain this conclusion."

f. The Depreciation Reserve.—The defendants (the commission) contended that the depreciation reserve should have been deducted as depreciation. The Court rejected this contention, saying: "I cannot accede to this view. The

† The words between these symbols are quoted verbatim from the *Smyth v. Ames* decision. The symbols are the authors'.

'depreciation reserve fund' is merely a sum supposed to be set apart to take care of depreciation with a margin over. It is not as a matter of fact actually set aside, but is really simply entered on the books and is in fact a mere matter of bookkeeping. It does not represent actual depreciation, only what observation and experience suggest as likely to happen, with a margin over. The law, however, requires actual depreciation to be deducted."

It may be remarked that it was not yet realized, in 1925, that this legal principle, that only actual depreciation can be deducted to get fair present value, establishes the complementary principle that the company is not entitled to continue annual depreciation charges greater than are necessary to maintain a depreciation reserve just equal to the accrued actual depreciation (see case 67, below).

g. The Fair Rate of Net Return.—The master and the Court held this to be 8 percent in this case.

8.6. The Period 1926 to 1930.—A plateau of construction-cost price levels about double pre-war prices was maintained from 1924 to 1929. The United States Supreme Court continued to uphold the *Smyth v. Ames* rule that all factors affecting value must be given due weights; this was despite the arguments of the supporters of the reproduction-cost-new-less-depreciation and the prudent-investment valuation formulas. Depreciation questions were prominent in valuation litigation; actual depreciation was upheld in preference to theoretical; our highest court called attention, in 1930, to the anomaly involved in permitting public utilities to charge for annual theoretical depreciation allowances greater than experience shows are necessary to maintain depreciation reserves just equal to total accrued actual depreciation; in 1930, also, our highest court ruled that the depreciation base for estimating actual depreciation should be *fair present value new*, instead of the customary *original cost new.* In numerous valuation litigations, the courts ruled on numerous other valuation questions during 1926 to 1930.

42. *Fort Smith v. Southwestern Bell Telephone Co.* 270 U.S. 627. 46 Sup. Ct. Rep. 206. 70 L. ed. 768. U.S. Sup. Ct., Jan. 25, 1926.

In this case, the United States Supreme Court, in a memorandum decision, sustained the decision of the United States District Court in granting a permanent injunction restraining the city from enforcing a telephone rate ordinance. *Southwestern Bell Telephone Co. v. City of Fort Smith.* 294 Fed.* 102. P.U.R. 1924E, 662. U.S. Dist. Ct., No. 407, Sept. 17, 1923. District Judge Youmans.

Judge Youmans' decision, sustained by the United States Supreme Court, is of special interest for its rulings on going value; because it followed so soon after the 1922 decision of the United States Supreme Court in *Galveston v. Galveston Electric Co.*, above. In that case (*a*) the question was raised of whether any going values should be allowed in rate cases (as distinguished from purchase cases); (*b*) a claim for going value based on "capitalization of past deficits" was rejected; (*c*) no going value separate from the overhead cost allowance was made.

The decision in the *Fort Smith Case*

a. Overruled the contention of the city, "that in determining a rate, going value should not be taken into consideration. It is admitted by the city that going

value is a proper element to be considered for the purchase and sale of a given utility."

b. Approved the company's claim of $115,005 going value, based on the "cost of establishing the business," as follows:[1]

(1) Cost of attaching subscribers	$ 12,540
(2) Cost of training employees of organization	7,985
(3) Cost of records	3,500
(4) Administration [10% of (1) + (2) + (3)]	2,403
(5) Maintenance and depreciation of idle plant	69,038
(6) Interest on above costs before exchange opens	2,984
(7) Interest and taxes on idle plant afterwards	16,555
	$115,005

Judge Youmans held that: "The above method does not undertake to capitalize past losses. It assumes a reconstructed plant without business, and gives the cost of securing business of the character and magnitude the company now has in Fort Smith."

Note.—*In the Southwestern Bell Telephone Company v. Fort Worth Case* (instituted in 1924 but decided later than the *Fort Smith Case*), the special master, whose report was approved by the United States District Court, allowed only $250,000 going concern value (about 6.4 percent the value of the other property), although the company, by an application of the Fort Smith method which the master discussed at much length, had claimed $807,394.

43. *Monroe Gas, Light & Fuel Co. v. Michigan Public Utilities Commission et al.* 11 F. (2d) 319. P.U.R. 1926D,* 13. U.S. Dist. Ct., No. 540, Feb. 27, 1926.

In a previous trial (292 Fed. 139), a preliminary injunction had been granted against a $1.54 per 1,000 cubic feet commission rate order, based on an original-cost fair value of $275,000; upon a rehearing, the commission found $350,000 fair value and ordered a $1.52 rate, which again was appealed to the Court; upon a second trial (the present case), the Court found this rate to be confiscatory and issued an injunction decree. The Court found the fair value to be $425,000, as of Jan. 1, 1923.

a. Weights Given to Original-cost Value and Reproduction-cost Value.—The Court fixed the fair value at $425,000, giving dominant weight to reproduction-cost value; " . . . we think that the Supreme Court has now adopted the rule that, at least in the absence of special circumstances controlling otherwise, and not present here, the dominant element in the fixing of a rate base in such a case as is now before us is the reproduction cost, less depreciation, of the property involved. This is not to doubt that there are many situations in which reproduction value would be a less controlling element; . . . "

b. Allowances for Physical Value, Working Capital, and Going Value.—As nearly as the available data show, the Court's findings seem to imply the following allowances:

	Depreciated physical value	Working capital	Going value	Total value
Original-cost value	$237,000	$30,000	$36,000	$303,000
Reproduction-cost value	359,000	30,000	36,000	425,000
Fair value	359,000	30,000	36,000	425,000

[1] The company claimed $739,972 reproduction cost new of its property.

About 10 percent actual depreciation was deducted by the commission from reproduction-cost values new.

c. Property Paid For out of the Depreciation Reserve.—In a preceding case, *Michigan Public Utilities Commission v. Michigan State Telephone Co.*, Mich. Sup. Ct., 228 Mich. 658. 200 N.W. 749. P.U.R. 1925C, 158, decided Oct. 30, 1924, the commission had found the depreciation to be equal to the depreciation reserve, $9,500,000, which it deducted to determine the present fair value, $49,000,000 (including $1,500,000 working capital); it then proceeded to deduct the depreciation reserve again, because it was reinvested in the property valued, but was overruled by the Michigan Supreme Court.

In the *Monroe Gas Case* the United States District Court ruled likewise that the value of property paid for out of the depreciation reserve must be included in the rate base.

d. Fair and Confiscatory Rates of Return.—The Court said that " . . . particularly with the declared willingness of the commission to allow a return of 8 per cent as this may be brought about by an increased volume of business and by more efficiency, we conclude that, under all the circumstances disclosed by the record, including the payment of the corporate Federal income tax, a 7 per cent return would not be confiscatory."

44. *Board of Public Utilities Commissioners v. New York Telephone Co.* 271 U.S. 23. 46 Sup. Ct. Rep.* 363. 70 L. ed. 809. P.U.R. 1926C, 740. U.S. Sup. Ct., April 12, 1926.

In this case (often referred to as the *New Jersey Telephone Case*), the United States Supreme Court affirmed a decree of the federal district court enjoining, as confiscatory, a telephone rate order of the New Jersey Board of Public Utility Commissioners. The opinion[1] was written by Mr. Justice Butler.

The New Jersey Commission, which valued the property as of June 30, 1924, at $76,370,000, found that the company's $16,902,530 depreciation reserve Dec. 31, 1923, was $4,750,000 greater than the correct accrued depreciation; and that the $3,452,000 charged by the company for depreciation in 1924 was excessive, $2,678,000 being sufficient.

On the basis of those findings, the commission ordered rates which admittedly would be insufficient to pay future actual annual depreciation charges in addition to fair net return, and held in justification that: "But having made such charges in the past, future charges beginning January 1, 1925, may be deducted from the normal charge until such time as at least $4,750,000 of the excess is absorbed, as hereinafter provided."

The United States Supreme Court overruled the commission; the Court held, in effect, that:

a. Depreciation is a matter of fact, not of past charges.

b. Future consumers cannot be penalized on account of past insufficient depreciation charges.

c. Future consumers are not entitled to profit on account of excess depreciation charges collected from past consumers.

Mr. Justice Butler said: "It may be assumed [even] as found by the Board, that in prior years the company charged excessive amounts to depreciation expense and so created in the reserve account balances greater than required adequately to maintain the property. [But] . . . constitutional protection against confiscation does not depend on the source of the money used to purchase the property. It is enough that it is used to render the serv-

[1] "This is a leading case with respect to annual depreciation charges and their relation to the depreciation reserve already accumulated." "Valuation of Public Service Corporations," by Whitten and Wilcox, rev. ed., 1928, p. 1687.

ice, . . . The relation between the company and its customers is not that of partners, agent and principal, or trustee and beneficiary. . . . The revenue paid by the customers for service belongs to the company. The amount, if any, remaining after paying taxes and operating expenses including the expense of depreciation is the company's compensation for the use of its property. If there is no return, or if the amount is less than a reasonable return, the company must bear the loss. Past losses cannot be used to enhance the value of the property or to support a claim that rates for the future are confiscatory. . . . And the law does not require the company to give up for the benefit of future subscribers any part of its accumulations from past operations. Profits of the past cannot be used to sustain confiscatory rates for the future."

45. *McCardle et al. v. Indianapolis Water Co.* 272 U.S. 400. 47 Sup. Ct. Rep.* 144. P.U.R. 1927A,* 15. U.S. Sup. Ct., Nov. 22, 1926.

The company filed, June 8, 1923, a petition for approval of a proposed higher rate schedule. After valuing the property at not less than $15,260,400, the commission issued an order for rates higher than the existing, but lower than the proposed schedule. The company appealed to the United States District Court and thence to the United States Supreme Court, claiming the proposed rates to be too low; the city of Indianapolis intervened, claiming them to be too high. Both courts valued the property at not less than $19,000,000 and found the proposed rates to be confiscatory.

This ruling opinion of the United States Supreme Court was delivered by Mr. Justice Butler. Mr. Justice Brandeis filed a dissenting opinion, in which Mr. Justice Stone concurred.

a. Non-operative Property.—Non-operative property, whose value should be deducted from the rate base was valued at $648,921 by Carter, the commission's engineer, $119,000 by Hagenah, a valuation engineer employed by the company, and only $68,000 by Metcalf, the company's consulting engineer.

b. Consideration of a Substitute Plant to Give Same Service Ruled Out.—Mr. Justice Butler ruled against the proposal of an engineer (testifying for the city) to reduce the "fair value" $785,013.11, by throwing out the value of the lower part of the company's supply canal and substituting therefor the estimated lower cost of a steam plant designed for the purpose of taking its place. Mr. Justice Butler said: "There is to be ascertained the value of the plant used to give the service and not the estimated cost of a different plant. Save under exceptional circumstances, the Court is not required to enter upon a comparison of different systems."

c. The Correct Rule as to Weights Due Original-cost Value and Reproduction-cost Value.—Mr. Justice Butler said: "It is well established that values of utility properties fluctuate, and that owners must bear the decline and are entitled to the increase. The decision of the Court in *Smyth v. Ames*, 169 U.S. 466, 547, 18, Sup. Ct. 418, 434 (42 L. ed. 819), declares that to ascertain value 'the present as compared with the original cost of construction' are, among other things, matters for consideration. But this does not mean that the original cost or the present cost or some figure arbitrarily chosen between these two is to be taken as the measure. The weight to be given to such cost figures and other items or classes of business is to be determined in the light of the facts in the case in hand."

Weight Due Original Cost.—"Undoubtedly the reasonable cost of a system of waterworks, well-planned and efficient for the public service, is good evidence of its value at the time of construction. And such actual cost will continue fairly well to measure the amount to be attributed to the physical elements of the property so long as there is no change in the level of applicable prices."

Weight Due Reproduction Cost.—"And, as indicated in the report of the commission, it is true that, if the tendency or trend of prices is not definitely upward or downward and it does not appear probable that there will be a substantial change of prices, then the present cost of constructing the plant less depreciation, if any, is a fair measure of the value of the physical elements of the property." . . .

"The validity of the rates in question depends on property value January 1, 1924, and for a reasonable time following. While the values of such properties do not vary with frequent minor fluctuations in the prices of material and labor required to produce them, they are affected by and generally follow the relatively permanent levels and trends of such prices. The fact that original cost was probably 12 to 20 per cent less than the estimate [of the value, Oct. 1, 1922] of the commission's engineer based on the average of prices for the ten years ending with 1921—two years before the rate order became effective—does not tend to support the commission's adoption of that estimate."

The Fair Cost-value Should Be Based on the Prices Most Likely to Prevail during a Reasonably Long Immediate Future Period of Years.—Mr. Justice Butler said: "The high level of prices and wages prevailing in 1922 and 1923 should be taken into account in finding value as of January 1, 1924, and in the years immediately following. Moreover, there is nothing in the record to indicate that the prices prevailing at the effective date of the order were likely to decline within a reasonable time—one, two, or three years—to the level of the average in the ten years ending with 1923. And we may take judicial notice [November, 1926] that there has been no substantial general decline in the prices of labor and materials since that time. The trend has been upward rather than downward."

d. Original-cost Physical Value Data.—The company's accounts were defective "in that there was not a careful division of expenditures between capital account and operating expenses." The commission stated that "it is believed that the fair original cost of the physical property was 12 to 20 per cent less than the $14,904,000 used as a basis herein" (its valuation dated Oct. 31, 1922, in a previous case). The later testimony showed net additions of $795,101 to Jan. 1, 1924. It is deduced that the original-cost fixed-capital physical value, Jan. 1, 1924, roughly approximated $13,200,000, besides about $100,000 for materials and supplies.

Reproduction-cost Physical Value Data.—Mr. Carter, the commission's chief engineer, and two pairs of consulting engineers employed by the company gave high-grade expert testimony as to the reproduction-cost fixed-capital physical value, Jan. 1, 1924, as follows:

Valuation engineers	Reproduction-cost prices			
	Jan. 1, 1924	Average of last 3 years	Average of last 5 years	Average of last 10 years
Carter	$19,400,000†	. . . .	. . . .	$16,900,000†
Hagenah and Erickson	22,669,026	$21,625,358	$22,652,799	19,624,354
Sanderson and Porter	21,898,662	20,968,127	21,863,858	18,931,979

† Deducting $100,000 for materials and supplies.

From the above, and other data given in the Supreme Court's opinion, the authors deduce implied approximate allowances as follows:

	Depreciated fixed capital	Working capital	Going value and water rights	Total values
Original-cost value................	$13,200,000	$235,000	$1,706,000	$15,141,000
"Spot" reproduction-cost value*...	21,323,000	235,000	1,706,000	23,264,000
"Period" reproduction-cost value†.	20,378,000	235,000	1,706,000	22,319,000
Fair value.......................	17,059,000	235,000	1,706,000	19,000,000

* Average of the three valuations.

† Assuming Carter's valuation would reduce the average the same proportion as for "spot" prices.

e. Actual Depreciation.—An engineer employed by the city deducted approximately 25 percent accrued depreciation, concerning which Mr. Justice Butler said: "The deduction was not based on an inspection of the property. It was the result of a 'straight-line' calculation based on age and the estimated or assumed useful life of perishable elements." On the other hand, Mr. Carter for the commission and Mr. Hagenah for the company, both inspected the property and estimated near 6 percent depreciation. Mr. Carter "estimated its condition by giving effects to the results of the examination and to the age of the property." Mr. Hagenah's estimate was "based on actual inspection and a consideration of the probable future life as indicated by the conditions found."

Mr. Justice Butler said: "The testimony of competent valuation engineers who examined the property and made estimates in respect of its condition is to be preferred to mere calculations based on averages and assumed probabilities."

f. Fair Rate of Return.—Mr. Justice Butler said: "The evidence is more than sufficient to sustain the rate of 7 per cent, found by the commission. And recent decisions support a higher rate of return."

The New York Gas Cases

46. *Ottinger v. Consolidated Gas Co.* 47 Sup. Ct. Rep. 198. P.U.R. 1927A, 37. U.S. Sup. Ct., Nov. 25, 1926.

47. *Ottinger v. Brooklyn Union Gas Co.* 47 Sup. Ct. Rep. 199. P.U.R. 1927A, 39. U.S. Sup. Ct., Nov. 25, 1926.

48. *Ottinger v. Kings County Lighting Co.* 47 Sup. Ct. Rep. 199. P.U.R. 1927A, 39. U.S. Sup. Ct., Nov. 25, 1926.

These suits (and others) were over the constitutionality of the New York state $1.00 gas rate of Act of 1923 (see *New York and Queens Gas Co. v. Prendergast*, above). Injunction decrees were granted by the United States District Court in all the suits; the attorney general of the state of New York appealed to the United States Supreme Court in these three cases.

They all were decided Nov. 25, 1926. The unanimous opinion of the United States Supreme Court, written by Mr. Justice McReynolds, confirmed the decrees of the lower courts (granting injunctions against enforcement) after modifying them so as to base the unconstitutionality of the law entirely on its confiscatory character under the Fourteenth Amendment. This confiscatory character seemed so apparent to Mr.

Justice McReynolds "that he did not deem it necessary to discuss the principles of valuation, or to cite authorities."[1]

Whitten and Wilcox,[1] have summed up the conclusions of the special masters and lower courts, as to the valuations in all three cases, as follows:

"In these New York Gas cases some differences of opinion as to fair value developed, but in the main, masters and judges found valuations below reproduction cost, although interpreting the rule to be that reproduction cost is the dominant element in the determination of value for rate purposes."

49. *United States v. Los Angeles & Salt Lake Railroad Co.* 47 Sup. Ct. Rep. 413. U.S. Sup. Ct., Feb. 21, 1927.

The Interstate Commerce Commission issued an order, June 7, 1923, fixing a single sum valuation of the above railroad for rate-making and other regulatory purposes. The United States District Court granted the railroad a decree annulling the order. Upon appeal, the United States Supreme Court reversed the lower court; the decision is cited here because of its rulings on water rights.

Water Rights.—The company owned water rights, to the use of water from springs and streams; in that arid country, these rights had a separate "present value," which the Commission set at $26,150; ascertained and reported " . . . as nearly as possible by the methods which we approved in the *Texas Midland Case* for determining the present value of lands. Original cost to date will be reported whenever possible to determine it from reliable evidence. It is no more possible to ascertain the cost of reproduction of water rights than the cost of the reproduction of land. These cannot be reproduced but only developed and put to beneficial use by human effort."

In determining the present market value of the water rights, the Commission took into account the amount of water consumed by the carriers from each source, the total quantity available therefrom, and the value for the next highest possible use of the water used by the carriers; the Commission ascertained the opinions on this last point of "many well-informed persons residing and engaged in business there."

50. *Idaho Power Co. v. Thompson et al.* 19F. (2d) 547. P.U.R. 1927D,* 388. U.S. Dist. Ct. for Idaho, No. 1148, April 20, 1927.

In this case, decided by one circuit and two district judges, it was held that, with corrections of particular rates, the electric rate schedule ordered by the Idaho Public Service Commission would not be confiscatory. Quotations are from the opinion by Judge Dietrich. Owing to his promotion to the circuit court, a concurring opinion was entered, written by Judge Cushman.

a. Allowances for Physical Value, Organization Expenses, Working Capital, and Going Value.—These were as follows:

Cost of physical property new	$18,020,302
Accrued depreciation	2,000,000
Fair present value of physical property	$16,020,302
Organization expenses	$ 400,000
Working capital	650,000
Going value	$ 1,500,000
Total fair value, as of June 30, 1924	$18,570,302

b. Contention Overruled that Sinking-fund Depreciation Should Not Be Deducted in Determining Fair Value.—Judge Dietrich said: "It is also urged by the

[1] Whitten and Wilcox, "Valuation of Public Utilities," 2d ed., The Banks Law Publishing Co., New York, 1928, pp. 189, 196.

plaintiff that any deduction for accrued depreciation of any kind is in necessary conflict with the sinking-fund theory established by the Idaho statutes (see section 2473) and adopted by the commission. For several reasons we are unable to concur in this view."

c. Functional Depreciation Should Be Deducted as Well as Physical.—Judge Dietrich said: "There remains the important process of relating the cost of the hypothetical new plant to the fair value of the old plant as it actually exists. Fair value implies a consideration of all factors which would be regarded as material in negotiating a sale and purchase of such property. Wear, decay, deterioration, obsolescence, inadequacy, and redundancy would all undoubtedly be considered as factors. It is suggested that obsolescence and inadequacy do not accrue but occur. But in essence how do they differ from wear and decay?"

d. Actual Depreciation Adopted in Preference to Theoretical.—Judge Dietrich said: "In respect to a complex system, some parts of which are older than others and general depreciation is wanting in uniformity, undoubtedly information procured by actual inspection is preferable to the so-called straight line or percentage method. But at best the ultimate findings must to some extent involve estimates and opinions, and in some particulars nothing better than estimated averages, based upon probabilities, are available."

The Court rejected the straight-line theoretical depreciations (respectively $3,086,091 and $3,965,785) estimated by the commission's two experts, and itself set the accrued actual depreciation at $2,000,000, taking into account all the information from actual inspection disclosed in the testimony of the witnesses for both the plaintiff (the company) and the defendant (the commission), "and weighing all considerations."

e. The Theory "As Good As New" when Deferred Maintenance Is Made Good Rejected.—Judge Dietrich said of the company's chief engineer: "By himself and through his assistants he made a detailed examination for the purpose of ascertaining what repairs and replacements should be made, either presently or in the immediate future, and the conclusion reached that such repairs and replacements could be made at an aggregate outlay of $438,734 was adopted as the full measure of depreciation. While he states that with this outlay the property could be put into a condition as good as new, or could be made as valuable as new, it is plain from all his testimony that he had in mind the standard of value now urged by the plaintiff, namely, value for present use—present service ability or efficiency. . . . When analyzed it is clear that plaintiff's estimates are of only a part of the elements that must be taken into account, and even if accepted as correct in respect to the elements considered it would furnish an inadequate measure of entire depreciation."

And the Court proceeded to adopt $2,000,000 as the amount of the accrued depreciation instead of the $438,734 for which the company contended.

f. Fair Rate of Return.—Judge Dietrich said: "The general range of allowance has been from 6 to 8 per cent, with comparatively rare instances of higher or lower rates. Each case is to a degree affected by its own distinctive circumstances. In addition to bare compensation we are of course to recognize that there is a measure of risk. In all investments there is risk." Considering the conditions of this case, he said: "At least we are unable to conclude that under such circumstances rates yielding a return of 7 per cent or over, so paid, would in any proper sense be confiscatory."

51. *United Fuel Gas Co. et al. v. Railroad Commission of Kentucky et al.* U.S. Sup. Ct. Rep. P.U.R. 1929A,* 433. U.S. Sup. Ct., Jan. 2, 1929.

The company's property is located mainly in West Virginia; its business is conducted in West Virginia, Kentucky, and Ohio, with a still wider potential market.

The litigation culminating in this case arose over a Kentucky Commission rate order that the maximum price for natural gas in Ashland, Catlettsburg, and Louisa, Ky., be 32 cents per 1,000 cubic feet. The company appealed to the United States District Court for an injunction decree against enforcement, or, in the alternative, against prevention of the discontinuance of service; the decree was denied in 13 F. (2d) 510; the denial was appealed to the United States Supreme Court. A decision by a different United States District Court, in parallel West Virginia litigation, is reported in 14 F. (2d) 209. P.U.R. 1927A,* 707.

The opinion of the United States Supreme Court was written by Mr. Justice Stone.

a. Property Allocated to the Kentucky Service.—Various witnesses testified to assignments[1] to the Kentucky service of percentages of total value ranging from 8.4 to 11 percent; the company used 11 percent in its calculations; the Supreme Court found that even with 12 percent the rate would not be confiscatory. The valuations discussed below are of the total property.

b. Gas Lands, Leases, and Rights.—The company claimed a valuation for these \$36,449,176;[2] the Supreme Court allowed only \$6,732,920.[2] "Appellants, as will more fully appear, reached their claimed value by an estimate by experts of the profits to be derived from the sale, in an unregulated market, of the quantities of gas estimated to underlie the proven and probable areas. The court below found that the value of appellant's gas field did not exceed its 'book cost,' which it took to be \$6,732,920. . . . "

Concerning the company experts' estimates, the Supreme Court said: "Such predictions can only be made on the basis of data which are not and cannot be known, and most of which are in the highest degree speculative. Such a process of estimating value is without any known sanction."

c. Depletion and Amortization.—The estimated life of the gas field was 18 years; over this period the company must provide, out of current income, in excess of operation costs, replacement costs, and profits, for the amortization of its entire investment, less net salvage at the end. The Court approved an annual amortization allowance of 4½ percent of the rate base.

d. Relative Weights Given Original Cost and Reproduction Cost.—The adopted reproduction cost new of the fixed-capital physical property except gas lands was 68.86 percent in excess of its original cost.[3] Finding that the rate would be nonconfiscatory, anyhow, the Supreme Court, "in the view that we take, and for the present purposes only," assumed, without deciding "that in the case as presented, present reproduction value of property used and useful in the business, if ascertainable, is to be taken as the rate base."

e. Allowances for Physical Value, Working Capital, Going Value and Gas Lands, Leases and Rights.—The Supreme Court opinion and the West Virginia United States District Court opinion are considered by the authors to mean that the true fair values of the property were held not to exceed the following allowances, all as of Dec. 31, 1923:

Original cost of fixed-capital physical property new[3]	\$19,945,582
Original-cost depreciation (22.7587%)[4]	4,539,355
Depreciated original-cost value of fixed-capital physical property	\$15,406,227

[1] The assignment should be on the basis of use.

[2] Including oil-production acreage amounting to \$389,591.

[3] See the parallel West Virginia District Court case. P.U.R. 1927A,* 707.

[4] Calculated from testimony of company's expert (Uebelacher), accepted by commission with minor changes.

Direct reproduction cost of fixed-capital physical property new..	$28,837,260
Overhead reproduction-cost new allowances (18%)............	5,190,707
Total reproduction cost of fixed-capital physical property new..	$34,027,967
Reproduction-cost depreciation (22.7587%)[1].................	7,744,323
Depreciated reproduction-cost value of fixed-capital physical property...	$26,283,644
Depreciated direct reproduction cost of fixed-capital physical property...	$22,274,274
Depreciated overheads reproduction-cost allowances (18%).....	4,009,370
Working capital...	999,000
Going value..	3,000,000
Gas lands, leases, and rights[2]..............................	6,732,920
Total reproduction-cost value of tangibles and intangibles......	$37,015,564

f. Discontinuance of Service by a Public Utility in Cities Which It Selects Subject to State Control.—The Supreme Court said: "The primary duty of a public utility is to serve on reasonable terms all those who desire the service it renders. This duty does not permit it to pick and choose and to serve only those portions of the territory which it finds most profitable, leaving the remainder to get along without the service which it alone is in a position to give. An important purpose of state supervision is to prevent such discrimination."

52. *Gilchrist et al. v. Interborough Rapid Transit Co. et al.* 279 U.S. 159. 49 Sup. Ct. Rep.* 282. P.U.R. 1929B,* 434. U.S. Sup. Ct., April 8, 1929.

The Interborough Company operates the New York City subways, leased from the city, which built and owns them; in conjunction, it also operates the elevated railway system, originally leased from the Manhattan Railway Co. and later improved and extended by the Interborough. The subway equipment is owned by the Interborough.

The Rapid Transit Commission, of three members, *Gilchrist et al.*, was established in accordance with the Rapid Transit Act, first passed by the New York legislature in 1891, and amended "some forty times" later. Through the agency of the commission, the city of New York, between the years 1895 and 1919, built and partially equipped a 74-mile subway system. The Rapid Transit Commission leased this subway property to the Interborough for operation by "Contract No. 1," Feb. 21, 1900, "Contract No. 2," July 21, 1902, "Contract No. 3," March 19, 1913, the "Third Track Certificate," March 19, 1913, the "Elevated Extension Certificate," March 19, 1913, and the "Supplementary Agreement," March 19, 1913. These contracts provided for a maximum 5-cent fare. The two certificates provided for improvements and extensions of the elevated railways; the supplementary agreement provided for the operation of elevated trains over designated portions of the subways.

In 1903, the Manhattan Railway leased its elevated railways to the Interborough Company for 999 years; they had been built under various franchises and at various times since 1875.

The United States Supreme Court says: "The record indicates that when this suit was begun [Feb. 14, 1928] the city had expended from its own treasury for the con-

[1] Calculated from testimony of company's expert (Uebelacher), accepted by commission with minor changes.

[2] Including $389,591 for oil-production acreage.

struction of subways $180,000,000; that the Interborough Company had advanced for such construction $58,000,000, and had expended for equipment not above $120,000,000—probably much less. The cost to the Interborough for laying third tracks on the elevated lines and building extensions thereto was $44,000,000. The original cost of the old elevated lines is not disclosed and perhaps cannot be definitely ascertained; it did not exceed $90,000,000. . . . The present value of the above mentioned properties is very large, but to determine this with fair accuracy would be exceedingly difficult."

In 1920 and again in 1922 and 1925, the Interborough Company applied unsuccessfully to the commission for higher rates, claiming that the great increase in price levels due to the World War had made the 5-cent fare contract rate confiscatory. By the suit begun Feb. 14, 1928, it obtained an interlocutory injunction from the District Court of the United States for the Southern District of New York, against interference with a proposed 7-cent fare. The United States Supreme Court stayed the order, pending further hearing, and finally reversed the lower court, saying:[1]

"Considering the entire record, we think the challenged order was improvident and beyond the proper discretion of the court."

The Court upheld the contention of the city that the subway contracts were profitable with the 5-cent fare.

"The following excerpts from an affidavit offered by the city are enlightening. The record supports the facts and figures used so far as herein important; also in general the stated conclusions.

'The operation under contract No. 3 has been highly profitable to the Interborough, as was the prior operation under contracts Nos. 1 and 2. . . . ' [The affidavit gave the earnings in the three years ending June 30, 1928.]

'These earnings are, of course, enormous and leave no room for claim that the five-cent fare fixed by contract No. 3 is inadequate, . . .

'The financial difficulties of the Interborough during the past eight years have been due to the elevated lease from the Manhattan Railroad Company, and not to the subway contract with the city . . . '."

Reversing the lower court's order, the Supreme Court said:

"The Transit Commission has long held the view that it lacks power to change the 5-cent rate established by contract; and it intended to test this point of law by an immediate orderly appeal to the courts of the state.[2] This purpose should not be thwarted by an injunction. Upon the record before us we cannot accept the theory that the subways and elevated roads constitute a unified system for rate-making purposes. Considering the probable fair value of the subways and the current receipts therefrom no adequate basis is shown for claiming that the five-cent rate is now confiscatory in respect of them."

53. *St. Louis & O'Fallon Railway Co. et al. v. United States et al. United States et al. v. St. Louis & O'Fallon Railway Co. et al.* Nos. 131, 132. 279 U.S.* 461. 49 Sup. Ct. Rep.* 384. P.U.R. 1929C,* 161. U.S. Sup. Ct., May 20, 1929.

This case came before the United States Supreme Court on cross appeals from a decree of the United States District Court for Eastern Missouri, three judges sitting, upholding a "recapture" order by the Interstate Commerce Commission, directing the company to place in a reserve fund one-half of its "excess profit," over 6 percent, for the years 1920 (10 months), 1921, 1922, 1923, and to pay the Commission the remain-

[1] The opinion was written by Mr. Justice McReynolds; Justices Van Devanter, Sutherland, and Butler dissented.

[2] The commission did institute such proceedings on the same date as those in this case. See *City of New York v. Interborough Rapid Transit Co.*, case 56, below.

ing one-half; all as prescribed in paragraph 6 of section 15*a* of the Interstate Commerce Act.[1]

The O'Fallon Railway was 9 miles of coal line in Illinois, near St. Louis. The main issue in the case was the correctness of the Interstate Commerce Commission's valuations of the railway property; $856,065 for 1920, to $997,236 for 1923. The valuations were made by a "split-inventory" method; this, in effect, valued the property, except land, at its original cost new less depreciation; the land was valued at present market value.

The United States Supreme Court, by a divided five-to-three decision, Mr. Justice Butler not participating, overruled the decree of the lower court (which had upheld the recapture order), on the ground that the Interstate Commerce Commission's valuations were incorrect; because, except as to land, the Commission had given little or no weight to current price levels. Mr. Justice McReynolds delivered the opinion of the Court. Mr. Justice Brandeis filed a long dissenting opinion, in which Justices Holmes and Stone concurred.

a. The Law of the Land as to the Elements of Value to Which the Commission Was Required to Give Due Consideration.—The opinion of the Supreme Court quoted *Smyth v. Ames* and *Southwestern Bell Telephone Co. v. Public Service Commission* in support of its ruling in this case that "the present as compared with the original cost of construction . . . the highly important element of present costs" . . . , as well as the other elements of value, must be given "such weight as may be just and right in each case," and added:

"The doctrine above stated has been consistently adhered to by this court."

b. Except as to Land, the Commission Gave Little or No Weight to Reproduction Costs.—"The report of the Commission is long and argumentative. . . . It carefully refrains from stating that any consideration whatsoever was given to present or reproduction costs in estimating the value of the carrier's property. Four dissenting Commissioners declare that reproductions costs were not considered; and the report itself confirms their view. Two of the majority avow a like understanding of the course pursued."

c. Valuations Made by Commission Invalid because of Failure to Give Due Weight to Reproduction-cost Value.—"In the exercise of its proper function, this court has declared the law of the land controlling valuations for rate-making purposes. The Commission disregarded the approved rule and has thereby failed to discharge the definite duty imposed by Congress. . . . "

"The question on which the Commission divided is this: When seeking to ascertain the value of railroad property for recapture purposes, must it give consideration to current, or reproduction, costs? The weight to be accorded thereto is not the matter before us. No doubt there are some, perhaps many, railroads the ultimate value of which should be placed far below the sum necessary for reproduction. But Congress has directed that values shall be fixed upon a consideration of present costs along with all other pertinent facts; and this mandate must be obeyed."

d. Depreciation.—The deduction of depreciation by the Commission in this case, from actual original costs new as well as from reproduction costs new, does not appear to have been contested; the subject of depreciation is not mentioned in the opinion of the Supreme Court.

e. Working Capital.—In its valuation of the O'Fallon property as of date June 30, 1919, the Commission allowed $50,000 for working capital as a part of the total valuation of $850,000 (of which about $50,500 was for land).

[1] The recapture clause was repealed in 1933, retroactively.

54. *New York Telephone Co. v. William A. Prendergast et al.* "36 F." (2d) 54. P.U.R. 1930B, 33. U.S. Dist. Ct., Nov. 7, 1929.

History of the Case

The *New York Telephone Case* litigation is a classic instance of the excessive delays, the excessive costs, the excessively long, tedious hearings, the excessively voluminous records of intricate technical testimony, which so often have made public regulation of public utilities ineffective; these might be obviated, in great part, by complete, continuous-inventory property-ledger accounts, on prescribed forms, kept up to date each year by the utilities, under the cooperative direction of the regulatory authority.

After 66 hearings, during a period of more than a year, in which 10,500 pages of testimony were taken and more than 400 exhibits received, the Public Service Commission of the state of New York made telephone rate orders on Jan. 25, 1923, covering the entire state. The rates so ordered were promptly put into effect, but in 1924 the company proceeded to appeal to the United States District Court for an injunction against their continued enforcement. The city of New York was permitted to intervene. On July 26, 1924 (300 Fed. 822. P.U.R. 1925A, 491), the Court granted an injunction,[1] pending final hearings; however, the Court limited rate increases in New York City to a maximum 10 per cent surcharge and forbade any increases at all elsewhere in the state. The Court referred the case to a special master to hold hearings and take testimony.

The company then applied to the federal court to have the 10 percent surcharge limit raised. This was denied, pending action by the commission. 11 F. (2d) 162. P.U.R. 1926C, 696. The commission had been proceeding with further hearings, and it promulgated new increased rate orders for New York city in 1926. *Re New York Telephone Co.*, case 1789, May 26, 1926. P.U.R. 1926E, 1. The company then filed a supplemental bill of complaint, which the district court referred to the special master.

The special master's hearings began in October, 1924, and extended over a period of four years; his records included about 37,000 pages of testimony and some 70,000 pages of exhibits. The final decision of the statutory three-judge district court (in the instant case) was rendered Nov. 7, 1929. It fixed the fair value of the property, enjoined the continued enforcement of the existing rates, and permitted an increase of rates sufficient to yield 7 percent net return.

Completing the narrative, the company tried to put an increased rate schedule into effect Feb. 1, 1930, without the commission's approval; the commission forestalled this action by an order, dated Jan. 31, 1930, that, effective till May 1, 1930, the increases in rates should be 20 percent less than those proposed by the company. *Re New York Telephone Co.*, case 6177, Jan. 31, 1930. P.U.R. 1930C, 439. The commission then proceeded to make further investigations, including the taking of testimony; on May 1, 1930, it rendered a decision in which the company's proposed rate schedule for Feb. 1, 1930, was modified. The fair value adopted for the property was $555,252,483, as of July 1, 1930.

In the meantime, the company, although it had won an injunction, appealed the district court's decision of Nov. 7, 1929, to the United States Supreme Court. The first briefs of the voluminous records submitted were so voluminous that the Court

[1] The Court rejected the commission's valuation for rate base: (1) it did not give due weight to reproduction cost; (2) it deducted depreciation equal to the entire depreciation reserve, instead of merely the actual depreciation; (3) it failed to include any allowance for going value.

returned them, and ordered that they be greatly condensed. The case remained pending in the Supreme Court until the spring of 1934; it was then dismissed, upon motion of the commission. Thus the district court's decision remained the final court adjudication of the case. Its opinion, by Circuit Judge Manton, will now be reviewed, as follows.

Review of the District Court's Opinion

a. The Weights Due Original Cost and Reproduction Cost.—"Reproduction cost, less actual depreciation, is not the legal equivalent of fair value, as the master stated it to be, for it is, as a matter of law, but evidence of value. . . . the weight of which is to be determined with all the evidence in the case, as tending to show both the value and the relative importance of all the evidence on the subject. Book cost, with deductions for depreciation, would be an index of present value if there were no fluctuations in the values of land, material or labor. But there are fluctuations, as this record discloses, and reproduction cost new is an estimate intended to reflect the upward or downward trend in prices or values which have occurred between the date of the actual investment and the date of investigation as to the present value. While the master says he followed the theory of reproduction cost new, less depreciation, or at least gave it controlling weight, nevertheless, the master's results as to present value are correct, except for the modifications which we shall later refer to."

The Court enumerated three reasons for giving controlling weight to reproduction cost for this particular property:

(1) Owing to the fact that the company's common stock was purchased by the American Telephone & Telegraph Co., it did not incur certain otherwise necessary overhead construction costs which would need to be considered in determining fair values.

(2) The master " . . . found that the plaintiff understated the true cost of its property in its book costs"; this was by incorrectly charging some construction costs to operation expense.

(3) There had been an "unquestioned increase in value of the property," since the construction dates of many of its units.

b. Fanciful and Speculative Allowances of Value Rejected.

(1) The master disallowed $18,000,000 claimed for promotion expenses.

(2) The master disallowed $9,500,000 claimed going-value allowance for speculative items.

(3) The Court reduced the master's $35,000,000 going-value allowance to $10,000,000.

(4) The Court disallowed the master's $14,609,459.76 "inexperience-factor" allowance; for the greater cost of equipment made by a manufacturer less experienced than the Western Electric Company (affiliated with the telephone companies).

(5) The Court disallowed the master's $22,943,109 "financing cost" allowance.

(6) The Court disallowed the master's $3,517,850.23 allowance for "bankers' checking engineers and interest on preliminary costs."

c. Depreciation Allowances.—There was the same conflict in this case between the company's claims for (1) very large annual "theoretical" depreciation-allowance additions to revenues, and (2) a comparatively small "actual" depreciation-rate-base deduction, which was found in the *Michigan Bell Telephone Case* (U.S. Dist. Ct., 1930[1]) and the *Illinois Bell Telephone Case,* (U.S.

[1] See case 58, below.

Sup. Ct., 1930, 1934.[1]). "The plaintiff deducted from gross earnings in each year since 1923 approximately 5 per cent of the cost of its depreciable property. . . . The commission approved this percentage for reserve depreciation." The company reported fixed capital and corresponding depreciation reserves at different dates as follows:

Dec. 31	Fixed capital	Depreciation reserve	Dec. 31	Fixed capital	Depreciation reserve
1924	$485,762,729	$121,514,739	1927	$561,097,131	$148,243,859
1925	547,586,978	138,191,814	1928	600,781,851	160,787,977
1926	614,488,824	156,371,844	1929	661,252,587	169,954,681

The above data are from the commission's decision of May 1, 1930. P.U.R. 1930C, *346.

How greatly the depreciation reserves exceeded the actual depreciations is illustrated by the master's finding that the actual depreciation on July 1, 1926, was $53,647,819. The Court found that the corresponding depreciation reserve was $125,000,000,[2] "making a difference of $71,000,000."

In the *Michigan Bell Telephone Case*,[3] the United States District Court allowed both the company's claims; for high annual depreciation revenue allowances, but a small actual depreciation rate-base deduction. In the *Illinois Bell Telephone Case*,[1] the United States Supreme Court disallowed the high annual depreciation revenue allowances as excessive. In the (instant) *New York Telephone Case*, the United States District Court held that the entire depreciation reserve must be deducted in determining the rate base. "The record satisfactorily shows that there is more reason to believe that the actual existing depreciation in the plaintiff's property is reflected by the amount of the reserve for depreciation than that it is shown by the estimate of the experts who stated observed depreciation. . . . "

Authors' Note.—Under this ruling, the company was permitted by the commission to increase its rates sufficiently to yield $37,734,154[4] net earnings in 1930, in addition to continuing its high theoretical annual depreciation allowances. The existing rates had yielded $27,894,219[4] net earnings in 1929.

As nearly as the authors can determine from the data at their command, a disposition of the case in line with the 1934 United States Supreme Court decision in the *Illinois Bell Telephone Case*[5] would have shown that the true rate of net earnings in 1926, would have been about 7.7 per cent if the surcharge rates had been in effect throughout the year, instead of 6.7 percent under the district court's ruling. The Court found "less than 6%" (it figures 5.84 percent) with the surcharge rates in effect one-half year.

d. The Allocation of Property, Revenues, and Expenditures to the Intrastate Business.—The value of the property was divided between intrastate and interstate business on a basis of actual use.

[1] See cases 59, 67, below.

[2] Note the discrepancy from the company's report, above.

[3] See case 58, below. The decision was soon withdrawn.

[4] See commission's decision of May 1, 1930. P.U.R. 1930C,* 358, 373.

[5] See Case 67, below.

Percentages Allocated to Intrastate Business

Year	Property value, %	Revenue, %	Operation expenses, %
1924	86.50	84.97	86.78
1926	85.75	82.25	83.91
1928	85.55	86.78	83.58

e. The 1926 Allowances for Fixed-capital Physical Values, Working Capital, and Going Value of the Property Allocated to Intrastate Business.—The master found that the book cost new for July 1, 1926, was $443,881,878; and that its reproduction cost new was $564,112,589. Deducting $46,003,005 (equal to 85.75 percent of $53,647,819) "observed depreciation," he found the depreciated reproduction-cost value to be $518,109,584. From this the Court deducted $56,658,385 (equal to 85.75 percent of $66,070,419) for speculative items (3), (4), (5), (6), listed under *c*, above; it also deducted $60,882,500 (equal to 85.75 percent of $71,000,000), additional depreciation for the difference between the master's observed depreciation and the total depreciation reserve. This left $400,571,700 for the Court's fair value, July 1, 1926, as follows:

Fixed-capital physical value new (including lands)	$481,737,075
Depreciation (85.75% of $124,647,819)	106,885,505
Depreciated fixed-capital physical value	$374,851,570
Working capital (85.75% of $15,964,000)	$ 13,689,130
Promotion expense	3,456,000
Going value (85.75% of $10,000,000)	8,575,000
The Court's fair value, July 1, 1926	$400,571,700

f. The Fair Rate of Net Return.—"A return of 7 per cent upon the fair value of plaintiff's property will yield plaintiff such fair return as will not confiscate its said property and such rate of return of 7 per cent is substituted for the rate of return of 8 per cent recommended by the special master."

55. *United Railways & Electric Company of Baltimore v. West, Chairman, et al. No.* 55. *West, Chairman, et al. v. United Railways & Electric Company of Baltimore, No.* 64. 280 U.S.* 234, 247. 50 Sup. Ct. Rep. 123. U.S. Sup. Ct., Jan. 6, 1930.

In this important decision, the United States Supreme Court established the ruling principle that, at least for public utilities, the legal depreciation base of each physical property unit is its present fair value new.

In spite of this decision, almost universal present (1935) practice continues to use original cost new as the depreciation base (but, see Sec. 6.6).

The Court divided six to three on this question. Mr. Justice Sutherland delivered the opinion of the majority.

"Upon application of the company to the commission, made in 1927, for an increase in fares, the commission passed an order making an increase, but not to the extent sought. Thereupon, suit was brought in a state circuit court on the grounds that the rate fixed by the commission was confiscatory and that the annual allowance for depreciation was calculated upon a wrong basis, namely, upon cost, instead of present value of depreciable property. The circuit court, in an able opinion, sustained the company upon both grounds and enjoined the enforcement of the commission's

order. On appeal, the court of appeals upheld the view of the circuit court in respect of depreciation, but held the rate of return not confiscatory. 155 Md. 572. Thereupon, the commission increased the depreciation allowance in accordance with the decree of the court and adjusted the rate of fare to the extent necessary to absorb the increased allowance. A second suit and appeal to the court of appeals, followed, and that court entered a decree, 157 Md. 70, sustaining the action of the commission; and it is that decree which is here for review."

a. The Value of Street Easements.—The Maryland Commission valued the company's property at $75,000,000, "and this amount was accepted without question by both parties" in the lower courts. It contained an allowance of $5,000,000 "for easements in the streets of Baltimore," made in compliance with a decision by the Maryland Court of Appeals in an earlier case. Before the United States Supreme Court, the commission challenged this allowance and made other objections to the valuation; but the Court ruled that: "We do not find it necessary to consider this challenge or these objections, for, if they ever possessed substance, they come too late."

b. The Legal Depreciation Base.—"The allowance for annual depreciation made by the commission was based on cost. The court of appeals held that this was erroneous and that it should have been based on present value. The court's view of the matter was plainly right. One of the items of expense to be ascertained and deducted is the amount necessary to restore property worn out or impaired, so as continuously to maintain it as nearly as practicable at the same level of efficiency for the public service. The amount set aside periodically for this purpose is the so-called depreciation allowance. Manifestly, this allowance cannot be limited by the original cost, because, if values have advanced, the allowance is not sufficient to maintain the level of efficiency. The utility 'is entitled to see that from earnings the value of the property invested is kept unimpaired, so that at the end of any given term of years the original investment remains as it was at the beginning.' *Knoxville v. Knoxville Water Co.* 212 U.S. 1, 13–14. This naturally calls for expenditures equal to the cost of the worn out equipment at the time of replacement; and this, for all practical purposes, means present value. It is the settled rule of this Court that the rate base is present value, and it would be wholly illogical to adopt a different value for depreciation. . . . "

c. The Fair Rate of Net Return.—The Court held that 6.26 percent rate of net return would be confiscatory in this case; and that " . . . it is not certain that 7½ per cent. or even 8 per cent. on the value of the property would not be necessary."

56. *City of New York v. Interborough Rapid Transit Co.* 240 N.Y. S* 316. P.U.R. 1030C*, 144. N.Y. Sup. Ct., Feb. 28, 1930.

For a review of the circumstances of this case see *Gilchrist et al. v. Interborough Rapid Transit Co. et al.*, above. In its opinion in that suit, the United States Supreme Court mentioned the intention of the New York Rapid Transit Commission to test the five-cent fare contract clause "by an immediate orderly appeal to the courts of the state."

The opinion of the New York Supreme Court in the instant case was by Judge Ingraham.

"The present action has been brought by the Rapid Transit Commission of the State of New York, for and on behalf of the city of New York, to compel a specific performance by the defendant of the so-called 'five-cent fare clauses' in two certain contracts to which the defendant is a party, which contracts are known, or at least have come to be known in the course of this litigation, as Contract No. 3 and the

Elevated Extension Certificate. The plaintiff also seeks to enjoin the defendant from carrying out its avowed intention of increasing the existing rate of fare on its subway and elevated lines from five to seven cents. . . .

"It is settled beyond successful dispute that a state may authorize a municipal corporation to establish by an inviolable contract the rates to be charged by a public service corporation for a definite term and thereby suspend during the life of such contract the governmental power of fixing and regulating rates. [Citing authorities.] And in such cases the courts may not relieve the utility from its obligation to serve at the agreed rates, however, inequitable. The enforcement of such rates is controlled by the obligation resulting from the contracts. . . . [Citing authorities.]

"The fare clauses appearing in the contracts here were specifically authorized and the obligation of both parties remains. The defendant cannot repudiate the portion which may have become onerous. The city is entitled to a specific performance on the part of the defendant of its contractual duty, thereby assuring to those traveling upon the lines covered by these agreements the uninterrupted continuance of the five-cent fare."

57. *Cumberland Glass Mfg. Co. v. United States.* No. J-315, 44 F. (2d)* 433. No. J-102, 44F. (2d)* 455. U.S. Ct. of Claims. Both cases decided Nov. 3, 1930.

These were suits to recover alleged overpayment of income and excess-profit taxes for the fiscal years ending June 30, 1917, 1918. The Court found for the plaintiff and ordered proper repayments. Excerpts from those portions of the decisions relating to depreciation (the main issue) are cited here because of their clear, authoritative pronouncements upon the issue of *actual* vs. *theoretical* depreciation.

The depreciation estimates affected the amount of taxes due in two ways: First, the total accrued depreciation, June 30, had to be deducted to get the "consolidated invested capital" for the following year, upon which the true rate of net return must be calculated to determine the excess-profit tax for the following year; second, the amount of depreciation during the following fiscal year had to be deducted to determine the net income on which to base the income tax.

a. The Company's Method of Estimating Actual Depreciation.—"There were certain officials who kept close contact with the business from day to day and who were familiar with the usage to which all the buildings and equipment had been put. At the end of the fiscal year these officials would go about the plants and discuss the depreciations that should be taken on the various items which might be depreciated and would decide how much to write off for wear and tear. In doing this they would consider the different items of the building and equipment separately. . . . There was a considerable difference in the nature of these items. [For example, some furnaces lasted two or three times as long as others. Some glass-making machinery lasted six years and some only two.] In determining the amount to be written off for wear and tear these officials gave consideration to the actual use and operation of the plant, the cost, age, and useful life of the various types of assets and the repairs and improvements made thereto."

b. The U.S. Commissioner of Internal Revenue's Method of Calculating Theoretical Depreciation.—"Subsequently, the Commissioner of Internal Revenue upon consideration and final audit of the returns for 1917 followed recommendations of the internal revenue agent, declined to accept the depreciation determined, and charged off by the plaintiff and its subsidiaries [there were three subsidiary plants besides the main plant] and determined depreciation reserve to June 30, 1916, and the allowance of depreciation as deductions from income for the taxable year ended June 30, 1917, by applying the following rates: Buildings, 5 per cent; machinery, 10 per cent; equipment, 7½ per cent; automobile trucks,

20 per cent; horses and wagons, 10 per cent; molds, 10 per cent."

c. The resulting estimates of depreciation were as follows:

Fiscal year ending	Actual depreciation, company's estimate	Theoretical depreciation, commissioner's estimate
June 30, 1911	$ 40,080.93	$ 72,308.03
June 30, 1912	70,305.73	75,343.30
June 30, 1913	34,393.25	79,006.63
June 30, 1914	39,790.11	84,888.06
June 30, 1915	49,364.82	88,574.91
June 30, 1916	55,405.56	91,695.37
Total 6 years	$289,340.40	491,816.30
June 30, 1917	108,443.77	98,683.56
Total 7 years	$397,784.17	$590,499.86

. . . .

The Court's Decision. Quotations from No. J-102

The case was heard and opinion rendered by Chief Justice Booth and Judges Green, Littleton, and Williams. The decision was written by Judge Williams.

d. The *actual* depreciation, as estimated by the company, was held correct, and the *theoretical* depreciation, as estimated by the commissioner, was rejected. The Court said: "What is reasonable allowance for 'wear and tear' to be charged off by a taxpayer as depreciation is one of fact to be determined in each case by the peculiar facts of such case. While the commissioner's determination is presumptively correct, it must give way if the proof shows his computation is erroneous and not in consonance with the actual facts. . . .

"In the instant case the depreciation charged off each year for the period from June 30, 1910, to June 30, 1916, was determined by officials of the company who had been connected with the management for many years and who possessed accurate and technical knowledge of the plant, its assets, and equipment. They were familiar with the actual use and operation of the plant, knew the age, cost, and probable useful life of its various items of equipment, and the current costs of repairs and replacements.

"That they acted in the utmost good faith and exercised their best judgment in estimating the depreciation charged off each year is not questioned.

"While the straight-line or fixed-percentage method used by the commissioner in determining plaintiff's allowable depreciation for the years from June 30, 1910, to and including the fiscal year ended June 30, 1917, is the one generally used in determining depreciation for tax purposes, and is quite generally accepted as the simplest and most accurate of the various methods used, computations made on that basis cannot stand where the facts in a particular case, as here, show that the result reached by the use of such methods would not be a reasonable allowance within the meaning of the statute."

58. *Michigan Bell Telephone Co. v. Odell et al.* No. 1322. 45 F. (2d)* 180. U.S. Dist. Ct., Nov. 25, 1930.

In this case, the Michigan Bell Telephone Co. applied for an injunction against the enforcement of a telephone rate order issued by the Michigan Public Utilities Com-

mission. The cities of Detroit and Grand Rapids intervened, by consent. The evidence was heard by a special master. While no statutory three-judge court was convened, the three judges of the district heard the final presentation of the case and joined in the decision, *per curiam.*

The Court approved the master's findings and issued the decree but, on Dec. 15, 1930, entered an order re-referring this cause to the special master for further proceedings; this was because of the United States Supreme Court's decision on the *Illinois Bell Telephone Case,* immediately below.

The master's findings as to the rate base and fair return in his report included the following:

a. Dominant weight was given to reproduction cost new less depreciation in determining the fair present value of the physical property. This was in view of the decision in *Monroe Gas, Light & Fuel Co. v. Michigan Public Utilities Commission,* above.

b. Only actual depreciation (11 percent) was deducted in determining the fair present value of the physical property.

c. Nevertheless, full theoretical straight-line fixed-percentage depreciation was allowed as the annual depreciation expense in determining the actual annual net return.

The Court, however, wavered upon the question of straight-line vs. the compound-interest modification of the sinking-fund theoretical depreciation method, contended for by the defendant; upon which his counsel "lavished a wealth of argument, illustration and reference," and which "is undoubtedly to him the crux of the case." "We should like to give further study to it if it were possible to do so without abandoning all other litigation now before the court," finally said the judge; and, in less than a month after his decision, he granted a motion for re-reference of the whole case.

d. Average fair value for year ending March 31, 1929.

Depreciated fair value of physical property	$136,232,372
Working capital (2.48% of present physical value)	3,378,563
Going value (8% of present physical value)	10,898,590
	$150,509,525

e. Fair rate of net return 7%

59. *Smith et al. v. Illinois Bell Telephone Co.* 51 Sup. Ct. Rep.* 65. U.S. Sup. Ct., Dec. 1, 1930.

The Circumstances and History of the Case

The American Telephone and Telegraph Company owned 99 percent of the stock of the Illinois Company; and substantially the same proportion of the stock of the Western Electric Co., manufacturers of telephone equipment, purchases from which concern made a large part of the expenses of the Illinois company. The American supplied general services to the Illinois company, receiving therefor, at first, 4½ percent of its gross revenues, reduced to 1½ percent in 1929.

On Aug. 16, 1923, the Illinois Commerce Commission issued a telephone rate order, effective within the city of Chicago, Oct. 1, 1923; thereupon, the company obtained, on Dec. 21, 1923, an interlocutory injunction by the United States District Court,[1] restraining enforcement of the rates until decision on a permanent injunction. The interlocutory decree required the company to set aside all excess collections over

[1] Affirmed by the United States Supreme Court, Oct. 19, 1925.

the disputed rates and to keep the segregated funds available for refunds if the injunction should be dissolved. The final hearing was delayed until April, 1929; the refund reserve then exceeded $11,000,000. After final hearing, the district court issued a final decree, Jan. 31, 1930, making the injunction permanent. 38 F. (2d) 77. P.U.R. 1930B* 148.

This district court final decree was appealed to the United States Supreme Court, whose decision in the present case set it aside and remanded the cause "to the district court, specially constituted as provided by the statute, for further proceedings in conformity with this opinion." Mr. Chief Justice Hughes delivered the opinion of the Court.

The Opinion of the Court

a. Further Findings of Fact.—The Supreme Court ruled that additional findings of fact should be made by the lower court on the following subjects:

(1) The value of the property devoted to intrastate business, and the amounts of the intrastate revenues and expenses, year by year; all segregated from the interstate business.

(2) The correct charges by the American Telephone & Telegraph Co., and the Western Electric Co., to the Illinois company for intrastate services and purchases, year by year.

(3) " . . . the amounts to be allowed in this case as an annual charge for depreciation in connection with the intrastate business," year by year.

(4) " . . . the rate of return which was realized from the intrastate business and the rate of return which it is fair to conclude would have been realized from that business under the prescribed rates," year by year.

b. Confiscatory Rates.—Mr. Chief Justice Hughes said: "A rate order which is confiscatory when made may cease to be confiscatory, or one which is valid may become confiscatory at a later date."

c. Property Paid For from the Depreciation Reserve.—The commission deducted from the rate base the value of the $26,000,000 depreciation reserve property paid for by the company out of its annual depreciation allowances; this action was reversed by the district court; the Supreme Court upheld the district court on this point.

d. The Actual Depreciation vs. the Depreciation Reserve.—The commission and the district court found the condition-percent of the property to be 90 percent; on the basis of the district court's fair value, this gives $13,089,236 actual depreciation, about one-half the company's $26,000,000 depreciation reserve.

e. The Relation of the Annual Depreciation Allowance to the Actual Depreciation and the Depreciation Reserve.—The Supreme Court said, illuminatingly:

"While it has been held by this Court that property paid for out of moneys received for past services belongs to the company, and that the property represented by the credit balance in the reserve for depreciation cannot be used to support the imposition of a confiscatory rate (*Board of Commissioners v. New York Telephone Co., supra*), it is evident that past experience is an indication of the company's requirements for the future. The recognition of the ownership of the property represented by the reserve does not make it necessary to allow similar accumulations to go on if experience shows that these are excessive. The experience of the Illinois company, together with a careful analysis of the results shown, under comparable conditions, by other companies which are a part of the Bell system, and thus enjoy the advantage of the continuous and expert supervision of a central technical organization, should afford a sound basis for judgment as to the amount which in fairness

both to public and private interest should be allowed as an annual charge for depreciation."

This language forcibly suggests that the annual depreciation allowance should be only sufficient to maintain a depreciation reserve just equal to the total accrued actual depreciation; and that public utilities are not to be permitted to continue to charge their customers for annual depreciation allowances higher than annual actual depreciation, while at the same time successfully fighting corresponding depreciation deductions in determining the fair values on which they charge for fair rates of return.

It is not surprising that, upon reading this opinion, a Michigan United States District Court, promptly withdrew its own already published decision,[1] upholding the Michigan Bell Telephone Co.'s contention that both these conflicting claims be allowed.

f. Allowances for Physical Value, Working Capital, and Going Value.—The authors interpret the findings of the district court as to total value, and their discussion in the Supreme Court's opinion, to imply the following approximate allowances, as of date June 30, 1923:

Original cost of fixed-capital physical property new	$101,626,014
Original cost depreciation (10%)	10,162,601
Depreciated original-cost value of fixed-capital physical property	$ 91,463,413
Reproduction cost of fixed-capital physical property new	$145,000,000
Reproduction-cost depreciation (10%)	14,500,000
Depreciated reproduction-cost value of fixed-capital physical property	$130,500,000
Fair value of fixed-capital physical property new	$130,892,364[2]
Fair value depreciation (10%)	13,089,236[2]
Present fair value of fixed-capital physical property	$117,803,128[2]
Working capital	3,000,000
Going value	4,196,872[2]
Total present fair value, June 30, 1923	$125,000,000[2]

g. The Fair Rate of Net Return.—The commission found this to be 7 percent.

8.7. The Period 1931 to 1934.—The outstanding fact during this period has been the continued prevalence of the great depression, which began late in 1929. Except for prior litigation carried over, the depression and the frantic governmental efforts to overcome it have colored and shaped most valuation litigation. Commodity prices fell below pre-war levels, but construction costs not nearly so soon or so far. Governmental efforts to increase prices included measures to regulate industry

[1] See the *Michigan Bell Telephone Case*, just above.

[2] Not less than these amounts.

to a previously unprecedented extent, limiting production, eliminating cut-throat competition, fixing minimum limits for wages and decreased maximum limits for hours of labor; the very measure of value was changed, at least as measured by gold, by the partial devaluation of the dollar, in terms of gold.

In recent valuation litigations, the courts have consistently upheld their former post-war rulings on valuation questions. The trend of their decisions seems to be to give more weight to actual facts, as disclosed by reliable, comprehensive accounting records; speculative expert witness assumptions and opinions, once so popular, seem to accomplish little or nothing in shaping opinions. This trend of decisions gives support to the present movement toward requiring continuous-inventory property-ledger accounts of industrial properties.

By far the most interesting decisions are those bearing on the numerous questions raised by the depression. The courts are exploring the line of demarcation between private industrial enterprises and public utilities; they are ruling on the right of the state to regulate publicly owned public utilities; they are beginning to decide how far the public can go in regulating private industry, by codes or by statutes, to control production, insure security, assure justice to labor and capital alike, to establish prices fair both to producers and to consumers; they are passing on the extent of governmental authority to change the obligations of existing contracts. The final adjudication of many of these questions by our highest courts remains for the future.

60. *Excess Income of Richmond, Fredericksburg & Potomac Railroad Co.* 170 I.C.C.* 451. I.C.C., April 7, 1931.

This was the Commission's first excess railway-income recapture-case decision after its valuations of the St. Louis & O'Fallon Railway had been overruled by the United States Supreme Court, for failure to give due weight to present reproduction costs.[1] In the instant decision the Commission[2] says: "In our findings of the value of the property used by the respondent in the service of transportation during the recapture years, we are according such weight to the present cost and original cost of construction as in our judgment is justified by the record. The values found reflect in substantial degree both elements of cost."

Note that the findings of the Interstate Commerce Commission in this case were never adjudicated in the courts.

a. The Commission's Method of Making the Valuations.—For each valuation year, the Commission's engineers determined the original costs, "period" reproduction costs, and "spot" reproduction costs, both new and depreciated, of all fixed-capital physical property except lands; the Commission then proceeded to adopt a physical value for the fixed-capital property (except land), giving what it considered due weights to original costs and reproduction costs. To this partial value, it added the present market value of lands used and useful and the average working capital during the year, to get the total fair value.

[1] See the O'Fallon decision, case 53, above.

[2] Commissioners Eastman and Mahaffie dissented, in part.

b. Allowances for Physical Value and Working Capital.—Analyzing the Commission's extended report, the authors conclude that their announced data and conclusions imply the following approximate allowances:

Year	Depreciated value of fixed-capital physical property except lands				Lands	Working capital	Total fair value
	Original cost	"Period" reproduction cost	"Spot" reproduction cost	Fair value			
1920	17,157,000	26,980,000	32,720,000	24,773,000	3,314,000	1,513,000	29,600,000
1921	17,600,000	26,435,000	26,530,000	24,452,000	3,428,000	1,820,000	29,700,000
1922	17,762,000	26,340,000	24,410,000	24,378,000	3,617,000	1,405,000	29,400,000
1923	18,314,000	26,870,000	27,630,000	25,084,000	3,721,000	1,295,000	30,100,000

c. Going Value.—Although the Commission did not make any separate allowance for going value, it claimed that its valuations were made "Upon consideration of the foregoing and all other facts of record pertaining to the value of the respondent's property as an economically developed, well maintained and seasoned railroad, in operation as a going concern. . . . "

d. The Use of Estimates in Supplying Omissions in the Records in Determining the Original Cost of the Property.—The Commission held that it is proper so to supply such omissions; and calls attention to the fact that the amounts and percentages of such omissions are constantly diminishing, as old units of property are retired and replaced.

e. Property Used but Not Owned.—The Richmond Terminal is owned by a separate company, whose capital stock is owned in equal shares by the Richmond, Fredericksburg & Potomac Railroad Co. and the Atlantic Coast Line Co., each of the two companies contributing to interest on bonded indebtedness, operating expenses (including taxes) and 6 percent dividends on the stock "in proportion to its use of the property as determined by the number of passenger train cars passing in and out of the station" (note the "use unit").

The Commission ruled that no part of the value of the Terminal Property should be included in the rate base of the Richmond, Fredericksburg & Potomac Railroad Co. Note that the Richmond, Fredericksburg & Potomac Railroad Co.'s terminal payments are regularly included in its expense and income accounts, and that the Terminal Company was held to be a separate "common carrier."

f. Overheads.—The Commission allowed overheads in estimating the reproduction costs as follows:

(1) General expenditures, 1½ percent of the road accounts, except land.

(2) Engineering, 4 percent of the road accounts, except land and engineering itself.

(3) Contingencies, nothing.

(4) Interest during construction (based, with one minor exception, on 6 percent interest and a three-year construction period), 10½ percent of road accounts and general expenditures, plus three months' interest on equipment costs.

g. The Depreciation Base.—In spite of the decision of the United States Supreme Court in the *Baltimore Street Railway Case* (see case 55, above), the Interstate

Commerce Commission held in favor of original cost new instead of present replacement cost new for the depreciation base in determining annual depreciation expense. The Commission argued that the Baltimore decision does not apply to "recapture" cases; on the theory that Congress, in passing the recapture law, had in mind the present depreciation-accountancy methods prescribed in the Commission's uniform systems of accounts.

h. The Deduction of Depreciation from Original Cost New.—The Commission held that accrued depreciation must be deducted in determining original-cost value, as well as in determining reproduction-cost value.

i. The "deferred-maintenance" theory of depreciation, argued for by the railroad company, was squarely overruled by the Commission.

61. *Georgia Public Service Commission et al. v. United States et al.* 51 Sup. Ct. Rep.* 619. No. 55, U.S. Sup. Ct., June 1, 1931.

62. *State of Alabama et al., Appellants, v. United States et al.* 51 Sup. Ct. Rep.* 623. No. 513, U.S. Sup. Ct., June 1, 1931.

The opinions in these two cases were both delivered by Mr. Justice Brandeis.

Both opinions uphold the right of the Interstate Commerce Commission to order and require state authorities to establish specific intrastate rates not lower than the corresponding rates properly ordered by the Commission for interstate business. The Interstate Commerce Commission had issued such orders in each of the two cases, after unsuccessful efforts to secure state cooperation. Both states made unsuccessful appeals for injunctions to the United States district courts.

The United States Supreme Court has repeatedly upheld the principle that the several states have the authority, subject to the United States Constitution, to control intrastate rates. The decisions in these two cases uphold the principle that state authority over intrastate rates must not be exercised in such a manner as to nullify interstate rates properly ordered by United States authorities.

63. *City of Logansport v. Public Service Commission et al.* 177 N.E. Rep.* 249. No. 25389. Indiana Sup. Ct., July 1, 1931.

This decision is of special importance on account of its rulings on the subject of state authority to control municipally owned public utilities; this disputed question bids fair to come up frequently in valuation litigation. The Court divided, two justices dissenting. Justice Martin delivered the majority decision.

In this case, the city of Logansport, Ind., appealed from a decision of the Cass County Circuit Court upholding an order, issued by the Indiana Public Service Commission, reducing the rates charged the public for electric current or service by an electric light plant owned and operated for 30 years by the city, " . . . paid for originally by funds raised by taxation, and enlarged and extended in part with funds raised by taxation and in part with surplus earnings of the plant. . . . " The reductions ordered varied from one-tenth to one cent per kilowatt-hour "for different classes of service and amounts of current consumed." The city instituted action to set aside the order, but the circuit court sustained demurrers by the commission to all four paragraphs of the city's complaint.

The City's Complaints

" . . . The theories of the several paragraphs of complaint are as follows":

"Paragraph I. That the right to manage the electric light property and fix the rates to be charged is vested in the city by reason of its inherent power as an independent body politic or by the right of self-government, and that such rates cannot be controlled by the legislature or by any commission appointed by it.

"Paragraph II. That the law creating the Public Service Commission of Indiana (The Spencer-Shively Act), Chapter 76, Acts 1913, §§12672–12802, Burns Ann. St. 1926, does not apply to municipally owned public utilities, and that the commission has no authority thereunder to fix the rates in question.

"Paragraph III. That the rates fixed by the commission are inadequate and confiscatory, and are therefore unlawful and unconstitutional, under section 21, art. 1, const., section 73, Burns' Ann. St. 1926, in that they will yield only sufficient revenue to pay operating and maintenance charges and that they do not provide and are not intended to provide for a reasonable return by the way of interest or earning power upon the investment [that is, a fair return upon the fair value of its property], the same as if it were a privately owned utility.

"Paragraph IV. That the rates are insufficient because they do not yield a sum sufficient to compensate the city for the taxes which the plant would have to pay if it were privately owned."

The Court's Rulings

The Indiana Supreme Court's rulings upon the city's complaints are as follows:

"I. *The State has the Power to Regulate the Rates to be Charged by Municipally Owned Public Utilities.*

"A city in the operation of an electric light utility, selling service to the public, acts in its private business capacity and not in its public governmental capacity, regardless of whether its power to so act is inherent, implied, or is granted by statute. . . . When a municipal corporation engages in an activity of a business, rather than one of a governmental nature, such as the supply of light or water or the operation of a railroad which is generally engaged in by individuals or private corporations, it acts as such corporation and not in its sovereign capacity. . . .

"Our own conclusion upon the question presented by the first paragraph of complaint is that the state has the power to regulate, either directly or through a Public Service Commission, the rates to be charged for utility service by a municipally owned public utility. And this we hold regardless of whether the city possessed its power to operate such plant by legislative grant or as an implied or inherent right and regardless also of whether the city, in operating the utility, is considered as acting in its governmental or in its private business capacity."

"II. *The Spencer-Shively Act, Creating the Public Service Commission, Expressly Applies to Municipally Owned Utilities and Such Commission Has Authority Thereunder to Fix the Rates to be Charged the Public by the City of Logansport for Electric Current. Sections* 101 *and* 102 *of the Act Do Not Limit Its Application Except in the Case of Utilities Operating Under Existing Franchises Which Regulate Rates.*"

Note.—It is obvious, however, that the legislature could amend the law to exclude municipally owned utilities from the jurisdiction of the Public Service Commission if it so desired. By a new act, passed in 1932, the legislature took away from the Commission the right to refuse permits to municipalities to construct plants, but left it the right to regulate rates for service by municipally owned utilities.

"III. *A Municipally Owned Utility is Entitled to Receive a Fair Return by Way of Interest Upon the Investment the Same as a Privately Owned Utility, and the Matter of Earning Such Return or Not is One of Policy for the Municipal Authorities.*

"It is argued by appellees that no return should be allowed as interest on the investment because the purpose of municipal government is to 'serve the public and not to produce revenue' and that 'local taxes should not be gathered in the guise of utility rates.' It appears to us that the purpose of municipalities which have decided to own utilities may have been to reduce the cost of the service to their users and at the same time to secure a reasonable return on the investments for the cities which

otherwise would go to private corporations. The taxpayer who is not an electric light consumer has his money tied up in a municipal plant the same as the taxpayer who is also a consumer. It is unfair that the money obtained from the former by taxation should be allowed to work only for the benefit of others. It should earn a reasonable amount which should go to the general fund of the city and thereby reduce its taxes. Local taxes should not be gathered in the guise of utility rates, but rates which provide for no more than a reasonable interest return on the investment are not local taxes in disguise." . . .

"IV. *A Municipally Owned Utility is Not Entitled in Addition to a Reasonable Return upon its Investment to Charge Rates to Yield a Sum Sufficient to Compensate the City for the Taxes Which Would be Paid upon the Plant if it were Privately Owned.*

"'The property of any county, city, town or township' which is exempted from taxation by clause 2, §14037, Burns. Ann. St. 1926, includes the electric light plant here involved. . . . Taxes not being actually levied and paid, they should not be included as a cost of service."

Note.—If, however, the property were not exempt, rates high enough to include taxes actually paid would be justifiable. Note further that:

"The Wisconsin Railroad Commission has in a number of cases held that municipally owned utilities which are exempt from taxation . . . may include as a part of the operating expenses local taxes, excluding state and county taxes, . . . on the theory that they would be collected and placed in the general fund of the city if the plant were privately owned and operated."

64. *New State Ice Co. v. Ernest A. Liebmann.* 52 Sup. Ct. Rep.* 371. No. 463. U.S. Sup. Ct., March 21, 1932.

The decision in this case is of special present interest because of the apparent present probability of wide extension of public regulation of industry; codes and other measures might give the majority of industrial enterprises a near public-utility status.

"The New State Ice Company, engaged in the business of manufacturing, selling and distributing ice under a license[1] or permit duly issued by the Corporation Commission of Oklahoma, brought this suit against Liebmann in the federal district court for the Western District of Oklahoma to enjoin him from manufacturing, selling, and distributing ice within Oklahoma City without first having obtained a like license[1] or permit from the commission." . . .

"The portion of the section immediately in question here is that which forbids the commission to issue a license to any applicant except upon proof of necessity for a supply of ice at the place where it is sought to establish the business, and which authorizes a denial of the application where the existing facilities 'are sufficient to meet the public needs therein.' The district court dismissed the bill of complaint for want of equity, on the ground that the manufacture and sale of ice is a private business which may not be subjected to the foregoing regulation. 42 F. (2d) 913. . . . The court of appeals affirmed. 52 F. (2d) 349." . . .

Upon further appeal, the United States Supreme Court affirmed the decree of the lower courts. Mr. Justice Cardozo took no part in the case. Justices Brandeis and Stone dissented. Mr. Justice Sutherland delivered the opinion of the Court.

a. Review of Former Decisions on What Industrial Enterprises Are Public Utilities.—Mr. Justice Sutherland said:

"It must be conceded that all businesses are subject to some measure of public regulation. And that the business of manufacturing, selling, or distributing

[1] The license in question was required by an Oklahoma statute, which declared that the ice business is a public business; provided for a hearing; required a showing of necessity; and required satisfactory qualifications and capacity.

ice, like that of the grocer, the dairyman, the butcher, or the baker, may be subjected to appropriate regulations in the interest of the public cannot be doubted; but the question here is whether the business is so charged with a public interest as to justify the particular restriction above stated."

The Justice then reviews the following border-line public-utility decisions:

Frost v. Corporation Commission. 278 U.S. 515, 519–521. 49 Sup. Ct. 235. 73 L. ed. 483. In this case, the Oklahoma statute, here questioned, was upheld in so far as it applies to cotton gins, in that state. See also *Chickashaw Cotton Oil Co. v. Cotton County Gin Co.* (C.C.A. 1930) 40 F. (2d) 846. 74 A.L.R. 1070.

Head v. Amoskeag Manufacturing Co. (1885.) 113 U.S. 9, 16–19. 5 Sup. Ct. 441. 28 L. ed. 889. *State v. Edwards.* (1893.) 86 Me. 102, 104–106. 29 A 947. 25 A.L.R. 504. 41 Am. St. Rep. 528. In these cases, grist mills, and some other kinds of mills, were held to be public utilities when the proprietors assumed to serve the general public.

Clark v. Nash. (1905.) 198 U.S. 361. 25 Sup. Ct. 676. 49 L. ed. 1085. 4 Ann. Cas. 1171. *Strickley v. Highland Gold Mining Co.* (1906.) 200 U.S. 527. 26 Sup. Ct. 301. 50 L. ed. 581. 4 Ann. Cas. 1174. These cases involved the validity of a Utah statute which declared that, in Utah, "irrigation of land, the mining, milling, smelting or other reduction of ores, and such use and application of such waters for electrical power to be applied as aforesaid are hereby declared to be for the public use, and the right of eminent domain may be exercised in behalf thereof," Chapter 95, §1, Laws of Utah, 1896. In the *Nash Case,* the United States Supreme Court sustained, under this statute, "The condemnation of a right of way across the lands of one private owner to convey water for the purpose of irrigating the lands of another private owner." In the *Strickley Case,* "the condemnation of a right of way for an aerial bucket line across private lands, for the purpose of transporting ores from a mine in private ownership, was upheld under the same statute."

Note.—Mr. Justice Sutherland might also have reviewed *Munn v. Illinois,*[1] in which public warehouses were held to be public utilities; *German Alliance Insurance Co. v. Lewis,*[2] in which a Kansas statute regulating insurance rates was upheld; and the various cases cited in these two decisions.

b. Enterprises for Manufacturing, Selling, and Distributing Ice Not Public Utilities.—But, Justice Sutherland goes on to say, he has discussed the above cases "in order to put them in contrast with the completely unlike circumstances which attend the business of manufacturing, selling and distributing ice. Here we are dealing with an ordinary business, not with a paramount industry, upon which the prosperity of the entire state in large measure depends. It is a business as essentially private in its nature as the business of the grocer, the dairyman, the butcher, the baker, the shoemaker, or the tailor, each of whom performs a service which, to a greater or less extent, the community is dependent upon and is interested in having maintained; but which bears no such relation to the public as to warrant its inclusion in the category of business charged with a public use."

. . . .

" . . . wherever gas or electricity is available (and one or the other is available in practically every part of the country), anyone for a comparatively moderate

[1] See case 3, above.
[2] See case 25, above.

outlay may have set up in his kitchen an appliance by which he may manufacture ice for himself. . . .

"Plainly, a regulation which has the effect of denying or unreasonably curtailing the common right to engage in a lawful private business, such as that under review, cannot be upheld consistent with the Fourteenth Amendment. . . . "

Accordingly, the United States Supreme Court ruled against the validity of the Oklahoma statute, as applied to the ice business.

Mr. Justice Brandeis' dissenting opinion, including footnotes, is more than twice as long as the majority opinion. Following his discussion of what may be termed the ordinary legal questions at issue, he says:

"Eighth. The people of the United States are now confronted with an emergency more serious than war. Misery is widespread, in a time, not of scarcity, but of over-abundance. . . . Some people believe that the existing conditions threaten even the stability of the capitalistic system. Economists are searching for the causes of this disorder and are re-examining the bases of our industrial structure. Business men are seeking possible remedies. Most of them realize that failure to distribute widely the profits of industry has been a prime cause of our present plight. But rightly or wrongly, many persons think that one of the major contributing causes has been unbridled competition. . . . Many insist there must be some form of economic control. There are plans for proration. There are many proposals for stabilization. And some thoughtful men of wide business experience insist that all projects for stabilization and proration must prove futile unless, in some way, the equivalent of the certificate of public convenience and necessity is made a prerequisite to embarking new capital in an industry in which the capacity already exceeds the production schedules."

But, Mr. Justice Brandeis goes on to say:

"Whether that view is sound nobody knows. The objections to the proposal are obvious and grave. The remedy might bring evils worse than the present disease. The obstacles to success seem insuperable. . . . We have been none too successful in the modest essays in economic control already entered upon. The new proposal involves a vast extension of the area of control. . . . each of the thousands of these judgments would call for some measure of prophecy. Even more serious are the obstacles to success inherent in the demands which execution of the project would make upon human intelligence and upon the character of men. Man is weak and his judgment is at best fallible."

65. *Wabash Valley Electric Co. v. Young.* 287 U.S. 488. 53 Sup. Ct. Rep.* 234. 77 L. ed. U.S. Sup. Ct., Jan. 9, 1933.

This suit was an appeal from a decision of the United States District Court for the Southern District of Indiana. 1 F. Supp.* 106. In the action, the plaintiff, Wabash Valley Electric Co., sought[1] "to enjoin the defendant, Public Service Commission of Indiana, from the enforcement of an order wherein a schedule of rates was fixed by it for electricity furnished by plaintiff to the city of Martinsville, Ind., and the inhabitants thereof." The Commission's rate order was to be effective as of date Feb. 1, 1929. The city of Martinsville was permitted to intervene in the suit. The district court refused an injunction; upon appeal, the United States Supreme Court affirmed the lower court's adverse decree in an opinion delivered by Mr. Justice Sutherland.

The appellant, one of seven affiliated Indiana public-utility corporations, "owns and operates an interconnected system in a territory comprising thirteen counties of the state, and sells and distributes electric current to approximately fifty cities and towns therein, including the inhabitants of the city of Martinsville, and also to a large

[1] Quoted from the district court's decision.

number of industrial plants and customers outside the limits of such cities and towns. Appellant's system consists in the main of general transmission and transformation properties, and local distributing plants. . . . Nearly all the electric current distributed by the appellant is 'purchased' by it from one of the other affiliated corporations which has had in operation since 1924 an extensive and modern generating plant known as the 'Dresser Plant.'"

a. Rate Base for Local Community Service Charges.—"The court below held that under the provisions of the state statute and in the light of the facts, not the entire property and system of the appellant, but the city of Martinsville alone, should be treated as the unit for the purpose of determining the schedule of rates to be charged therein."

In adopting this view, the United States District Court, in the absence of a ruling by the Indiana Supreme Court, followed a Wisconsin Supreme Court decision, construing a similar act, by which the Wisconsin Commission was "required to treat the municipality as a unit and to base its rate upon the cost of serving the individual municipality rather than the average cost of serving many distinct and scattered municipalities."

Of this ruling, the United States Supreme Court said: " . . . and with that view we see no reason to disagree. . . . "

"Normally, the unit for rate-making purposes, we may assume, would be the entire interconnected operating property of a utility used and useful for the convenience of the public in the territory served, without regard to particular groups of consumers or local subdivisions. But conditions may be such as to require or permit the fixing of a smaller unit."

b. Method of Making the Valuation.—Referring to the district court's procedure, Mr. Justice Sutherland said: " . . . in fixing the value of the property used and useful for supplying electric current to the city, the court determined the value of the local property, to which it added that proportionate part of the value of the system property which it found to be fairly attributable to the Martinsville service."

c. Allocation of "Power-system" Property to Local Community Rate Base.—This allocation was made on the basis of use, measured by proportionate sales of kilowatt-hours of electricity. The lower court decided that 3.30 percent would be a fair average proportionate allocation.

d. Fair Value vs. Reproduction Cost New Less Depreciation.—The lower court found, and the Supreme Court affirmed, that, in this case, the fair physical value did not exceed the reproduction cost new less depreciation; and that it was not necessary to determine the true fair value; the rate ordered was not confiscatory, even for a rate base including reproduction cost new less depreciation.

e. Allowances for Working Capital, Going Value, and Reproduction-cost Physical Value.—These were as follows:

	Martinsville Property	"Power System"
Depreciated reproduction-cost value of fixed-capital physical property	$ 84,589	$2,569,395
Working capital (including materials and supplies)	9,858	247,000
Going value	8,500	250,000
Total reproduction-cost values	$102,947	$3,066,395
3.30% of "power-system" values	101,191	
Fair Martinsville rate base not greater than	$204,138	(May, 1930).

f. Annual Depreciation.—Four percent was allowed.

g. Allocation of Power-system Operating Costs.—The lower court found that the power-system operating cost of electricity, delivered at Martinsville, was $0.0133 per kilowatt-hour, "exclusive of any allowance for depreciation and return."

h. Allocation of Costs of Rate-case Expenses.—The company claimed a Martinsville operating expense of more than $60,000 for rate-case costs; the courts allowed $400 per year for 10 years. There was evidence "justifying the conclusion that a large part of the expenditure was made" for an unrelated merger proceeding and for appraising the local properties in the other municipalities. The lower court said that to allow the company's claim "will serve as a penalty for the filing of a petition by a municipality, or its citizens, for a reduction of rates, because it will invariably culminate in an increase, rather than a reduction."

i. The Fair and the Actual Rate of Net Return.—Both courts held that rates yielding 7 percent net return would not be confiscatory in this case. The lower court found that the proposed rates would have yielded $81,868.72 gross income during the year ending May 31, 1930. As shown below, the corresponding net return would have been $20,440.18, which is 10.01 percent on the $204,138 rate base.

Power, 2,528,511 kilowatt-hours at $0.0133	$33,629.20
Rate expense	400.00
Other expenditures	25,544.43
	$59,573.63
Deduct construction expenditures	4,920.25
Total operating expenses	$54,653.38

Gross income	$81,868.72
Operating expenses	54,653.38
Operation return	27,215.34
Depreciation (4%)	6,775.16
Net return	$20,440.18

66. *Los Angeles Gas & Electric Corp. v. Railroad Commission of California et al.* 53 Sup. Ct. Rep.* 637. 412 U.S. Sup. Ct., May 8, 1933.

This suit was an appeal from the District Court of the United States for the Southern District of California. From an adverse decree [58 F. (2d) 256], the Los Angeles Gas & Electric Corp., plaintiff, appeals.

Mr. Chief Justice Hughes delivered the opinion of the Court. It is quoted herein at considerable length because it contains the most comprehensive, authoritative collection yet made of authoritative rulings on present valuation law in the United States.

"The company, organized in 1909, supplies both gas and electric current. Its rates for the latter are not in controversy. The two departments, both with respect to investment and operation, are distinct, and have been separately treated for rate-making purposes for many years. From 1913, when natural gas in substantial quantities was first made available in Los Angeles, until 1927, the company distributed a mixture of natural and manufactured gas, and since 1927 straight natural gas has been distributed. The company's service extends over the greater part of Los Angeles and neighboring cities and unincorporated territory. It has over 2,900 miles of mains and 385,000 meters. From 1917 the company's gas rates have been fixed by the California Railroad Commission. Rate orders were made in 1917, 1919, 1921, 1923, 1926 and 1928. During this period the company's business greatly increased.

The rate base for its gas department, as fixed by the commission, grew from approximately $12,500,000 in 1916 to about $59,000,000 in 1929. . . .

"Under the commission's order of 1928, the gas rates were estimated to yield a return slightly in excess of 7.5 per cent. 32 C.R.C. 374, 386. Concluding that these rates actually yielded a much higher return, the commission reduced the rates by the order now under review. It was intended to effect a reduction of 9 per cent in gross revenue. 35 C.R.C. 463, 469. The reduction amounted to about $1,300,000 in gross revenue and about $1,080,000 in net revenue."

a. Synopsis of the Supreme Court's Principal Rulings.—The most important pronouncements in the opinion delivered by Mr. Chief Justice Hughes may be summarized as follows:

(1) Consistently continuing to uphold the *Smyth v. Ames* rule, that "such weight as is just and right in each case" must be given to all factors affecting value, by considered judgment, not by "formula," the Court found that in this case original cost ("historical cost") was entitled to dominant weight, in determining physical value.

(2) Confirming the rule that actual depreciation must be deducted from fair value new to determine the present fair rate base, the Court upheld the California Commission's use of a fair-value new rate base, in combination with an annual depreciation sinking-fund annuity allowance.

(3) Upholding the principle that the law requires a fair allowance for going value to be included in the rate base, the Court found that the California Commission's failure to make a straightforward going value allowance was sufficiently offset, in this particular case, by its errors in determining the original-cost (historical cost) value new, combined with its addition of $4,796,000 to its erroneous result.

(4) Finding that—owing to this coincidence of compensating errors—the Commission's erroneously reached rate base was sufficiently close to the true fair value, the Court refused to delay justice by remanding the suit for retrial because the rates were "arrived at by arbitrary methods condemned by our decisions."[1]

b. The Function of the Courts in Rate Cases.—"We approach the decision of the particular questions thus presented in the light of the general principles this Court has frequently declared. We have emphasized the distinctive function of the Court. We do not sit as a board of revision, but to enforce constitutional rights. [Citations omitted.] The legislative discretion implied in the rate-making power necessarily extends to the entire legislative process, embracing the method used in reaching the legislative determination as well as that determination itself. We are not concerned with either, so long as constitutional limitations are not transgressed."

c. The Fair-value Rate Base and Its Determination.—"This Court has repeatedly held that the basis of calculation is the fair value of the property; that is, what the complainant is entitled to demand, in order that it may have 'just compensation' is 'a fair return upon the reasonable value of the property at the time it is being used for the public.' . . . And mindful of its distinctive function in the enforcement of constitutional rights, the Court has refused to be bound by any artificial rule or formula which changed conditions might upset. We have said that the judicial ascertainment of value for the purpose of deciding whether rates are confiscatory 'is not a matter of formulas, but there must be a

[1] The quotation is from the minority dissenting opinion by Mr. Justice Butler, in which Mr. Justice Sutherland joined.

reasonable judgment having its basis in a proper consideration of all relevant facts.' "

d. The Just and Right Weights Due Original Cost.—"The actual cost[1] of the property—the investment the owners have made—is a relevant fact. [Citations omitted.] But, while cost must be considered, the Court has held that it is not an exclusive or a final test. The public have not underwritten the investment. The property, on any admissible standard of present value, may be worth more or less than it actually cost. The time and the circumstances of the outlay, and the effect of altered conditions, demand consideration. Even when cost is revised so as to reflect what may be deemed to have been invested prudently and in good faith, the investment may embrace property no longer used and useful for the public. This is strikingly illustrated in the present case where the company has a large gas manufacturing plant which, in view of the supply of natural gas, has not been used for several years and is not likely to be used for many years to come, if at all. But no one would question that the reasonable cost of an efficient utility system 'is good evidence of its value at the time of construction.' We have said that 'such actual cost will continue fairly well to measure the amount to be attributed to the physical elements of the property so long as there is no change in the level of applicable prices.' [Citations omitted.] And, when such a change in the price level has occurred, actual experience in the construction and development of the property, especially experience in a recent period, may be an important check on extravagant estimates."

e. The Just and Right Weights Due Reproduction Cost.—"This Court has further declared that, in order to determine present value, the cost of reproducing the property is a relevant fact which should have appropriate consideration. [Citations omitted.] In *Southwestern Bell Telephone Co. v. Public Service Commission*, *supra*, this Court said that 'it is impossible to ascertain what will amount to a fair return upon properties devoted to public service, without giving consideration to the cost of labor, supplies, etc., at the time the investigation is made. An honest and intelligent forecast of probable future values, made upon a view of all the relevant circumstances, is essential. If the highly important element of present costs is wholly disregarded, such a forecast becomes impossible.' [Citations omitted.] But, again, the Court has not decided that the cost of reproduction furnishes an exclusive test. [Citations omitted.] We have emphasized the danger in resting conclusions upon estimates of a conjectural character. We said, in Minnesota rate cases, supra [citations omitted]: 'The cost-of-reproduction method is of service in ascertaining the present value of the plant, when it is reasonably applied and when the cost of reproducing the property may be ascertained with a proper degree of certainty. But it does not justify the acceptance of results which depend on mere conjecture. , . . ' "

f. Original Cost Entitled to Dominant Weight in the Los Angeles Gas Case.—"The weight to be given to actual cost, to historical cost,[2] and to cost of reproduction new, is to be determined in the light of the facts of the particular case. [Citations omitted.]

"In determining the weight to be ascribed in the instant case to historical cost as shown by the evidence, the outstanding fact is that the development of the

[1] See p. 308 for the distinction between "actual cost" and "historical cost." Original cost is the same as historical cost.

[2] The Supreme Court's "historical cost" is termed "original cost" in this treatise.

property had, for the most part, taken place in a recent period. . . ." The company's estimate of capital and the rate base as fixed by the Commission for 1919 were under $19,000,000. "Thus the additions and betterments which brought the historical cost of the fixed property (with land at current values) up to $58,842,187, as found by the commission at the end of 1929, took place in the ten preceding years and two-thirds of the latter amount appears to have been the cost of additions and betterments after January 1, 1922, as the rate base taken at that time was approximately $20,000,000. 20 C.R.C. P. 97, 98. We have had occasion to take judicial notice of the high level of prices of labor and materials prevailing not only from 1917, as incident to the war, but also in 1922 and 1923, and that there was no 'substantial general decline' in such prices from that time to 1926. [Citations omitted.] During these years the historical cost of the company's fixed property increased by additions and betterments to over $52,000,000. 29 C.R.C. P. 181. There can be no question that the cost of additions and betterments from 1926—in the period just preceding the commission's order under review—was good evidence of their value at that time. And, so far as prices of labor and materials are concerned, we find no warrant for a conclusion that there had been any change in levels during the years that intervened from the first valuation in 1917 which made it unfair to the company, in fixing rates for the future, to take the historical cost as found by the commission as evidence of the value of the company's structural property at the time of the rate order. On the contrary, it clearly appears that, by reason of the downward trend, the prices for labor and materials, which were reflected in that historical cost were higher than those which obtained during the later period to which the prescribed rates apply."

And, accordingly, the Supreme Court proceeded to give original-cost (historical cost) dominant weight in determining the approximate, true fair present value of the company's fixed-capital physical property.

g. Obsolescent Plant Should Be Excluded as Not Used or Useful.—"There is no evidence which discredits the commission's conclusion that the supply of natural gas will be abundant and constant. The commission found in effect that at least two of the artificial gas manufacturing plants of plaintiff were no longer needed and might well be retired. Nevertheless it included them in its valuation as a live necessary part of the operative property. It appears that, had these plants been eliminated, the fair value base would have been reduced by approximately $3,000,000."[1] The Supreme Court held that "the finding of the district court is amply sustained that, if the manufacturing facilities no longer needed had been eliminated, the fair value base would have been reduced by about $3,000,000."

h. Overheads Allowances.—Although it was the opinion of the commission's engineers that the company could reasonably have charged 11.25 percent for overheads in its historical-cost accounts, it actually charged only 6.35 percent. Accordingly, the commission allowed only 6.35 percent. The Supreme Court ruled that 11.25 percent should have been allowed, thereby increasing historical cost $2,177,475.

i. Accrued Depreciation.—"No ground appears for challenging the finding of the Commission, made upon inspection and appraisal, that the accrued depreciation of the property amounted to $7,650,000. . . . In determining present value, deduction must be made for accrued depreciation. [Citing the *Knoxville Water Case*, and the *Minnesota Rate Cases*, both above.] But the commission

[1] Quoted from the district court's opinion. 58 F. (2d) 259.

made its calculation of the company's return, under the rates prescribed, upon the rate base it fixed, undepreciated."

j. Going Value. (The Supreme Court's discussion of going value is quoted at much length in Sec. 12.11, which see.)—Circuit Judge Wilbur said, in his district court opinion [58 F. (2d) *270], that: "The railroad commission expressly refused to make any specific allowance for going concern value." The Supreme Court held that a fair going-value allowance must be included in the rate base. It found, however, that the commission's errors in determining the historical cost, together with its allowance of $4,796,000 more than historical cost, combined—by coincidence—to compensate for its failure to make a going-value allowance.

k. The California Railroad Commission's Findings as to Historical Cost New, Reproduction Cost New, and Fair Value New, All as of June, 1930.

(1) The commission found that the historical cost new was $60,704,290, including overheads at 6.35 percent. This included $3,500,888 for land, at current value; $429,126 for organization and franchise expense; $450,000 for materials and supplies; $645,000 cash working capital; but no specific allowance for going value.

(2) The corresponding estimate of the commission's engineer for reproduction cost new was $72,467,928 with 21.64 percent overheads.

(3) The commission's adopted fair-value-new rate base was $65,500,000.

l. The Supreme Court's Analysis of the Commission's Findings.—This was based on the conclusion that, in this case, historical cost (original cost) should be given dominant weight in determining fair physical value.

The commission's historical cost new		$60,704,290
The commission added to historical cost		4,795,710
The commission's fair value new		$65,500,000
Deduct obsolescent plants	$3,000,000	
Add difference between 6.35 and 11.25% overheads	2,177,475	
Net deduction	$ 822,525	
Corrected historical costs new		$59,881,765
Balance (available to consider as going value)		$ 5,618,235

m. The Supreme Court's Approximate Allowances for Physical Value, Working Capital, Preliminary Expense, and Going Value.—On the basis of the findings and analysis reported in paragraphs *i*, *k*, and *l*, above, the Supreme Court allowances would be as follows:

Lands at current values		$ 3,500,888
Fixed-capital physical property except land:		
Direct costs fair value new (overhead base)	$49,309,439	
11.25% overheads	5,547,312	
Fair value new	$54,856,751	
Accrued actual depreciation (13.954512%)	7,650,000	
Depreciated fixed-capital fair physical value, besides lands		$47,206,751
Total depreciated fair value of fixed-capital physical property		$50,707,639

Working capital ($450,000 materials and supplies, $645,000 cash)	1,095,000
Preliminary expenses	429,126
Going value	5,618,235
Depreciated fair value, the true rate base	$57,850,000
Accrued depreciation	7,650,000
Fair value new	$65,500,000

n. The Sinking-fund Depreciation Annuity Allowance for a Nominal Rate Base Equal to Value New.—The commission, using the sinking-fund theoretical depreciation method, made a "depreciation-annuity" allowance of $1,072,000, coupled with the use of a "nominal" rate base equal to fair value of depreciable property new; instead of true rate base equal to depreciated fair value. The Supreme Court noted this procedure without disapproval. The use of the fair-value-new nominal rate base is to compensate for the fact that the sinking-fund annuity is less than the real annual depreciation. See Secs. 5.25, 5.26.

o. The Fair Rate of Net Return.—The commission sought to fix rates which would yield 7 percent net return. The Supreme Court said: " . . . we find it impossible to hold that 7% is so low as to be confiscatory."

67. *Lindheimer et al. v. Illinois Bell Telephone Co. Illinois Bell Telephone Co. v. Lindenheimer et al.* 54 Sup. Ct. Rep. * 658. Nos. 440, 548. U.S. Sup. Ct., April 30, 1934.

In this notable decision, the United States Supreme Court cut incisively through a maze of technical evidence and lower court rulings to find and expedite justice. The legal principle deciding justice was found to be that correct annual depreciation allowances must equal, but not exceed, those necessary to maintain a depreciation reserve which constantly equals, but does not exceed, the true, total accrued actual depreciation. The opinion is classic upon this subject.[1]

The case had come to the United States Supreme Court for the second time. In the suit, an interlocutory injunction had been secured, in 1923, from the United States District Court, Northern District of Illinois, against the enforcement of an Illinois Commerce Commission order reducing telephone rates in the city of Chicago. The injunction order required that all collections in excess of the disputed rates should be kept impounded for possible refunds to subscribers upon final decision of the suit. Owing to delays due to the city, final hearing was not had till 1929; upon its conclusion, the district court granted a permanent injunction decree. Upon appeal, this decree was reversed by the United States Supreme Court, in a decision which has already been reviewed herein.[2] The case was remanded for further procedure.

After taking much further testimony, the district court again granted a permanent injunction decree. 3 F. Supp. 595, April 29, 1933. The opinion in this case contained a long list of findings and conclusions on fair values, total and annual depreciations, fair operating-cost charges, incomes, actual and fair returns, etc.; all segregated, for each year 1923 to 1932, between interstate business, intrastate toll business, and intrastate subscriber business. The essence of the district court's conclusions was that even the unreduced rates were confiscatory, each and every year, yielding only from 4.8 percent net return in 1932 to 5.8 percent in 1929.

[1] The 1934 *Illinois Bell Telephone Case* opinion and the 1933 *Los Angeles Gas Case* opinion, both delivered by Mr. Chief Justice Hughes, together constitute a classic, comprehensive, authoritative statement of present valuation law.

[2] See *Smith et al. v. Illinois Bell Telephone Co.*, Nov. 25, 1930, case 59 above.

These confiscatory rates of return were calculated upon adopted annual depreciation allowances sufficient to maintain depreciation reserves far in excess of the highest estimates of the total accrued depreciation. Thus the district court failed to heed the plain admonition[1] of the Supreme Court when it remanded the case.

a. The Financial History of the Illinois Bell Telephone Co. Refutes the Conclusion that It Had Been Operating under Confiscatory Rates.

"The financial history of the Illinois Company repels the suggestion that during all these years it was suffering from confiscatory rates. . . . During this period appellee paid the interest on its debt [about $50,000,000 since 1923] and 8 per cent dividends on its stock. . . . The book cost of the plant in service and general equipment in intrastate business in Chicago increased from $95,582,666 at the end of 1923 to $174,160,314 at the end of 1930 and to $177,384,628 at the end of 1931. . . . 'The business expanded with great rapidity. The number of telephones in Chicago increased from 690,000 at the end of 1923 to 940,000 at the end of 1931, and was 987,000 at the peak in 1929.' . . . The company informs us that the property was kept 'at a high and even standard of maintenance throughout the years involved' . . . and 'was at all times capable of giving adequate telephone service abreast of the art.' The property has been efficiently and economically operated, and the company has enjoyed excellent credit.

"This actual experience of the company is more convincing than tabulations of estimates. In the face of that experience, we are unable to conclude that the company has been operating under confiscatory rates."

b. The United States Supreme Court's Opinions on Depreciation and the Depreciation Reserve.—"Broadly speaking, depreciation is the loss, not restored by current maintenance, which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy, and obsolescence. Annual depreciation is the loss which takes place in a year. In determining reasonable rates for supplying public service, it is proper to include in the operating expenses, that is, in the cost of producing the service, an allowance for consumption of capital in order to maintain the integrity of the investment in the service rendered. The amount necessary to be provided annually for this purpose is the subject of estimate and computation. In this instance, the company has used the 'straight line' method[2] of computation, a method approved by the Interstate Commerce Commission. 177 I.C.C. pp. 408, 413. . . . According to the principle of this accounting practice, the loss is computed upon the actual cost of the property as entered upon the books, less the expected salvage, and the amount charged each year is one year's pro rata share of the total amount. . . . the company employs averages, that is, average service life, average salvage of poles, of telephones, etc. While property remains in the plant, the estimated depreciation rate is applied to the book cost and the resulting amounts are charged currently as expenses of operation. The same amounts are credited to the account for depreciation reserve, the 'Reserve for Accrued Depreciation.' When property is retired, its cost, less salvage, is taken out of the depreciation reserve account. According to the practice of the company, the depreciation reserve is not

[1] See case 59, above. Note that the authors' interpretation of the Supreme Court's admonition was published in 1932.

[2] But note that this was only for determining the annual depreciation allowance. The company "sharply criticised" the deduction of "straight-line" accrued depreciation to get the rate base.

held as a separate fund, but is invested in plant and equipment. As the allowances for depreciation, credited to the depreciation reserve account, are charged to operating expenses, the depreciation reserve invested in the property thus represents, at a given time, the amount of the investment which has been made out of the proceeds of telephone rates for the ostensible purpose of replacing capital consumed. If the predictions of service life were entirely accurate and retirements were made when and as these predictions were precisely fulfilled, the depreciation reserve would represent the consumption of capital, on a cost basis, according to the method which spreads that loss over the respective service periods. But if the amounts charged to operating expenses and credited to the depreciation reserve are excessive, to that extent subscribers for the telephone service are required to provide, in effect, capital contributions, not to make good losses incurred by the utility in the service rendered and thus to keep its investment unimpaired, but to secure additional plant and equipment upon which the utility expects a return.

" . . . The calculations are mathematical, but the predictions underlying them are essentially matters of opinion. They proceed from studies of the 'behavior of large groups' of items. These studies are beset with a host of perplexing problems. Their determination involves the examination of many variable elements and opportunities for excessive allowances, even under a correct system of accounting, are always present. The necessity of checking the results is not questioned.[1] The predictions must meet the test of experience.

"In this instance, the evidence of expert computations of the amounts required for annual allowances does not stand alone. In striking contrast is the proof of the actual condition as maintained. . . .[2] The existing depreciation as thus asserted by the company, and the amounts it shows as the depreciation reserve allocated to the intrastate business at Chicago (taking in each case the average amounts per year), are as follows:

Years	Existing depreciation*	Depreciation reserve	Years	Existing depreciation*	Depreciation reserve
1923	$11,992,000	$26,797,000	1928	$16,241,000	$42,769,000
1924	12,865,000	29,316,000	1929	15,300,000	44,515,000
1925	13,775,000	32,155,000	1930	15,863,000	45,829,000
1926	14,621,000	35,572,000	1931	15,828,000	48,362,000
1927	15,360,000	39,352,000			

* These are actual depreciations. The district court estimated 16 percent straight-line accrued theoretical depreciation 1923 to 1928, and 15 percent thereafter.

"In explanation of this large difference, the company urges that the depreciation reserve in a given year does not purport to measure the actual depreciation

[1] Note that, in the authors' method of estimating actual depreciation (Secs. 5.15 to 5.20), the estimates are checked yearly by field examinations of the property items; and do check out exactly for each item at its actual retirement. Note further that the depreciation-accountancy procedure taught in Secs. 6.12 to 6.25, herein, will maintain depreciation reserves always just equal to the corresponding, true, total accrued actual depreciation, as continuously checked by actual experience.

[2] The company introduced evidence to prove that the actual depreciation was only 9 percent, 1923 to 1928, and only 8 percent thereafter. The commission estimated only 10 percent in 1923.

at that time; that there is no regularity in the development of depreciation; that it does not proceed in accordance with any fixed rule; that as to a very large part of the property there is no way of predicting the extent to which there will be impairment in a particular year. . . .

"Giving full weight to these considerations, we are not persuaded that they are adequate to explain the great disparity which the evidence reveals."

c. The Supreme Court ruled that the excess of the company's and the district court's annual depreciation allowances over proper allowances was so great as entirely to vitiate the District Court's finding that the reduced rates ordered would be confiscatory.

"Rather it appears that the depreciation reserve to a large extent represents provision for capital additions, over and above the amount required to cover capital consumption. This excess in the balance of the reserve account has been built up by excessive annual allowances for depreciation charged to operating expenses. . . .

"We find this point to be a critical one. The questionable amounts charged to operating expenses for depreciation are large enough to destroy any basis for holding that it has been convincingly shown that the reduction in income through the rates in suit would produce confiscation. . . .

"The decree below is reversed, and the cause is remanded, with direction to dissolve the interlocutory injunction, to provide for the refunding,[1] in accordance with the terms of that injunction and of the bonds given pursuant thereto, of the amounts charged by the company in excess of the rates in suit, and to dismiss the bill of complaint."

68. *Harold E. West et al. v. Chesapeake & Potomac Telephone Co. of Baltimore.* U.S. Sup. Ct., June 3, 1935. This decision is noteworthy because it was the first U.S. Supreme Court ruling on the proper use of cost indexes (Chap. IX) in making valuations. Saying, "This is not to suggest that price trends are to be disregarded; quite the contrary is true," the court nevertheless rejected the commission's valuation because it was based upon an empirically weighted average of some sixteen construction cost indices. The Supreme Court also rejected the lower court's valuation; it was made by merely "deducting the company's depreciation reserve from book cost and adding to the difference an allowance for working capital." The Supreme Court's decision is in harmony with the *Smyth v. Ames* rule, that *all* factors affecting value must be accorded consideration in determining fair value, giving each factor "such weight as may be just and right in each case," as determined by sound judgment, *not* by "formula."

The litigation arose over an order by the commission "directing the company to put into effect Jan. 1, 1934, reductions in its rates, sufficient to diminish annual net income by $1,000,000." The company resisted the order and obtained an injunction decree in the Federal District Court. The case was then appealed to the U.S. Supreme Court. Mr. Justice Roberts delivered the opinion.

The Commission's Commodity-index Valuation.—"The company's books accurately show installations and retirements of plant, and from them historical cost is ascertained to be $50,025,278 as of Dec. 31, 1933, with a depreciation reserve of $11,483,357. The commission made no appraisal of the physical plant and property but attempted to determine present value by translating the dollar value of the plant as it was found by the district court in the earlier case at Dec. 31, 1923, plus net additions in dollar value in each subsequent year, into an equivalent of dollar value at Dec. 31, 1932. . . . "

[1] The refunds were about $21,000,000, *N.Y. Times*, May 1, 1934.

To accomplish this: "The commission thought it found the answer in commodity indices, prepared to show price trends. It selected sixteen of these, one covering as many as 784 commodities, falling into different classes, and weighted for averaging; others much less comprehensive; and its witness calculated by the use of each index the reduction in value of the company's assets considered as a conglomerate mass of dollar value from 1923, or subsequent date of acquisition, to 1932. As might be expected, the results varied widely. The lowest value found by the use of any index was $24,983,624; the highest $36,056,408—48 per cent higher. The commission then weighted these sixteen indices upon a principle not disclosed, giving them weights of from one to four, and thus got a divisor of thirty-one for the total obtained by adding the weighted results of all. This gave what the commission styled its 'fair value index,' which it applied to the 1923 value of the property then owned and to cost of all net additions in subsequent years, to obtain value as of 1932. The result, after adding some $660,000 for working capital, was a rate base of $32,621,190. . . .

"This method [the commission's] is inappropriate for obtaining the value of a going telephone plant. An obvious objection is that the indices which are its basis were not prepared as an aid to the appraisal of property. They were intended merely to indicate price trends. . . .

"Again, the wide variation of results of the employment of different indices, already mentioned, impugns their accuracy as implements of appraisal. . . .

"To substitute for such factors as historical cost and cost of reproduction, a 'translator' of dollar value obtained by the use of price-trend indices, serves only to confuse the problem and to increase its difficulty, and may well lead to results anything but accurate and fair. This is not to suggest that price trends are to be disregarded; quite the contrary is true. And evidence of such trends is to be considered with all other relevant factors.

"A more fundamental defect in the commission's method is that the result is affected by sudden shifts in price level. It is true that any just valuation must take into account changes in the level of prices. . . .

"But it is to be remembered that such a property as that here under consideration is a great integrated aggregate of many and diverse elements; it is not primarily intended for sale in the market, but for devotion to the public use now and for the indefinite future; and has, so far as its market value is concerned, no real resemblance to a bushel of wheat or a ton of iron. While, therefore, the owner of such a property must assume and may not pass on to the public the risk involved in a general decline in values, and may have the advantage also of a general rise in such values, it would not only be unfair but impracticable to adjust the value and the consequent rate of return to sudden fluctuations in the price level. . . .

"We agree, therefore, with the view of the district court that the method was inapt and improper, is not calculated to obtain a fair or accurate result, and should not be employed in the valuation of utility plants for rate-making purposes. . . ."

CHAPTER IX

WAGE AND PRICE TRENDS AND INDEXES; CONSTRUCTION-COST INDEXES

This chapter deals with the various sources of statistical material on price trends of construction materials, labor, and unit costs; and with the nature of the data provided by the various commodity, labor, and construction-cost indexes.

In making every engineering valuation, a large mass of data of the correct unit costs to use for the particular property must be collected; they are necessary for determining its original cost, reproduction cost, and fair cost-values. So far as practicable, these data must be secured from the book records of the enterprise, as described in Chap. XI; almost always, however, a number of omissions in the book records must be supplied by the aid of past and present price data secured from other sources.

It is becoming increasingly apparent that much help in correct determinations of fair cost-values may be found by the scientific study and correct interpretation of price trends.

SOURCES OF PRICE AND COST DATA

The main sources of price and cost data are: (1) published data; (2) the valuator's private collection of data; (3) the book records of the particular property to be valued.

9.1. Published Cost and Price Data.—These are to be found in many books, bulletins, and professional and trade journals.

Books and Bulletins.—The numerical data published in books and bulletins indicate past rather than present prices and costs; their special value to valuators is due to their methods of presentation and to their accompanying analyses and discussions.

Professional and Trade Journals.—Extensive data of prices and costs are published currently in numerous professional and trade journals.

Engineering News-Record "Construction Costs."—Many of the tables and diagrams in this chapter are reproduced, by permission, from *Engineering News-Record* "Construction Costs." This 9- by 12-inch paper-bound bulletin, of about 100 pages, is published annually, usually in April.

9.2. The Valuator's Private Collection of Price and Cost Data.—The valuation engineer generally possesses a mass of cost and price data which apply to his particular work. These have usually been

collected over a period of years; each valuation completed adds new data to the files.

9.3. The Book Records of the Particular Property to Be Valued.—The book records of the particular property to be valued must always be the principal source of the price and cost data used in determining its fair cost-values.

The methods of obtaining, studying, and using these book-records data are described and discussed in detail in Chap. XI.

DATA OF MATERIALS PRICES AND CONSTRUCTION WAGES

Some selected examples of published data of the unit prices of construction materials and wages are presented below.

9.4. Selected Data of the Unit Prices of Construction Materials.

Basic materials, New York City, 1874 *to* 1934. Data of the 1874 to 1934, average, yearly, New York, unit prices of common brick, cement, lime, pine lumber, and steel shapes are shown in Table 9.1, Fig. 9.1 (both from *Engineering News-Record,* "Construction Costs," 1935). Cast-iron pipe prices are shown from 1850 to 1934.

Basic materials, various American cities, 1910 *to date.* On pages 48 to 64 of "Construction Costs," 1935 (and on corresponding pages of later editions), the high, low, and average, yearly unit prices, each year since 1909, at a number of American cities, are given for: bars, brick, cast-iron pipe, cement, crushed stone, gravel, lead, lime, linseed oil, lumber, sand, steel shapes, and hollow tile.

1915 *to* 1934 *price trends of steel products.* These are shown in Fig. 9.2, below. The prices are from the *Iron Age.*

1915 *to* 1934 *price trends of metals.* These are shown in Fig. 9.3, below. The prices are from the *Iron Age.*

1913 *to* 1934 *price trends of building tile, sewer pipe, cast-iron pipe.* These are shown in Fig. 9.4, below. The prices are from *Engineering News-Record.*

1913 *to* 1934 *price trends of lumber, cement, brick, structural steel.* These are shown in Fig. 9.5. The prices are from "Construction Costs," 1935.

9.5. Data of Construction-cost Wages.—Tables 9.2, 9.3, and 9.4 are from *Engineering News-Record* "Construction Costs," 1935.

Building wages and manufacturing earnings, rates per hour, 1910 *to* 1934. These are shown in Table 9.2; in Fig. 9.6 they are diagrammed for the years 1925 to 1934.

New York City wage rates per hour, 1874 *to* 1934, *for carpenters, bricklayers, mason's laborers, common laborers, structural ironworkers.* These are shown in Table 9.3, diagrammed in Fig. 9.7.

Yearly average wages of six trades in seven cities, 1910 *to* 1934. These are shown in Table 9.4. The trades are bricklayers, hod carriers, carpenters, structural ironworkers, plasterers, common laborers.

PRICE AND LABOR INDEXES

The data of Secs. 9.4 and 9.5 show the prices of *particular commodities,* and the wage rates for *particular classes of labor,* at *particular locations.* The price trends of *groups of commodities* and the wage trends of *labor*

Table 9.1

Basic Building-materials' Prices—1874 to 1934 in New York City

Average yearly prices at New York, except in the case of steel, which is f.o.b. Pittsburgh mill; wrought-iron I-beams for years, 1879, 1885, and 1886. Brick, wholesale, f.o.b. Lime, common lump, wholesale, f.o.b., 180 lb. net, 1910 to 1915, inclusive; afterward, 280 lb. Cement, net, delivered; Rosedale, 1874 to 1878, inclusive; afterward Portland. Lumber, No. 1 common, rough, wholesale, at dock, for 1-in. yellow pine, 1874 to 1909, inclusive; afterward 3 in. long-leaf.

Year	Brick, common, per 1,000	Cement per barrel	Lime per barrel	Pine per thousand ft. b.m.	Steel shapes per 100 lb.
1874	$ 7.41	$1.48	$1.50	$26.00	
1875	7.12	1.25	1.40	21.00	
1876	5.87	1.20	1.00	21.00	
1877	4.93	1.10	0.80	20.00	
1878	4.71	0.95	0.70	20.00	
1879	4.08	2.70	0.80	20.96	$2.78
1880	7.00	2.88	0.87	19.00	
1881	7.50	2.72	0.95	19.00	
1882	7.12	2.82	1.01	19.00	
1883	8.28	2.75	1.05	19.00	
1884	6.65	2.60	0.95	19.00	
1885	6.10	2.28	1.00	18.66	3.12
1886	7.51	2.28	1.00	19.00	3.00
1887	7.43	2.07	0.82	19.00	
1888	6.50	2.16	0.83	16.75	
1889	7.06	2.23	0.92	20.31	
1890	6.65	2.25	0.95	21.75	
1891	5.53	2.38	0.87	18.00	
1892	5.77	2.27	0.93	18.50	
1893	5.83	2.15	0.93	18.50	
1894	5.00	2.03	0.85	18.50	1.20
1895	5.31	1.97	0.78	16.92	1.35
1896	5.06	2.00	0.69	16.42	1.60
1897	4.94	1.97	0.72	16.44	1.24
1898	5.75	2.00	0.70	15.75	1.27½
1899	5.69	2.05	0.75	16.25	1.45
1900	5.25	2.16	0.68	24.50	1.95
1901	5.77	1.89	0.77	17.50	1.60
1902	5.39	1.95	0.81	21.00	1.60
1903	5.91	2.03	0.79	23.50	1.70
1904	7.50	1.67½	0.80	21.50	1.60
1905	8.10	1.43	0.89	24.92	1.62
1906	8.55	1.55	0.95	29.33	1.60
1907	6.16	1.55	0.95	30.50	1.75
1908	5.10	1.46	1.05	30.50	1.76
1909	6.39	1.41	1.05	33.04	1.50
1910	5.70	1.07	0.75	36.00	1.46
1911	5.31	1.04	0.75	36.00	1.36
1912	6.75	0.97	0.75	39.50	1.33
1913	6.75	1.18	0.91½	28.50	1.50
1914	5.50	1.17½	0.97	29.25	1.16
1915	5.50	1.00	0.97	27.75	1.28
1916	8.00	1.32	1.50	31.58	2.45
1917	9.10	1.76	2.02	37.33	3.62
1918	10.98	2.48	2.16	42.66	3.00
1919	15.79	2.63	2.65	49.60	2.52
1920	18.00	3.52	3.15	63.73	2.45
1921	17.40	2.82	3.47	48.16	2.00
1922	16.91	2.43	2.79	47.05	1.70
1923	20.00	2.66	2.98	55.22	2.39
1924	18.00	2.50	2.88	59.00	2.19
1925	15.00	2.50	2.38	59.67	1.95
1926	17.00	2.50	2.18	62.17	1.93
1927	15.00	2.40	2.49	61.46	1.86
1928	14.00	2.30	2.30	61.20	1.87
1929	11.71	2.23	2.25	58.00	1.92
1930	10.88	2.19	2.25	55.58	1.71
1931	10.29	1.89	2.25	51.18	1.63
1932	9.75	1.62	2.25	41.33	1.56
1933	9.50	2.26	2.50	53.00	1.64
1934	10.87	2.48	2.50	46.46	1.78

Cast-iron Pipe Prices—Average Yearly—1850 to 1934[1]

Prices per ton of 2,000 lb.; f.o.b. Delaware River Foundries

Includes all classes bell and spigot pipe

Year	Price	Year	Price
1850	$35.72	1884	$26.79
1855	40.18		
1860	32.15	1885	25.00
1865	58.05	1886	26.79
		1887	28.58
1870	50.01	1888	25.00
1871	53.58	1889	23.04
1872	51.79		
1873	58.05	1890	24.65
1874	43.76	1891	23.04
		1892	21.43
1875	37.51	1893	21.30
1876	31.26	1894	18.47
1877	25.00		
1878	23.22	1895	17.95
1879	29.47	1896	17.64
		1897	15.48
1880	29.47	1898	15.48
1881	33.04	1899	21.17
1882	35.68		
1883	32.15	1900	22.33
		1901	21.63

6″ and larger, class B or heavier, bell and spigot pipe

Year	Price	Year	Price
1902	$26.62	1919	$53.87
1903	27.56		
1904	22.92	1920	71.77
		1921	50.58
1905	26.54	1922	46.42
1906	30.12	1923	57.75
1907	31.52	1924	54.33
1908	24.69		
1909	24.19	1925	49.16
		1926	48.50
1910	22.99	1927	41.42
1911	19.99	1928	35.92
1912	21.22	1929	36.66
1913	21.96		
1914	20.37	1930	37.00
		1931	34.71
1915	22.52	1932	30.58
1916	30.25	1933	34.58
1917	52.33	1934	41.00
1918	58.68		

[1] From "Price History of Cast Iron Pipe," United States Cast Iron Pipe and Foundry Co.

groups are best shown by price and labor *indexes*, of *national scope;* of these, the most important, of recognized soundness, are described and shown in Secs. 9.6 to 9.12.

TABLE 9.2.—CONSTRUCTION WAGE RATES—YEARLY AVERAGES, 20 CITIES[1]

Year	Rates per hour			
	Wages, construction		Manufacturing (earnings per hour) (Nat. Ind. Conf. Bd.)	
	EN-R skilled, average[2]	Common	Skilled and semiskilled	Unskilled
1910	$0.525	$0.179		
1911	0.534	0.179		
1912	0.542	0.182		
1913	0.555	0.19		
1914	0.565	0.177		
1915	0.57	0.182		
1916	0.58	0.192		
1917	0.61	0.281		
1918	0.68	0.38		
1919	0.78	0.466		
1920	1.05	0.579	$0.692	$0.548
1921	1.06	0.54	0.594	0.446
1922	1.00	0.442	0.562	0.409
1923	1.10	0.518	0.616	0.451
1924	1.19	0.555	0.638	0.473
1925	1.22	0.538	0.641	0.469
1926	1.27	0.548	0.647	0.477
1927	1.32	0.554	0.652	0.490
1928	1.35	0.5558	0.658	0.494
1929	1.36	0.5465	0.670	0.503
1930	1.38	0.5607	0.667	0.495
1931	1.27	0.5002	0.634	0.461
1932	1.015	0.4269	0.559	0.401
1933	1.006	0.4562	0.55	0.401
1934	1.097	0.5319	0.641	0.478

[1] From *Engineering News-Record.*

[2] Average for 20 cities of rates for bricklayers, carpenters, iron workers.

9.6. Price and Labor Indexes Defined.—Price indexes and labor indexes are devices for showing the relative changes in the prices of particular groups of commodities or wages over considerable periods of time.

TABLE 9.3.—WAGE RATES PER HOUR, NEW YORK CITY

Year	Carpenters, 10-hr. day	Bricklayers, 8-hr. day	Masons' laborers, 8-hr. day	Common, laborers, 10-hr. day	Structural iron workers
1874	$0.23	$0.43	$0.18	$0.14	
1875	0.22½	0.37	0.18	0.14	
1876	0.21	0.31	0.17	0.14	
1877	0.20	0.31	0.15	0.12	
1878	0.18	0.31	0.15	0.11	
1879	0.17	0.31	0.16	0.11	
1880	0.18	0.34	0.18	0.12	
1881	0.19	0.46	0.21	0.13	
1882	0.21	0.50	0.22	0.15	
1883	0.21	0.50	0.23	0.13	
1884	0.21	0.50	0.22	0.13	
1885	0.20	0.50	0.22	0.13	
1886	0.21	0.50	0.23	0.13	
1887	0.21	0.50	0.20	0.14	
1888	0.22	0.50	0.16	0.12	
1889	0.22	0.50	0.23	0.13	
	8-hr. day			9-hr. day	
1890	0.43	0.47	0.28	0.14	
1891	0.43	0.47	0.28	0.14	
1892	0.44	0.50	0.29	0.14	
1893	0.43	0.50	0.30	0.14	
1894	0.44	0.50	0.30	0.14	
1895	0.43	0.50	0.30	0.14	
1896	0.44	0.50	0.30	0.14	
1897	0.44	0.50	0.29	0.14	
1898	0.44	0.50	0.30	0.14	
1899	0.45	0.55	0.33	0.14	
1900	0.50	0.56	0.33	0.14	
1901	0.50	0.59	0.33	0.19	
1902	0.51	0.65	0.36	0.17	
1903	0.56	0.65	0.36	0.16	
1904	0.53	0.65	0.36	0.19	
1905	0.54	0.70	0.36	0.20	
1906	0.58	0.70	0.38	0.21	
				8-hr. day	
1907	0.62½	0.70	0.37½	0.22	$0.60
1908	0.62½	0.70	0.37½	0.22	0.60
1909	0.62½	0.70	0.37½	0.22	0.60
1910	0.62½	0.70	0.37½	0.22	0.62½
1911	0.62½	0.70	0.37½	0.22	0.62½
1912	0.62½	0.70	0.37½	0.23	0.62½
1913	0.62½	0.70	0.37½	0.22½	0.62½
1914	0.62½	0.75	0.37½	0.22½	0.62½
1915	0.62½	0.75	0.37½	0.25	0.62½
1916	0.62½	0.75	0.37½	0.25	0.66
1917	0.69	0.75	0.42½	0.30	0.68
1918	0.69	0.81	0.47	0.40½	0.80
1919	0.75	0.87½	0.57½	0.40½	0.87½
1920	1.12½	1.25	0.77½	0.75	1.12½
1921	1.12½	1.25	0.87½	0.60	1.12½
1922	1.12½	1.25	0.87½	0.60	1.12½
1923	1.12½	1.50	0.95	0.61½	1.12½
1924	1.31¼	1.50	1.00	0.68¾	1.29
1925	1.31¼	1.50	1.00	0.65¼	1.46¾
1926	1.45	1.64½	1.07	0.81½	1.60
1927	1.50	1.75	1.12½	0.90⅝	1.75
1928	1.50	1.75	1.12½	0.90⅝	1.75
1929	1.55	1.84	1.15	0.98½	1.81
1930	1.65	1.92½	1.03⅛	1.03⅛	1.92½
1931	1.45	1.71¼	1.02½	0.65⅝	1.92½
1932	1.33	1.47	1.00	0.64½	1.67½
1933	1.37½	1.57½	0.91	0.72½	1.65
1934	1.40	1.53	0.82½	0.84¾	1.62

TABLE 9.4.—YEARLY AVERAGE WAGES OF SIX BUILDING TRADES IN SEVEN CITIES, 1910 TO 1934

	Bricklayers							Hod carriers							Structural iron workers						
Year	Boston	New York	Chicago	Birmingham	St. Louis	San Francisco	Seattle	Boston	New York	Chicago	Birmingham	St. Louis	San Francisco	Seattle	Boston	New York	Chicago	Birmingham	St. Louis	San Francisco	Seattle
1910	$0.60	$0.70	$0.675	......	$0.70	$0.875	$0.75	$0.35	$0.375	$0.45	......	$0.375	$0.50	$0.50	$0.50	$0.625	$0.65	......	$0.60	$0.625	$0.563
1911	0.60	0.70	0.675	......	0.70	0.875	0.75	0.35	0.375	0.45	......	0.425	0.50	0.50	0.563	0.625	0.66	......	0.65	0.625	0.60
1912	0.60	0.70	0.725	......	0.70	0.875	0.75	0.35	0.375	0.45	......	0.425	0.50	0.50	0.563	0.625	0.68	......	0.65	0.625	0.60
1913	0.65	0.70	0.75	$0.70	0.70	0.875	0.75	0.35	0.375	0.40	......	0.4875	0.50	......	0.625	0.625	0.68	$0.625	0.65	0.75	0.625
1914	0.65	0.75	0.75	0.70	0.75	0.875	0.75	0.35	0.375	0.40	......	0.4875	0.50	......	0.625	0.625	0.68	0.625	0.65	0.75	0.625
1915	0.65	0.75	0.75	0.70	0.75	0.875	0.75	0.35	0.375	0.40	......	0.4875	0.50	......	0.625	0.625	0.68	0.625	0.65	0.75	0.625
1916	0.65	0.75	0.75	0.70	0.75	0.875	0.75	0.35	0.375	0.425	......	0.4875	0.50	......	0.625	0.66	0.68	0.625	0.675	0.75	0.625
1917	0.70	0.75	0.75	0.70	0.75	0.875	0.81	0.40	0.45	0.45	......	0.4875	0.50	......	0.68	0.68	0.69	0.625	0.70	0.75	0.75
1918	0.80	0.81	0.75	0.875	0.85	1.00	1.00	0.425	0.45	0.50	......	0.51	0.625	......	0.80	0.80	0.70	0.75	0.80	0.875	0.875
1919	0.80	0.875	0.875	0.875	1.00	1.125	1.125	0.50	0.575	0.575	......	0.6375	0.75	......	0.80	0.875	0.875	0.80	0.925	1.00	1.00
1920	1.00	1.25	1.25	1.00	1.25	1.25	1.25	0.70	0.775	1.00	......	0.70	0.93	......	1.00	1.125	1.25	1.00	1.25	1.125	1.125
1921	1.00	1.25	1.25	1.00	1.25	1.25	1.125	0.70	0.875	1.00	......	0.85	1.00	0.875	1.00	1.125	1.25	......	1.25	1.25	1.125
1922	1.00	1.25	1.10	1.00	1.25	1.25	1.125	0.70	0.875	0.725	......	0.85	0.71	0.875	1.00	1.125	1.05	......	1.06	1.125	1.00
1923	1.25	1.50	1.10	1.125	1.50	1.375	1.25	0.70	0.95	0.725	......	1.00	0.77	0.93	1.05	1.125	1.05	1.05	1.25	1.125	1.125
1924	1.25	1.50	1.35	1.125	1.75	1.25	1.25	0.82	1.00	0.725	$0.385	1.20	0.8125	0.95	1.16	1.29	1.25	1.00	1.45	1.125	1.125
1925	1.25	1.50	1.50	1.25	1.75	1.25	1.35	0.7625	1.00	0.825	0.6075	1.20	0.8125	1.00	1.1375	1.4675	1.36	1.135	1.50	1.125	1.125
1926	1.35	1.645	1.56	1.25	1.75	1.25	1.375	0.77	1.07	0.88	0.625	1.20	0.8125	1.00	1.23	1.60	1.405	1.25	1.50	1.125	1.125
1927	1.40	1.75	1.625	1.25	1.75	1.375	1.375	0.79	1.125	0.925	0.625	1.20	0.875	1.00	1.25	1.75	1.50	1.55	1.50	1.375	1.155
1928	1.425	1.75	1.625	1.45	1.75	1.375	1.4275	0.805	1.125	0.93125	0.52	1.20	0.875	1.00	1.3125	1.75	1.50	1.75	1.50	1.375	1.215
1929	1.50	1.84	1.65	1.50	1.75	1.375	1.50	0.85	1.15	0.93	0.50	1.20	0.875	1.00	1.375	1.81	1.57	1.63	1.57	1.375	1.25
1930	1.50	1.925	1.70	1.50	1.75	1.375	1.50	0.85	1.03⅛	0.975	0.50	1.20	0.875	1.00	1.375	1.925	1.625	1.25	1.75	1.375	1.25
1931	1.50	1.7125	1.70	0.8125	1.75	1.375	1.1875	0.85	1.025	0.975	0.35	0.875	0.875	0.70	1.375	1.925	1.625	0.75	1.75	1.375	1.1875
1932	1.105	1.47	1.40	0.825	1.50	1.175	0.975	0.70	1.00	0.825	0.40	1.00	0.70	0.80	1.205	1.675	1.375	0.75	1.50	1.13	1.13
1933	1.10	1.575	1.406	0.812	1.50	1.125	1.01	0.70	1.1875	......	......	1.00	0.70	0.80	1.14	1.65	1.35	0.75	1.47	1.20	1.10
1934	1.12	1.53	1.50	1.00	1.50	1.50	1.30	0.70	0.904	0.825	......	0.875	0.70	1.00	1.20	1.62	1.35	1.00	1.47	1.375	1.13

	Carpenters							Plasterers							Common laborers						
Year	Boston	New York	Chicago	Birmingham	St. Louis	San Francisco	Seattle	Boston	New York	Chicago	Birmingham	St. Louis	San Francisco	Seattle	Boston	New York	Chicago	Birmingham	St. Louis	San Francisco	Seattle
1910	$0.50	$0.625	$0.60	......	$0.60	$0.625	$0.563	$0.65	$0.69	$0.69	......	$0.75	$0.875	$0.75	$0.225	$0.22	$0.375	......	$0.225	$0.3125	$0.3125
1911	0.50	0.625	0.60	......	0.625	0.625	0.563	0.65	0.69	0.69	......	0.75	0.875	0.75	0.225	0.22	0.375	......	0.225	0.3125	0.3125
1912	0.50	0.625	0.65	......	0.625	0.625	0.563	0.65	0.69	0.75	......	0.75	0.875	0.75	0.225	0.23	0.375	......	0.225	0.3125	0.3125
1913	0.50	0.625	0.65	$0.525	0.625	0.625	0.56	0.65	0.69	0.75	......	0.75	0.875	0.75	0.35	0.225	0.40	......	0.25	0.27	0.375
1914	0.55	0.625	0.65	0.45	0.625	0.625	0.56	0.65	0.69	0.75	......	0.75	0.875	0.75	0.35	0.225	0.40	......	0.25	0.31	0.375
1915	0.55	0.625	0.65	0.45	0.625	0.625	0.56	0.65	0.69	0.75	......	0.75	0.875	0.75	0.35	0.25	0.40	......	0.25	0.31	0.375
1916	0.57	0.625	0.70	0.45	0.625	0.625	0.56	0.70	0.75	0.75	......	0.75	0.875	0.75	0.35	0.25	0.425	......	0.25	0.31	0.375
1917	0.60	0.69	0.70	0.45	0.65	0.68	0.65	0.70	0.75	0.75	......	0.75	0.875	0.875	0.375	0.30	0.45	......	0.30	0.375	0.4375
1918	0.65	0.69	0.70	0.55	0.70	0.75	0.825	0.70	0.75	0.81	......	0.875	1.00	1.00	0.40	0.405	0.50	......	0.365	0.43	0.56
1919	0.75	0.75	0.80	0.65	0.825	0.875	0.93	0.80	0.90	0.875	......	1.00	1.125	1.125	0.40	0.405	0.575	......	0.425	0.625	0.6875
1920	1.00	1.125	1.25	0.75	1.00	1.06	1.00	1.00	1.25	1.25	......	1.25	1.25	1.25	0.6875	0.75	1.00	......	0.6075	0.75	0.75
1921	1.00	1.125	1.25	0.75	1.25	1.125	0.875	1.25	1.25	1.25	......	1.375	1.375	1.25	0.6875	0.60	1.00	......	0.6075	0.81	0.75
1922	1.00	1.125	1.10	0.75	1.10	1.04	0.875	1.125	1.25	1.10	......	1.375	1.275	1.125	0.6875	0.60	0.725	......	0.5575	0.625	0.625
1923	1.05	1.125	1.25	0.75	1.25	1.04	1.00	1.125	1.25	1.50	......	1.50	1.275	1.25	0.6875	0.615	0.725	......	0.6075	0.625	0.625
1924	1.05	1.3125	1.17	0.76	1.525	1.00	1.00	1.25	1.50	1.50	......	1.75	1.275	1.375	0.6625	0.6875	0.8175	$0.335	0.70	0.525	0.625
1925	.0875	1.3125	1.3475	0.6025	1.50	1.00	1.0825	1.25	1.50	1.50	......	1.75	1.50	1.375	0.6025	0.6525	0.8175	0.31	0.6525	0.525	0.5625
1926	1.20	1.45	1.39	0.6575	1.50	1.00	1.125	1.50	1.75	1.50	......	1.75	1.50	1.375	0.615	0.815	0.86	0.30	0.6525	0.525	0.5675
1027	1.25	1.50	1.30	0.95	1.50	1.125	1.125	1.50	1.75	1.625	......	1.75	1.50	1.375	0.595	0.90625	0.8875	0.325	0.5875	0.55	0.64
1928	1.279	1.50	1.50	1.00	1.50	1.125	1.125	1.50	1.75	1.625	......	1.75	1.50	1.50	0.6025	0.90625	0.90	0.325	0.60	0.55	0.678
1929	1.38	1.55	1.47	1.00	1.50	1.125	1.125	1.50	1.75	1.625	......	1.75	1.375	1.50	0.625	0.985	0.92	0.29	0.61	0.57	0.70
1930	1.375	1.65	1.625	1.00	1.50	1.125	1.125	1.625	1.925	1.70	......	1.75	1.375	1.50	0.625	1.03125	0.975	0.275	0.6375	0.6875	0.70
1931	1.175	1.45	1.625	0.60	1.50	1.125	0.875	1.50	1.7125	1.70	$1.00	1.75	1.375	1.1875	0.60	0.65625	0.7375	0.20	0.6125	0.59⅜	0.60
1932	1.04	1.33	1.40	0.60	1.275	0.96	0.695	1.625	1.50	1.375	1.25	1.50	1.10	1.20	0.575	0.645	0.665	0.225	0.56	0.56	0.485
1933	1.065	1.375	1.3125	0.60	1.25	0.90	0.73	1.375	1.50	1.375	......	1.50	1.10	1.20	0.548	0.725	0.784	0.248	0.6223	0.64	0.48
1934	0.9375	1.40	1.3125	1.00	1.25	1.05	0.83	1.231	1.50	1.50	1.00	1.50	1.38	1.18	0.55	0.848	0.825	0.30	0.7875	0.64	0.53

Indexes are based on the prices of groups of commodities because it is impracticable to obtain the weighted average price for all commodities. Because wholesale and retail prices have characteristics independent each of the other, separate indexes are computed for each, but the valuator is concerned principally with the indexes of wholesale prices—or base prices. The number of commodity prices used in the best known indexes ranges from 25 to 404, although certain indexes have been constructed that are based on more than 1,000 commodities. In selecting the items for the construction of any index, an attempt is made to insure that those selected shall constitute a fair criterion for the whole field of prices for which it is intended that the index shall be representative.

The *simple indexes* are prepared by taking the sum of the unit prices of a group of commodities for each date or period, and dividing them by the sum for the same group of commodities for the base period. These indexes are generally less reliable than are those in which the relative importance of the respective commodities is taken into account; these are termed weighted indexes.

A *weighted index* may be prepared as follows: The total money paid for all the index commodities sold during the base period (usually one year) is called 100 percent; then the index for any other date or period is obtained by dividing this total into the amount which the *same quantities of goods* would have cost at prices of the new date or period. It is important to note that a constant quantity of goods is used throughout the calculation in an index of this type.

The superiority of the quantity-weighted index over the simple index is less than may at first appear; a simple index based upon a large number of carefully selected commodities often gives results very closely approximating those of the corresponding weighted index.

Indexes are most commonly stated as percentages of the average price on some given date—or for some given period—which is arbitrarily established as 100 percent. It has been quite common to take the average price for the year 1913 as 100 percent, that being the last year for which prices were unaffected by conditions due to the World War. As the year 1914 shows but little war effect, it also is not infrequently used. More recently there is to be noted an increasing tendency to use the prices of the year 1926 as a base.

An index based on the prices for any year may be changed over to any other base year by dividing each year's index number by the index number of the year it is desired to use as a base.

9.7. The U.S. Bureau of Labor Statistics' Wholesale Price Index.—The Bureau has divided all the important wholesale commodities into nine groups. An index is computed for each of the nine groups and a composite one for all the commodities. Indexes based on yearly averages

are available from 1860 to the present; since January, 1913, the Bureau has also published the monthly averages as well as the yearly averages for each of the nine groups. In the earlier years these groups were based on the volume of sales of about 327 commodities; at the present time the list is somewhat larger, varying from time to time. The commodities selected include all of the more important articles of wholesale trade. The Bureau states:

> Difficulties in obtaining satisfactory units of comparisons have kept out of the index such things as machinery and many other sorts of manufactured goods; but the large proportion of the nation's total transactions deal with the commodities entering into the index, and the tendency of price fluctuations in the omitted manufactured articles to follow the general tendency leaves the index as a reasonably accurate picture of general variations in wholesale prices.
>
> For each commodity group, the base is established by multiplying the total quantity of each article marketed in 1913 by the average price of that article in the year 1913, adding all the products so obtained for the group and calling the totals 100 percent. The sum of the totals of the nine groups gives the base of 100 for all commodities. For all other index dates similar calculations are made, with prices as of those dates and total quantities the same as were used for 1913, so that the total of any group divided by the corresponding total for 1913 gives a true weighted average price expressed as a percentage of the weighted average price of 1913.[1]

The Bureau began in 1930 to base its index on 1926 averages and therefore presents the figures with 1913 as a base and also with 1926 as a base. Prices used in calculating the index are obtained weekly both from official and from private sources.

The general commodity index figures of the Bureau are given in Table 9.5 and are shown graphically in Fig. 9.8. In an article that appeared in *The Annalist*, July 4, 1921, Ralph G. Hurlin presented index numbers for wholesale prices for the period 1810 to 1920. These are given in Fig. 9.8 for the period 1810 to 1860. After 1860, the Hurlin Index follows the Bureau of Labor Statistics Index rather closely; it therefore is not shown in the figure.

9.8. Dun's Index.—One of the well-known indexes of wholesale commodity prices is Dun's Review Index. The basis of this index is the wholesale prices of about 300 commodities, divided into seven groups. Quotations for each commodity are secured for the first business day in each month. Many of these quotations are those of the New York market; the prices of nearly all the steel products are as of Pittsburgh market, and some other markets are quoted. All these quotations are separately multiplied by figures determined upon as the estimated

[1] Statistical Abstract, U.S. Bureau of Labor Statistics.

annual per capita consumption of the commodity. Therefore this also gives a truly weighted index.

The tabulation gives the worth in dollars of the estimated per capita consumption of each group of commodities for the year. Thus the sum of the index figures for the seven groups for any given date gives the wholesale cost of the total per capita consumption of the selected commodities, on the date. Fortunately, the total per capita consumption of $120.89 estimated for the year 1913 is near enough to $100 to permit of ready comparisons with the Bureau of Labor Index, which is computed with a base of 100 for the same year. Dun's Index figures may be converted to a base of 100 for 1913 by dividing by 1.2089.

The seven groups included in Dun's Index of wholesale commodity prices are: Breadstuffs, meats, dairy and garden, other foods, clothing, metals, and miscellaneous. Building materials are included in the miscellaneous group; it, therefore, will be of interest in connection with valuation problems.

From the standpoint of valuation work, too much weight is given to provisions and food-stuffs in Dun's Index. The index seems to have some of the characteristics of a cost-of-living index as well as of a wholesale price index. Dun's Index figures are given in Table 9.5 and are presented graphically in Figure 9.8.

9.9. Bradstreet Index.—The Bradstreet Index of wholesale prices is regarded as a reliable barometer of the prices of producers' goods. The index is an unweighted one and is based on 96 commodities and prices quoted on the first day of each month. The index number is the sum, in dollars and cents, of the wholesale prices of 1 pound of each of 96 commodities, in common use in the United States. From 1892 until 1898 the index was published quarterly and since that time it has been issued monthly.

9.10. Table and Diagram of Commodity-price Indexes.—Table 9.5 has been prepared to show in a convenient manner Dun's, Bradstreet's, and the U.S. Department of Labor Indexes to the end of 1934; using the year 1913 as a base. These are shown graphically in Fig. 9.8.

9.11. The U.S. Bureau of Labor Statistics Union-wage Index.—For some time the U.S. Department of Labor Statistics has compiled a Union Wages Index.[1] This index constitutes a minimum union wage scale, based on data from 58 cities located in 38 different states. The index numbers are computed for hourly rates, as shown in Table 9.6, and for weekly full-time rates. The number of trades taken into account lies in the neighborhood of 91. As most of the labor listed is skilled, the index is practically one of skilled-labor rates. The trend of this index is shown in Fig. 9.8.

[1] *Monthly Labor Rev.*, U.S. Dept. Labor, Washington, D.C.

TABLE 9.5.—DUN'S, BRADSTREET'S AND U.S. BUREAU OF LABOR STATISTICS WHOLESALE-COMMODITY-PRICE INDEX NUMBERS, 1860 TO 1934

Year	Index numbers				Year	Index numbers			
	Dun's	Brad-street's	U.S. Bur. Labor Statistics			Dun's	Brad-street's	U.S. Bur. Labor Statistics	
			1913 = 100	1926 = 100				1913 = 100	1926 = 100
1860	115.19	......	88		1900	93.53	7.8839	81	56
1861	101.92	......	88		1901	95.85	7.5746	79	55
1862	118.51	......	103		1902	100.41	7.8759	84	59
1863	173.18	......	130		1903	99.04	7.9364	86	59
1864	278.09	......	167		1904	100.17	7.9186	86	60
1865	194.44	......	190		1905	100.57	8.0987	86	60
1866	207.98	......	168		1906	105.35	8.4177	89	62
1867	188.52	......	151		1907	111.80	8.9045	94	65
1868	182.82	......	142		1908	109.87	8.0096	90	63
1869	164.63	......	135		1909	117.78	8.5153	97	68
1870	148.78	......	125		1910	118.80	8.9881	101	70
1871	151.51	......	110		1911	116.80	8.7129	93	65
1872	150.48	......	122		1912	124.40	9.1867	99	69
1873	143.09	......	121		1913	120.89	9.2115	100	70
1874	143.13	......	117		1914	122.21	8.9034	98	68
1875	134.70	......	112		1915	125.39	9.8530	101	70
1876	116.48	......	104		1916	148.81	11.8237	127	86
1877	109.55	......	97		1917	204.12	15.6385	177	118
1878	96.27	......	89		1918	229.22	18.7117	194	131
1879	97.29	......	85		1919	230.85	18.6642	206	139
1880	108.65	......	94		1920	247.89	18.8095	226	154
1881	111.90	......	93		1921	170.80	11.3696	147	98
1882	123.23	......	95		1922	172.22	12.1185	149	97
1883	107.25	......	93		1923	189.79	13.4028	154	101
1884	99.71	......	87		1924	189.32	12.7838	150	98
1885	90.70	......	82		1925	196.26	13.9445	159	104
1886	89.23	......	81		1926	187.63	13.0207	151	100
1887	93.62	......	81		1927	186.50	12.7787	147	95
1888	95.13	......	83		1928	193.44	13.2990	140	98
1889	89.69	......	83		1929	189.50	12.6560	138	95
1890	91.55	......	80	56	1930	......	10.7451	124	86
1891	96.09	......	80	56	1931	......	....	...	73
1892	90.11	7.7769	75	52	1932	......	....	...	65
1893	90.61	7.5324	77	53	1933	......	....	...	66
1894	83.29	6.6846	69	48	1934	......	....	...	75
1895	81.52	6.4346	70	49					
1896	74.32	5.9124	67	47					
1897	72.45	6.1159	67	47					
1898	79.39	6.5713	70	49					
1899	86.51	7.2100	75	52					

At the close of the World War, in 1918, there was a decrease in the volume of building construction; nevertheless, the shortage of labor at that time permitted wages to continue to increase, although the efficiency of labor declined. A revival of building activity by the middle of 1919 served to strengthen both trends until the collapse in 1920. The cycle was repeated for 1922 to 1923. From that date until 1930, wages for skilled workers tended to increase, while efficiency remained constant. A new trend set in during 1929, the effect of which has not yet become entirely clear; doubtless it has been affected by the strenuous governmental efforts during 1929 to 1932 to maintain wages, and since 1932 to restore them.

9.12. Construction-labor Indexes.—Table 9.2 and Fig. 9.6 show index numbers for skilled and common labor and manufacturing earnings in 20 cities for the years 1910 to 1934.

Figure 9.10 shows the Burns and McDonnell Engineering Co. labor-index numbers for power-plant building construction.

Figure 9.16 shows the Associated General Contractors of America's labor-index numbers for general construction work.

TYPE CONSTRUCTION-COST INDEXES

In Secs. 9.4 and 9.5, data have been presented of actual prices for materials and construction-wage rates; general price and labor indexes have been presented in Secs. 9.6 to 9.12. In engineering valuation, however, such general cost data are not sufficient; the valuation engineer requires cost indexes especially computed for specific types of property; a number of such cost indexes have been prepared by different authorities and published; often, however, the valuation engineer must compute and use cost indexes of his own, computed from data collected from the records of the particular property he is valuing.

9.13. The Calculation of Type Construction-cost Index Numbers.—Such calculations may proceed approximately as follows:

1. *Basic-type Examples and Period.*—Select the basic period, for which the index number is to be 100, and a sufficient number of basic examples of the particular type of property, actually constructed during the basic period.

2. *Classified Computation Groups of Items.*—Classify the construction items of these basic examples into groups convenient for cost analysis (see Table 9.7).

3. *Basic-group Percentages of Total Cost.*—Determine these by making detailed estimates of the total actual cost of each construction item of each of the several basic examples selected; from these, calculate the corresponding "mean" cost percentages for a "composite" basic example of the particular type of construction (see Table 9.7).

4. *Yearly Index Numbers Based on Cost Analyses of Actual Typical Constructions.*—Determine the prices prevailing for this type of construction in any particular year by analyzing the actual costs of typical constructions during the year; compute the

corresponding index number for the year by applying these prices to the quantities of different cost items in the "composite" basic example.

5. *Yearly Index Numbers by Price and Labor Indexes.*—In the absence of sufficient representative constructions in any particular year, its index number may be esti-

Table 9.6.—Index Numbers of Wages per Hour, 1840 to 1934[1]
Compiled by U.S. Bureau of Labor Statistics

Year	Index[2] number	Year	Index[2] number	Year	Index[2] number	Year	Index[2] number	Index[3]
1840	33	1865	58	1890	69	1915	103	
1841	34	1866	61	1891	69	1916	111	
1842	33	1867	63	1892	69	1917	128	
1843	33	1868	65	1893	69	1918	162	
1844	32	1869	66	1894	67	1919	184	
1845	33	1870	67	1895	68	1920	234	
1846	34	1871	68	1896	69	1921	218	
1847	34	1872	69	1897	69	1922	208	
1848	35	1873	69	1898	69	1923	217	
1849	36	1874	67	1899	70	1924	223	
1850	35	1875	67	1900	73	1925	226	
1851	34	1876	64	1901	74	1926	229	
1852	35	1877	61	1902	77	1927	231	103
1853	35	1878	60	1903	80	1928	232	105
1854	37	1879	59	1904	80	1929	233	107
1855	38	1880	60	1905	82	1930	229	97
1856	39	1881	62	1906	85	1931	217	85
1857	40	1882	63	1907	89	1932	186	64
1858	39	1883	64	1908	89	1933	. . .	67
1859	39	1884	64	1909	90	1934	. . .	76
1860	39	1885	64	1910	93			
1861	40	1886	64	1911	95			
1862	41	1887	67	1912	97			
1863	44	1888	67	1913	100			
1864	50	1889	68	1914	102			

[1] Exclusive of agriculture. On currency basis during Civil War.
[2] 1913 = 100.
[3] Average hourly earnings, factory workers (25 industries). 1923 = 100.

mated by using cost prices based on other data of prevailing prices and wages, including price and labor indexes.

Table 9.7 shows the application of the above method to the determination of the component cost percentage for "composite" Class A skyscraper-type office buildings, built 1927 to 1930, ranging in height

TABLE 9.7.—CLASS A SKYSCRAPER-TYPE OFFICE BUILDINGS. COST ANALYSIS—PERCENT OF TOTAL[1]

Schedule A

Item	Description	Building number						Composite building			
		No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	High	Low	Mean	Median
		Height 40 stories	Height 22 stories L	Height 34 stories	Height 26 stories	Height 46 stories H	Height 36 stories	Height 46 stories	Height 22 stories	Height 34 stories	Height 35 stories
		%	%	%	%	%	%	%	%	%	%
I	Excavation and foundations	13.4H	5.4	8.0	12.2	7.5	1.0L	13.4	1.0	8.0	7.8
	Structural frame, stack, fireproofing, cement work	22.6	17.8L	22.2	27.6H	22.0	20.2	27.6	17.8	22.1	22.1
	Subtotal	**36.0**	**23.2**	**30.2**	**39.8**	**29.5**	**21.2**	**41.0**	**18.8**	**30.1**	**29.9**
II	Exterior facia, masonry, windows, glass, weather stripping	11.6	9.8	12.7H	10.5	8.1L	9.3	12.7	8.1	10.3	10.2
	Roof coverings and flashings	0.5	10.5	0.4L	0.4L	0.6H	0.5	0.6	0.4	0.5	0.5
	Subtotal	**12.1**	**10.3**	**13.1**	**10.9**	**8.7**	**9.8**	**13.3**	**8.5**	**10.8**	**10.7**
III	Interior finishes, plastering, surfacing	14.8L	20.7	19.0	20.6	24.9H	16.3	24.9	14.8	19.4	19.8
IV	Mechanical trades, electrical and elevators	23.3	23.1	21.3	20.1L	24.6	28.1H	28.1	20.1	23.4	23.2
V	Special equipment, tenant changes, winter conditions, bond fee, general conditions, etc.	13.8	22.7	16.4	8.6L	12.3	24.6H	24.6	8.6	16.3	15.1
		100.0	100.0	100.0	100.0	100.0	100.00			100.0	

Schedule B

Item	Description	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	High	Low	Mean	Median
	Date built and period of construction	1929–1930	1927–1928	1927–1928	1928–1929	1928–1930	1928–1930	1927–1930			
		14 mo.	14 mo.	14 mo.	16 mo.	18 mo.	15 mo.	18 mo.	14 mo.	15 mo.	15 mo.
	Cubic-foot volumes	6,338,000	3,082,544	4,381,245	11,389,000	13,806,741	5,396,000	13,806,741	3,082,544	7,398,920	5,867,000
	Cubic-foot costs	$0.7101,	$0.777,	$0.783,	$0.714,	$0.784,	$0.787H,	$0.784,	$0.710,	$0.759,	$0.780,
		%	%	%	%	%	%	%	%	%	%
I	Foundations and Frame:										
	Excavation, foundations, caissons, demolition	13.4H	5.4	8.0	12.2	7.5	1.0L	13.4	1.0	8.0	7.8
	Concrete fireproofing and cement work	9.9	6.2	9.8	13.3H	8.6	5.7L	13.3	5.7	8.9	9.2
	Structural steel, stack, erection	12.7	11.6	12.4	14.3	13.4	14.5H	14.5	11.6	13.2	13.0

II	Exterior:										
	Granite, cut stone, models, ornaments, terra cotta and spandrels	5.4	2.3	6.7H	6.5	5.3	2.0L	6.7	2.0	4.7	5.4
	Masonry stone mortar, cleaning and pointing, waterproofing	3.3	4.3	2.9	2.7	1.4L	4.5H	4.5	1.4	3.2	3.1
	Subtotal	**8.7**	**6.6**	**9.6**	**9.2**	**6.7**	**6.5**	**11.2**	**3.4**	**7.9**	**8.5**
	Metals and wood sash glass, weather strips, caulking	2.9	3.2H	3.1	1.3L	1.4	2.8	3.2	1.3	2.4	2.8
	Roofing, quarry tile, flagstone, sheet metal, flashings	0.5	0.5	0.4L	0.4L	0.6H	0.5	0.6	0.4	0.5	0.5
III	Interior:										
	Hollow metal work, elevator enclosures, kalamein	1.1	2.6H	1.5	0.4L	0.8	2.0	2.6	0.4	1.4	1.3
	Lathing, plastering, acoustical work	2.0L	4.3H	4.1	3.7	3.3	3.5	4.3	2.0	3.5	3.6
	Carpentry, millwork, cabinet work, finish, hardware	4.6	2.6	5.3	6.1	8.3H	1.8L	8.3	1.8	4.8	4.9
	Ornamental iron, bronze, stairs, mail chutes, fire escapes, miscellaneous	3.4	4.8	2.7L	5.2	5.7H	4.3	5.7	2.7	4.3	4.5
	Marble, tile, terrazzo, structural glass, mastic, rubber	3.1L	5.7	4.7	4.6	6.1H	3.5	6.1	3.1	4.6	4.6
	Painting and decorating	0.6L	0.7	0.7	0.6L	0.7	1.2H	1.2	0.6	0.8	0.7
IV	Mechanical and Electrical:										
	Electrical work, fixtures, flood lighting, beacons	5.1	6.5	4.1L	7.8	9.1H	8.8	9.1	4.1	6.9	7.2
	Heating, ventilating, insulation, boilers, stokers, etc.	5.5	3.8L	5.3	4.5†	6.7H	3.8L†	6.7	3.8	4.9	4.9
	Plumbing and sprinklers	3.5	5.8	4.6	4.2	3.3L	6.0H	6.0	3.3	4.6	4.4
	Elevators, signals and closers, dumbwaiters, etc.	9.2	7.0	7.3	3.6L	5.5	9.5H	9.5	3.6	7.0	7.2
V	Miscellaneous:										
	Vault doors, safety cabinets, steel shutters	......	0.2H	......	0.1L	0.1L	......	0.2	0.1	0.1	0.1
	Special equipment, decorations, directional signs	0.8	......	......	0.3	3.0H	0.5	3.0	0.0	0.7	0.8
	Tenant changes[2]	......	13.8H	5.1	......	1.0	11.6	13.8	0.0	5.2	3.1
	Winter conditions, sidewalk, bridge, bond, fee, general conditions, tests	13.0H	8.7	11.2	8.2L	8.2	12.5	13.0	8.2	10.3	10.0
	Total cost	100.0	100.0	100.0	100.0	100.0	100.0			100.0	

[1] From *Engineering News-Record* "Construction Costs," 1934.

[2] Depend on rental conditions during construction.

H = high; L = low.

† Buy steam.

Table 9.8.—American Appraisal Company Building Indexes

1913 costs = 100. Compiled by the American Appraisal Company, Research Department. Discontinued in this form in 1933.

Year	New England	New York, New Jersey	Middle Atlantic	Pittsburgh	Central Western	Northwestern	Southeastern	Texas	Pacific Coast	Average, United States
Cost of All-frame Construction										
1913	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1914	99.0	97.0	96.0	97.0	97.0	96.0	95.0	98.0	95.0	96.7
1915	99.5	97.5	98.0	98.0	100.0	96.5	94.5	102.5	100.0	98.5
1916	108.5	107.5	106.5	111.0	112.0	106.0	104.5	115.0	113.0	109.3
1917	134.5	132.5	133.0	135.0	135.5	130.0	128.5	138.5	135.5	133.7
1918	167.0	159.0	165.5	161.5	162.5	159.5	158.5	175.5	169.5	164.3
1919	218.0	204.0	214.0	208.5	210.5	204.0	211.5	221.5	217.0	212.1
1920	264.5	263.7	274.5	272.5	270.5	263.0	268.7	273.7	267.0	268.7
1921	193.2	197.2	196.7	199.5	198.7	197.5	196.0	197.0	187.5	196.0
1922	190.7	188.5	189.2	190.5	189.0	186.5	192.2	193.0	186.5	189.6
1923	210.0	207.2	209.5	212.0	207.0	201.2	210.7	209.5	212.7	208.9
1924	205.5	205.2	206.7	213.7	208.0	195.7	205.5	205.2	197.5	204.8
1925	204.2	205.7	206.2	210.2	205.2	189.7	203.2	197.0	196.5	202.0
1926	206.7	209.2	209.8	211.5	206.3	186.6	206.7	203.6	193.3	203.8
1927	206.3	208.1	211.2	212.8	208.4	186.8	209.0	204.6	191.2	204.3
1928	209.5	208.3	211.5	212.2	207.2	185.5	206.1	200.4	190.6	203.5
1929	212.6	209.8	214.2	209.8	204.9	191.7	201.1	196.4	198.6	204.5
1930	208.5	206.0	206.4	200.3	196.3	183.5	186.5	184.8	171.9	195.4
1931	178.0	179.0	176.0	170.0	165.0	153.0	158.0	159.0	153.0	166.0
1932	155.0	155.0	156.0	148.0	144.0	140.0	135.0	137.0	135.0	145.0
Cost of Brick, Wood-frame Construction										
1913	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1914	102.0	100.0	100.0	99.0	101.0	100.0	100.0	104.0	98.0	100.4
1915	104.5	101.5	102.5	101.0	104.0	102.5	99.5	107.0	103.0	102.9
1916	116.0	113.5	113.5	114.0	116.5	112.0	109.5	120.0	113.0	114.2
1917	145.0	139.0	143.5	139.5	142.0	139.0	133.5	142.5	131.0	139.5
1918	175.5	171.5	182.0	171.5	172.5	165.5	174.5	170.5	159.0	171.4
1919	219.5	216.5	229.5	218.0	222.5	209.0	231.5	218.0	203.0	218.7
1920	288.0	288.5	298.2	293.5	289.2	281.5	288.7	273.2	250.2	283.5
1921	222.5	225.7	220.0	221.7	219.2	209.0	215.5	207.2	200.2	215.7
1922	205.2	204.7	203.5	199.7	197.2	189.0	194.7	183.7	184.5	195.8
1923	227.5	234.0	232.0	226.7	222.0	210.0	219.5	199.7	198.7	218.9
1924	225.2	230.5	230.2	231.5	223.7	204.2	217.5	200.5	196.7	217.9
1925	216.2	222.0	223.0	221.7	218.7	195.0	209.5	194.2	192.0	210.2
1926	217.3	227.0	226.6	224.5	218.1	192.5	219.0	200.2	190.3	212.9
1927	219.4	227.4	230.2	223.5	217.5	194.8	219.0	202.2	189.4	213.7
1928	218.7	225.6	229.5	223.5	217.5	195.0	217.9	197.1	188.3	212.6
1929	219.4	225.9	230.2	225.3	217.7	194.6	219.3	199.9	189.9	213.6
1930	217.6	224.1	221.8	218.7	212.5	190.0	210.0	194.9	183.2	208.0
1931	188.0	196.0	192.0	188.0	182.0	166.0	177.0	173.0	158.0	180.0
1932	165.0	171.0	170.0	164.0	159.0	147.0	149.0	149.0	141.0	157.0

TABLE 9.8.—AMERICAN APPRAISAL COMPANY BUILDING INDEXES.—(*Concluded*)

Year	New England	New York, New Jersey	Middle Atlantic	Pitts-burgh	Central Western	North-western	South-eastern	Texas	Pacific Coast	Average, United States
Cost of Brick, Steel-frame Construction										
1913	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1914	99.0	96.0	97.0	96.0	97.0	99.0	98.0	99.0	97.0	97.6
1915	102.5	99.5	100.5	98.5	100.5	102.0	99.5	105.0	102.0	101.1
1916	125.0	121.0	122.5	121.0	121.5	122.5	118.0	123.5	118.5	121.6
1917	163.0	156.5	161.5	157.5	155.0	155.0	150.0	151.0	146.0	155.1
1918	185.0	182.0	191.5	181.0	176.5	174.0	182.5	171.5	170.0	179.3
1919	207.5	210.5	221.5	209.0	207.5	200.5	222.5	203.0	200.5	209.2
1920	260.2	262.2	272.0	262.2	254.7	252.7	263.2	246.2	236.7	256.7
1921	202.7	209.7	207.0	204.5	201.5	195.7	202.2	192.5	194.5	201.2
1922	191.0	195.0	191.0	186.5	184.0	180.0	182.2	171.2	179.2	184.7
1923	218.5	226.7	228.7	217.2	211.7	204.5	209.5	191.0	198.7	212.1
1924	217.0	221.5	226.5	221.2	214.0	198.0	207.0	191.7	193.7	210.1
1925	204.7	211.2	216.2	212.2	207.7	191.2	197.5	185.7	188.5	201.7
1926	201.6	208.0	208.6	208.1	202.9	185.8	198.3	187.5	186.7	198.7
1927	199.4	206.0	208.0	204.6	200.0	184.4	197.3	197.3	182.9	199.6
1928	200.2	206.5	208.3	204.6	199.9	184.7	197.6	190.3	182.6	197.2
1929	200.8	206.9	208.6	205.3	199.7	182.7	198.0	190.3	183.3	197.3
1930	197.0	203.7	199.7	197.6	193.2	176.7	189.5	183.8	177.3	190.9
1931	174.0	182.0	176.0	173.0	168.0	158.0	160.0	160.0	156.0	168.0
1932	156.0	162.0	161.0	154.0	150.0	145.0	141.0	142.0	142.0	150.0
Cost of Reinforced-concrete Construction										
1913	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1914	100.0	98.0	98.0	97.0	99.0	97.0	98.0	99.0	98.0	98.2
1915	104.0	100.0	101.5	99.5	103.0	100.0	98.5	105.0	103.5	101.7
1916	121.5	118.0	120.0	120.5	124.5	115.0	114.5	126.0	116.5	119.6
1917	154.0	145.5	155.0	146.5	153.5	145.0	138.5	149.0	139.5	147.4
1918	177.5	168.5	187.0	169.5	175.5	163.0	169.5	165.5	166.0	171.4
1919	213.0	201.0	219.0	206.0	216.5	196.0	222.5	209.5	204.0	209.7
1920	268.2	259.7	275.7	268.5	275.0	257.0	268.7	251.7	254.2	264.4
1921	212.5	213.0	205.0	208.7	215.7	197.5	211.0	201.0	201.7	207.4
1922	198.7	187.5	188.2	188.0	193.7	178.5	185.5	183.5	184.7	187.6
1923	217.2	216.0	215.7	217.5	218.7	197.2	208.0	198.2	197.2	209.6
1924	212.5	212.0	208.5	219.7	216.5	192.5	202.7	189.5	191.7	205.8
1925	207.2	210.2	204.2	210.7	212.5	187.0	194.5	182.7	189.7	199.9
1926	205.7	212.8	205.2	213.2	210.6	184.1	201.0	188.1	186.2	200.8
1927	205.3	213.2	207.6	211.0	209.3	184.3	200.3	186.8	182.2	200.0
1928	206.7	213.7	208.2	210.6	209.4	184.8	199.9	187.1	182.5	200.3
1929	207.5	215.6	209.5	211.5	209.1	183.5	200.4	186.4	184.8	201.0
1930	207.7	214.8	203.3	203.9	203.5	178.9	191.6	180.4	177.4	195.7
1931	180.0	188.0	178.0	176.0	172.0	158.0	160.0	159.0	154.0	170.0
1932	161.0	164.0	163.0	156.0	151.0	147.0	138.0	142.0	142.0	152.0
Average of Four Types										
1932	159.0	163.0	163.0	156.0	151.0	145.0	141.0	143.0	140.0	151.0

from 22 to 46 stories. The percentages were computed from the total costs in dollars; these are omitted from the table.

9.14. Cost-time Factors.—As already stated in Chap. VI, there is a strong move at present (1935) to require continuous property-ledger accounts of industrial properties, showing the costs of the different property items in sufficient (Secs. 6.12, 6.28) detail to permit the computation of "cost-time factors" for the different kinds of property. Such factors are similar in principle, though simpler and more definite in application, to the construction-cost index numbers discussed in Sec. 9.12, above. They would be very valuable, when available.

It is never safe to give great weight to general cost indexes in valuing particular properties without first proving definitely that they apply to the particular circumstances and conditions.

9.15. American Appraisal Company Index Numbers for Buildings.—Table 9.8 shows American Appraisal Company index numbers for all-frame, brick and wood-frame, brick and steel-frame, and reinforced-concrete buildings; while Table 9.9 and Fig. 9.9 show the company's general building-cost index numbers for the entire United States.

Table 9.9.—American Appraisal Company. Building-cost Index[1]
1913 = 100

Year	Average	Year	Average	Year	Range	Average
1913	100.0	1923	212.4	1926	173–245	206
1914	98.2	1924	209.6	1927	174–245	206
1915	101.0	1925	203.4	1928	174–245	205
1916	116.2	1926	204.0	1929	175–244	206
1917	143.9	1927	204.4	1930	157–245	199
1918	171.6	1928	203.4	1931	140–227	171
1919	212.4	1929	204.0	1932	121–194	151
1920	268.3	1930	197.5	1933	117–183	146
1921	205.1	1931	171.0	1934	135–187	157
1922	189.4	1932	151.0			

[1] From *Engineering News-Record* "Construction Costs," 1935.

9.16. Fruin-Colnon Cost Index, St. Louis Buildings.—Table 9.10 and Fig. 9.9 show the Fruin-Colnon, St. Louis building-cost index numbers. These were "determined by actual and estimated costs of material and labor on a typical industrial plant consisting of five buildings, erected in St. Louis."

9.17. Turner Building-cost Index.—Table 9.11 and Fig. 9.9 show the building-cost index prepared by the Turner Construction Co., New York. It is "based on actual costs encountered in its own work.

Table 9.10.—Fruin-Colnon, St. Louis Building-cost Index[1]
Yearly averages, 1913 = 100

1913	100	1921	197	1928	213
1914	100	1922	188	1929	211
1915	102	1923	217	1930	207
1916	108	1924	223	1931	196
1917	128	1925	222	1932	178
1918	173	1926	218	1933	181
1919	174	1927	214	1934	191
1920	227				

[1] From *Engineering News-Record* "Construction Costs," 1935.

The following factors have been considered: labor rates; materials prices; productivity of labor; efficiency of plant and management; competitive conditions."

Table 9.11.—Turner Building-cost Index[1]
Yearly averages, 1913 = 100

1913	100	1922	165*	1930	165
1914	100	1922	175†	1931	145
1915	103	1923	196	1932	136
1916	120	1924	194	1933 to March	130
1917	147	1925	195	1933 to Oct.–Nov.	150
1918	166	1926	195	1934 to July	164
1919	196	1927	190	1934 to December	165
1920	252	1928	190		
1921	183	1929	195		

[1] From *Engineering News-Record* "Construction Costs," 1935.
* Low point during year.
† Average for year.

9.18. Aberthaw Index of Cost of Building.—This is shown in Table 9.12 and Fig. 9.9 for each month, January, 1915, to June, 1929, inclusive; and for each quarter, July, 1929, to January, 1935. The original building was constructed in 1914. The index numbers have been computed by repricing from current labor rates and material costs. 1914 = 100.

9.19. Tuttle Index of Cost of Building.—This is shown in Table 9.13 and Fig. 9.9 for each month, January, 1915, to May, 1933, and March, 1934 and 1935. The index was established by recomputing cost of 1932 building on the basis of current prices. 1914 = 100.

9.20. Comparison of Construction- and Building-cost Indexes.—Figure 9.9 shows how the American Appraisal, Fruin-Colnon, Turner, Aberthaw, and Tuttle building-cost indexes compare with each other and with the *Engineering News-Record* general construction-cost index (Sec. 9.24). See also the constructograph (Sec. 9.26).

Table 9.12.—Aberthaw Index of Cost of Building
1914 = 100

Year	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Average
1915	100	100	100	100	100	100	100	100	100	100	100	100	100
1916	100	101	102	103	105	107	108	109	110	111	113	115	107
1917	117	118	119	120	122	125	126	129	130	132	135	137	126
1918	138	139	140	142	144	145	147	150	151	153	155	157	147
1919	158	161	162	165	168	171	173	176	179	181	183	187	172
1920	190	196	210	220	230	240	253	265	258	252	248	241	234
1921	237	226	197	186	176	172	170	161	160	157	153½	153½	170
1922	151½	151½	151½	151½	157	169	171	174	190	192	191½	191½	179
1923	192	196½	196½	204	204	207	206	206	206	204	202	199	202
1924	199	200	202	202	200	199	198	196	196	195	195	197	198
1925	197	197	197	197	195	194	194	194	194	194	194	194	195
1926	195	195	195	199	199	199	199	198	197	197	197	197	196
1927	196	194	193	193	193	193	192	192	192	191	191	191	193
1928	191	192	192	192	191	191	191	191	191	191	191	190	191
1929	190	190	190	190	190	190	.....	...	...	188½	.....	.....	189
1930	192	.....	.....	189	...	...	186	...	...	185	.....	.....	188
1931	185	.....	.....	181	...	...	176	...	...	174	.....	.....	179
1932	172	.....	.....	172	...	...	167	...	...	166	.....	.....	169
1933	166	.....	.....	165	...	...	167½	...	...	173	.....	.....	168
1934	175	.....	.....	176	...	...	176½	...	...	176½	.....	.....	176
1935	176½												

Whole building repriced every quarter from then current labor rates and material costs. Original building constructed in 1914.

The comparatively wide variations between the different indexes show clearly that great care is absolutely necessary in using construction-cost indexes as aids in valuing particular properties.

9.21. Power-plant Building Costs.—Figure 9.10 shows the Burns and McDonnell Engineering Co. indexes of the materials' costs, labor costs, and total costs of power-plant buildings, or pump stations, 1913 to 1934, inclusive.

The items entering into the value of buildings are cement, both in plain and reinforced concrete, sand, crushed rock, brick, reinforcing steel, structural steel, roofing, sheet metal, windows, doors, heating material, electric lighting, common labor, carpenters, brick layers, structural steel men, metal workers, roofers, and electricians. The original proportions were taken from an average of 25 buildings. These have been added to and now include about 50 buildings.

9.22. Index Numbers for Five Utilities.—Table 9.14 shows the Burns and McDonnell Engineering Co. index numbers, for 1913 to 1934, inclusive, for waterworks plants, electric-light plants, street-railway systems, natural-gas plants, and artificial-gas systems; for each of these five utilities, Table 9.14 also shows the relative yearly values of the

Table 9.13.—Tuttle Index of Cost of Building
1914 = 100

Year	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Average
1915	100	100	100	100	100	100	100	100	100	100	100	100	100
1916	100	101½	103	104½	106	107½	109	110½	112	113½	115	116½	108
1917	118	120	121½	123½	125	127½	129½	131	133	134½	136½	138	139
1918	140	141½	143½	145	147	149	151	153	155	157	159	161	150
1919	163	165	167	169	171	173	175½	178	180½	183	185½	188	175
1920	192	196	200	204	208	212	216	220	224	228	232	236½	214
1921	241	220	197	186	176	172	167	161	160	157	153½	153	179
1922	151½	151	153	153	157	169	171	174	190	192	191½	191½	170
1923	191½	196½	196½	204	204	208	207	204	204	204	204	202	202
1924	202	202	204	204	203½	203½	202	201	200½	200½	200½	201	202
1925	201	201	202	202	199½	199½	199½	199½	199	199	199	199	200
1926	200	201	201	205½	205½	205½	205½	205½	205½	205½	205½	205½	204
1927	205½	204	204	200	195½	195½	195	193½	193½	193½	193½	193½	198
1928	193½	196	196½	195	195	194	194½	194½	194½	195½	195½	195½	195
1929	195½	195½	196	196	196	196	196	196	196	196	196	196	196
1930	196	195½	194½	194½	194	166	166	166	164½	164½	164½	164½	178
1931	164½	164½	164½	164	162½	160½	160½	160	160	161	161	159½	162
1932	160	160	160	151½	151½	151½	151½	154	154	154	153½	153½	155
1933	153½	153½	153½	153½	153½	Index not computed, June, 1933–February, 1934							153
1934	.....	.....	173	Index resumed, March 1, 1934									
1935	.....	.....	173										

1934 Index established by recomputing cost of 1932 building on basis of 1934 cost experience and determining relative change in cost.

constituent items, divided into five groups: land, buildings, equipment, distribution and miscellaneous.

Figure 9.11, redrawn from the *Engineering News-Record* figure, shows the relative values of the five utilities graphically; the *Engineering News-Record* cost index has been added for comparison.

9.23. Interstate Commerce Commission Railroad Construction-cost Indexes.—These indexes are based on 1910 to 1914 = 100. They were released by the Commission in mimeograph form, Jan. 19, 1934, but are taken here from *Engineering News-Record* "Construction Costs," 1935.

Figure 9.12 is a map showing the division of the country into Regions I to VIII, for each of which indexes have been established.

Figure 9.13 shows the variations of the index numbers for total "road"-construction and total "equipment" costs for the years 1915 to 1933; also the relative values of track laying, bridges, and grading; for comparison, the *Engineering News-Record* general construction-cost index numbers are shown for the same years.

The road-construction costs are the totals for accounts 1 to 46, except account 2, "land." These accounts are enumerated in Table

Table 9.14.—Relative Yearly Value of Five Main Items Entering Into Physical Value of Five Utilities, 1913–1934[1]

Waterworks: 25 systems; 68 items, of which the main ones are land, buildings, boiler-room equipment, pumps, steam piping, miscellaneous equipment, cast-iron pipe, wrought-steel pipe, wood pipe, valves and hydrants, meters, standpipes, reservoirs, furniture and fixtures, tools, autos, and miscellaneous. The land value is entirely a local matter, and it was found that the most accurate relation of the land to the other items was obtained by keeping the unit for land the same throughout the period.

Electric-light Plants: 25 plants; 84 items, of which the main ones are land, buildings, boiler-room, engine-room and substation equipment, transformers, lighting equipment, poles, conductors, distribution transformers, meters, underground conduits, furniture and fixtures, tools, autos, and miscellaneous.

Street Railways: 10 systems; 82 items, of which the main ones are land, buildings, (car barns and shops) tangent track, special layouts, grading, paving, bridges, span construction, rails, spikes and bolts, ties and labor, poles, conductors, cars and equipment, furniture and fixtures, tools, autos, and miscellaneous. Power-plant buildings and equipment excluded because power is so frequently bought from a separate department or company.

Natural Gas: 15 systems; 58 items, of which the main ones are land, buildings, holders, small pumps, miscellaneous equipment, cast-iron pipe, wrought-steel pipe, regulators, governors and valves, meters, furniture and fixtures, tools, autos, and miscellaneous.

Manufactured Gas: 25 plants; 63 items, including land, buildings, boilers, gas machinery, holders, miscellaneous equipment, cast-iron pipe, wrought-steel pipe, regulators, governors and valves, meters, furniture and fixtures, tools, automobiles, and miscellaneous.

1913 = 100

	1913	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934
Waterworks Plant																						
Land	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9
Buildings	11.2	10.8	11.1	13.3	16.5	18.5	20.6	25.6	20.9	19.0	20.8	20.7	20.6	20.7	20.2	20.1	20.4	19.7	18.2	16.6	16.9	18.7
Equipment	12.1	12.0	14.0	17.3	22.6	23.	24.3	26.0	23.4	21.8	22.9	23.0	22.9	22.7	22.5	23.0	23.2	22.3	20.9	19.5	21.0	22.9
Distribution	72.7	67.1	71.3	94.1	143.2	161.1	165.8	197.8	155.8	130.1	149.8	152.2	143.2	140.5	127.5	123.8	131.8	130.0	123.3	112.6	116.8	129.7
Miscellaneous	1.1	1.1	1.2	1.4	1.8	2.0	2.3	2.8	2.2	2.	2.2	2.0	2.1	2.0	1.9	1.9	1.9	1.9	1.6	1.4	1.6	1.7
Total	100.0	93.9	100.5	129.0	187.0	207.9	215.9	255.1	205.2	175.8	198.6	200.8	191.7	188.8	175.0	171.7	180.2	176.8	166.9	153.0	159.2	175.9
Electric-light Plant																						
Land	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8
Buildings	10.1	9.7	10.0	12.0	14.9	16.7	18.5	23.1	19.9	17.1	18.8	18.6	18.5	18.6	18.2	18.2	18.5	17.8	16.5	14.9	15.2	16.8
Equipment	38.9	36.7	36.7	47.9	73.5	79.5	81.0	86.7	84.6	75.1	73.4	73.5	73.6	73.0	72.0	72.9	75.4	71.0	66.6	60.9	66.2	75.5
Distribution	44.8	43.5	47.1	56.1	63.4	69.6	75.2	82.6	77.4	70.4	70.4	69.6	70.9	71.2	71.1	72.9	75.8	71.7	67.1	59.4	61.8	70.3
Miscellaneous	3.4	3.4	3.4	4.1	5.1	6.0	6.8	8.6	6.8	6.2	6.2	6.1	6.1	6.0	5.7	5.6	5.6	5.5	5.0	4.5	4.6	4.9
Total	100.0	96.1	100.0	122.9	159.7	174.6	184.3	203.8	191.5	171.6	171.6	170.6	171.9	171.6	169.8	172.4	178.1	168.8	158.0	142.5	150.6	169.3

Street-railway System																						
Land	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
Buildings	4.7	4.5	4.6	5.6	6.9	7.8	8.6	10.8	8.8	8.0	8.8	8.7	8.6	8.6	8.5	8.5	8.6	8.3	7.6	6.9	7.1	7.8
Track construction	64.3	63.1	63.4	71.9	83.3	104.8	113.8	144.1	119.9	105.4	113.6	112.8	110.2	110.2	108.6	108.5	106.9	104.7	100.1	92.9	94.1	97.9
Span construction	5.8	5.4	6.3	8.7	9.9	9.7	10.0	11.1	9.0	8.5	8.7	8.8	8.1	8.0	8.1	8.4	8.7	8.4	7.8	7.0	7.0	7.5
Cars and equipment	20.9	22.4	22.8	25.1	27.2	31.1	44.4	61.1	59.1	53.0	47.0	42.9	40.7	40.7	40.7	39.9	39.9	39.9	34.0	32.4	30.8	35.8
Miscellaneous	1.7	1.6	1.6	2.2	2.9	3.2	3.5	4.3	3.6	3.2	3.3	3.2	3.2	3.2	3.1	3.0	3.0	3.0	2.7	2.5	2.5	2.7
Total	100.0	99.6	101.3	116.1	132.8	159.2	182.9	234.0	203.0	180.7	184.0	179.0	173.4	173.3	171.6	170.9	169.7	166.9	154.8	144.3	144.1	154.3
Natural-gas Plants																						
Land	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
Buildings	1.6	1.5	1.6	1.9	2.4	2.6	2.9	3.7	3.0	2.7	2.9	2.9	3.0	3.0	2.9	2.9	2.9	2.8	2.6	2.4	2.4	2.7
Equipment	1.7	1.6	1.8	2.6	4.0	3.6	3.5	4.1	3.1	3.1	3.1	3.1	3.0	3.0	3.0	3.0	2.9	2.8	2.6	2.4	2.7	2.9
Distribution	92.2	89.9	92.0	111.0	155.7	182.9	186.5	202.4	187.3	158.4	168.7	173.2	175.0	174.6	169.9	170.3	171.1	168.8	164.7	155.0	157.8	169.0
Miscellaneous	2.9	2.8	2.9	3.3	4.4	5.1	5.8	7.8	6.4	5.6	5.9	5.7	5.5	5.4	5.3	5.2	5.2	5.2	4.8	4.3	4.2	4.5
Total	100.0	97.4	99.9	120.4	168.1	195.8	200.3	219.6	201.4	171.4	182.2	186.5	188.1	187.6	182.7	183.0	183.7	181.2	176.3	165.7	168.7	180.7
Artificial-gas Systems																						
Land	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2
Buildings	6.8	6.5	6.7	8.1	10.0	11.2	12.5	15.5	12.7	11.5	12.6	12.5	12.4	12.5	12.2	12.1	12.4	12.0	11.1	10.1	10.3	11.3
Equipment	30.4	29.1	33.1	49.9	76.4	68.7	67.2	75.9	66.1	62.8	63.4	61.2	60.5	60.6	60.3	59.4	58.7	53.9	49.2	47.2	51.4	57.4
Distribution	56.5	54.4	55.6	68.1	98.3	114.8	116.5	130.4	115.6	98.2	107.5	108.6	107.3	106.7	101.5	101.0	103.8	102.6	99.7	92.5	93.8	102.1
Miscellaneous	2.1	2.1	2.1	2.5	3.1	3.6	4.1	5.5	4.6	4.1	4.2	4.0	3.9	3.8	3.8	3.7	3.7	3.7	3.5	3.0	3.1	3.2
Total	100.0	96.3	101.7	132.8	192.0	202.5	204.5	231.5	203.2	180.8	191.9	190.5	188.3	187.8	182.0	180.4	182.8	176.4	167.7	157.0	162.8	178.2

[1] From *Engineering News-Record* "Construction Costs," 1935.

9.16. To the "road"-construction costs there must be added "equipment," the total of accounts 51 to 58, and "general expenditures," the total of accounts 71 to 77. The sum is the total cost, exclusive of land. The variations of these sums, Regions I to VIII, are shown in Table 9.15.

Table 9.15.—Percentages of Total Railroad Construction Costs[1]

	United States entire	Region							
		I	II	III	IV	V	VI	VII	VIII
I. Road, weighted average, accounts 1–46, except land.....	68.75	71.15	63.13	63.06	66.03	72.96	73.67	71.61	74.51
II. Equipment, weighted average, accounts 51–58..............	23.69	20.85	29.52	29.43	25.93	19.19	18.80	20.77	18.50
III. General expenditures, accounts 71–77, except 76......	0.97	1.07	0.79	0.77	0.99	1.10	1.11	1.09	1.13
76. Interest during construction	6.59	6.93	6.57	6.75	7.04	6.75	6.42	6.53	5.86
Weighted average, accounts 71–77	7.56	8.00	7.35	7.51	8.03	7.85	7.53	7.62	6.99
Weighted average accounts 1–77, except land....................	100	100	100	100	100	100	100	100	100

[1] From *Engineering News-Record* "Construction Costs," 1935.

In addition to the above data, Table 9.16 shows the separate road-construction index numbers for each separate account each year, 1915 to 1933.

9.24. Highway Construction-cost Trends.—Figure 9.14 has been redrawn from a chart published in *Public Roads* for May, 1933.[1] This figure shows separate index numbers for highway construction costs of grading, structures, surfacing, and a "composite mile," for each year, 1922 to 1932, inclusive; the base period is 1925 to 1929, inclusive.

The general or composite mile curve is an index in which the weights assigned to the prices of the various items used are the amounts of these items which would have been used in building one mile of road, if that mile had contained an average amount of grading and footage of structures and had the surface been composed of gravel, macadam, concrete, etc., in the amounts proportioned according to the average usage of these surfaces.[1]

GENERAL CONSTRUCTION-COST INDEXES

The construction-cost indexes presented and discussed in Sec. 9.14 to 9.23, respectively, apply only to particular types of construction. Two agencies, the *Engineering News-Record* staff and the Associated General Contractors of America, working independently, have developed general construction-cost indexes. Each of these is intended to show the

[1] See also *Engineering News-Record* "Construction Costs," 1934.

general trend of the total costs of all kinds of private and public construction in the entire United States.

9.25. The Engineering News-Record General Construction-cost Index Number.—This "is a national index of general construction cost in the United States. It has four components, of which three are prices of materials and one is wages. The materials are structural steel at Pittsburgh, cement at Chicago, and lumber at New York. The labor element is the average wage for common labor in twenty cities. These four factors are weighted according to their relative importance, resulting in the following quantities: structural steel, 2,500 lb.; cement, 6 bbl.; lumber, 600 ft. b.m.; common labor, 200 man-hours. These are repriced each month."[1]

"Computation for January, 1935, *Engineering News-Record* cost index follows:

Steel	2,500	× $0.018	=	45.00
Lumber	0.6	× 57.00	=	34.20
Cement	6.0	× 2.00	=	12.00
Labor	200.0	× 0.5376	=	107.52
Index				198.72

"The purpose of this Cost Index is to show the movement of general construction costs. It is not intended to apply to a specific type of structure, nor to a class of construction, nor to a particular locality. It is used in financing operations, in estimating and in valuation. Its possible application in a special problem must be determined by the individual user."[1]

Engineering News-Record also publishes monthly construction-volume index numbers. These are designed to indicate the relative total physical volumes of construction in progress in the United States, including both private and public work.

Table 9.17 shows the *Engineering News-Record* construction-cost index numbers, for each year, 1903 to 1913, and each month, January, 1914, to December, 1934.

Table 9.18 shows the *Engineering News-Record* construction-volume index numbers, for each month, January, 1914, to December, 1934.

Figure 9.15 shows graphically both the construction-cost and the construction-volume index numbers (the latter by quarterly averages). The Aberthaw index of reinforced-concrete buildings is shown also, for comparison.

Note that a graphical comparison of the *Engineering News-Record* cost index with other building-cost indexes is shown in Fig. 9.9. Its relation to utility costs is indicated in Fig. 9.11; to railroad costs in Fig.

[1] From *Engineering News-Record* "Construction Costs," 1935.

TABLE 9.16.—RAILROAD COST INDEXES BY YEARS FOR "ROAD" CONSTRUCTION[1]
Applicable to entire United States and to established regions
Accounts 1–46, "Road," Engineering Section, Bureau of Valuation, Interstate Commerce Commission, Jan. 19, 1934
1910–1914 = 100

Regions	Percent	Index by years 1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933
I to VIII	68.75	101	110	134	159	178	214	175	157	171	171	166	166	164	161	160	152	143	131	127
I	71.15	103	112	138	162	184	220	178	163	177	178	172	173	171	167	166	158	148	134	131
II	63.13	102	111	134	160	180	214	176	160	174	174	168	168	167	165	164	157	147	133	130
III	63.06	102	111	135	160	180	214	176	160	173	173	169	168	167	164	163	155	145	132	129
IV	66.03	101	110	132	157	180	214	173	152	167	166	164	163	159	155	154	145	135	126	122
V	72.90	101	109	133	158	179	218	171	153	168	167	163	164	160	156	156	148	139	129	125
VI	73.67	102	110	133	157	177	215	175	155	169	170	163	164	162	158	158	150	141	129	125
VII	71.61	100	109	131	157	175	212	177	158	170	170	165	166	163	160	158	151	142	131	128
VIII	74.51	105	109	133	160	176	208	173	157	171	171	166	167	164	161	161	154	145	133	130

TABLE 9.16a.—RAILROAD COST INDEXES BY YEARS AND BY ACCOUNTS APPLICABLE TO THE ENTIRE UNITED STATES—REGIONS I TO VIII, INCLUSIVE
Engineering Section, Bureau of Valuation, Interstate Commerce Commission, Jan. 19, 1934
(The Indexes represent territorial index factors and are not necessarily applicable for use in the determination of unit reproduction costs upon individual roads)

"Road" construction	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933
Engineering	101	110	134	159	178	214	175	157	171	171	166	166	164	161	160	152	143	131	127
Grading	100	110	130	165	190	250	170	143	160	164	149	153	143	135	133	123	118	106	98
Underground power tubes																			
Tunnels and subways	103	109	128	150	183	208	179	165	179	179	179	178	169	155	155	143	130	119	111
Bridges, trestles, culverts	105	111	146	162	178	206	165	160	176	173	171	170	168	164	163	150	134	122	122

Elevated structures	102	124	169	177	184	210	150	153	173	171	168	165	163	163	162	154	144	129	122
Ties	100	100	112	133	170	201	189	157	177	175	172	173	175	176	175	170	155	144	139
Rails	101	106	121	148	152	168	158	144	145	145	144	144	144	144	144	144	144	140	134
Other track material	99	129	198	210	203	209	192	161	182	179	177	177	177	177	170	169	165	163	158
Ballast	103	107	114	140	150	207	191	176	175	175	174	175	176	176	176	168	159	146	146
Track laying and surfacing	100	100	130	163	175	218	174	165	188	188	188	188	188	188	188	182	175	164	157
Right-of-way fences	100	122	142	178	194	204	189	177	179	179	176	175	175	175	173	171	164	147	135
Snow, sand fences, snowsheds	103	108	119	165	199	280	197	194	212	200	201	201	204	204	204	198	188	125	126
Crossings and signs	104	108	137	161	182	208	171	164	178	175	171	169	166	165	165	161	153	131	127
Station and office buildings	101	115	135	154	185	215	192	180	194	193	188	184	189	188	187	182	165	141	145
Roadway buildings	100	115	136	156	185	216	192	178	196	196	189	187	192	191	190	186	166	140	145
Water stations	101	120	159	170	191	213	185	178	187	187	186	182	185	186	184	177	161	147	151
Fuel stations	101	120	153	160	190	212	181	166	185	185	182	180	183	183	183	174	159	144	149
Shops and engine houses	102	118	141	159	188	216	191	180	193	192	188	185	189	188	187	176	161	137	142
Grain elevators	100	110	128	150	185	214	190	184	197	197	193	190	195	193	193	182	165	137	142
Storage warehouses	100	115	135	155	185	210	193	178	198	198	193	189	193	191	191	184	165	137	142
Wharves and docks	100	114	133	152	178	204	167	158	175	175	174	177	178	178	178	172	158	136	141
Coal and ore wharves	101	117	145	155	184	204	170	159	176	176	174	174	176	176	176	172	157	136	142
Gas-producing plants	108	122	148	175	194	213	194	176	188	189	186	185	188	189	189	178	163	145	148
Telegraph and telephone lines	103	124	147	158	164	192	191	162	187	179	163	157	163	165	165	150	138	121	119
Signals and interlockers	94	106	132	152	165	175	163	158	165	164	162	169	158	155	154	147	138	130	130
Power dams, canals, pipe lines																			
Power-plant buildings	104	122	141	158	189	218	197	184	196	196	191	186	191	191	189	177	162	138	143
Power-substation buildings	101	117	137	156	187	218	194	180	197	197	192	188	193	191	190	176	161	137	142
Power-transmission systems	115	166	190	181	186	176	145	132	142	136	140	141	137	142	150	136	116	98	98
Power distribution systems	109	148	178	192	189	205	172	163	178	172	175	176	175	178	181	173	148	144	144
Power-line poles, fixtures	106	116	145	169	194	230	208	179	209	203	185	183	198	199	209	200	172	147	147
Underground conduits	101	110	119	172	206	250	228	214	220	215	220	216	219	219	217	215	175	175	175
Miscellaneous structures	101	117	137	156	186	217	192	179	195	195	190	186	191	190	189	182	164	141	146
Paving	104	124	153	177	205	217	191	190	191	191	191	190	190	190	190	190	181	156	150
Roadway machines	105	113	127	146	158	170	162	149	151	151	151	151	151	149	148	147	144	138	138
Roadway small tools	100	100	179	179	184	202	181	170	173	185	190	190	191	191	190	160	155	155	150
Shop machinery	115	126	155	192	200	210	198	173	183	185	185	186	187	189	191	176	166	155	155
Power-plant machinery	115	126	155	192	200	210	198	173	183	185	185	186	187	189	191	176	166	155	155
Power-substation apparatus	115	126	155	192	200	210	198	173	183	185	185	186	187	189	191	176	166	155	155
Weighted average, accounts 1–46	101	110	134	159	178	214	175	157	171	171	166	166	164	161	160	152	143	131	127

[1] From *Engineering News-Record* "Construction Costs," 1935.

Table 9.17.—Engineering News-Record Construction-cost Index[1]

1913 = 100

Yearly averages prior to 1914

Year	1913 = 100	1926 = 100
1903	93.90	45
1904	87.40	42
1905	90.55	43½
1906	95.10	46
1907	100.55	51
1908	97.20	47
1909	90.92	44
1910	96.33	46
1911	93.43	45
1912	90.70	43½
1913	100.00	48

	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934
J	89.22	87.12	112.66	167.75	184.51	198.05	206.55	230.87	168.72	191.70	217.90	210.40	207.15	211.50	203.90	209.40	208.96	194.48	162.48	158.44	191.26
F	89.22	87.36	116.66	167.75	184.51	201.50	225.10	230.67	168.72	197.40	220.30	209.70	206.55	210.15	204.65	210.40	206.46	196.61	161.82	159.30	194.06
M	88.97	87.66	125.16	175.66	184.51	198.05	240.85	224.27	162.04	205.25	224.70	210.20	207.65	208.80	204.65	207.78	206.80	194.51	157.24	158.44	194.06
A	87.97	90.16	129.51	183.41	186.01	191.25	265.20	213.07	164.72	213.50	221.60	209.55	207.05	209.00	206.40	203.40	207.12	191.63	153.12	160.16	195.86
M	88.62	90.16	131.06	187.61	186.01	191.25	268.90	210.82	164.62	216.70	222.38	207.20	207.30	206.80	207.00	205.15	205.86	189.33	152.78	164.39	199.61
J	87.87	88.96	128.71	199.26	186.31	191.85	273.80	209.82	166.62	220.70	216.85	204.60	204.80	205.55	206.15	205.65	203.36	187.23	152.20	163.41	199.61
J	88.12	90.51	127.96	204.01	188.65	193.65	265.65	203.82	169.70	222.10	214.40	204.60	207.80	203.68	206.65	204.77	200.95	174.37	153.36	165.50	199.65
A	90.11	91.76	128.76	198.41	193.85	196.65	252.00	193.47	173.40	221.50	213.15	204.60	208.30	205.50	207.29	205.91	200.95	171.38	156.80	167.00	198.40
S	90.36	93.01	131.76	190.31	193.85	202.85	255.20	188.27	185.00	221.50	211.25	202.10	208.30	203.60	207.29	207.57	199.58	171.40	157.96	175.48	200.58
O	88.51	96.16	135.11	167.11	193.85	202.25	255.20	182.57	188.60	220.30	207.55	205.35	209.80	204.40	207.71	206.32	198.72	169.78	159.16	187.74	200.86
N	87.26	101.06	138.16	166.51	193.55	206.85	255.32	166.32	188.60	220.90	205.70	205.95	210.80	201.98	209.46	208.46	198.54	169.28	158.20	190.14	201.36
D	86.51	107.06	149.41	167.11	194.75	206.85	251.62	167.82	192.60	217.30	208.58	205.95	210.80	203.90	210.16	209.46	196.86	166.23	158.46	192.14	201.86
Av.	88.56	92.58	129.58	181.24	189.20	198.42	251.28	201.81	174.45	214.07	215.36	206.68	208.03	206.24	206.78	207.02	202.85	181.35	156.97	170.18	198.10
1926 = 100 (converted from 1913 = 100)																					
Av.	42.57	44.50	62.29	87.12	90.95	95.38	120.74	97.01	83.86	100.29	103.52	99.35	100.00	99.14	99.40	99.51	97.51	87.17	75.45	81.85	95.22

[1] From *Engineering News-Record* "Construction Costs," 1935.

TABLE 9.18.—ENGINEERING NEWS-RECORD CONSTRUCTION-VOLUME INDEX NUMBER[1]
1913 Construction cost and contract value = 100

	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934
January	71	48	96	69	57	38	138	52	100	151	125	141	198	187	208	325	199	208	121	129	149
February	83	76	95	47	151	91	130	55	68	129	144	145	187	182	225	378	204	198	105	82	116
March	142	75	90	55	90	52	113	90	186	162	161	161	240	226	318	382	408	278	97	62	130
April	113	82	137	65	58	78	134	109	161	186	159	211	233	257	316	332	320	289	135	65	109
May	155	96	129	50	67	80	126	108	221	149	191	209	254	299	299	395	354	251	149	103	101
June	128	105	129	57	106	129	92	119	178	173	149	214	246	297	308	347	324	243	131	109	119
July	139	129	136	87	107	122	83	115	146	151	161	178	248	252	266	320	226	248	166	65	128
August	118	126	130	63	111	135	86	116	137	100	162	266	252	306	306	290	259	187	111	73	94
September	95	116	145	166	145	113	80	109	149	166	159	262	227	273	296	340	232	225	124	122	101
October	108	101	97	87	67	207	79	130	154	154	164	233	244	269	363	273	211	229	137	162	98
November	84	150	105	76	66	71	57	106	114	121	128	194	211	272	251	245	207	180	137	133	115
December	62	89	106	49	27	41	63	117	134	123	145	206	188	319	244	166	192	133	113	116	105
Yearly average	107	100	119	67	86	98	99	96	147	146	154	201	228	263	287	317	260	220	127	102	114

[1] From *Engineering News-Record* "Construction Costs," 1935.

9.13; to the Associated General Contractors' Constructograph in Fig. 9.16. In general, the *Engineering News-Record* cost index is higher since 1920 than most of the other indexes.

9.26. The Associated General Contractors of America's Constructograph.—The Constructograph Index is closely allied to the *Engineering News-Record* cost index. The Constructograph is published regularly in the *Constructor*, the national publication of the Associated General Contractors. The data for the preparation of this index are collected and analyzed by the association's engineer, H. E. Foreman.

Figure 9.16 shows the Constructograph for each year, 1914 to 1934, and for each month, January, 1931, to February, 1935. The *Engineering News-Record* Construction-cost Index has been added for comparison.

The Constructograph is especially valuable because it shows wages and materials' prices indexes in addition to the index of total construction costs.

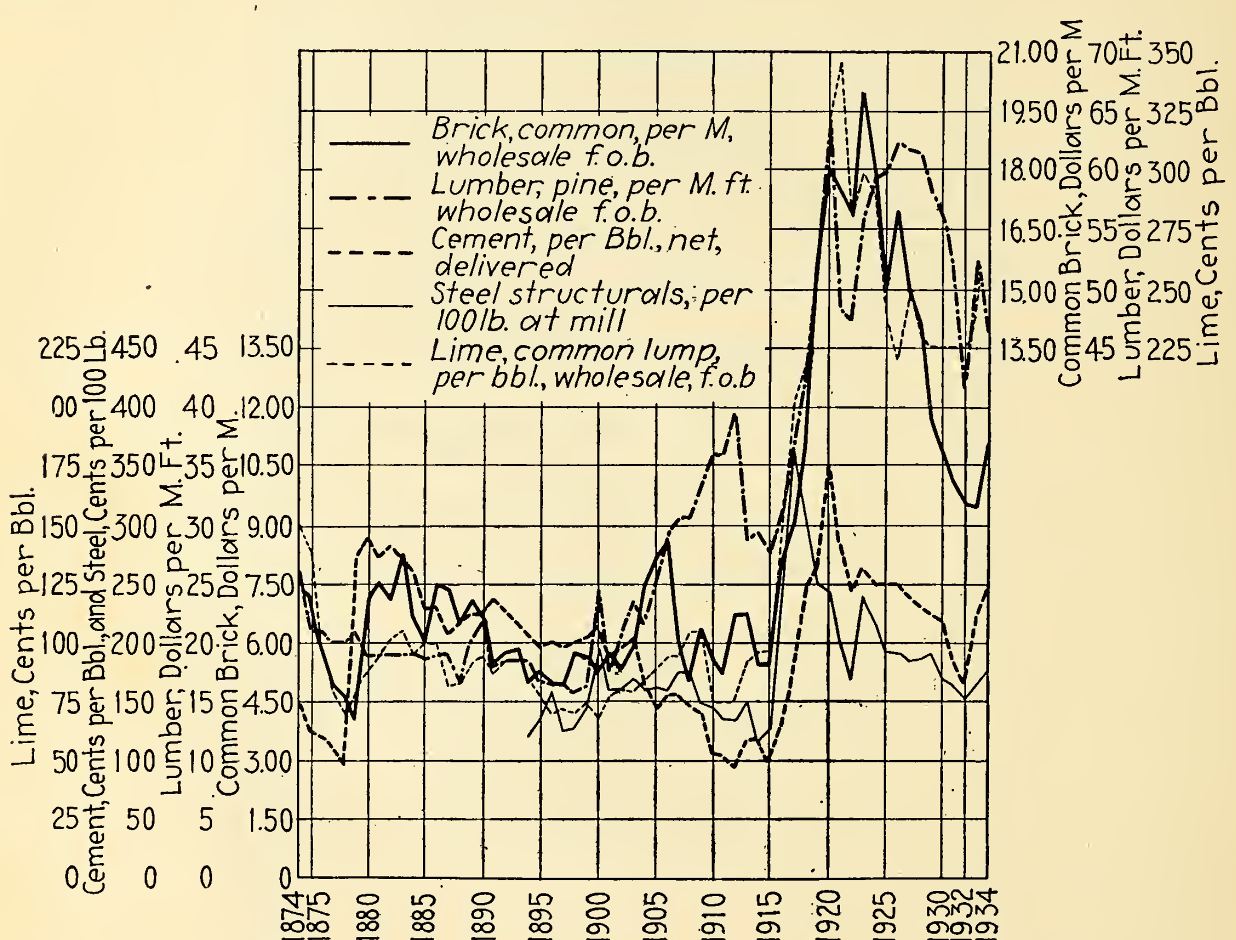


Fig. 9.1.—Basic building materials' prices—1874 to 1934—in New York City.

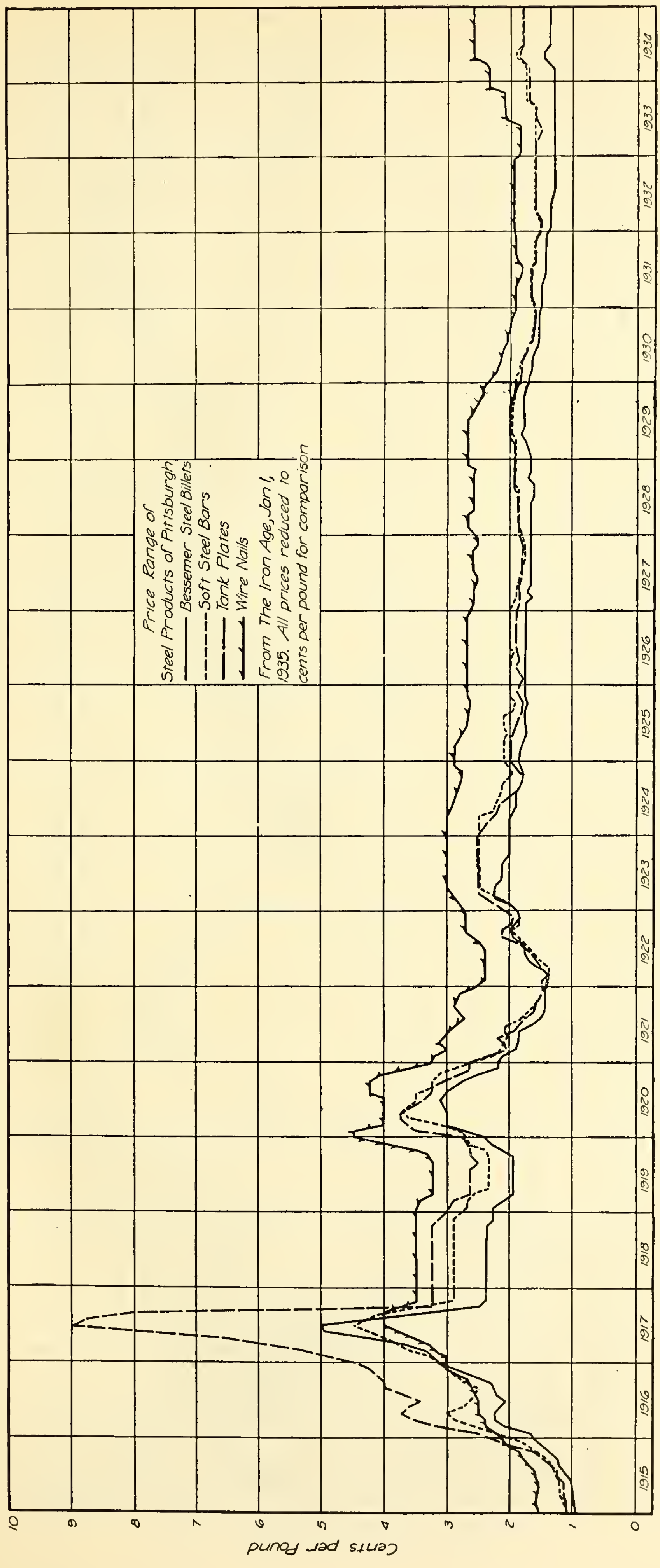


FIG. 9.2.—Trend of prices of steel products, 1915–1934.

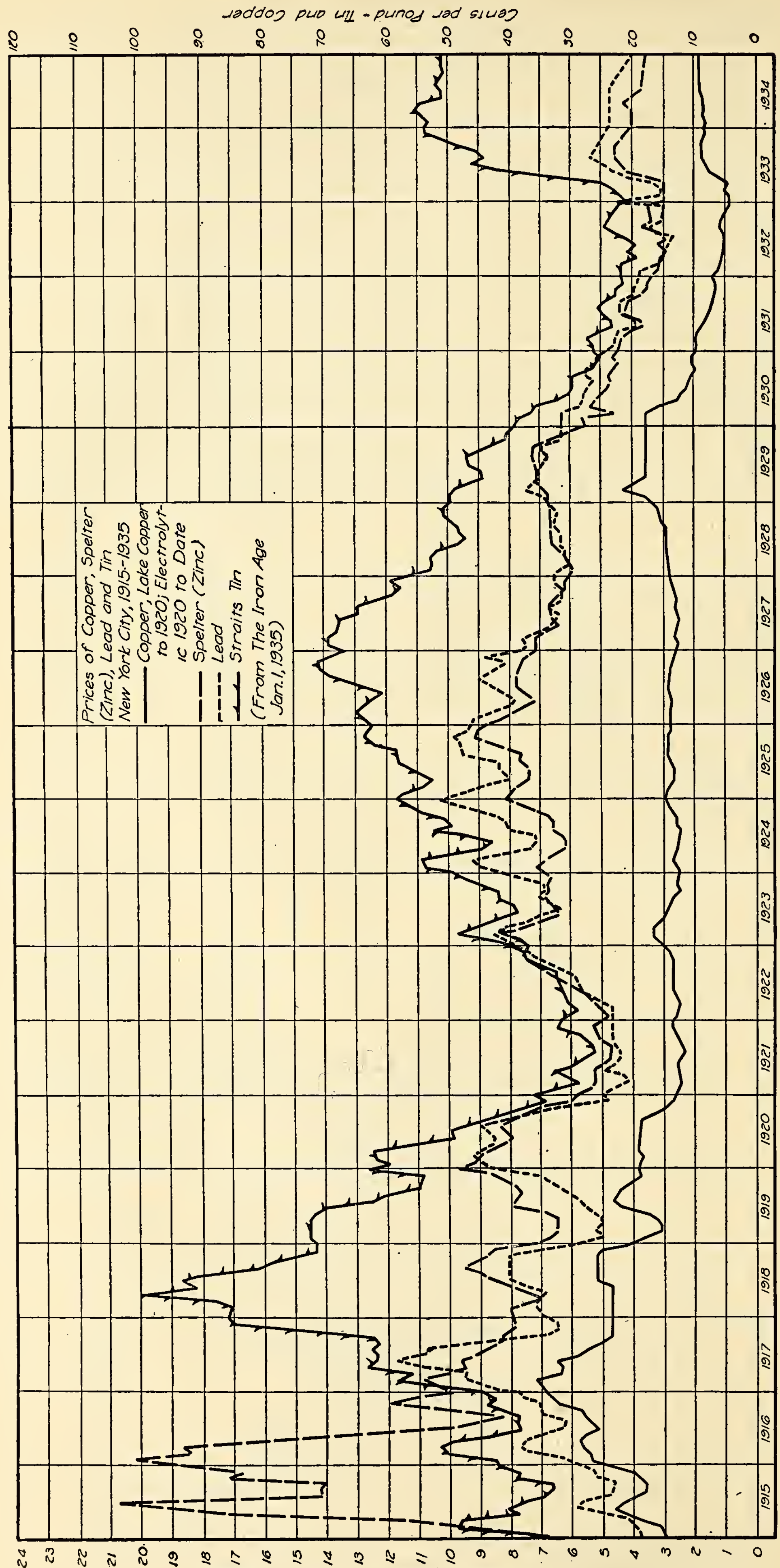


Fig. 9.3.—Trend of prices of metals, 1915–1934.

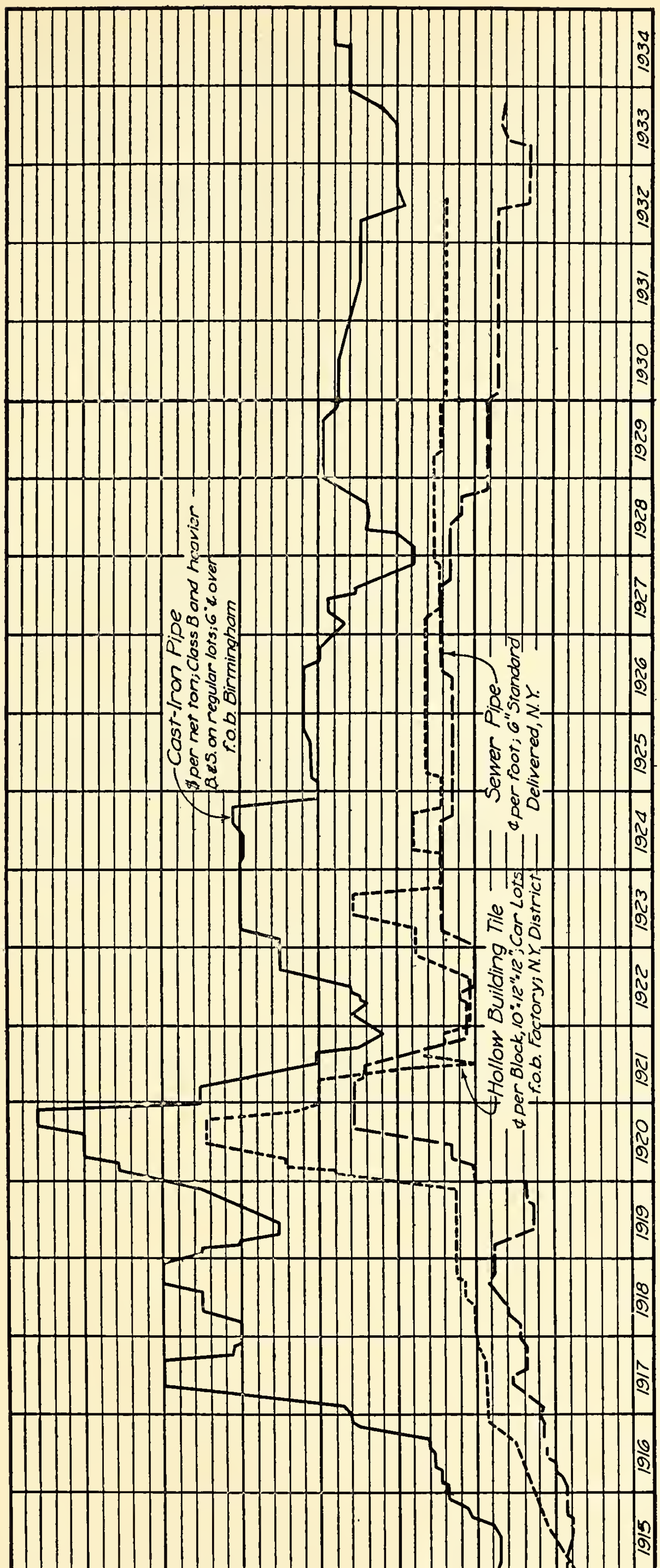


FIG. 9.4.—Price trends of building tile, sewer pipe, and cast-iron pipe.

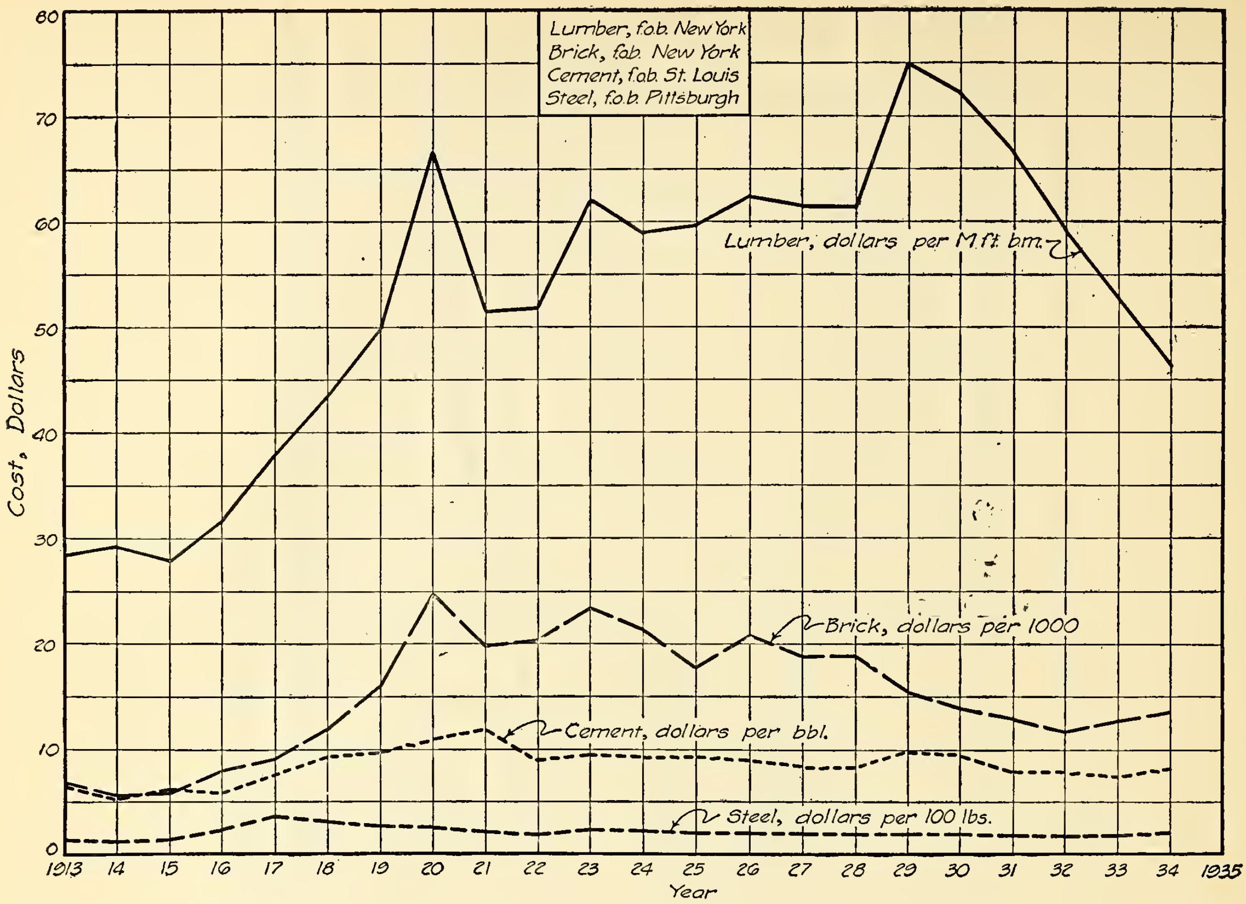


FIG. 9.5.—Price trends of lumber, brick, cement, and steel.

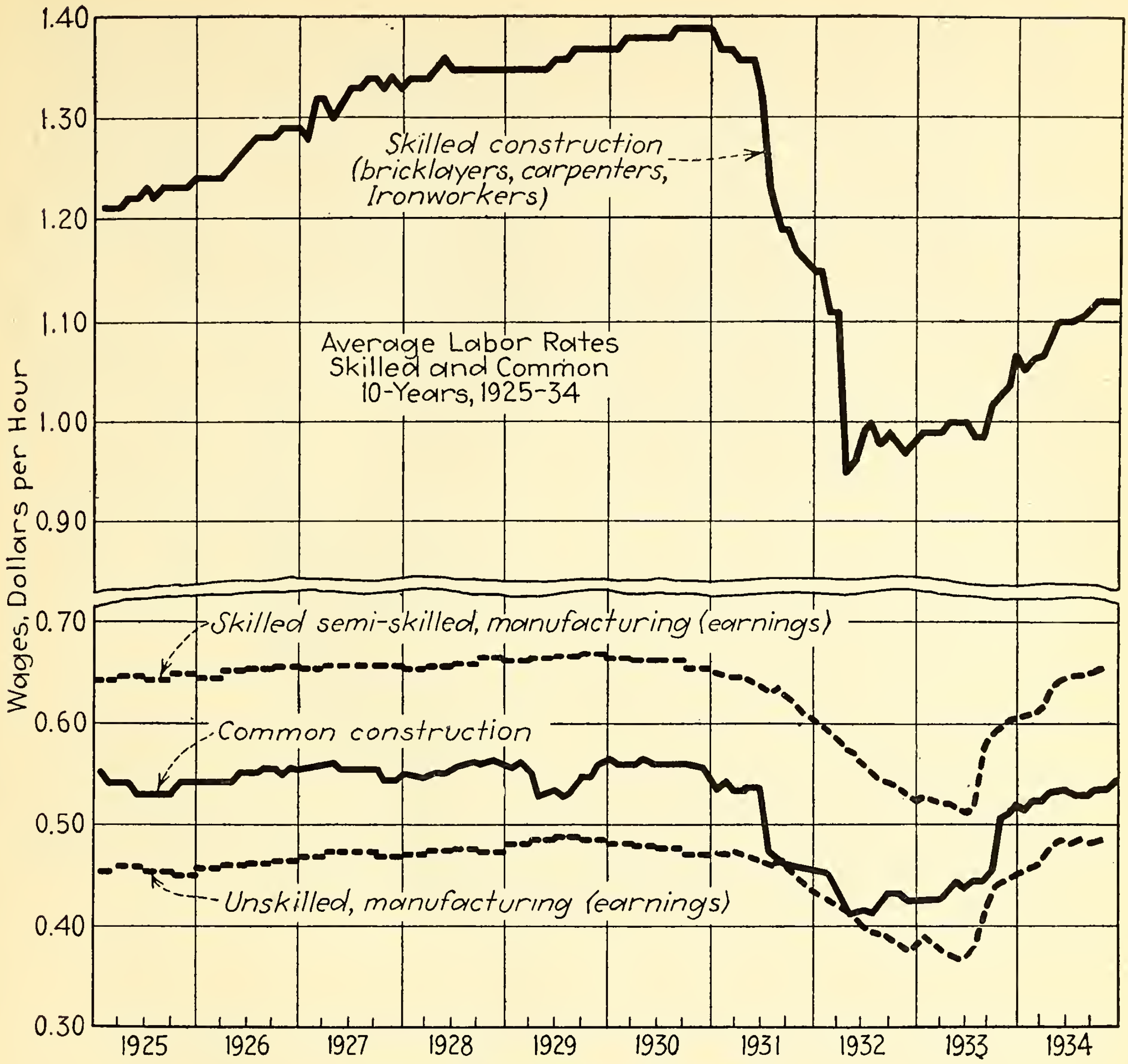


FIG. 9.6.—Construction wages.

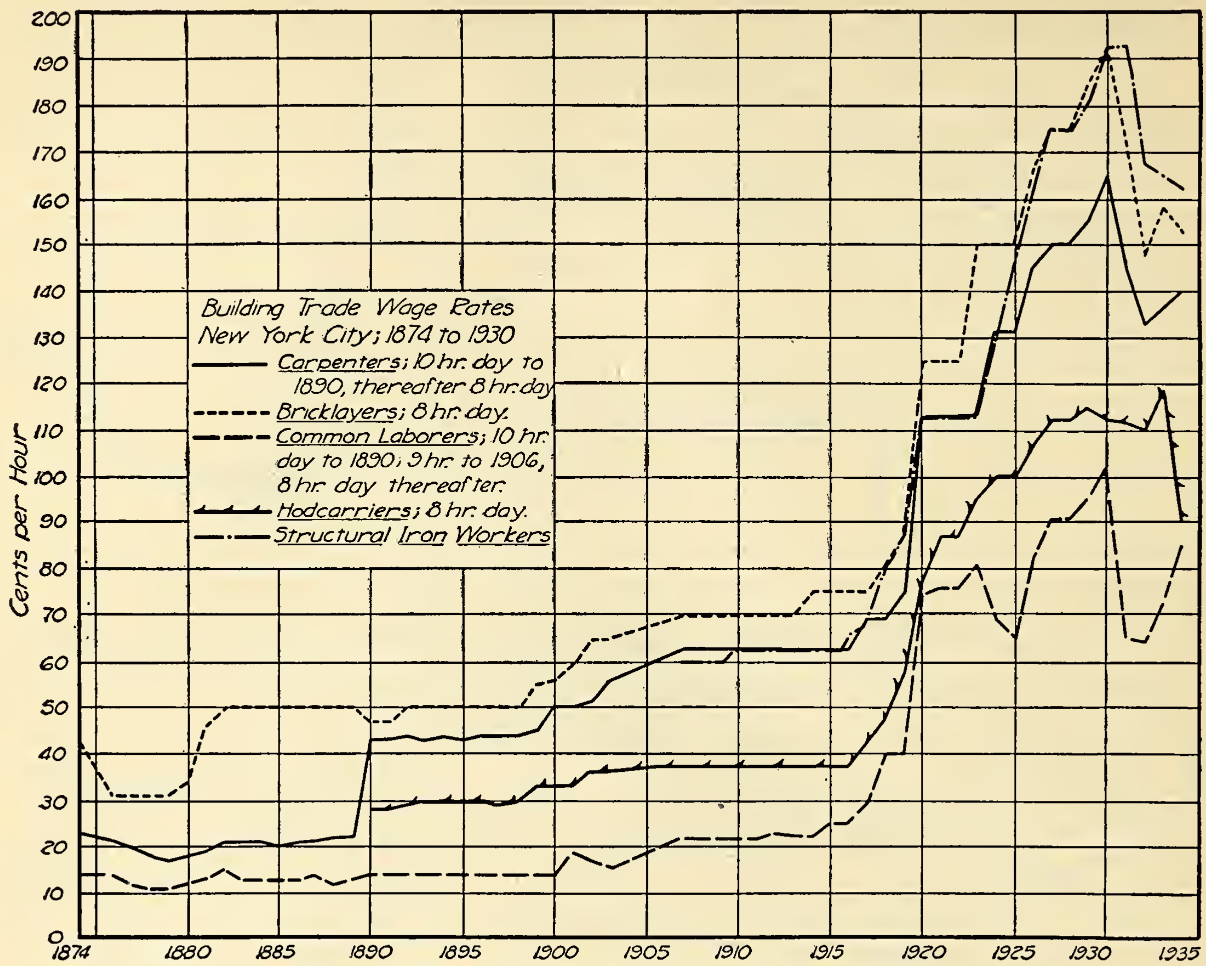


Fig. 9.7.—Trend of wage rates in New York City.

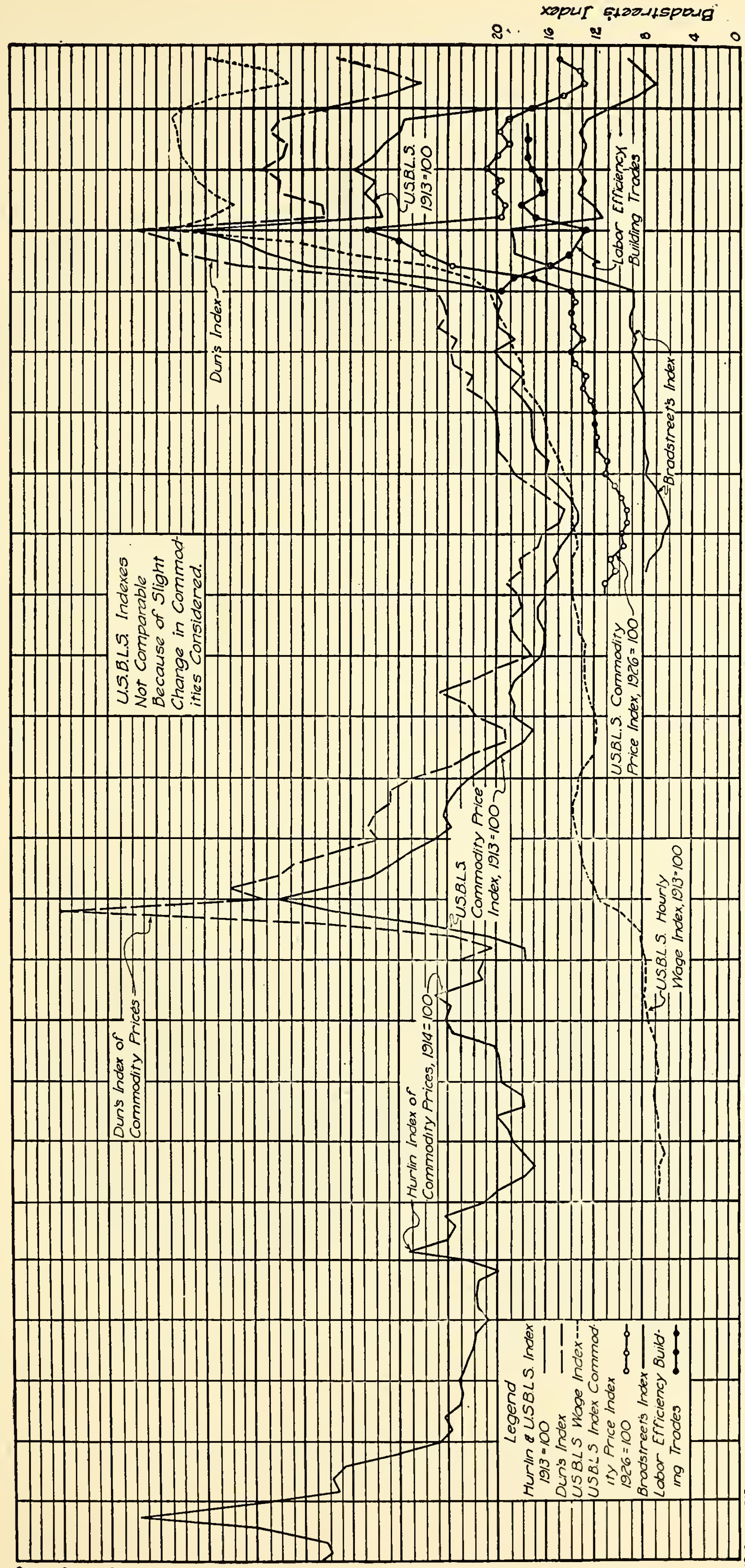


FIG. 9.8.—Graphical presentation of Dun's, Bradstreet's, Hurlin, and U.S.B.L.S. indexes.

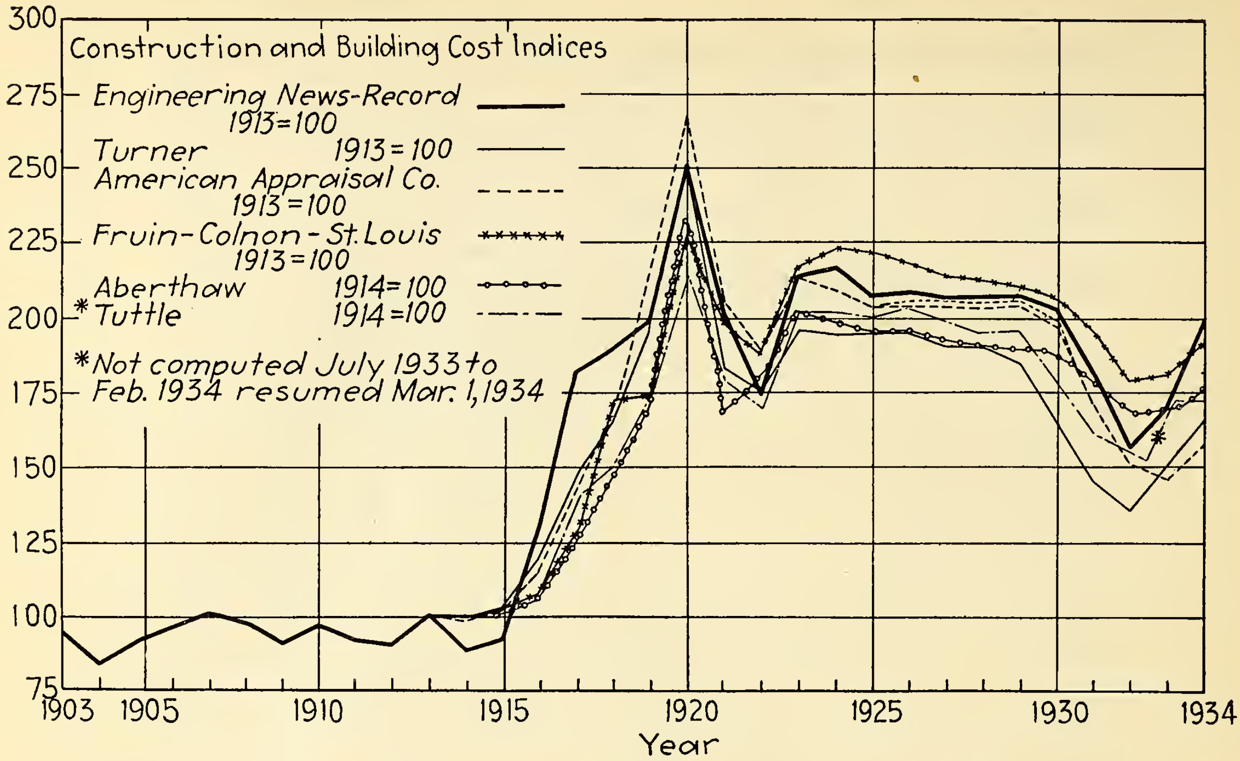


FIG. 9.9.—Construction- and building-cost indexes.

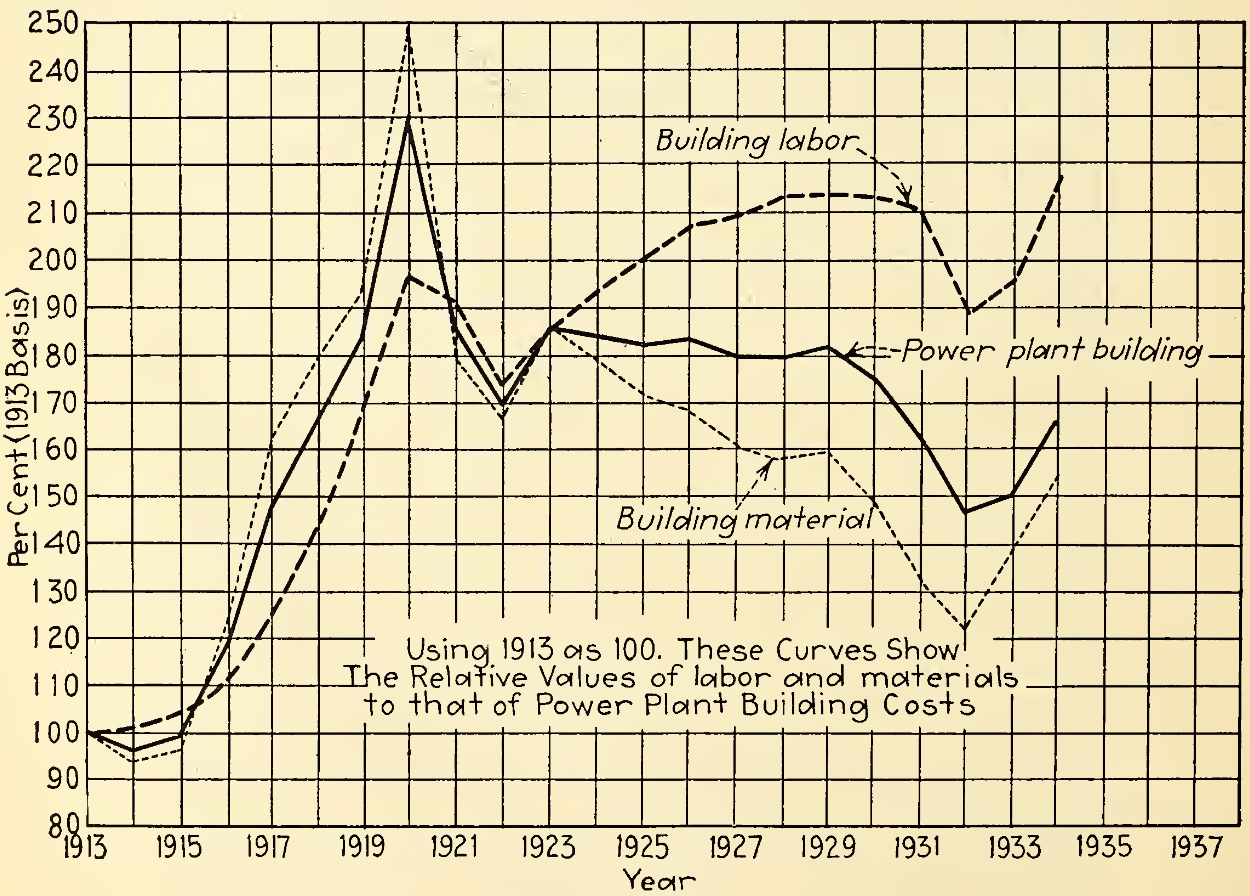


FIG. 9.10.—Power-plant building costs.

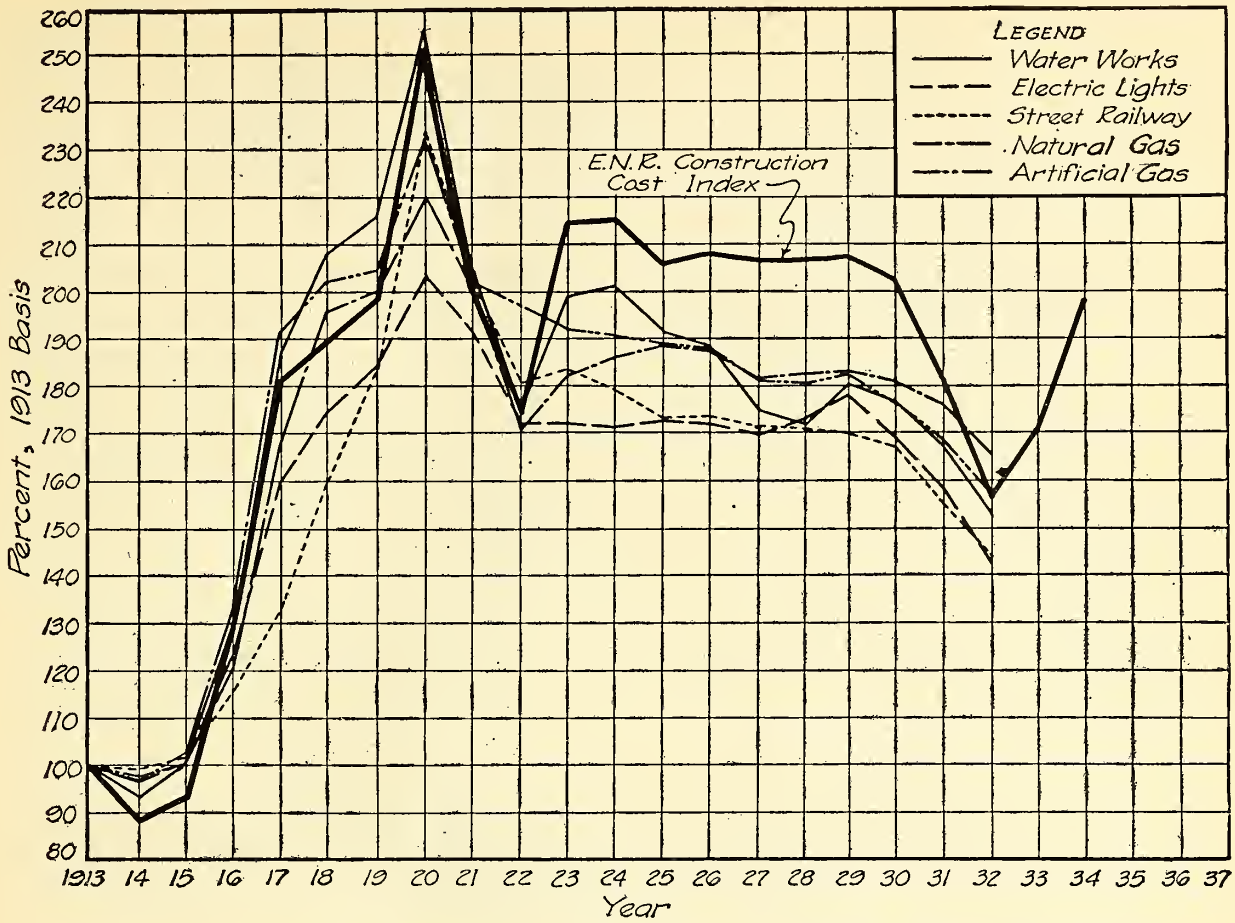


Fig. 9.11.—Relative yearly value of five types of utilities using 1913 as 100.

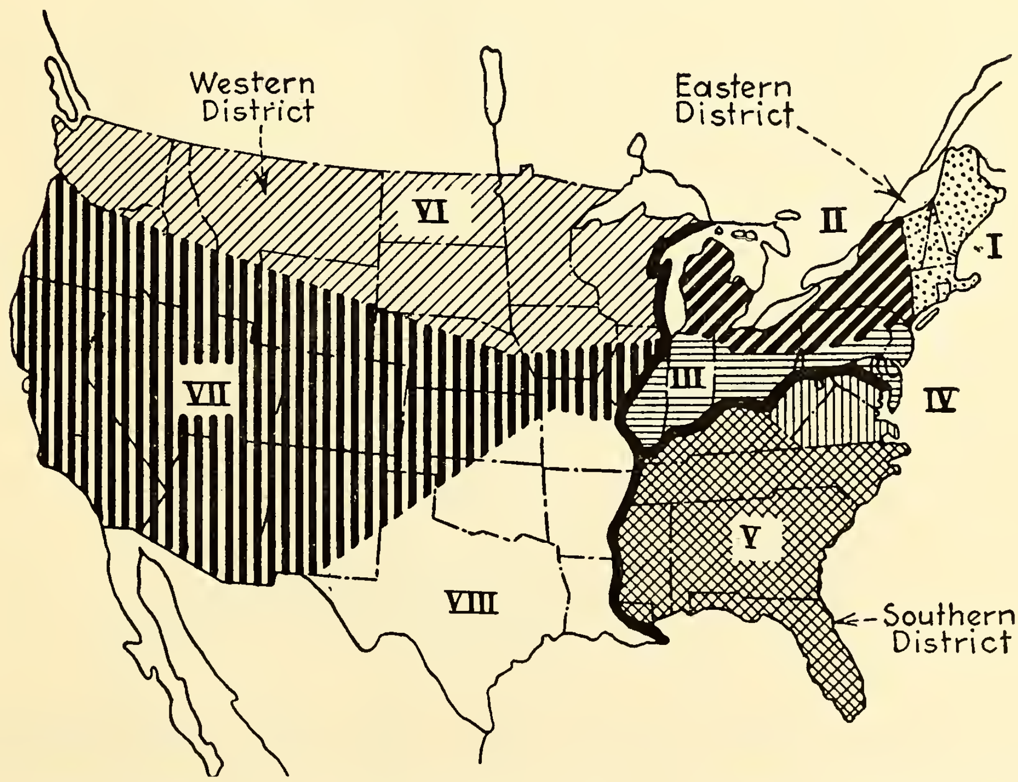


Fig. 9.12.—Regional groups for which railroad construction-cost indexes have been established by the Interstate Commerce Commission.

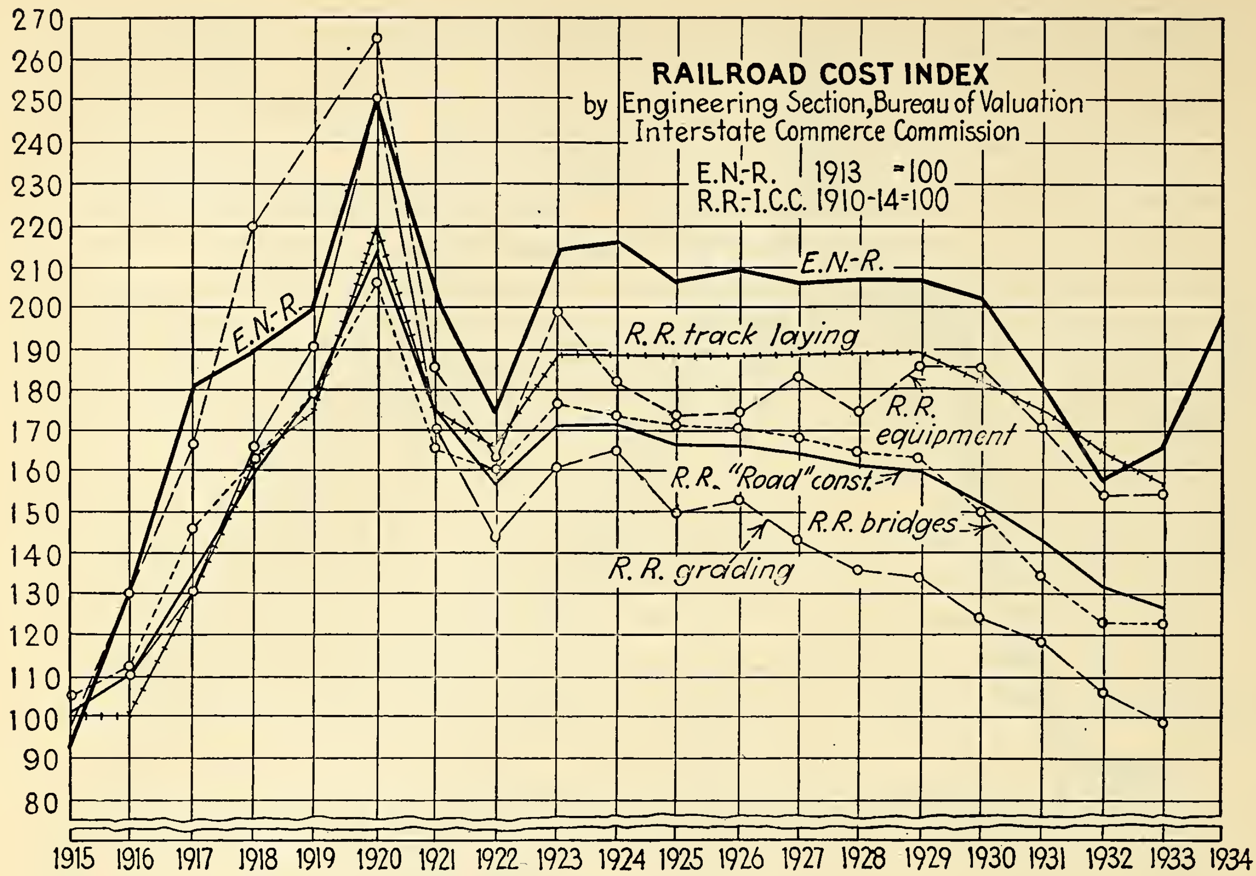


Fig. 9.13.—Railroad cost index.

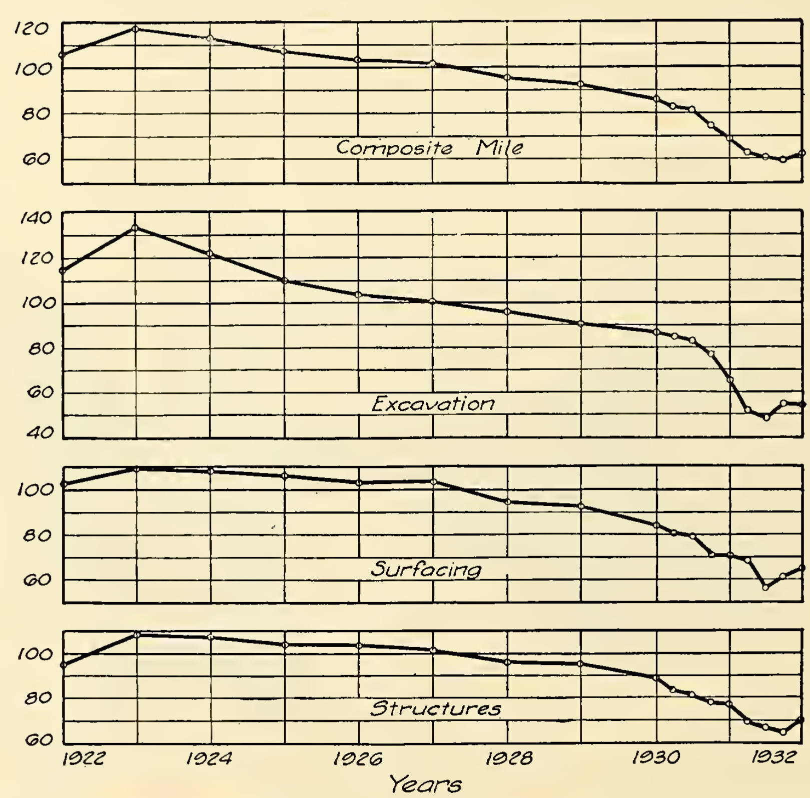


Fig. 9.14.—Trend of highway-construction cost. Base, 1925 to 1929 federal aid costs.

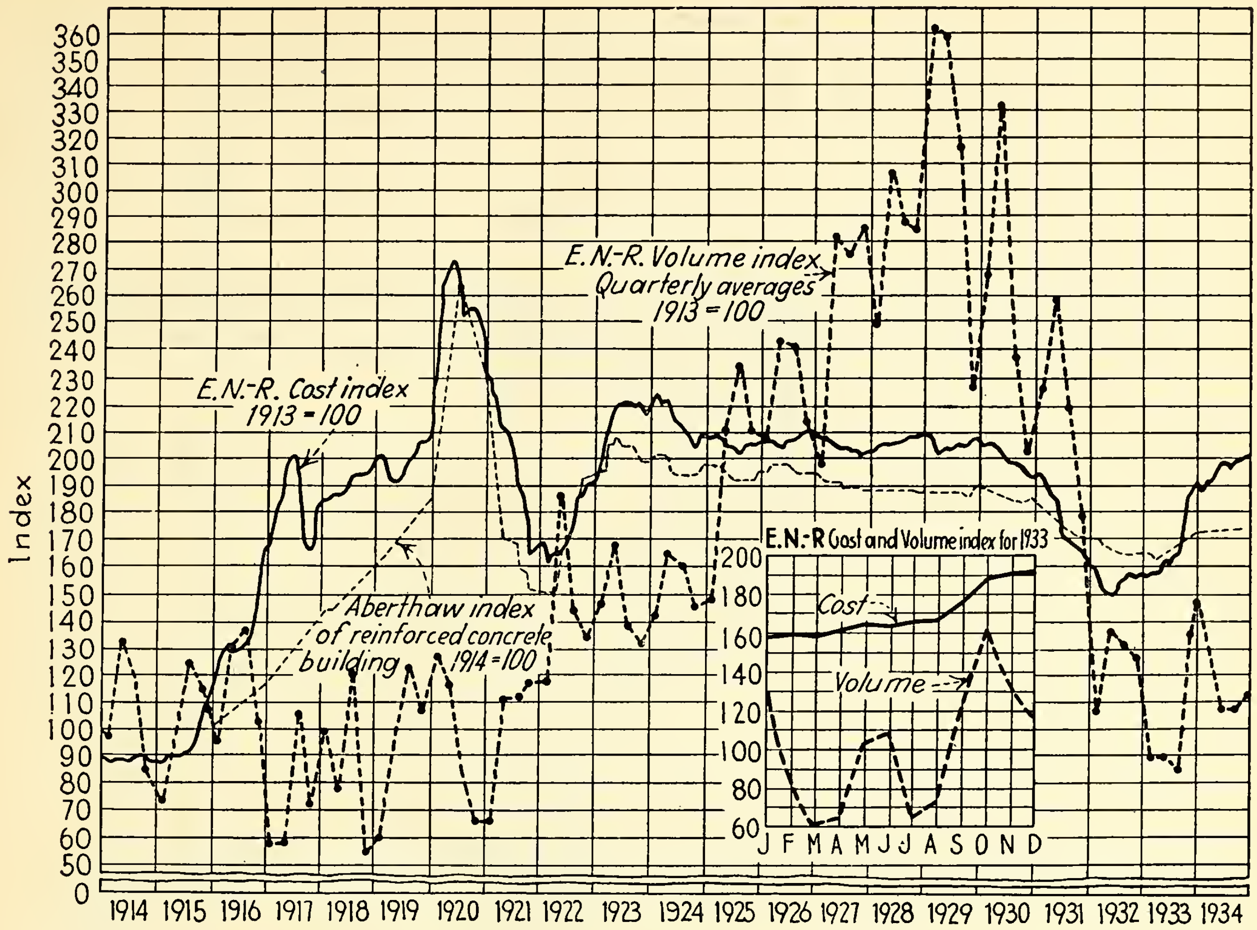


FIG. 9.15.—*Engineering News-Record* construction-cost and -volume indexes, 1914–1934.

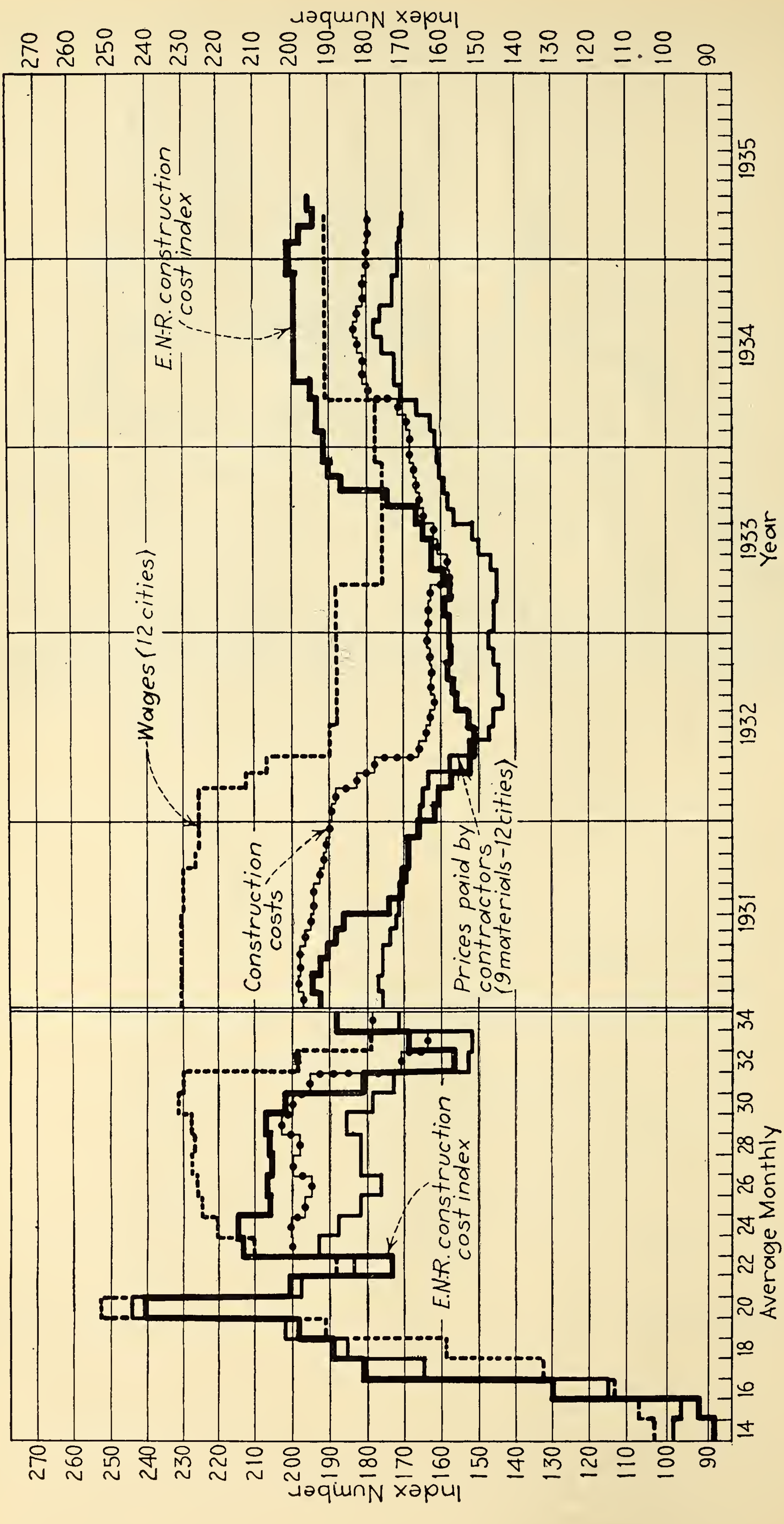


FIG. 9.16.—The constructograph, 1914–1935.

CHAPTER X

PRELIMINARY VALUATION EXAMINATIONS; INVENTORIES; STUDIES OF CORPORATE STRUCTURE, HISTORY, AND ACCOUNTS

The methods used in making engineering valuations vary in their details to suit the characteristics of different properties and the ideas of different practitioners; a general process has been gradually evolved to which most engineering valuations conform, in a general way, as already outlined in Sec. 1.15. The preliminary and preparatory valuation work stated in that outline is as follows:

1. Make a preliminary examination of the property, and plan the valuation.
2. Make a complete detailed inventory of the present existing items of the property by field work, aided by a thorough study of the book records, and carefully checked by final field examinations.
3. By the aid of the book records of the enterprise, study its inception, corporate structure, preliminary expenses, construction costs, annual incomes, annual operation costs, annual allowances for depreciation, annual disposition of net returns, annual replacements, annual improvements and enlargements, past reorganizations (if any), other historical data.

PRELIMINARY EXAMINATION OF THE PROPERTY; PLANNING THE VALUATION

The first step toward making an engineering valuation of a particular property must be to visit and study it, in order to plan its systematic valuation.

10.1. General Examinations of Properties Preliminary to Valuation.— Before the detailed work of the valuation is undertaken, the engineer who is to be responsible for the valuation must make a preliminary general examination and study of the whole property; this is to gain a correct idea of its general character, and sufficient familiarity with its features to enable him to plan his work and organize his staff properly. The character and the sources of the data available and required must be ascertained. The property records should be examined to ascertain their character, and their value in making a complete inventory or in checking such inventory if already made. Clients must be consulted and owners and managers of the property and all others concerned interviewed. Appraisal boards will often hold public hearings, at which all interested parties can appear and express their views. The general

history of the inception, the financing, the construction, and the operation of the property must be examined and studied.

10.2. Planning Valuations.—The valuation work should be planned systematically; consideration should be given to the magnitude and character of the property, to the completeness and reliability of its accounts and other book records, and to the purposes of the valuation. If complete valuations of subdivisions of the property are required, it must be divided into definite valuation sections.

10.3. Valuation Sections.—A valuation section of an industrial property is a definitely delimited geographical subdivision of which a separate, complete valuation is required.

In valuing industrial properties of ordinary size, the value of the entire property is the main objective, but it is often necessary to ascertain the separate values of those portions of the property located in different tax districts with sufficient accuracy for tax purposes. Absolutely precise tax-district valuations would require that the detailed property inventory should be distributed to the several districts, a costly operation; highly condensed tax-district inventories will usually suffice, and tax districts are not considered to be separate valuation sections.

Great industrial properties, such as those of railway systems built up by combining different lines and extending through several states, may often require complete separate valuations of extensive valuation sections of property.

THE PLANNING, PREPARATION, AND ANALYSIS OF THE VALUATION INVENTORY

A complete and correct inventory of the entire property is the first essential in making an engineering valuation. This inventory must be planned carefully and made systematically.

10.4. The Relation of Property Ledgers to Valuation Inventories.—There are many industrial properties which do not yet have and maintain complete, up-to-date property ledgers. Hence two classes of inventory work must be recognized.

1. *Valuation Inventories of Properties Maintaining Up-to-date Property Ledgers.*—For such a property, a tentative inventory should be prepared from the book records in advance. The valuation-inventory field work consists in checking the tentative inventory thoroughly by actual examination of each property unit; omissions must be supplied and other errors corrected; such additional data as are needed for valuation must be obtained.

2. *Valuation Inventories of Properties without Property Ledgers.*—For these properties, the data for valuation inventories must be obtained primarily by field examinations of the property, listing the inventory and other valuation data of each unit as found. The inventories prepared from the field data must be checked in the field after made.

If desired by the owners, such valuation inventories can be made the basis for establishing property ledgers.

10.5. Inventory Sections.—To prevent omissions and duplications, and to systematize the inventory work, the entire property to be valued should be divided into carefully planned inventory sections.

An *inventory section* is a definitely defined unit area of an industrial property, selected and delimited for convenience and accuracy in inventory work.

The size, shape, and other features of inventory sections vary greatly and should always be those best adapted to the particular properties to be inventoried. In the case of a factory, a single building may often constitute an inventory section; in the case of a great telegraph system, a single inventory section might readily be 100 miles long.

An inventory section should always be confined within the limits of a single valuation section (Sec. 10.3).

The division of the Ames electric utility property into inventory sections is shown in Fig. 21.1.

Inventory Field Work

10.6. Inventory Field Books and Forms.—Ordinary engineer's field books are often used to record rough notes of inventory field data; sometimes rough, sectional large-scale field maps can be used to advantage.

Loose-leaf notebooks, with leaves ruled specially for inventory work, have many advantages for recording inventory field data. The authors consider the 8½- by 11-inch size the best for most cases, and suggest the ruling shown in Fig. 10.1 for general inventory work.

Only the data of actual observations and measurements are to be recorded in the inventory field books and forms, leaving classification and grouping of items for the office work, along with all computations and pricing. The data provided for in Fig. 10.1, of the age, the physical condition for its age, and the past and future service conditions of each property unit are needed in forecasting its probable service life (Secs. 3.19 to 3.22). The condition data must be determined by actual observation by a trained and experienced engineer.

Although the simple inventory field data form shown in Fig. 10.1 (or some similar form) is well adapted to general industrial-property inventory work, such work can often be greatly facilitated by the use of additional, more complex, special forms for some of the different special classes of the property. As an example, the form used for obtaining the field data on the distribution of pole lines in the valuation of a certain small electric power and light property is shown in Fig. 10.2.

Such special forms have been devised and used in very great variety and number for the many different classes of property found in different great industrial properties.

Different valuators prefer different forms, and the same valuator may use different forms for different properties. Consulting valuation engineers, appraisal companies, and the valuation departments of utility commissions and of great properties usually have their own preferred forms.

The 1931 valuation of the Western Union Telegraph Co.'s property furnishes an illustration of the field inventory forms needed for a large property. Some 11 different forms, all loose-leaf sheets approximating 8½ by 11 inches, were used in the field inventory work of this valuation.

Form 364 was unruled, horizontally or vertically.

Form 361 was ruled horizontally, but not vertically.

Form ____ Valuation of ____ (Property) ____ Sheet ____
By (Valuation Agency) ____ (City) (State) (Date) of ____
Inventory Field Notes for Inventory Section ____

(1) Physical Condition -- E, Excellent, G, Good, A, Average, P, Poor, B, Bad -- FOR THE AGE
(2) Service Conditions (Past & Future) -- M, Mild, F, Favorable, A, Average, U, Unfavorable, S, Severe

Mfgr's No. and Plant No.	Quantities	Location	Description	Age	Cond. Phys. (1)	Cond. P. Ser. (2)	Cond. F. Ser. (2)	Remarks

Fig. 10.1.—Suggested inventory form for field notes. Loose leaf, 8½ by 11 in.

Form 363 had vertical rulings for (1) unit cost; (2) cost new; (3) cost new less depreciation; this last should usually be left for the office computations, for which it may be advisable to collect some field data when making the inventory.

Forms 355, 319, and 356 were for pole-line inventories; 319 served for summarizing the data on 355, for transfer to 356.

Form 357 was for aerial wire.

Form 358 was for aerial cable.

Form 359 was for underground conduit.

Form 360 was for underground cable.

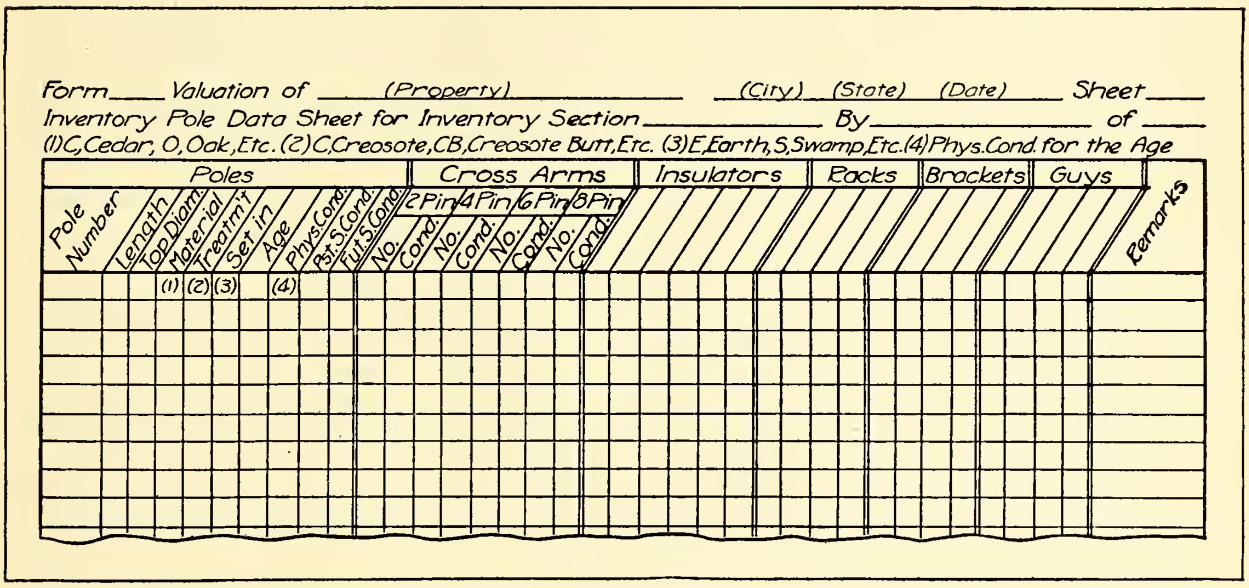
Form 362 was for switchboards and other station equipment.

10.7. Inventory Field Parties.—The number of men required for a single inventory field party varies; usually it will range from one to enough to run survey lines or do cross-section work.

In much of the work of making inventories of industrial properties, two men are sufficient, one of whom need often be only an assistant, employed all or a part of the time, mainly to help in making measurements. In the 1931 Western Union Telegraph Co.'s property valuation, the field parties varied in numbers but were often two men, an inspector and a note keeper; they rode along the railway line in a gas car, as

most of the work was observing and recording pole data. Large parties are required for such work as the valuation of the roadbeds and structures of railways; and of other classes of property requiring field surveys, including the cross-sectioning of excavations and embankments.

At least one man on each inventory field party should be especially trained and otherwise qualified for inventory work. Regular employees in the valuation departments of governmental regulatory commissions, or appraisal companies, or consulting engineering firms, or great industrial corporations are used for inventory work, if available. Comparatively recent engineering graduates are readily trained for inventory work.


Form____ Valuation of ____(Property)____ (City) (State) (Date) ____ Sheet____
Inventory Pole Data Sheet for Inventory Section____ By____ of____
(1)C,Cedar, O,Oak,Etc. (2)C,Creosote,CB,Creosote Butt,Etc. (3)E,Earth,S,Swamp,Etc. (4)Phys.Cond. for the Age

Poles										Cross Arms								Insulators	Racks	Brackets	Guys	Remarks
Pole Number	Length	Top Diam.	Material	Treatm't	Set in	Age	Phys.Cond.	Pst.S.Cond.	Fut.S.Cond.	2 Pin No.	Cond.	4 Pin No.	Cond.	6 Pin No.	Cond.	8 Pin No.	Cond.					
			(1)	(2)	(3)		(4)															

Fig. 10.2.—Pole data sheet. An example of special forms for inventory field notes. Loose leaf, 8½ by 11 in.

Other employees of industrial plants make good helpers; so also do engineering students and sometimes even high-school students.

10.8. Inventory Field Work.—In the field work of making an inventory, the attention and effort should be centered upon the *main objective;* this is *a complete, correct list of all the units of the property.* Every care within reason should be taken to insure that *every* property unit is listed *once and once only.*

Distraction of attention and effort from the main objective should be avoided by leaving, for the later office work, all the operations of distributing the different units between different property groups and subgroups, of preparing classified group summaries for convenience in making valuation computations and of finally pricing the units for valuation.

The boundaries of inventory sections (Sec. 10.5) should be so definitely delimited as to leave no excuse[1] for omissions or duplications by the

[1] It is easy to make absurd errors in inventory work. For use in a highly controverted waterworks valuation case, one of the authors was once furnished an inventory agreed to by both the city and the waterworks company. The standpipe, the most prominent feature of the property, was omitted.

inventory parties working in adjacent sections. For example, at the boundary between the power-plant section and a distribution section of an electric power and light property, definite instructions should be given as to the exact place (say the outside surface of the power-plant building, or the first point of attachment outside the building) where the distribution-system section begins.

Prior to starting his work in any particular place or area, the leader of the inventory party should carefully study all the available pertinent maps and other engineering drawings, property ledgers, and other book records. This is especially necessary when part of the items may be concealed from view, as in the case of water mains and valves; or conduits in buildings, between the ceilings and the floors of the next higher stories.

Field inventory work should be done on a systematic plan. In a single room of a building, the individual items had best be inventoried as actually found located; in, say, clockwise order. The rooms on a single floor and the different stories of the building should be inventoried in systematic order.

The inventory field work should be both *thorough* and *complete* as it progresses, leaving no uninventoried units behind the party.

Finally, the inventory should be *checked* in the field after it is first completed, either by a competent engineer not on the original field party, or by the leader of the field party some time after the original work in each area is finished.

Inventory Office Work

Like the inventory field work, the office work of making an inventory of an industrial property should be carefully planned and systematized.

10.9. Inventory Work and the Book Records.—The nature and importance of the many valuable valuation data obtainable from the book records of industrial properties are discussed in detail in Secs. 10.12, 11.8 to 11.15, 14.2, 14.3, 14.6, 15.1, 15.3, and 15.7. It need only be said here that the book records of the property should be fully utilized in securing, checking, classifying, and summarizing the inventory data of the property for the final valuation.

Inventory office work with the property book records will usually begin before the inventory field work starts; and will continue until the valuation-inventory data have been grouped in valuation groups, and in each group have been arranged in classified summaries, ready for pricing the units and computing their original costs, reproduction costs, and fair values.

10.10. Valuation Groups and Subgroups.—Haphazard methods have no proper place in valuation work. Prior to attempting to value them, the different inventory property units, found in the field work, must

be distributed among different valuation groups; the entire property should be divided into such groups to facilitate and systematize the valuation work.

A valuation group of industrial-property units is a subdivision of the property comprising all units of a particular general character or use. Examples: Land; structures; boiler-plant equipment; distribution mains; hydrants; general equipment; electric plant; overhead conductors; poles, towers, and fixtures; line transformers and devices; bridges, trestles, and culverts; grading; ties; water stations.

A valuation subgroup is a subdivision of a valuation group, restricted to property units so similar that grouping them together on the same computation form facilitates the work of pricing the units and computing their values. Examples of subgroups: Of distribution mains—cast-iron mains; of poles, towers, and fixtures—wooden poles of different woods, lengths, diameters, and settings; of bridges, trestles, and culverts—culverts of a given material, of different sizes and lengths.

Valuation Groups Should Correspond to the Fixed-capital Primary Accounts.—The best basis for the division of an industrial property into valuation groups is the system of fixed-capital primary accounts actually used for the property.

1. For utility properties, the uniform systems of accounts prescribed by the Interstate Commerce Commission[1] and by state railroad or utility commissions, or those recommended by the National Association of Railways and Utilities Commissioners,[2] are in general use (Secs. 2.10, 2.11).

From 10 to 61 primary fixed-capital accounts are prescribed in the above uniform classification systems for different types of property.

2. In the cases of industrial properties which are not utilities, and in the cases of utilities not using the above uniform systems of accounts, the basis for the subdivision of the property into valuation groups should be the actual fixed-capital primary accounts used for the property, as shown by the books.

3. In the cases of industrial properties not using any acceptable system of fixed-capital primary accounts, the best basis for the subdivision of the property into valuation groups would be such system of accounts as the valuator, after due consideration, would judge to be best for the particular property.

NOTE.—It was formerly the practice of valuators to divide industrial properties, which they valued, into valuation groups without much regard to the actual system of accounts used by the enterprise. Example: The valuation groups used in a certain waterworks valuation in 1912 by an appraisal board on which one of the authors served were as follows:

1. Preliminary costs.
2. Real estate and water rights.
3. Distribution system.
4. Pumping station.
5. Purification plant.

[1] Government Printing Office, Washington, D.C.

[2] State Law Reporting Company, New York, N.Y.

6. The water supply.
7. Stock, tools, and other property.
8. Meters.
9. Engineering.
10. Administration and legal costs.
11. Contingencies during construction.
12. Interest on investment lost during construction.
13. Going-concern value.
14. Working capital.

10.11. Valuation-group Inventory-summary Forms.—The ultimate object of the inventory office work is the preparation of complete and correct summaries of the entire inventory, by valuation groups and subgroups of the property, in the forms most convenient for the final

Form____ Valuation of ____(Property)____ _(City)_ (State) (Date) Sheet____
Acct ____ (Description of Property Group) By(Valuation Agency) of____

Classified Summary for Valuation Computations

(1) M.T.C. = Mortality Type Curve ($L_0, L_1, L_2, L_3, L_4, L_5, S_0, S_1, S_2, S_3, S_4, S_5, S_6, R_1, R_2, R_3, R_4, R_5$)
(2) E.A.L. = Estimated Average Life of Similar Units. (3) P.S.L. = Probable Service Life of This Unit

Quantities	Items	M.T.C. / E.A.L.	Age / P.S.L.	Condition % / PFORR	Salv. Val. %	Unit Prices			Values New			Present Values		
						Orig. Cost	Repr. Cost	Fair Value	Orig. Cost	Repr. Cost	Fair Value	Orig. Cost	Repr. Cost	Fair Value
		(1) (2)	(3)											

FIG. 10.3.—Suggested inventory form for classified summaries for property group valuation computations. Loose leaf, 8½ by 11 in.

pricing of the several property units and for the systematic computation of their values new and present values.

Figure 10.3 is presented herewith as a general form, suggested by the authors, for valuation-group and subgroup inventory summaries.

The data to be inserted in the third column of Fig. 10.3 are the "mortality type curve" (MTC) of each property item and the "estimated average life" (EAL) of similar items. From these the "probable service life" (PSL) of the item can be estimated (Secs. 3.20 to 3.22) for insertion in the fourth column, together with the age of the item.

From the age of the item and its probable service life, its "condition-percent" for the fifth column can readily be taken from the condition-percent tables in Appendix B, using the "actual present-worth," the "straight-line," or the "sinking-fund" method, whichever is adopted.

NOTE.—When considered advisable, the condition-percents obtained as above can be corrected to allow for decreases in service efficiency and capacity by multiplying by the estimated PFORR as described in Sec. 5.17.

The salvage value percents to insert in the sixth column of Fig. 10.3 are those estimated from past experience with similar property items, or by analysis of the probable net salvages at the dates of retirement.

For the columns provided in Fig. 10.3 for original-cost and reproduction-cost, unit prices must be selected by the valuator himself (not by the inventory office force), after a careful study of all the pertinent data thereon obtainable from the book records of the particular property, as well as of data of the costs of similar items elsewhere and of general price trends (Sec. 10.12; also Chaps. IX, XI).

The unit prices must include the proper allowances for overhead as well as for direct prices. The methods of determining the correct direct-cost prices and the proper overhead-cost allowances are discussed at length in Chap. XI.

The fair-value unit prices must be determined by giving due weights to both original costs and reproduction costs; the date the original costs were incurred, and the probable trend of prices during the next few years, must be given consideration in deciding what relative weights are due the original costs and reproduction costs. This must be done separately for the different property units; the object in each case must be the probable prices which will prevail during the next three to five future years.

While forms like Fig. 10.3 give the most complete data for valuation, somewhat simpler forms are more commonly used. In the 1931 valuation of the Western Union Telegraph Co.'s property, the forms corresponding to Fig. 10.3 were as follows:

Form 561. No horizontal rulings. Vertical rulings for eight columns, as follows: (1) character of property and description; (2) condition-percent; (3) percent of cost new; (4) unit; (5) number of units; (6), (7), (8), reproduction cost (new, total, and less depreciation).

Form 562. Practically the same as 561, with horizontal rulings added.

Form 563. No horizontal or vertical rulings.

These forms make no provisions for showing the original costs of the units, or their fair values.

Collection Forms.—Completed forms like Fig. 10.3, or like the simpler forms mentioned just above, are the ultimate objective of valuation office work; in reaching this objective it is usually found advisable, in the valuation of a large property, to systematize the work of collecting the data from the inventory field sheets for transference to the group inventory-summary sheets by the use of intermediate "collection sheets." In the 1931 valuation of the Western Union Telegraph Co.'s property, the following collection sheets were used:

Form 314. General collection sheet.
Form 315. Collection sheet for ____ feet poles.
Form 316. Collection sheet for crossarms.
Form 317. Collection sheet for pole fittings.
Form 318. Collection sheet for aerial wires.

STUDIES OF THE HISTORY, CORPORATE STRUCTURE, ACCOUNTS, AND OTHER CIRCUMSTANCES OF THE ENTERPRISE

The history and the circumstances of each particular property must be ascertained and given due consideration in its valuation. The book records of the enterprise should supply most of the data required.

10.12. Studies of the History and Circumstances of the Enterprise.—The pertinent facts to be considered in these studies are to be ascertained from all available sources, especially the book records. The original inception, organization, and promotion of the enterprise must be studied. The financial history includes all sales of stocks and bonds, the annual incomes and expenditures, the net returns after deducting annual depreciations, the disposition of each year's net returns and depreciation allowances. Besides the original organization, any reorganizations of the enterprise must be studied.

10.13. Valuation Studies of the Book Accounts of the Enterprise.—The book accounts should supply correct data of

The actual organization and promotion expenditures (Secs. 12.1 to 12.8).

The actual costs of lands (Sec. 11.5).

The direct original costs of existing property units (Secs. 6.13, 6.15, 6.28, 11.10).

The actual original-cost overheads of existing property units (Secs. 6.13, 6.15, 6.28, 11.11 to 11.15).

Data for estimating the direct reproduction costs of existing property units (Secs. 6.13, 6.15, 6.28, 11.18, 11.19).

Data for estimating the reproduction-cost overheads of existing property units (Secs. 11.20 to 11.25).

Data necessary in determining the going value of the property (Secs. 12.13 to 12.18).

Data necessary in determining good-will value and other-intangibles' values (Chap. XIII).

Data necessary in determining working capital, and other funds wisely tied up in the enterprise (Chap. XIV).

Data necessary in determining earning value, service-worth value, and stock-and-bond value (Chap. XV).

CHAPTER XI

THE ORIGINAL-COST VALUE, REPRODUCTION-COST VALUE, AND FAIR COST-VALUE OF FIXED-CAPITAL PHYSICAL PROPERTY

The determination of the cost-values of a property is always a major part of its engineering valuation; it should be started as soon as the preliminary and preparatory work described in Chap. X has progressed sufficiently.

The process of determining the cost-values is outlined in paragraphs 4 to 7, inclusive, of Sec. 1.15, as follows:

4. Determine the present fair values of all lands inventoried.
5. Determine the present original-cost values of all inventoried fixed-capital physical-property items except lands.
6. Determine the present reproduction-cost values corresponding to 5, above, using the same inventory.
7. Determine the present fair cost-values of all inventoried fixed-capital physical-property items, giving such weights to 5 and 6, above, as are just and right in this case for each respective unit, then add 4, above.

COST-VALUE DEFINITIONS AND INVENTORIES

As treated herein, cost-values, and inventories therefor, apply only to the present existing units of the properties valued.

11.1. Definition and Explanation of Original-cost Value.—The original-cost value (Secs. 1.14, 11.6, 11.7, 11.27) of each existing property unit is its value as estimated entirely upon the basis of its actual original cost.

1. The *original-cost value new* is equal to the actual original cost of the property unit installed new, ready for service; and was both its true value at that time and the total original investment then made in it.

2. The *present original-cost value* is equal to the original cost new of the unit less its total accrued (original-cost) actual depreciation at the date of valuation; and shows the amount of the original investment which still remains in the unit when valued.

NOTE.—The owner of the property operates it subject to a binding obligation to set aside each year, out of income before net return, a sum sufficient to repay him a portion of his investment in each property unit equal to its actual depreciation during the year.

3. Owing (*a*) to the continual retirements and replacements of existing property units, and usually (*b*) to repeated additions of new units on account of growth, the present original-cost value of the entire property constantly tends to approach closer

to its present reproduction-cost value as time elapses; except during periods of rapidly rising or falling prices.

4. The present (depreciated) original-cost value of the entire property shows the price level of existing competitive investments in the particular enterprise.

11.2. Definition and Explanation of Reproduction-cost Value.—The reproduction-cost value of each existing property unit is its value as estimated entirely upon the basis of its reproduction cost.

1. The *reproduction-cost value new* is equal to the now estimated reproduction cost new, ready for service; and shows what both its true value and the total investment in it would have been, when it was new, if it had been installed at the reproduction prices now estimated.

2. The *present reproduction-cost value* is equal to the reproduction cost new of the unit less its total accrued (reproduction-cost) actual depreciation at the date of valuation; and shows what the total remaining investment in the unit would still be, if it had been installed, when new, at the reproduction prices now estimated.

3. The present (depreciated) reproduction-cost value of the entire property shows the price level now forecasted, upon the basis of reproduction cost alone, for additional competitive investments in the immediate future.

11.3. Definition and Explanation of Fair Cost-value.—The fair cost-value of each existing property unit is its fair value as estimated entirely upon the two bases of actual original cost and estimated reproduction cost; giving to each "such weight as may be just and right in each case."

1. The *present fair cost-value new* of the property unit is its fair value new, as *now* estimated entirely upon the basis of its costs; giving due weights to both original cost and reproduction cost.

2. The *present depreciated fair cost-value* of the property unit is equal to its present fair cost-value new less its total accrued (fair cost-value) actual depreciation at its age at the date of valuation; and shows what the remaining investment in it would now be if its total accrued actual depreciations and depreciation reserve charges had all been determined upon depreciation bases equal to its fair cost-values new from year to year; as required by the decision of the United States Supreme Court in the *Baltimore Street Railway Case* (Sec. 8.6).

3. The present (depreciated) fair cost-value of the entire property shows the price level now forecasted for future additional competitive investments, upon the basis of a fair balancing of both actual original costs and estimated reproduction costs.

11.4. Inventory for Original- and Reproduction-cost Value and Fair Cost-value.—The same inventory serves for determining the original-cost value, the reproduction-cost value, and the fair cost-value, of a property. The inventory must contain only present existing property units; it must be prepared very carefully, with every reasonable effort to avoid omissions of any kind. For a detailed description of acceptable methods see Chap. X.

THE VALUES OF THE LANDS OWNED BY INDUSTRIAL ENTERPRISES

Since lands are in a class by themselves as to depreciation and appreciation, it is customary in engineering valuation to determine the fair values of lands separately from those of other fixed-capital physical property. For a general discussion of land valuation see Chap. XVI. Only a brief résumé will be given here.

11.5. The Valuation of Lands Owned by Industrial Enterprises.—Present market value is usually entitled to greater weight than any other consideration in determining the present fair value of land; this is because it usually reflects the effects of all the various factors affecting value (all of which must be given due weight); and because, when not subject to excessive temporary fluctuations due to public hysteria in times of booms and depressions, market value comes near to the legal concept of fair value. Moreover, owing to the comparative frequency of bona fide sales of land, its present market value can usually be determined with greater facility and reliability than any other indicator of value.

Two cases of land values must be distinguished in engineering valuation:

1. The value of *lands owned by enterprises of a private character.* In general, the value of land, as of other property, includes many elements,[1] and "for each separate [possible] use of one's property by others the owner is entitled to a reasonable compensation. . . . "[1] However, the combined effect of these separate elements of value is best represented in most cases by the present market value of similar near-by lands.

2. The value of *lands owned by public utilities* is by law the present market value of similar near-by lands. This principle was established by the United States Supreme Court in 1913, in the *Minnesota Rate Cases* decision (Sec. 8.4); prior to that decision, it was common to offer expert-opinion evidence in valuation cases as to estimated reproduction-cost land values far in excess of present market value; by reason of conjectural "railway values," "severance damages," "plottage values," etc., all of which are now rejected.

So far as practicable, the present market values of lands should be determined by the actual prices found to have been paid in recent bona fide sales of similar near-by lands. Frequently the opinions of competent disinterested real-estate dealers are given material weight; sometimes, by agreement, the lands are appraised by a board of such realtors.

In times of depressions, or real-estate booms, the temporary public hysteria should be discounted by sound judgment as to conservative future prices.

The actual costs incurred in securing titles to the lands owned should always be ascertained, so far as practicable, and reported; especially where the purchases are recent.

Lands not owned and used for agricultural production are not usually subject to physical depreciation; in the past, they more commonly have appreciated in value.

[1] See *Monongahela Navigation Co. v. United States* (Sec. 8.3).

At the present time (1935), widespread recent decreases in land values quite generally have reversed the former trend.

THE ORIGINAL-COST VALUES OF FIXED-CAPITAL PHYSICAL PROPERTY EXCEPT LAND

Various names for the same meaning, and various meanings for the same term, have appeared in the past in engineering-valuation nomenclature for the value factor herein designated "original cost."

11.6. What Is Meant by Original Cost New.—The original cost new of the property owned by an industrial enterprise is herein defined as the sum of the total actual original costs, incurred at their respective dates of installation, of its present existing property units; completed new, ready for service.

Thus the original costs new have been incurred not at one but at many different dates; they include both the actual direct costs and the actual overhead costs of the several units.

Note.—The terms "actual cost," "historical cost," and "historical reproduction cost" have all been used more or less frequently to mean original cost as defined above; the term "original cost" has sometimes been used to mean either the cost of the original units of the original property, or the cumulative book costs of the original and later property units, or, more frequently, the cost to the present owner.

In the *Los Angeles Gas Case*, the United States Supreme Court defined actual cost to be "the investment the owners have made" (Sec. 8.7). Apparently, this means the present owners.

11.7. Meaning of Present Original-cost Value.—As explained in Sec. 1.17 and 11.1, the total accrued actual depreciations to date must be deducted from the original costs new of all property units to get their present original-cost values at the date of valuation.

The present original-cost values thus obtained are the present actual investments in the several property units; they will be the present prudent investments if the original investments were prudent.

Note.—In early valuation practice, depreciation was not deducted in getting the original-cost value; this was an anomaly in logic the absurdity of which is now apparent.

11.8. The Process of Determining Present Original-cost Value.—The present original-cost values of the several inventory property units can be determined in the following manner:

1. From the book records of the property, supplying omissions by the use of data of past prices and price trends of similar units, ascertain (or carefully estimate if necessary) the total actual original cost new of each inventory property unit, installed ready for service; including (*a*) its direct construction costs, and (*b*) its fair share of all actual overhead construction costs.

2. By the methods explained in Chap. V, determine and deduct the total accrued actual depreciation of each property unit from its total original cost new to get its present original-cost value.

The original-cost depreciation base for each unit must be its total original cost new, ready for service, including overheads. The condition-percent for each unit will be the same for original-cost value and for reproduction-cost value.

11.9. Property Accounting Systems Showing Original Costs.—For use in ascertaining the original costs of property units, the book records of industrial properties may be classified as follows:

1. Book records which include complete up-to-date property-ledger accounts (Secs. 6.12 to 6.20) of all property units. Such property-ledger accounts constitute continuous inventories, from which the correct direct and overhead original costs of all units can readily be ascertained. Such complete records are comparatively rare as yet; this is an unfortunate situation which is constantly improving.

2. Book records in which the accounts of the original costs of property units are more or less imperfect. With imperfect records, extensive and careful work is often required to supply omissions with sufficient accuracy.

11.10. Direct Original-cost Construction Prices.—The direct[1] original cost new of each property unit is the sum of all the actual original costs directly chargeable to it when installed new, ready for service.

To the utmost extent feasible, all direct original costs must be ascertained by a

TABLE 11.1.—AMES ELECTRIC UTILITY VALUATION. LABOR RATES PER HOUR[1]

Classes of labor	1932	1931	1930	1929	1928	1927	1926	1925	1924	1923	1922	1921	1920	1919	1918	1917
Line foremen	$0.725	$0.725	$0.725	$0.70	$0.70	$0.70	$0.65	$0.65	$0.65	$0.65	$0.65	$0.65	$0.50	$0.50	$0.50	$0.50
Linemen	0.675	0.675	0.675	0.65	0.65	0.65	0.60	0.60	0.60	0.60	0.60	0.60	0.45	0.45	0.45	0.45
Groundmen	0.40	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.40	0.40	0.40	0.40
Line truck driver	0.65	0.65	0.65	0.60	0.60	0.60	0.55	0.55	0.55	0.55	0.55	0.55				
Teamster	0.40	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.40	0.40	0.40	0.40
Day labor	0.40	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.40	0.40	0.40	0.40
Meter tester	0.70	0.70	0.70	0.675	0.675	0.675	0.625	0.625	0.625	0.625	0.625	0.625	0.475	0.475	0.475	0.475
Meter installer	0.675	0.675	0.675	0.65	0.65	0.65	0.60	0.60	0.60	0.60	0.60	0.60	0.45	0.45	0.45	0.45

[1] From vol. IV, Ames electric utility valuation.

[1] When property units are constructed by contract, the direct original costs are the "contract costs" paid the contractor.

Table 11.2.—Ames Electric Utility Valuation.[1] Direct Original Costs of 30-ft. Western Red Cedar Poles, 6-in. Top

Items of unit cost	Cost per pole on dates of installation															
	1932	1931	1930	1929	1928	1927	1926	1925	1924	1923	1922	1921	1920	1919	1918	1917
Costs of Poles, f.o.b. Ames:																
No treatment	$ 4.00	......	......	......	......	......	......	......	......	......	$ 6.87	$ 7.00	$ 9.02	$ 6.87	$5.92	$5.25
Brush treatment	4.82	......	......	......	......	......	......	......	$ 7.35	$ 7.50	7.55	8.00	8.95	6.87		
½-in. penetration treatment	6.50	$ 6.75	$ 6.75	$ 7.55	$ 7.35	$ 7.45	$ 7.15	$ 6.55	7.35	7.50						
Costs of Handling to Storage:																
By 3 men, 1 team, wagon	......	......	......	......	......	......	......	0.333	0.333	0.333	0.333	0.333	0.275	0.275	0.275	0.275
By 3 men, 1 tractor, wagon	......	......	......	0.336	0.336	0.336	0.314									
By 4 men, 1 truck, wagon	0.235	0.235	0.235													
Costs of Hauling to Location:																
By 3 men, 1 team, wagon	......	......	......	......	......	......	......	0.92	0.92	0.92	0.92	0.92	0.75	0.75	0.75	0.75
By 3 men, 1 tractor, wagon	......	......	......	0.78	0.78	0.78	0.74									
By line crew, truck	0.59	0.59	0.59													
Costs of Erection:																
Digging 90 in.	1.01	1.01	1.01	0.95	0.95	0.95	0.875	0.875	0.875	0.875	0.875	0.875	0.675	0.675	0.675	0.675
Setting 30 in.	......	......	......	......	......	......	......	......	0.875	0.875	0.875	0.875	0.675	0.675	0.675	0.675
Setting 50 in.	0.667	0.667	0.667	0.95	0.95	0.95	0.875	0.875								
Foreman	0.603	0.603	0.603	0.70	0.70	0.70	0.65	0.65	0.65	0.65	0.65	0.65	0.50	0.50	0.50	0.50
Truck or wagon	0.667	0.667	0.667	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.30	0.30	0.30	0.30
Total Costs of Handling and Erection	3.77	3.77	3.77	4.22	4.22	4.22	3.95	4.15	4.15	4.15	4.15	4.15	3.18	3.18	3.18	3.18
Total Costs, Installed:																
No treatment	7.77	......	......	......	......	......	......	......	......	......	11.02	11.15	12.20	10.05	9.10	8.43
Brush treatment	8.59	......	......	......	......	......	......	......	11.50	11.65	11.70	12.15	12.13	10.05		
½-in. penetration treatment	10.25	10.52	10.52	11.77	11.57	11.67	11.10	10.70	11.50	11.65						

[1] From vol. IV, Ames electric utility valuation.

TABLE 11.3.—AMES ELECTRIC UTILITY VALUATION. DIRECT ORIGINAL COSTS AND REPRODUCTION COSTS OF TRANSFORMER PLATFORMS[1]

Location	Materials and labor for location No. 4	Cost	Date built	Original cost: Materials	Original cost: Labor	Original cost: Total	1932 cost: Materials	1932 cost: Labor	1932 cost: Total
No. 4	4—3″ × 12″ × 4′	$ 2.88							
	6—3″ × 12″ × 16′	17.30							
	23—4″ × 6″ × 4′	11.00							
	2—4″ × 6″ × 16′	3.84							
	1—2″ × 4″ × 56′	1.90							
	4 18-in. through bolts	0.45							
	2 18-in. D.A. bolts	0.35							
	4 12-in. through bolts	0.39							
	24 D. washers	0.29							
	32 lb. nails	1.60							
	2 gal. red paint	2.70							
	2 1-in. clamps and bolts	0.08							
	8 1½-in. clamps and bolts	0.47							
	2 2-in. clamps and bolts	0.09							
	18 ft. of 1-in. conduit	3.00							
	70 ft. of 1½-in. conduit	17.20							
	36 ft. of 2-in. conduit	11.90							
	1 1-in. entire fitting	0.25							
	4 1½-in. entire fitting	2.40							
	2 2-in. entire fitting	2.80							
	1 1-in. fitting and cover (A)	0.78							
	4 1½-in. fitting and cover (A)	4.60							
	2 2-in. fitting and cover (A)	4.16							
No. 4	Labor, 9½ gang-hours	27.50	1923	$90.43	$27.50	$117.93	$79.60	$33.40	$113.00
No. 10	....	......	1923	83.56	27.50	111.06	73.70	33.40	107.10
14	....	......	1923	71.90	27.50	99.40	63.30	33.40	96.70
48	....	......	1923	79.67	27.50	107.17	70.30	33.40	103.70
233	....	......	1923	31.23	21.80	53.03	29.80	26.40	56.20
5	....	......	1926	97.41	27.50	124.91	86.00	33.40	119.40
40	....	......	1926	50.42	23.20	73.42	46.00	28.20	74.20
225	....	......	1926	35.80	23.20	59.00	32.70	28.20	60.90
49	....	......	1927	77.32	29.90	107.12	68.00	33.40	101.40
139	....	......	1928	30.42	25.20	55.62	32.00	28.20	60.20
207	....	......	1928	34.93	25.20	60.13	37.00	28.20	65.20
230	....	......	1928	36.95	25.20	62.15	39.00	28.20	67.20
501	....	......	1928	38.46	29.50	67.96	40.00	30.00	70.00
514	....	......	1928	42.77	29.50	72.27	42.50	30.00	72.50
88	....	......	1930	56.60	28.15	84.75	54.60	28.15	82.75
250	....	......	1930	52.41	28.15	80.56	51.00	28.15	79.15
281	....	......	1930	31.79	28.15	59.94	30.70	28.15	58.85
307	....	......	1930	52.41	28.15	80.56	50.00	28.20	78.20
31	....	......	1932	32.38	28.15	60.53	32.38	28.15	60.53

[1] From vol. IV, Ames electric utility valuation.

thorough study of the book records of the property. This work is simple when the book records include (as they always should) complete property-ledger accounts; in the absence of such complete records, a large amount of work is usually necessary to find the original costs of the several property units, and to supply omissions in the records.

There usually are certain construction costs of a rather general character which should be included in direct costs rather than in the overhead costs. Examples are allowances for the shrinkage of embankments; for unavoidable losses during construction of track spikes and plates; and of building materials, such as nails, cement, lumber, and glass. To the utmost extent feasible, all allowances for such costs should be included in either the inventory quantities or the inventory direct-cost prices; overhead cost allowances should be confined entirely to costs which cannot be allocated directly to property units of one kind.

11.11. Overhead Construction-cost Allowances.—Overhead construction costs are those which cannot be allocated directly to specific units.

In this treatise, overhead construction costs are classified in four groups: (1) engineering; (2) general expenditures; (3) omissions and contingencies; (4) interest lost during construction. These groups are defined and discussed in Secs. 11.12 to 11.14, inclusive.

It is necessary to distribute overhead costs to specific property units on a percentage basis. For convenience in calculation, it is customary to use *percentages of direct cost* for this purpose.

Although overhead general expenditures are likely to be proportional to the sum of direct costs and engineering, it is simpler and sufficiently accurate to use the somewhat higher general overhead distribution percentages computed on direct costs alone. Similarly, interest lost during construction (on direct costs, plus engineering, general, and omissions and contingencies overheads) is allowed as a percentage (higher than the interest rate) of direct costs only.

The *overhead base* is the direct-cost sum upon which the overhead cost percentages are computed.

The lands owned by public utilities should not be included[1] in the property of which the direct costs constitute the overhead base.

This is because it is the law of the land that the total values of such lands are equal to the market values of similar near-by lands. The same principle would seem to apply to many, if not all, industrial properties of private character.

There should often be two or more overhead bases, especially in cases of large properties; different overhead percentages should often apply to different classes of property.

For example, correct overhead-allowance percentages are much higher for road-account railroad property than for railroad equipment; and in other industrial properties should often differ materially for structures, equipment and "other property."

[1] See *Minnesota Rate Cases* decision, Sec. 8.4.

11.12. Original-cost Allowances for Engineering and for General Overheads.—These two classes of overheads include all recorded money outlays for overhead construction costs. "Omissions and contingencies" allowances, if made, are for unrecorded expenditures; "interest lost during construction" represents income foregone, not money paid out.

1. *Engineering original-cost overheads* include all actual original-cost overhead expenditures for engineering services. Engineering services include making plans and specifications; assisting in letting contracts; laying out and directing work; inspecting and testing materials; making estimates; keeping engineering records; making engineering reports.

2. *General expenditures original-cost overheads* include all actual original-cost overhead expenditures except those for engineering, and may usually be classified as: (*a*) superintendence, office, and miscellaneous salaries, wages, and expenses; (*b*) insurance; (*c*) injuries and damages; (*d*) legal costs; (*e*) taxes.

NOTE.—Organization expenses are sometimes included in the general overhead expenditures allowances; since they depreciate with the organization instead of with the lives of the physical-property units, organization expenses should be capitalized as intangible value, not as overhead costs (Chap. XII).

To the utmost extent feasible, all data for engineering and for general expenditures original-cost overhead allowances should be obtained direct from the book records of the particular properties concerned. Two classes of such records are found:

First: Standard accounting systems, with separate accounts, constantly kept up to date, for the various classes of overhead expenditures. From such book records (still unfortunately too rare), adequate and reliable data for engineering and for general expenditures overhead allowances are readily obtainable.

For example, the Interstate Commerce Commission prescribes Account 1 for engineering expenditures; and the following numbered accounts for general expenditures:

71. Organization expenses.
72. General officers and clerks.
73. Law.
74. Stationery and printing.
75. Taxes.
76. Other expenditures—general.

The National Association of Railway and Utilities Commissioners, in its Uniform Classification of Accounts for Electrical Utilities, provides Account 351 for engineering and superintendence expenditures; and the following numbered accounts for general overheads:

352. Law expenditures during construction.
353. Injuries and damages during construction.
354. Taxes during construction.
356. Miscellaneous construction expenditures.

Second: Book records which are more or less imperfect as to actual overhead expenditures. With imperfect records, a large amount of work is usually required to find all cost data actually existent therein, and to get whatever additional data are needed to enable the omissions to be

supplied with sufficient accuracy; general overhead percentages must ordinarily be used for property units for which individual overhead records are missing.

Illustrative examples of original-cost overheads data are presented below:

1. *Interstate Commerce Commission Data of Railway Overheads.*—For use in its railway-valuation work, the Interstate Commerce Commission made a detailed investigation of the actual engineering and general expenditures outlays in some 121 railway projects, scattered over the United States, constructed between 1903 and 1918, with results as follows:[1]

Engineering.—There were only a few cases where the engineering costs fell below 2 or were above 5 percent. The weighted average was 3.6 percent. On the basis of this investigation, the valuation engineers "were instructed to compute engineering as a percentage of the road accounts excluding Nos. 1 (engineering) and 2 (land), and that the percentage so included shall not be less than 2 nor more than 5. If an abnormal case arises in which the engineer believes a greater or less sum should be estimated, he will list that case for special discussion and determination."

General Expenditures (including organization expense).—The investigation showed the following percentages (of overhead bases which included engineering, but from which land and equipment were excluded):

Districts of the United States

Eastern..........	1.766%	Central.........	1.362%	Pacific..........	4.086%
Southern.........	2.251	Western........	0.838	*Average*.........	1.930

Owing to abnormal conditions, the percentage for the Western District was considered too low and that for the Pacific District too high. The Commission concluded that 1½ percent of all road accounts except land (but including engineering) "will do full justice to the carrier," except where more than a nominal charter fee has been required.

Table 11.4.—Data of Engineering and of General Overheads. Iowa State Highway Commission

Year	Total expenditures	Overhead base	Overhead expenditures			
			Total		Administration	Engineering*
			Amount	Percent	Percent	Percent
1932	$ 17,526,808	$16,322,940	$1,203,868	7.375	1.574	5.801
1931	31,675,401	29,589,153	2,086,248	7.051	1.142	5.909
1930	46,073,658	43,652,676	2,420,982	5.546	0.828	4.718
1929	33,078,779	31,252,219	1,826,560	5.844	0.941	4.903
1928	34,243,816	32,469,787	1,774,029	5.464	0.874	4.590
Grand total..	162,598,462					

* The engineering included extensive testing of materials and very thorough inspection of all materials and workmanship.

[1] See Report on Valuation of Texas Midland Railroad, Sixty-fifth Congress, 2d session, Aug. 22, 1918, pp. 28–31.

2. *Iowa State Highway Commission Data of Highway Overheads.*—Table 11.4 shows the actual overhead outlays in Iowa for highway construction in 1928 to 1932, inclusive, costing $162,598,462. The data are from very accurate and complete accounts. The increase in the percentages in 1931 and 1932 illustrates the difficulty in reducing overhead costs as rapidly as the volume of construction decreases, especially when falling prices are increasing the volume of construction per unit of total cost.

3. *Ames, Ia., Electric Utility Overhead Expenditures.*—Table 11.5 shows the actual overhead outlays of the $800,000 Ames, Ia., municipally owned electric utility during 1929 to 1932, inclusive. The same overhead organization served the municipal water, sewerage, and street utilities also, and it is possible that somewhat more than its correct share of general overheads may have been allocated to the electric utility.

The data for 1929 and 1930 show the effects of relatively small and large additions of yearly construction expenditures to those for the regular operation costs.

Table 11.5.—Ames Electric Utility Property. Data of Engineering and of General Overheads

Items	Fiscal years ending March 31							
	1932		1931		1930		1929	
	Amount	Per cent	Amount	Per cent	Amount	Per cent	Amount	Per cent
Annual Disbursements								
Operation costs.....	$107,127	.....	$114,069	....	$127,994	....	$126,495	
Construction costs...	40,939	.....	69,380	....	138,716	....	33,308	
Increase in stock....	2,900	.....	−122	....	3,260	....	−1,858	
Total disbursements.	150,966	.....	183,327	....	269,970	....	157,945	
Overhead base......	136,964	.....	167,961	....	254,823	....	138,697	
Overhead costs......	14,002	10.22	15,366	9.15	15,147	5.94	19,248	13.88
Analysis of Engineering Overheads								
Salaries and wages*..	4,500	3.28	5,000	2.98	5,000	1.96	5,000	3.61
Expenses*..........	1,312	0.96	1,480	0.88	1,570	0.62	1,335	0.96
Total engineering.	5,812	4.24	6,480	3.86	6,570	2.58	6,335	4.57
Analysis of General Overheads								
Salaries and wages*..	3,058	2.23	6,369	3.79	6,354	2.49	6,202	4.47
Expenses*	1,313	0.96	1,480	0.88	1,570	0.62	1,333	0.96
Insurance...........	3,595	2.62	618	0.37	341	0.13	1,906	1.37
Injuries and damages	147	0.11	419	0.25	312	0.12	3,474	2.51
Taxes..............	77	0.06						
Total, general.....	$ 8,190	5.98	$ 8,886	5.29	$ 8,577	3.36	$ 12,915	9.31

* Salaries, wages, and expenses were not separated between engineering and general overheads in the original accounts. They have been divided by the city manager as above.

4. *Overhead Expenditures during the Construction of a* $2,500,000 *Electric Power Plant in Eastern Iowa,*[1] 1923 *to* 1925.

			Land	Insurable Property
Engineering overheads			2.00%	5.00%
General overheads:	Land	Insurable Property		
Salaries and expenses	1.87%	1.87%		
Law	0.02	0.02		
Injuries and damages	....	0.82		
Taxes	0.12	0.12		
			2.01%	2.83%
Interest during construction			9.38	9.38
Total original-cost overheads			13.39%	17.21%

5. *Overhead Expenditures Actually Charged and Accumulated Historically on the Books of a Large California Electric Utility.*[2]

Engineering and superintendence		4.26%
General overheads:		
Miscellaneous general expenditures	4.11%	
Legal expenses	0.07	
Injuries and damages	1.19	
Taxes	0.03	5.40
Interest during construction		3.74
Total original-cost overheads		13.40%

6. *"Historical"-Cost*[3] *Overhead Data of California Utilities.*[2]—In Table 11.8, properties J, K, L, and M are water utilities; N is a gas utility; O and P are telephone utilities; Q and S are electric railways.

7. *Original-cost Overhead Data of a Large Utility Property in Several Central States.*—The overhead data in Table 11.6 are from the book records of a number of operating plants, located in several states, but owned and managed by a single company, whose central office property devoted to the service of these plants is listed under the heading, "Realty department."

The above overhead percentages are based on estimated percentages for existing property units installed prior to about 1920, plus actually recorded overheads for units installed later.

8. *Miscellaneous Original-cost overhead data*[4] of the actual expenditures during the construction of a number of notable water-supply, sewerage, power, and subway works are given in Table 11.7. These works were of such character as to require

[1] Data furnished by the company.

[2] Data furnished by L. E. Torrey, assistant engineer, California Railroad Commission.

[3] "Historical" cost is the same as "original" cost.

[4] Data from the 1917 Report of the Valuation Committee of the American Society of Civil Engineers. See *Trans.*, **81,** 1562 *et seq.*

TABLE 11.6.—ORIGINAL-COST OVERHEADS DATA OF A LARGE UTILITY PROPERTY IN FOUR CENTRAL STATES[1]

Item	Electric department		, Gas department		Heat department		Realty department	All departments	
Present value lands		$ 373,254		$ 533,845		$ 6,746	$ 87,873		$ 1,001,718
Present direct-cost value of fixed-capital physical property, except land		15,091,479 100.00%		8,781,027 100.00%		343,541 100.00%	495,565 100.00%		24,711,612 100.00%
Original-cost Overheads									
Engineering	......	5.51%	......	5.42%	......	5.89%	6.50%	......	5.51%
General:									
Miscellaneous	2.03%	....	3.40%	....	3.39%	....	......	2.50%	
Legal expenses	0.46	....	0.62	....	0.88	....	......	0.51	
Injuries and damages	0.84	....	0.65	....	0.45	....	......	0.75	
Taxes	0.28	3.61	0.46	5.13	0.55	5.27	......	0.34	4.10
Interest during construction	......	4.43	......	3.86	......	4.03	2.99	......	4.19
Total original-cost overheads	......	13.55%	......	14.41%	......	15.19%	9.49%	......	13.80%

[1] Data collected in 1933, by James P. McKean, valuation engineer.

comparatively high engineering costs—for the services of high-grade consulting engineers.

11.13. Original-cost Allowances for Omissions and Contingencies.—The making of any original-cost overhead allowance whatsoever for "omissions and contingencies" can be justified only when there are sound reasons to conclude: (1) that there are errors in the inventory of such character as to cause a net shortage instead of a surplus; and/or (2) that contingencies, not now discoverable, of a nature to cause higher costs than present fair estimates of their original costs would indicate, were encountered in the construction of inventory items whose costs are not given in the book records of the property.

If the inventory, the search of the cost records, and the study of the property have been made with painstaking care, no original-cost allowance at all should be made for "omissions and contingencies"; no valuation should be considered satisfactory in which more than a very small allowance is justifiable.

11.14. Original-cost Allowances for Interest Lost during Construction. The present existing property units of typical industrial properties have been installed at many different dates, and the times elapsing between the dates of payment for them and the dates of their beginning service have varied materially. The proper allowance in the cost of *each unit* for "interest lost during construction" should be ascertained at

Table 11.7.—Data of General and of Engineering Overheads
From Report of Valuation Committee, American Society of Civil Engineers, 1917[1]

Dates	Locality and character of works	Overhead base	Overhead percentages		
			Engineering	General*	General and engineering*
1885	Washington Bridge, Harlem River, New York City	$ 2,648,785	6.1	1.50	
1867–1901	St. Louis, Mo., waterworks	8,472,576	11.2		
1897–1907	Cincinnati, Ohio, waterworks	9,279,203	9.1	4.90	14.0
1903–1905	Washington, D.C., slow sand filters	3,356,000	7.4		
1904–1908	Columbus, Ohio, improved waterworks	1,189.210	8.1		
1906	Kennebec Water District, Maine	267,408	7.35	2.53	9.88
1909	Charles River Basin, Boston, dam locks, embankments, conduits	3,215.656	15.5	3.20	18.7
1910	Springfield, Mass., waterworks	2,000,000	....	......	14.30
1911	Des Moines, Ia., New Source water supply	....	8.6		
1895–1912	Boston Transit Commission, subways, tunnels, Cambridge connection	15,303,708	7.29	3.80	11.09
1906–1912	Louisville, Ky., sewers	3,289,331	{ 1.70 † 10.23 }	3.46	15.39
To 1912	Metropolitan Waterworks, Boston	23,427,000	9.55	2.43§	11.98‡
1905–1914	Los Angeles, Calif., aqueduct	22,540,580	8.8		
To 1914	New York City Catskill water supply	108,144,129	11.12		
To 1915	Pacific Gas & Electric Co.:				
	Big Creek	11,339,308	6.23	7.93	14.16
	San Joaquin	1,563,485	9.50	3.00	12.50
	Spaulding Drum Development	3,498,894	6.00	5.70	11.70

[1] See *Trans. A.S.C.E.*, 81, pp. 1563–1565.
* Not including taxes.
† Preliminary engineering 1.70%.
‡ No allowance included for legal services of the attorney general's office.

the date of its installation; it should be entered in the property ledger at that time.

However, with the incomplete property records still unfortunately so common, it is often necessary in valuation work to allow general original-cost "interest-lost" percentages for many units, or even most. Such percentages should be determined by careful study of the property and its book records. They will be found to vary from year to year, and will be materially less than reproduction-cost interest-lost percentages, estimated on the assumption of a single "construction period" long enough to build the entire property.

In the case of the valuation of the $800,000 Ames electric utility (Chap. XXI), the authors studied the average time required for routine replacement and improvement units to pass through storage into actual service; in addition, they compared the dates of payment for current new construction costing $125,000 with the date when service by the new property began. It was concluded that, in this case, 2½ percent (4½ months) of original-cost average interest lost should be allowed.

Examples of original-cost general percentage allowances for interest lost during construction in the cases of property units for which individual records are not available will be found, for 13 different utilities, on pages 416 and 318, and in Tables 11.6, and 11.8; the range is from 1.60 percent to 9.38 percent.

11.15. Total Original-cost Overhead Allowances.—Normally, the original-cost overhead percentages of a typical industrial property which has been in operation a considerable number of years will be less than either those actually incurred in constructing the original property or those which would now be estimated as necessary to incur in reproducing the present existing property in one construction project. This is because: (1) the engineering and the general overhead construction expenditures during operation (for replacements, improvements, and enlargements) are by a permanent overhead organization, which serves operation as well as construction needs; (2) usually no allowance should be made for "omissions and contingencies"; (3) the interest lost during construction should be less because the units do not have to wait the completion of an entire property before going into use.

In the case of the $800,000 Ames electric utility property (Sec. 21.16), the authors found the original-cost total overheads percentage to be 12½ percent.

In the *Los Angeles Gas Case* (Sec. 8.7), the original-cost total overheads for the $57,850,000 utility were 11.25 percent, as compared with 22.32 percent reproduction-cost overheads; both estimated by the California Railroad Commission's engineers. The United States Supreme Court approved 11.25 percent for fair value.

In the case of the large California electric utility[1] the original-cost overheads of which are given on page 317, the corresponding reproduction-cost overheads were as follows:

Engineering and superintendence		5.00%
General overheads:		
Miscellaneous general expenditures	3.50%	
Legal expenses	0.25	
Injuries and damages	1.50	
Taxes	0.25	5.50
Interest during construction		5.20
Total reproduction-cost overheads		15.70%
Total original-cost overheads		13.40%

In Table 11.8,[1] the "historical"-cost overheads of properties J to P, inclusive, ranged from 8.60 to 10.67 percent; while the reproduction-cost overheads of properties A to I, inclusive, ranged from 13.60 to 18.00 percent. For properties Q and S, original-cost and reproduction-cost overheads compared as follows:

Q Original-cost overheads 8.21%; reproduction-cost overheads 14.33%.
S Original-cost overheads 7.68%; reproduction-cost overheads 10.17%.

[1] Data from L. E. Torrey, assistant engineer, California Railroad Commission.

THE REPRODUCTION-COST VALUES OF FIXED-CAPITAL PHYSICAL PROPERTY EXCEPT LAND

The great dearth in early property accounts of any real records of the actual original costs of the individual property units was undoubtedly one important reason for the widespread use and acceptance of the reproduction-cost measure of value during 1900 to 1915; property cost records still are found to be inadequate in the majority of cases.

Nevertheless, the one great purpose for which reproduction costs of properties are determined is to enable "such weight as is just and right in each case"[1] to be given to "the present as compared with the original cost of construction."[1] The determination of reproduction cost new less depreciation for this purpose is nearly always one of the major features of making an engineering valuation; valuations in which reproduction cost has not been given due weight have always been disapproved by the United States Supreme Court.

"An honest and intelligent forecast of future values, made upon a view of all the relevant circumstances, is essential. If the highly important element of present costs is wholly disregarded, such a forecast becomes impossible. Estimates for tomorrow cannot ignore prices of today."[2]

11.16. The Reproduction-cost-new-less-depreciation Concept.—The object in view in determining the reproduction-cost-new-less-depreciation value of fixed-capital physical property is to ascertain what its value would be on the basis of present costs of construction alone. The general reproduction-cost concept is about as indicated below:

1. The reproduction cost sought is that of constructing the entire present fixed-capital physical property in one project, starting at the date of valuation.
2. The property to be reproduced is a duplicate of the actual present fixed-capital property used for production purposes (but see 7, below).
3. So far as effect on construction is concerned, the present plant is assumed to be nonexistent.
4. In general, present conditions are assumed to prevail during reproduction (but see 7, below).

This assumption of present conditions applies to topography, soil, timber and other culture (as on adjacent lands), transportation facilities (other than those of the plant itself), materials, construction, equipment and methods, adjacent population, prevailing standards of living, business conditions, etc.

5. The concept of the purely imaginary process of reproducing the property is:
   a. Estimate the construction period which would be needed; and set up what is considered to be the best, simple construction program.
   b. Estimate the construction costs of this program; use contract prices, or the costs of construction by the owners, or part one and part the other, as may be best, everything considered.

[1] See the *Smyth v. Ames* decision, 1898, Sec. 8.3.
[2] See *Southwestern Bell Telephone Case,* Sec. 8.5.

6. In the whole process outlined in 1 to 5, above, avoid speculative and/or conjectural assumptions or reasoning. "The cost-of-reproduction method is of service in ascertaining the present value of the plant, when it is reasonably applied, and when the cost of reproducing the property may be ascertained with a reasonable degree of certainty. But it does not justify the acceptance of results which depend on mere conjecture."[1]

7. Modify rules 1 to 5, above, in whatever fashion may be found necessary in the interests of fairness and sound reasoning, whenever strict technical adherence to them would give unfair or unreasonable results.

Examples: Rule 2, above, should be modified by substituting present available equivalent units for such existing property units as can no longer be procured at reasonable cost.

Rule 4, above, has been modified, by the United States Supreme Court, by excluding from valuations all costs of cutting and relaying pavements built after the installation of such existing constructions as water mains and other conduits and street railways.

It must always be borne in mind that the whole object of using the cost-of-reproduction method is to find the fair "present as compared with the original cost of construction"; ingenious technical assumptions, slavishly adhered to, are not sound supports for estimated present costs widely at variance with actual original costs.

Elaborately detailed construction programs extending over a considerable number of years sometimes justify such characterizations as: "The fanciful estimates that often characterize reproduction cost appraisals";[2] "the hypothetical vagaries that are likely to creep into reproduction cost appraisals when based on opinion evidence."

11.17. The Process of Estimating Reproduction-cost-new-less-depreciation Value.—The present reproduction-cost values of the several inventory[3] property units can be estimated in the following manner:

1. Estimate the *construction period* which would be required to construct the entire present existing fixed-capital property in one project, starting at the date of valuation; and set up what is considered to be the best simple construction program, specifying the percentage of capital expenditure during each of a limited number of construction-period subdivisions.

2. Estimate the *fair present direct-cost construction prices* of all inventory units, upon the assumption that they are to be built by the method selected for the program set up.

3. Estimate the *fair reproduction-cost overhead percentages* which would be incurred in carrying out the construction of the property in accordance with the program assumed.

[1] See the *Minnesota Rate Cases* decision, 1913, Sec. 8.4.

[2] WHITTEN and WILCOX, "Valuation of Public Utilities," 2d ed., The Banks Law Publishing Company, New York, pp. 536, 539.

[3] The inventory is the same used for determining the original-cost values.

4. Compute the *total reproduction costs*[1] new of the respective inventory units, by adding the proper overhead percentages to their direct costs.

5. By the methods explained in Chap. V, estimate and deduct the total actual depreciation accrued to the date of valuation from the total reproduction cost new of each unit[1] to get its present reproduction-cost-new-less-depreciation value.

11.18. "Spot" Reproduction Direct-cost Construction Prices.—One or both of two kinds of reproduction-cost estimates of value are made, based respectively on "spot" and on "period" prices.

Spot reproduction-cost prices are hereby defined to be the current-year average construction prices. Two investigations of current-year average prices should be made:

First: The construction prices actually paid for piecemeal construction during the current year by the enterprise; and/or which would have to be paid in making renewals of other existing property units.

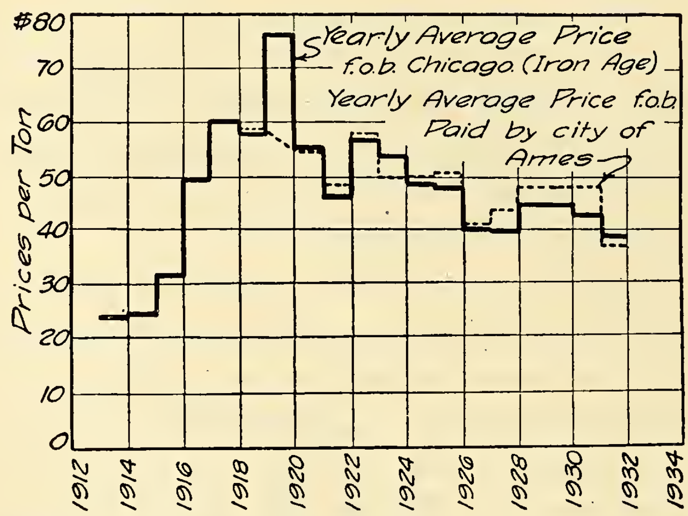


Fig. 11.1.—Diagram from Ames electric utility valuation on prices of cast-iron pipe.

The data for estimating such prices are the identical data obtained for estimating original costs, as described in Sec. 11.10, above. See especially Fig. 11.1, and Tables 11.1, 11.2, and 11.3 as illustrations of such data and of methods of arranging them for study. Costs in former years can be transformed to the current year by the help of cost-time factors, and special or general construction cost indexes, as described in Chaps. VI and IX.

Second: The estimated construction prices which would have to be paid for construction on a large scale during the current year to contractors, and/or for the costs of construction by the owner.

The selection of such estimated spot prices requires careful study of actual current construction prices, with the help of special and/or general construction-cost indexes (Chap. IX) of published past and current cost and price data,[2] of similar data collected direct by the valuator, and of data obtained by current correspondence with construction materials and equipment manufacturers and dealers.

[1] Sometimes the depreciation of the overheads is calculated in one separate amount for each group of units to which a uniform total overhead percentage applies.

[2] See *Engineering News-Record*, "Construction Costs."

In general, it would seem that construction prices for work on a large scale ought to be lower than the actual costs incurred in piecemeal construction during the history of the enterprise.

The "difficulty factor," allowing for increased costs due to present increased difficulties from traffic, subterranean obstructions, etc., was excluded by the Commission's engineer and by the courts in the valuation of the property of the Los Angeles Gas & Electric Corporation (Sec. 8.7).

11.19. "Period" Reproduction Direct-cost Construction Prices.—*Period reproduction-cost prices* are hereby defined to be average construction-cost prices over a period of years. The period for which the reproduction-cost prices used in making valuations are estimated should be a future period of three to five years, beginning at the date of valuation. The average prices over past periods of three and five years, respectively, ending at the date of valuation, should often be estimated and reported also.

In estimating period reproduction-cost values, the fairly stable construction prices which are most likely to prevail during the future period adopted should be sought. In forecasting them, due consideration must be given to spot prices, past-period prices, present and probable future price trends, and business conditions.

As in the case of spot reproduction-cost prices, two investigations of period reproduction-cost prices are needed: (1) of the prices actually paid by the enterprise for piecemeal construction; (2) of the estimated prices which the enterprise would have to pay for the construction of the entire property in one project (Sec. 11.18).

The data to be collected and studied for period reproduction-cost data are the same as those described and discussed in Sec. 11.10 for original costs, and in Sec. 11.18 for spot reproduction costs.

11.20. Reproduction-cost Overhead Allowances.—The classification and definitions (Secs. 11.11, 11.12, 11.13, 11.14) of overhead-cost construction allowances apply to reproduction costs as well as to original costs; but it must be kept in mind that original-cost overheads are actual past expenditures, which should be ascertainable from the book accounts, while reproduction-cost overheads are estimated future expenditures, thought by the valuator to be necessary in reproducing the property in one construction project, beginning at the date of valuation.

The overhead base (or overhead bases) should include the same classes of construction direct costs for reproduction-cost overheads as for original-cost overheads.

As stated in Sec. 11.15, estimated reproduction-cost overhead allowances often materially exceed actual original-cost overhead expenditures.

TABLE 11.8.—OVERHEAD COST PERCENTAGES OF CALIFORNIA UTILITIES[1]

Property	Purpose	Basis	Amount	Utility	Engineering and superintendence	General overheads: Miscellaneous expenditures	General overheads: Legal	General overheads: Injuries and damages	General overheads: Taxes	General overheads: Total general	Omissions and contingencies	Sub-total	Interest: Period, months	Interest: Rate	Interest: Total interest	Total overheads
					%	%	%	%	%	%	%	%		%	%	%
A	Condemnation	Reproduction cost	$ 2,383,993	Water	4.00	2.00	0.50	1.00	0.50	4.00	2.00	10.00	12	6.25	6.88	16.88
B	Condemnation		256,039	Water	4.00	3.50	0.50	1.00	0.25	5.25	2.00	11.25	(?)6	5.00	5.56	16.80
C	Rates		2,007,993	Water	4.00	1.00	0.25	0.75	0.20	2.20	2.00	8.20	12	5.00	5.40	13.60
D	Sale, finance		444,493	Water	5.00	2.25	1.00	1.50	0.25	5.00	2.00	12.00	6	3.00	3.40	15.40
E	Condemnation		165,132	Electric	5.00	2.50	1.00	2.00	0.50	6.00	3.00	14.00	...	2.00	2.30	16.30
F	Condemnation		55,675	Electric	5.00	3.50	0.50	1.69	....	5.69	2.00	12.69	6	3.00	3.38	16.07*
G	Condemnation		1,227,975	Electric	6.00	3.50	0.75	1.50	0.25	6.00	2.00	14.00	...	3.50	3.99	17.99†
H	Condemnation		337,383	Electric	6.00	3.50	0.75	1.50	0.25	6.00	2.00	14.00	...	3.50	3.99	17.99†
I	Condemnation		61,607	Gas	5.00	3.50	0.50	1.43	....	5.43	2.00	12.43	6	3.00	3.37	15.80
Q	Financial		52,129,360	Electric Ry.	5.00	2.50	2.50	1.50	0.50	7.00	Inclusive	12.00	...	....	{3.75, 11.25}	14.33‡
R	Sale, finance		22,332,430	Electric Ry.	5.00	2.50	2.50	0.50	0.50	6.00	Inclusive	11.00	...	....	6.00	9.58‡
S	Rates		38,192,410	Electric Ry.	4.00	(2.50, inclusive)				2.50	Inclusive	6.50	...	....	{3.25, ?}	10.17‡
J	Sale, finance	Original cost	222,620	Water	4.00	1.00	0.50	1.00	....	2.50	2.00	8.50	4	2.00	2.17	10.67
K	Sale, finance		27,532	Water	2.00	2.00	0.50	1.00	....	3.50	2.00	7.50	3	1.50	1.61	9.11
L	Sale, finance		166,515	Water	3.00	2.00	....	....	....	2.00	2.00	7.00	3	1.50	1.60	8.60
M	Sale, finance		234,964	Water	3.00	2.00	0.50	0.50	....	3.00	2.00	8.00	3	1.50	1.62	9.62
N	Rates		281,757	Gas	4.00	....	0.50	0.50	....	1.00	3.00	8.00	3	1.50	1.62	9.62
O	Rates		200,050	Telephone	3.20	1.50	0.50	1.20	....	3.20	2.00	8.40	4	2.00	2.20	10.60
P	Sale, finance		269,435	Telephone	3.00	2.00	0.50	1.40	....	3.90	2.00	8.90	3	1.50	1.60	10.50
Q	Financial		30,639,222	Electric Ry.	5.00	2.50	2.50	1.00	0.50	6.50	Inclusive	11.50	...	....	{3.00, 6.00}	8.21‡
S	Rates		28,221,682	Electric Ry.	4.00	(2.50, inclusive)				2.50	Inclusive	6.50	...	....	{3.00, 4.50}	7.68‡

[1] Data supplied 1932 by L. E. Torrey, assistant engineer, California Railroad Commission.
* Used 16.10 %.
† Used 18.00 %.
‡ Percentages shown applied to various accounts. Total is approximate average over all accounts.

11.21. Methods of Estimating Reproduction-cost Overhead Allowances.—Two methods are in use for estimating reproduction-cost overheads:

1. *The Synthetic Method.*—In this, the estimator makes a supposititious setup of the complete overhead organization; he then estimates its salaries, wages, and other expenses in detail throughout the assumed construction period.

Example: The Public Service Commission of Indiana,[1] in its valuation of the Terre Haute, Indianapolis and Eastern Traction Co., worked up an exhibit of "administration and legal expense" and an exhibit of "estimated cost of superintending construction"; in each of these, the supposititious numbers of employees of different classes, their salaries and wages, the durations of their respective services, and the other items of overhead expenses were set forth in detail throughout the construction periods assumed for the respective geographical "divisions" of the property.

2. *The Analogous Percentage Method.*—In this, the estimator makes direct estimates of the respective percentages of the different elements of reproduction-cost overhead expenditures, by analogy with collected data of actual historical overhead expenditures in similar past construction projects.

Examples: The California Railroad Commission[2] has supplied the data for Table 11.8, which shows in detail the percentages used for different elements of reproduction-cost overhead allowances in the valuations of 12 utilities; and similar data of the original-cost overhead allowances in the valuations of two of these same utilities and seven others.

11.22. Reproduction-cost Allowances for Engineering and General.

1. *Engineering reproduction-cost overheads* include all estimated expenditures for engineering services during the (imaginary) reproduction of the existing property in the assumed (future) construction period. (See Sec. 11.12 for a list of the engineering services which would be required.)

2. *General expenditures reproduction-cost overheads* include all estimated overhead reproduction-cost actual payments of money, other than for engineering, during the assumed (future) construction period. General overhead expenditures may usually be classified as for (*a*) superintendence, office, and miscellaneous salaries and expenses; (*b*) insurance; (*c*) injuries and damages; (*d*) legal costs; (*e*) taxes (Sec. 11.12).

As already explained in Sec. 11.15, estimated reproduction-cost engineering and general overhead percentages are likely to be greater than the original-cost percentages actually incurred.

Compare the original-cost and reproduction-cost data for a large California electric utility, pages 316 and 319; also note, in Table 11.8, the near equality of the original-cost and reproduction-cost general expenditure percentages for electric-railway utilities Q and S.

[1] Information furnished in 1932 by Harry V. Wenger, chief engineer, Public Service Commission of Indiana.

[2] Information furnished in 1933 by L. E. Torrey, assistant engineer, California Railroad Commission.

It seems reasonable to conclude that the engineering and the general overhead past actual expenditures should have greater weight in determining the values of the several property units than estimated future expenditures.

The data in Secs. 11.12, 11.15, and 11.21, and in Tables 11.4 to 11.8 illustrate the kind of overhead data needed in estimating engineering and general reproduction-cost overhead percentages. Valuators collect and use additional similar data.

11.23. Reproduction-cost Allowances for Omissions and Contingencies.—As stated in Sec. 11.13, very small, if any, original-cost construction overhead allowances should be made for omissions and contingencies; engineering valuators are becoming more and more conservative about making reproduction-cost allowances for assumed overheads of this speculative class. The case is not that of a nonexistent property, yet to be built. The valuation is of an existing property, already constructed. It must be based on a carefully prepared and completely detailed inventory, a careful search must be made for "hidden construction," and every reasonable effort must be made to ascertain the actual facts about all "contingencies" really encountered during the actual historical construction of the property.

The Interstate Commerce Commission has concluded[1] that no separate allowance at all should be made for "omissions and contingencies," and other valuators more and more frequently reach the same conclusion. As adequate property-ledger accounts of all property units become more and more the rule, there is less and less excuse for conjectural allowances of any kind in making valuations.

When any omissions-and-contingencies reproduction-cost overhead allowances at all are made, they should be very small in any valuation made with sufficient care to be acceptable. Table 11.8, for example, shows no such allowance at all for utilities Q, R, and S; only 2.00 percent for A, B, C, D, F, G, H, and I; and only 3.00 percent for E.

11.24. Reproduction-cost Allowances for Interest Lost during Construction.—This is the class of overhead percentages for which, even more than for omissions and contingencies, reproduction-cost allowances often materially exceed actual original costs. This is because the dates when property units begin service are assumed to wait the completion, in one construction period, of the entire property, or at least of a major division thereof; whereas, in the actual historical construction of a long-established property, many, or even most, of the existing property units have been put into service promptly on the completion

[1] See Texas Midland Railway Report, House Document 1261, Sixty-fifth Congress, pp. 24, 142.

of their installations, in the regular course of making renewals, improvements, and enlargements when needed.

For example, compare, on page 316, the 3.74 percent average percentage of historical interest-lost charges on the books of an established large California electric utility with the 9.38 percent interest lost during the construction in a continuous three-year period of a $2,500,000 electric power plant in Iowa. Also, note the much higher reproduction-cost than original-cost allowances in Table 11.8 for interest lost.

The Interest Rate for Estimating Reproduction-cost Interest-lost Allowances.—Although other rates may be correct in some instances, it is quite customary to use a 6 percent interest rate in estimating reproduction-cost interest-lost overhead-percentage allowances.

Note.—The 6 percent is the rate of interest which is assumed lost on both the overhead base and all overheads except interest lost. The corresponding percentage of the overhead base alone is larger. It must be calculated (by obvious means) and then multiplied by the interest-lost time period (in years) to get the interest-lost total percentage allowance.

Six percent is the legal rate of interest in a large part of the United States when no other rate is specified; the Interstate Commerce Commission has decided[1] that it should be used for railroad properties in estimating interest lost, although a strong railroad with the best of credit could sell 4½ percent bonds at par in normal times. The 6 percent rate covers various extra costs in securing funds promptly for construction needs; such funds often must be supplied before it is wise to sell bonds.

The interest rate used should always be that which is fair for a strong owner company, with good credit.

Overhead costs are a part of the costs of the physical-property units; their prices and values are in no way affected by risks incurred by financial agencies in supplying funds to pay for them. The risks of investments in particular properties must be provided for entirely from net return, the fair rate of which is high when the risks are high.

The Length of the Interest-lost Reproduction Period.—Evidently, the length of the assumed period before completed property units begin service is a major consideration in determining the theoretical percentage to allow for reproduction-cost interest lost; in order to avoid the speculative, conjectural allowances which the courts reject, the assumed construction period for reproducing the property should be as short and the assumed construction program as simple as practicable.

See, on page 346, the rejection by the United States Supreme Court, as "too conjectural," of the assumed seven-year reproduction period

[1] See Texas Midland Railway Report, House Document 1261, Sixty-fifth Congress, p. 154.

and program proposed by the company's engineers for the Los Angeles gas property.

In many cases of large properties the reproduction period should be shortened by assuming construction to proceed simultaneously on a number of "divisions"; and by further assuming service to begin, in each division, without waiting the completion of the entire property.

Methods of Estimating Reproduction-cost Interest-lost Percentages.—Two methods are in common use for estimating reproduction-cost interest-lost percentages:

1. *A uniform rate of construction-cost expenditure assumed:* This is the simplest assumption. There are two variations:
   a. Interest assumed lost for an average time equal to one-half the assumed construction period.
   b. Interest assumed to be lost for an average time equal to one-half the assumed construction period, plus an assumed uniform time[1] required to secure capital before beginning to pay it out.

2. *A program of construction expenditures in successive subperiods assumed;* with a definite percentage of the total allotted to each. Only the simplest and most reasonable assumptions are permissible. The length of subperiod should usually be at least one-half year. No great refinements should be attempted in selecting the percentage allotments to different subperiods.

Example: Application of Methods 1 and 2 to the Ames electric utility property (Chap. XXI).

Overhead base = \$589,231. This is the estimated present depreciated total direct cost of all fixed-capital physical property except land.

Other overheads = 10% (engineering, 4.00; general, 5.75; omissions and contingencies, 0.25%).

Construction period = $1\frac{1}{2}$ years (Oct. 1, 1932, to April 1, 1934).

Method 1

a. Interest lost = (0.06) ($1\frac{1}{2}/2$ years) (1.10) = 4.95% of overhead base.

b. Interest lost = (0.06) [($1\frac{1}{2}/2 + \frac{1}{8}$) years](1.10) = 5.77% of overhead base.

Method 2

Capital available:
$\frac{1}{4}$, Oct. 1, 1932; $\frac{1}{2}$ more, April 1, 1933; the last $\frac{1}{4}$, Oct. 1, 1933.

Construction expenditures:
$\frac{1}{16}$, first 6 months; $\frac{7}{16}$, second 6 months; $\frac{1}{2}$, last 6 months.

Earnings on capital:
6% till available; 3%, 6 months or + accounts; 1%, shorter accounts.

a. Earnings first 6 months:

$$
\begin{aligned}
(\tfrac{3}{4}\text{ capital})\ (0.06)\ (\tfrac{1}{2}\text{ year})\ (1.10) &= 2.475\% \\
(\tfrac{3}{16}\text{ capital})\ (0.03)\ (\tfrac{1}{2}\text{ year})\ (1.10) &= 0.309 \\
(\tfrac{1}{32}\text{ capital})\ (0.01)\ (\tfrac{1}{2}\text{ year})\ (1.10) &= 0.017 \\
\hline
&\ 2.801\%
\end{aligned}
$$

[1] The Interstate Commerce Commission allows three months. See Texas Midland Railway Report, House Document 1261, Sixty-fifth Congress.

b. Earnings second 6 months:

($\frac{1}{4}$ capital) (0.06) ($\frac{1}{2}$ year) (1.10)	=	0.825%
($\frac{1}{4}$ capital) (0.03) ($\frac{1}{2}$ year) (1.10)	=	0.413
($\frac{7}{32}$ capital) (0.01) ($\frac{1}{2}$ year) (1.10)	=	0.120
		1.358%

c. Earnings third 6 months:[1]

($\frac{1}{8}$ capital) (0.03) ($\frac{1}{2}$ year) (1.10)	=	0.206%
($\frac{1}{8}$ capital) (0.01) ($\frac{1}{2}$ year) (1.10)	=	0.069
Third 6 months	=	0.275%
Second 6 months	=	1.358
First 6 months	=	2.801
Total earnings	=	4.43%

Interest on capital during construction	=	(0.06) ($1\frac{1}{2}$ years) (1.10)	=	9.90%
Earnings of capital during construction	=			4.43
Reproduction-cost interest-lost overhead allowance	=			5.47%

NOTE.—The actual original-cost interest lost was (0.06) ($4\frac{1}{2}$/12 years) (1.10) = 2.48 (used, 2.50%)

11.25. Total Reproduction-cost Overheads Percentage Allowances.—As already stated in this chapter, the theoretical reproduction-cost overheads percentages estimated by customary methods are usually materially greater than the actual overheads percentages which have been incurred in constructing existing industrial properties. The differences between the theoretical and the actual percentages are due to the assumptions made; these not infrequently are so far fetched that they are termed "speculative" or "conjectural" in court decisions and valuation discussions. Valuators are coming more and more to question the justice of giving any material weight in making valuations to such excess theoretical overheads.

The authors, in their 1932 valuation of the Ames electric utility (Chap. XXI) used the actual (12.50 percent) historical overhead expenditures on the property in estimating both reproduction-cost and original-cost values.

In its 1933 decision in the *Los Angeles Gas Case* (Sec. 8.7), the United States Supreme Court approved, for that property, adding the *actual* (11.25 percent) original-cost overheads percentage to the direct costs of the physical property. The *theoretical* reproduction-cost overheads were estimated at 22.32 percent.

[1] Checking balances assumed maintained at averages of one-eighth capital after second six months, by transfers from time deposits till they are exhausted.

OVERHEAD COST ALLOWANCES ACTUALLY APPROVED OR IMPLIED IN VALUATION COURT DECISIONS

It is obvious that the percentages of overhead allowances approved or implied in valuation court decisions must give valuable indications of what the courts will consider "just and right" in different circumstances. However, in studying their decisions the proper function of the courts in making valuation decisions must always be kept clearly in mind. The United States Supreme Court has said recently:[1]

> . . . it is the function of the court in deciding whether rates are confiscatory not to lay down a formula, much less to prescribe an arbitrary allowance, but to examine the result of the legislative action [for example, by a commission in a valuation for rate making] in order to determine whether its effect is to deny the owner of the property a fair return on its use.

Hence, the courts do not always, or even most often, find it necessary actually to decide upon definite valuation figures. For this reason, the data in Tables 11.9, 11.10, 12.4, and 14.1, hereinafter, show in most cases the conclusions of the authors as to what the respective court decisions imply rather than exact allowances definitely adopted; other students of the decisions might not agree closely with the authors in all the cases.

11.26. Overhead Cost Allowances in Court Decisions.—In Tables 11.9 and 11.10, the authors present their conclusions as to the approved or implied overhead cost allowances in some 25 court valuation decisions studied and analyzed. Table 11.10 contains data of the separate allowances for different classes of overheads in eight of these decisions—all in which separate allowances could be found.

Study of Table 11.9 shows a range of 8.72 to 21.93 percent total overheads, in 25 decisions. In none of these decisions since 1924 was more than 18 percent overheads allowed; the authors consider it to be possible that the increasing availability of reliable book records of actual overheads may be causing less weight to be given to the usually higher estimated reproduction-cost percentages, which often were the only basis available for consideration in early cases.

Study of Table 11.10 indicates that high total overhead allowances may sometimes be due mainly to estimated interest lost. While only two separate allowances appear for omissions and contingencies, it must be borne in mind that these may have been considered in the direct costs adopted for the overhead bases in the other 23 cases, Table 11.9, or in deciding the total overheads in the 17 cases in Table 11.9 which do not

[1] In 1933, in the *Los Angeles Gas Case* decision.

Table 11.9.—Approximate Approved or Implied Allowances Made in Valuation Court Decisions for Total Overhead Costs[1]

Dates of: Valuation	Dates of: Decision	Valuation court case	State	Court	Approximate overhead base	Approximate overhead allowance: Amount	Approximate overhead allowance: Percent	Other fixed physical property so far as known
1912	June 14, 1915	Des Moines Gas Co. v. Des Moines	Iowa	U.S. Supreme Court	$ 1,975,026*	$ 296,254*	15.00	$ 156,603
1914	Mar. 4, 1918	Denver v. Denver Union Water Co.	Colo.	U.S. Supreme Court	8,539,056*	1,872,382*	21.93	403,128
1919	Apr. 10, 1922	Galveston Elec. Co. v. Galveston	Texas	U.S. Supreme Court	1,832,776*	254,449*	13.88	58,836
Jan. 1, 1924	June 11, 1923	Bluefield W.W. & Imp. Co. v. P.S. Comm	W.Va.	U.S. Supreme Court	720,920*	72,092*	10.00	‡
Jan. 1, 1924	Nov. 22, 1926	McCardle v. Indianapolis Water Co.	Ind.	U.S. Supreme Court	15,778,322*	2,366,748*	15.00	‡
May 1, 1930	Jan. 9, 1933	Wabash Valley Elec. Co. v. Young et al., Martinsville	Ind.	U.S. Supreme Court	73,888†	10,701†	14.48	‡
Nov. 1930	May 8, 1933	Los Angeles Gas & Elec. Corp. v. Calif. R.R. Comm. et al.	Calif.	U.S. Supreme Court	49,309,439*	5,547,312*	11.25	3,500,888
Jan. 1, 1924	June 11, 1923	Geo Ry. & Power Co. v. R.R. Comm. of Georgia	Ga.	U.S. Supreme Court	6,697,366*	1,205,526*	18.00	
May 1, 1928	Apr. 17, 1929	Vincennes Water Supply Co. v. P.S. Comm.	Ind.	U.S. Circ. Ct. of Appeals	789,422†	124,000†	15.71	16,000
June 30, 1926	Sept. 22, 1930	Bd. of P.U. Commrs. v. Elizabethtown Water Co., Consol.	N.J.	U.S. Circ. Ct. of Appeals	5,615,853†	954,695†	17.00	842,165
June 30, 1922	July 18, 1923	Arkansas Water Co. v. City of Little Rock	Ark.	U.S. Dist. Ct.	2,374,063*	376,329*	15.85	¶
May 1, 1919	Apr. 21, 1921	Winona v. Wis.-Minn. Light & Power Co.	Minn.	U.S. Dist. Ct.	350,000*	52,500*	15.00	8,960
Sept. 1, 1921	June 4, 1923	Reno Power, Light & Water Co. v. P.S. Comm.	Nev.	U.S. Dist. Ct.	324,819*	48,723*	15.00	1,650
Dec. 31, 1922	Dec. 13, 1924	Westinghouse Elec. & Mfg. Co. v. Denver Tramway Co.	Colo.	U.S. Dist. Ct.	18,724,723*	3,800,652*	20.30	¶
Dec. 31, 1921	Dec. 27, 1924	Duluth St. Ry. Co. v. R.R. & Warehouse Comm.	Minn.	U.S. Dist. Ct.	4,556,771*	445,222*	9.77	128,474
Dec. 31, 1923	June 30, 1925	Kings Co. Lighting Co. v. Prendergast	N.Y.	U.S. Dist. Ct.	6,350,727*	891,819*	14.04	2,576,523
Oct. 31, 1924	Nov. 24, 1925	N.Y. & Richmond Gas Co. v. Prendergast	N.Y.	U.S. Dist. Ct.	3,246,707*	564,744*	17.39	483,869
Sept. 30, 1924	Jan. 7, 1926	Middlesex Water Co. v. Board of P.U. Commrs.	N.J.	U.S. Dist. Ct.	2,097,070*	366,987*	17.50	¶
Jan. 1, 1923	Feb. 7, 1926	Monroe Gas Light & Fuel Co. v. P.U. Comm.	Mich.	U.S. Dist. Ct.	350,000*	49,000*	14.00	
Aug. 31, 1925	Oct. 2, 1926	Brooklyn Borough Gas Co. v. Prendergast	N.Y.	U.S. Dist. Ct.	4,867,835*	807,721*	16.59	1,542,630
Dec. 31, 1926	Sept. 28, 1929	Louisville Ry. Co. v. Louisville	Ky.	U.S. Dist. Ct.	21,611,976*	3,101,319*	14.35	‡
Dec. 31, 1927	Feb. 11, 1929	Worcester Elec. Light Co. v. Atwill	Mass.	U.S. Dist. Ct.	11,337,814*	1,590,294*	14.03	435,063
Mar. 31, 1927	Sept. 10, 1929	West Palm Beach Water Co. v. W. Palm Beach	Fla.	U.S. Dist. Ct.	3,349,699*	292,039*	8.72	445,000
May 1, 1930	Oct. 20, 1932	Elko-Lamoille Power Co. v. P.S. Comm.	Nev.	U.S. Dist. Ct.	....	....	12.20	
Sept. 1, 1931	Jan. 5, 1933	Wichita Gas Co. et al. v. P.S. Comm.	Kans.	U.S. Dist. Ct.	60,007,606†	9,719,405†	16.20	‡

[1] Data collected mainly by Prof. J. C. Hempstead, Iowa State College.

* New.

† Depreciated.

‡ No data of land value available.

§ Including $1,543,030 additions June 1, 1923 to Aug. 31, 1925.

¶ Land included in overhead base.

TABLE 11.10.—APPROXIMATE APPROVED OR IMPLIED ALLOWANCES MADE IN VALUATION COURT DECISIONS FOR DIFFERENT CLASSES OF OVERHEAD COSTS[1]

Date of decision	Valuation court case (see Table 11.9)	Engineering, percent	General overheads*				Omissions and contingencies, percent	Interest during construction, percent	Total overheads, percent
			Administration* and miscellaneous, percent	Legal, percent	Taxes, percent	Total general, percent			
Mar. 4, 1918	Denver v. Denver Union Water Co.			Total. . . .	12.50 %		.	9.43	21.93
Apr. 17, 1929	Vincennes Water Supply Co. v. P.S. Comm.	4.44	2.28	0.63	2.03	4.94	. . . .	6.33	15.71
Dec. 31, 1924	Westinghouse Elec. & Mfg. Co. v. Denver Tramway Co.	5.18	4.49		1.74	6.23	. . . .	8.89	20.30
Apr. 30, 1925	Kings Co. Lighting Co. v. Prendergast	5.00	2.00		0.40†	2.40	. . . .	6.64	14.04
Nov. 24, 1925	N.Y. & Richmond Gas Co. v. Prendergast	9.86	2.14		0.02	2.16	0.85	4.52	17.39
Jan. 7, 1926	Middlesex Water Co. v. Bd. of P.U. Commrs.	4.00	1.50		1.50†	3.00	3.50	7.00	17.50
Feb. 11, 1929	Worcester Elec. Light Co. v. Atwill	5.00	1.06	0.22	0.40	1.68	. . . .	7.35	14.03
Jan. 5, 1933	Wichita Gas Co. v. P.S. Comm.	5.54	1.75		1.46†	3.21	. . . .	7.45	16.20

[1] Data collected mainly by Prof. J. C. Hempstead, Iowa State College.

* "Injuries and damages" not indicated separately; doubtless included in "Administration and miscellaneous."

† Included in "Administration and miscellaneous."

appear in Table 11.10. Only one allowance of more than 5 percent for general overheads and only one of more than 6 percent for engineering appear in Table 11.10.

It is obvious that correct overhead allowances for any particular property must be determined by careful study of its own records and circumstances; the allowances which have been approved by the courts for other properties can be of help only to such extent as similarity of conditions may justify.

FINAL DETERMINATION OF THE FAIR COST-VALUES OF ALL FIXED-CAPITAL PHYSICAL-PROPERTY UNITS EXCEPT LAND

After separate determination[1] has been made of the original-cost and reproduction-cost values of all fixed-capital physical-property units except land, their fair present cost-values must also be determined, giving "such weight as may be just and right in each case"[1] to "the present as compared with the original cost of construction."[2]

11.27. Weights Due Original- and Reproduction-cost Value in Determining Fair Cost-value.—The present original-cost values of physical-property units are equal to the present investments (Secs. 1.14, 1.16, 11.1) therein; the present reproduction-cost values of the same units are equal to what the present investments (Secs. 1.14, 1.16, 11.1) therein would be if the units had been installed, when new, at the prices now used in estimating their reproduction-costs new.

It is the law of the land that the present original-cost values and the present reproduction-cost values of physical-property units shall both always be given due consideration; to each, such weights as may be just and right in each case must be accorded in determining fair present cost-values. A detailed discussion of what weights are just and right under different circumstances will be found in Secs. 7.11, 7.15, and 7.16.

11.28. Determination of Fair Cost-value of Physical Property.—Distinction should be made between the determination of the total fair present (depreciated) cost-value of the entire physical property (except lands), and the determination of the separate cost-values (new and depreciated) of the respective property units thereof.

1. *The Entire Fixed-capital Physical Property (except Lands).*—By applying the principles discussed in Sec. 11.27, direct determination may be made of the total fair present (depreciated) cost-value of the entire fixed-capital physical property (except lands). This has been the usual custom heretofore.

2. *The Individual Physical-property Units.*—By applying the principles discussed in Sec. 11.27, determination may be made of the present fair cost-values (new and depreciated) of each separate unit of the fixed-capital physical property (except lands). The authors believe this to be feasible, and that it is the better practice.

[1] By the methods explained under the third and fourth center heads of this chapter.
[2] See *Smyth v. Ames*, Sec. 8.3.

The total fair present (depreciated) cost-value will then be the sum of the depreciated cost-values of the separate units; the sums of the accrued actual depreciations (total and annual) of the separate units will be the total actual depreciations (total and annual) of the entire property, estimated on the basis of fair present value, as required by the decision of the United States Supreme Court in the *Baltimore Street Railway Case* decision (Sec. 8.6).

As stated in Sec. 6.6, by readjusting the fair cost-values new of the several property units from time to time during their service lives (and readjusting their total accrued actual depreciations accordingly), it is feasible to maintain depreciation reserves just sufficient to renew all units at their dates of actual retirements by identical units installed at the then prevailing prices, thus constantly keeping the property unwasted without obtaining new or retiring old capital.

CHAPTER XII

THE INTANGIBLE VALUES KNOWN AS PRELIMINARY-EXPENSE AND GOING VALUE

To determine the total cost-value of a property, the values of its intangible elements must be ascertained and added to its physical values.[1] The intangible values of an industrial property are (1) preliminary-expense value; (2) going value; (3) good-will value; (4) "other-intangibles" values. Of these, (1) and (2) will be treated in this chapter.

Although the intangible values of industrial properties are just as real as their physical values, intangible values are much less evident and definite; care in determining them must always be exercised to avoid estimates based on conjectural and speculative reasoning. It is customary to determine the proper allowance for each kind of intangible value by itself, giving due weight to original-cost and reproduction-cost price levels.

PRELIMINARY-EXPENSE VALUE

The inception of industrial enterprises almost always involves some necessary and legitimate preliminary expenditures for organization and promotion, in addition to expenditures for the planning and construction of units of physical property.

12.1. Definition of Preliminary-expense Value.—The preliminary-expense value of an industrial property is the fair present capital allowance which should be made for the actual, necessary, legitimate expenses incurred for the organization and promotion of the enterprise.

Necessary franchise costs, and necessary, customary, and fair costs of selling the securities, are to be included. All unnecessary, imprudent, and/or not legitimate preliminary expenditures must be excluded. Changes in price levels and other conditions since the expenditures were actually made must be given due consideration.

12.2. Organization Expenses.—Organization expenses, as herein defined, are those incurred in the inception and organization of the enterprise, as distinguished from the expenses of selling its securities.

Organization expenses include only those for the actual time and actual necessary expenses of the organizers, and for all other necessary organization costs; including professional fees for general engineering and legal advice, incorporation fees and other costs, and franchise costs when a franchise is required.

[1] This is as stated in paragraph 8 of the outline, in Sec. 1.15, of the general method of making an engineering valuation.

12.3. Franchise Costs.—Only the actually paid, necessary costs of securing a franchise should be included in allowances for organization expenses. Franchise fees, if charged, and legal fees for drawing up and recording the franchise are examples of allowable franchise costs.

Franchise Value.—In general, all allowances for franchise value, as distinguished from franchise cost, are disallowed by the courts in public-utility valuation litigations. See *Pacific Gas & Electric Co. v. San Francisco.* U.S. Sup. Ct. (Sec. 8.5). Because of special circumstances, a franchise may in rare cases have the nature of a contract, whose value should be determined and allowed as in the case of other contracts. See *Newton v. Consolidated Gas Co.* U.S. Sup. Ct. (Sec. 8.5); and *Gilchrist et al. v. Interborough Rapid Transit Co.* U.S. Sup. Ct. (Sec. 8.6).

12.4. Promotion Expenses.—Promotion expenses, as herein defined, are those incurred in promoting selling, and in selling, the securities of the enterprise to secure the capital required for the construction and development of the property.

Promotion expenses include only wise, necessary, actual general management, office, and advertising expenditures, plus fair, customary brokers' fees and/or salesmen's commissions. Financing costs greater than those customary and fair must not be capitalized.

12.5. Historical and Reproduction Organization and Promotion Expenses.—Both historical and reproduction costs must be given proper consideration and due weights in determining correct fair allowances for preliminary-expense values.

Historical organization and promotion expenses are those actually incurred preliminary to construction, but not including the costs of engineering plans and specifications, plus similar later expenditures at the times of enlargements of the property, reorganizations of the enterprise, and sales of additional securities. All historical organization and promotion expenses should be items of record in the book accounts. Omissions must be supplied only with great caution. In the cases of reorganizations, and sales of refunding securities, former allowances for preliminary-expense values should be depreciated sufficiently to keep the total allowance from exceeding a fair total for the *present* property alone.

Reproduction organization and promotion expenses are those which would be necessarily incurred by a new corporation, organized at the present time, to construct the present property under present conditions. Estimates of reproduction organization and promotion expenses should be based on the actual expenses of actual similar enterprises, modified only as may be necessary to conform to the conditions of the particular property, and to present price levels and business conditions. Conjectural and/or speculative estimates must be avoided.

12.6. Discounts and Premiums.—Discounts and/or premiums on securities sold must not be capitalized. In effect, they are increases or decreases, respectively, in the actual interest and/or dividend rates yielded, as compared with the "nominal" rates specified. Interest and dividends are part of the net returns from the property, year to year, to be paid out of actual yearly net earnings.

Discounts on bonds and/or similar obligations, having definite maturity dates, should be amortized over their respective lives; the amortization payments should be made out of the net returns from the property. In some instances, such equal, annual amortization payments for this purpose as are actually made during construction, before operation begins, may be capitalized as "interest lost during construction."

12.7. Unamortized Expenses.—Frequently claims which would be rejected under the principles set forth in Secs. 12.1 to 12.6, above, are made for large intangible-value allowances based on "unamortized financing costs and other unamortized debits." Examples are the asset balance-sheet items on page 26 for: "Brokerage on preferred stock of predecessor companies," "premiums and expenses of retirement of funded debts of predecessor companies," and "excess of predecessor companies' securities over book value of property acquired therefor." Sometimes claims running into millions of dollars are made for "promoters' remuneration."

Financing costs of the above character have frequently been incurred by holding companies and otherwise, in effecting large consolidations of formerly independent operating companies; they may be warranted by the large profits at stake. Thus, in the case of an enterprise 60 percent of the capital for which was obtained by the sale at par of 5 percent bonds, 15 percent by the sale at par of 6 percent preferred stock, and only 25 percent by the sale of common stock, the owners of the common stock will realize 16.4 percent on their own contribution to the capital if the enterprise earns 8 percent fair net return.

In the case of *Los Angeles Gas & Electric Corp. v. Railroad Commission of California et al.*,[1] the company claimed intangible allowances of $5,921,470, for cost of financing, and $2,500,000 for promoters' remuneration; both of these claims were rejected by the United States Supreme Court, as "too conjectural to be allowed."

In the opinion of the authors, financing costs of the character discussed in this section, even if really incurred, should be amortized by the owners themselves, out of their fair net returns on property values which exclude allowances for such costs.

12.8. Preliminary-expense Values and Physical Values.—The actual preliminary-expense values allowed in a considerable number of litigated cases are shown in Table 12.4 in percents of present physical-property values. Such data will be of some use as showing what preliminary-expense allowances are actually made under normal conditions.

DEFINITIONS AND ILLUSTRATIONS OF GOING VALUE

The principle that an assembled plant, already earning an established income, has a real going value which must be allowed in addition to its

[1] See case 66, Sec. 8.7. The commission's engineer had allowed only $429,126 for "organization and franchises."

other values in both sales and rates valuations, has been well settled in law for more than forty years by many court decisions; but the principle is so difficult to elucidate clearly, and apply, that even yet opinions often conflict greatly on going value and its correct determination.

12.9. Definition of Going Value.—Going value is the "element of value in an assembled and established plant, doing business and earning money, over one not thus advanced" (Sec. 12.11).

The United States Supreme Court (Sec. 12.11) has declared it to be self-evident that there is such an element of value.

Note.—Going value is sometimes termed "going-concern value"; it must not be confused with "good-will value," from which it is entirely distinct.

12.10. Illustrations of the Going Values of Properties.—A simple illustration of going value is that of a new residence, already rented and yielding a net income above all expenses plus depreciation of, say, $36.50 per month. Any would-be investor in a residence to rent could buy an equally valuable lot and build thereon a duplicate house at a total cost of, say, $5,000; but it would take, say, six months to do this and rent the property. Evidently an investor would gain six months' net rent by buying the residence already built and rented, whose going value would therefore be about $139 (deducting $79 for difference in 6 percent interest on purchase price and on construction costs, adding $3 interest on rent, and dividing by 1.03 to get present worth).

Note.—If the residence considered were not new, its depreciation would have to be taken into account.

Another illustration of going value is that of an existing assembled electric utility, having a well-established income sufficient to yield about $24,000 (approximately 7 percent) annual net return, above all expenses plus depreciation. Upon investigation by competent experts, giving due consideration to the utility's history and circumstances, including its outlook for the next three years, it appears that it would require one year to "assemble" a duplicate plant; during this year, a purchaser of the established plant would receive $24,000 net return; the builder of a duplicate plant would be reimbursed $18,076 for interest during the construction year, at 6 percent, partly by earnings on capital before paid out and partly by an overhead cost allowance for "interest lost during construction."

It further appears that, after a duplicate plant was assembled, it probably would secure enough business to pay expenses plus depreciation, and in addition yield $4,000 net return during the first and $16,000 during the second year of operation; after this, it would earn the same income as the established plant.

Under these circumstances, the established income of the existing utility would yield net returns in excess of the net returns probably yielded by a duplicate plant, yet to be assembled and established, as follows:

First year hence	$ 5,924	greater profit; present worth	$ 5,536
Second year hence	20,000	greater profit; present worth	17,469
Third year hence	8,000	greater profit; present worth	6,531

Hence, the going value of the assembled and established plant over one not thus advanced is about $29,500

GOING-VALUE LEGAL PRINCIPLES

The United States Supreme Court (1933) (Sec. 12.11) has said that the principle of going value "is obviously difficult of application." Because of this difficulty, various really consistent court decisions in cases involving going values have seemed contradictory. The latest, best, and most authoritative exposition of going-value legal principles is contained in the decision of the United States Supreme Court in the *Los Angeles Gas Case,* May 8, 1933, as quoted at length in Sec. 12.11.

12.11. The United States Supreme Court on Going-value Legal Principles.—*Los Angeles Gas & Electric Corp. v. Railroad Commission of California et al.* 53 U.S. Sup. Ct. Rep. 637. U.S. Sup. Ct., May 8, 1933. Opinion written by Mr. Chief Justice Hughes. Only that part of the decision discussing the general legal principles of going values quoted below.

8. As an item additional to the estimates of value thus far considered, the company claims to be entitled to an allowance of $9,228,667 for "going value." This court has declared it to be self-evident "that there is an element of value in an assembled and established plant, doing business and earning money, over one not thus advanced," and that this element of value is "a property right" which should be considered "in determining the value of the property, upon which the owner has a right to make a fair return." *Des Moines Gas Co. v. Des Moines,* 238 U.S. 153, 165, 35 S. Ct. 811, 815, 59 L. ed. 1244; *Denver v. Denver Union Water Co.,* 246 U.S. 178, 191, 192, 38 S. Ct. 278, 283, 62 L. ed. 649; *McCardle v. Indianapolis Water Co., supra,* 272 U.S. 414, 47 S. Ct. 144, 71 L. ed. 316. The going value thus recognized is not to be confused with good will, in the sense of that "element of value which inheres in the fixed and favorable consideration of customers, arising from an established and well-known and well-conducted business," which, as the court has repeatedly said, is not to be considered in determining whether rates fixed for public service corporations are confiscatory. *Des Moines Gas Co. v. Des Moines, supra.* See *Willcox v. Consolidated Gas Co.,* 212 U.S. 19, 52, 29 S. Ct. 192, 53 L. ed. 382, 48 L.R.A. (N.S.) 1134, 15 Ann. Cas. 1034; *Cedar Rapids Gas Co. v. Cedar Rapids,* 223 U.S. 655, 669, 32 S. Ct.

389, 391, 56 L. ed. 594; *Galveston Electric Co. v. Galveston, supra,* 258 U.S. 396, 42 S. Ct. 351, 66 L. ed. 678. Nor does this recognition of going value countenance a mere attempt to recoup past losses. *Galveston Electric Co. v. Galveston, supra,* 258 U.S. 394, 395, 42 S. Ct. 351, 66 L. ed. 678. Deficits in the past do not afford a legal basis for invalidating rates, otherwise compensatory, any more than past profits can be used to sustain confiscatory rates for the future. *Board of Commissioners v. New York Telephone Co.,* 271 U.S. 23, 31, 32, 46 S. Ct. 363, 70 L. ed. 808. The concept of going value is not to be used to escape the just exercise of the regulatory power in fixing rates, and, on the other hand, that authority is not entitled to treat a living organism as nothing more than bare bones.

The principle as thus recognized and limited is obviously difficult of application. *Cedar Rapids Gas Co. v. Cedar Rapids, supra.* It does not give license to mere speculation; it calls for consideration of the history and circumstances of the particular enterprise, and attempts at precise definition have been avoided. It is necessary again, in this relation, to distinguish between the legislative and judicial functions. It is the appropriate task of the commission to determine the value of the property affected by the rates it fixes, as that of an integrated, operating enterprise, and it is the function of the court in deciding whether rates are confiscatory not to lay down a formula, much less to prescribe an arbitrary allowance, but to examine the result of the legislative action in order to determine whether its total effect is to deny to the owner of the property a fair return for its use.

Thus, in *Cedar Rapids Gas Co. v. Cedar Rapids, supra,* this court noted that, in the decision under review, the fact "that the plant was in successful operation," had expressly been taken into account and that a value had been fixed which "considerably exceeded its cost," and hence the court found no warrant for changing the result. In *Des Moines Gas Co. v. Des Moines, supra,* the court, dealing with the master's report and the exclusion of a special item for going value, observed that the master, "applying the rule of the *Cedar Rapids Case*" had already valued the property in the estimate of what he called its physical value, upon the basis of a plant in actual and successful operation." As the master had included overheads at 15 percent in that valuation, in addition to organization expenses, the court was unable to hold that "the element of going value" had not been given the consideration it deserved. In *Denver v. Denver Union Water Co., supra,* the court, premising that "each case must be controlled by its own circumstances," pointed out that the master had "expressly declared that his detailed valuation of the physical property and water rights included no increment because the property constituted an assembled and established plant, doing business and earning money," and that an examination of his elaborate report convinced the court that this was true. And in that case the court found that the return allowed by the ordinance in question was clearly confiscatory. In *Lincoln Gas Co. v. Lincoln, supra,* 250 U.S. 267, 268, 39 S. Ct. 454, 63 L. ed. 968, the court questioned the propriety of the master's treatment of going value, but, noting compensatory errors in favor of the complainant, could not conclude that the master was wrong in holding that the ordinance was not shown to be confiscatory. In *Galveston Electric Co. v. Galveston, supra,* the court took occasion

to say that the expressions in the *Denver Case* and in the *Lincoln Case* were not to be taken as modifying in any respect the rule declared in the *Des Moines Case* as to the exclusion of good will. In *Georgia Railway & Power Co. v. Railroad Commission of Georgia, supra,* the finding below as to going value was not disturbed. In *Bluefield Water Works Co. v. Public Service Commission, supra,* while 10 percent had been added for going value, the total result was a valuation which could not be sustained. In *McCardle v. Indianapolis Water Co., supra,* where the rates were held to be confiscatory, the court found that the evidence was "more than sufficient to sustain 9.5 per cent for going value" and that the commission's engineer had made no appraisal of that element.

12.12. Summary of Going-value Legal Principles.—The going-value legal principles so ably stated by Mr. Chief Justice Hughes in Sec. 12.11, above, may be summarized as follows:

1. The United States Supreme Court "has declared it to be self-evident 'that there is an element of value in an assembled and established plant, doing business and earning money over one not thus advanced.'" This element of value is what is termed "going value" herein.

2. Fair going-value allowances must be included in valuations made for sales or condemnations purposes. No serious attack seems to have been made upon this principle; which was enunciated as early as 1894 by Mr. Justice Brewer of the United States Circuit Court of Appeals, in the *Kansas City Water Works Case.*[1] The principle was upheld later by the U.S. Supreme Court in *Omaha v. Omaha Water Company,* 218 U.S. 180, May 31, 1910. In its decision, Mr. Justice Lurton said: "The difference between a dead plant and a live one is a real value. . . . "

3. Fair going-value allowances must also be included in valuations made for rate-making purposes. Although questioned at much length by Whitten and Wilcox,[2] this principle has been upheld repeatedly by the United States Supreme Court, in the cases cited by Mr. Chief Justice Hughes (Sec. 12.11) in the *Los Angeles Gas Case* decision in 1933; in which he reiterates that: "This court has declared it to be self-evident that . . . this element of value is a 'property right' which should be considered 'in determining the value of the property, upon which the owner has a right to make a fair return.'"

4. Going value must not be confused with good-will value; from which it is entirely distinct. Good-will values are not usually allowable in public-utility valuations.

5. The determination of a fair allowance for the going value of a particular property requires due consideration of the "history . . . of the particular enterprises"; but recognition of going value does not countenance "a mere attempt to recoup past losses."

6. The determination of a fair allowance for the going value of a particular property also requires due consideration of the "circumstances of the particular enterprise." These circumstances doubtless include recent, present, and probable near future incomes, operation costs, depreciations, price trends, and business conditions; and even the estimated probable costs of developing the established business during a reasonably limited, immediately future time; but it must always be remembered that the going-value principle "does not give license to mere speculation."

[1] See case 10, Sec. 8.3.

[2] See "Valuation of Public Service Corporations," The Banks Law Publishing Co., New York, 1928, Chap. XXIX.

7. The courts have not laid down any "formula" for determining fair going-value allowances; these must be determined by sound judgment in each case, based upon due "consideration of the history and circumstances of the particular enterprise."

8. It is the function of the courts to determine whether the *total* valuation of the property made by a valuation or regulatory body is or is not in error sufficiently to render the proposed valuation and/or rates confiscatory. Thus the courts may refuse to decide particular valuations and/or rates to be confiscatory, even when the proposed allowances for going value are entirely wrong. In the *Los Angeles Gas Case,* for example, the United States Supreme Court upheld the rates ordered by the California Railroad Commission, although at the same time holding that the commission was in error in not (avowedly) allowing any going value at all; and also in not (avowedly) conforming in other important respects to the law of the land on valuations of public-utility property as repeatedly upheld by the Court.

AIDS TO JUDGMENT IN DETERMINING GOING VALUES

Although fair allowances for going values can be determined only by sound judgment, based upon due consideration of the "history and circumstances" of each particular case, and although the going-value principle "is obviously difficult of application," it also seems obvious that judgment must not be exercised blindly. There are some systematic methods of arranging and studying the data of the history and circumstances of particular enterprises which will aid materially in giving them the due consideration called for by law; and in forming sound judgments based upon them.

12.13. The Best Measure of Going Value.—The Supreme Court (Sec. 12.11) of the United States "has declared it to be self-evident 'that there is an element of [going] value in an assembled and established plant, doing business and earning money, over one not thus advanced'"; it seems to the authors that it must also be self-evident that the best measure of going value is the excess of the present worths of the probable near-future net returns from the income of an assembled and established plant over the present worths of those from the same income yet to be developed by a duplicate plant "not yet thus advanced."

12.14. Simple Estimates of Present Worth of Probable Income.—The excess of the present worths of the probable near-future returns from the income of an "assembled and established plant, doing business and earning money, over those from the same income yet to be developed by a plant not thus advanced," may often be approximated, as nearly as the history and the circumstances of the enterprise warrant, by comparatively simple methods, such as those illustrated in Sec. 12.10, above.

12.15. Formal Estimates of Present Worth of Probable Income.—It is often advisable to arrange the data of the "history and circumstances" of particular enterprises in some formal, systematic way, on diagrams and in tables, so as to make their relations to each other and to

probable near-future price trends and business conditions more readily and clearly understandable. Figure 12.1 and Table 12.1, below, are examples of such a diagram and such a table.

In general, studies of the going values of particular properties are made in connection with their valuations; so that prior determinations may be presumed of the present original-cost and reproduction-cost values, and the fair cost-values of their fixed-capital physical-property units, as described in Chap. XI. With the data of such determinations available, going-value studies may well proceed about as follows:

1. The "assembled and established plant, doing business and earning money," herein designated Plant E, is the actually existing and operating plant which is being valued.

2. The "plant not thus advanced," herein designated Plant D, is a duplicate (depreciation and all) of the actually existing and operating plant which is being valued, but is yet to be assembled; after it is assembled it has yet to develop a business duplicating the established business of the plant which is being valued.

3. The construction period required to assemble Plant D is the construction period already adopted and used in determining the present reproduction-cost value of the fixed-capital physical-property units of Plant E (Chap. XI).

4. The development-of-business period must be the least reasonably feasible time for Plant D, after assembled, to develop a business duplicating the business of Plant E; the principle of going value "does not give license to mere speculation." In determining the length of time required, the present character of the customers must be presumed, including their present desires and needs for the commodities, and/or services, supplied by the existing plant.

5. Past and Probable Near-future Plant E Incomes and Operating Expenses.—These may be forecast by diagramming the past data of the existing Plant E as in Fig. 12.1, and then extending the income and expense curves through the next few future years, giving careful consideration to past plant changes (such as enlargements); to probable population number and character changes; and to actual past and present and probable near-future price trends and business conditions.

6. Probable Near-future Plant E Annual Depreciations.—These should be "actual" depreciations, estimated by the methods of Chap. V. Legally, the depreciation should be based on the fair cost-values new of the inventory units; actually, reproduction-cost depreciations may most often be sufficiently nearly correct.

7. Probable Plant D Incomes during the Construction Period.—Usually, the only Plant D income during the construction period will be the interest throughout it on the capital required for the construction of the fixed-capital physical-property units of duplicate (depreciation and all) Plant D. This interest may actually be earned in part on funds kept otherwise invested part of the time, and/or deposited in banks; interest not so earned is allowed in the valuation of the physical property, as "interest lost during construction."

The rate of interest used should be that at which an investor with good credit can get money on secured paper. Six percent is most customary.

Note.—In cases where the construction period is quite long, the investor may sometimes leave part of the capital, part of the time, earning returns higher than "interest rates" in other investments.

The *capital* required is the total present (depreciated) reproduction-cost value (or present fair cost-value) of the fixed-capital physical property, less interest lost during construction (which does not have to be paid in money).

8. Probable Plant D Incomes and Operation Expenses during the Development-of-business Period.—The starting points for the Plant D income and operation-expense curves in Fig. 12.1 must be obtained by making careful detailed analyses of the actual present income and operation expenses of the existing and operating Plant E. Usually, some important income and expense items would start in full when Plant D started operating. Intelligent forecasts must be made for the other items.

The Plant D income and expense curves of Fig. 12.1 are drawn through the starting points thus obtained; they must be tangent to the corresponding Plant E curves at the end of the development-of-business period.

Intermediate points on the curves should be checked by studying the probable rates of increase, year by year, of the separate items of the actual, existing Plant E income and expense accounts.

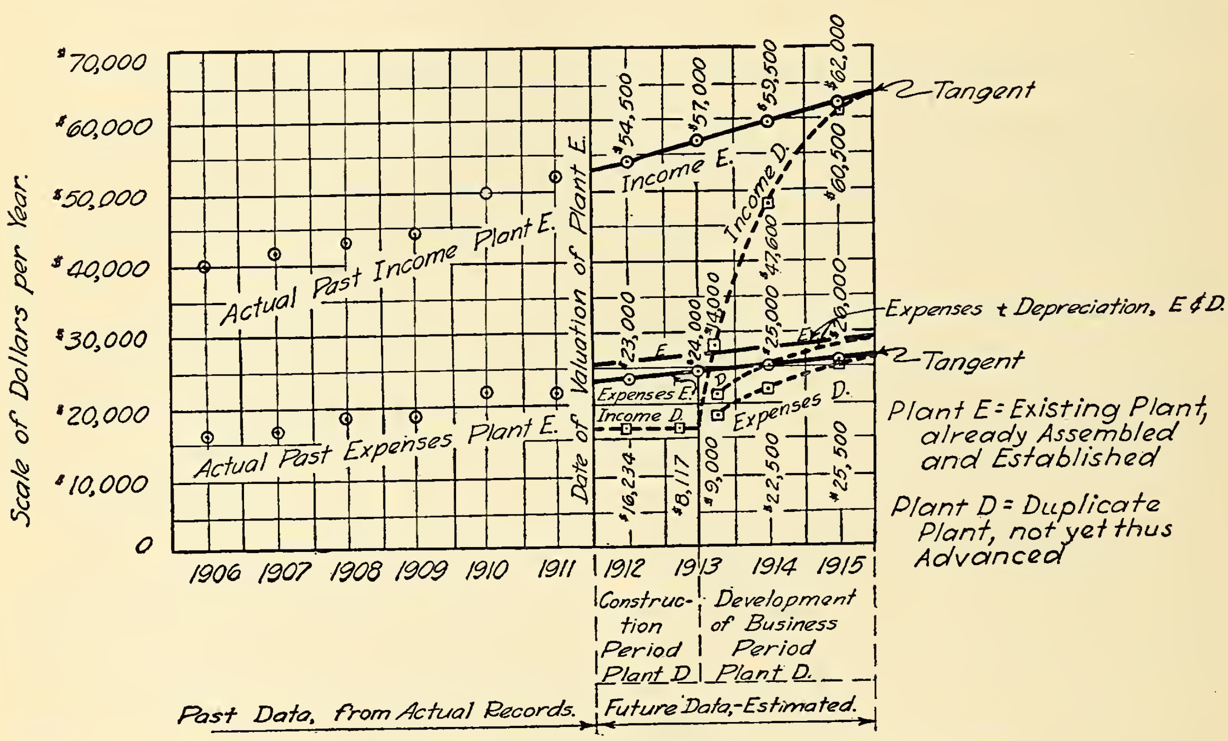


Fig. 12.1.—Diagram illustrating going-value calculations.

9. Plant D Annual Depreciations.—Plant D has no depreciation during the construction period. During the development-of-business period, the annual depreciations of duplicate Plant D are the same, year by year, as those of the actual, existing Plant E.

10. Tabulation of Final Going-value Calculations.—The data of the "history and circumstances" of the enterprise, studied as above and diagrammed in Fig. 12.1, may be arranged to advantage in some such form for final calculation as is illustrated in Table 12.1.

12.16. Example of Formal Estimate of Present Worth of Income.—An example is given in this section of the application of the method described in Sec. 12.15 to an actual waterworks valuation, as of Jan. 1, 1912, made by a valuation board of three men, on which one of the authors served.

1. Plant E was a waterworks plant which had been in operation about 29 years, serving a Middle-west manufacturing city of about 18,000 population.

2. Plant D was a duplicate (depreciation and all), but not yet assembled, of Plant E.

3. The construction period extended from Jan. 1, 1912, to July 1, 1913. (Fig. 12.1).

4. The development of business period extended from July 1, 1913, to Dec. 31, 1915 (Fig. 12.1).

5. For data of Plant E probable incomes and operation expenses Jan. 1, 1912, to Dec. 31, 1915, see Fig. 12.1.

6. and 9. The annual depreciations of Plant E, Jan. 1, 1912, to Dec. 31, 1915, and of Plant D, July 1, 1913, to Dec. 31, 1915, as shown in Fig. 12.1, were taken from the reproduction-cost valuation studies, extended to include 1912 to 1915.

7. The Plant D income during the construction period was the interest at 6 percent on $270,572. The present (depreciated) reproduction-cost value (not including going value) was $281,395; $10,823 of this was for "interest lost during construction," and did not have to be paid in money.

8. Plant D incomes and expenses during the development-of-business period. Detailed analyses of the present actual income and expense accounts indicated that

Table 12.1.—Calculations of Excess of Present Worths of Probable Future Net Returns from an Assembled and Established Plant

Year	Yearly accounts	Actual assembled and established Plant E	Duplicate Plant D, not yet thus advanced	Excess of net return of Plant E	Present worths of excess of Plant E net returns
1912	Gross incomes	$54,500	$16,234		At 8%
	Operation expenses	23,000	0		
	Depreciations	2,356	0		
	Net returns	$29,144	$16,234	$12,910	$11,954
1913	Gross incomes	$57,000	$22,117		
	Operation expenses	24,000	9,000		
	Depreciations	2,450	1,225		
	Net returns	$30,550	$11,892	$18,658	$15,996
1914	Gross incomes	$59,500	$47,600		
	Operation expenses	25,000	22,500		
	Depreciation	2,549	2,549		
	Net returns	$31,951	$22,551	$ 9,400	$ 7,462
1915	Gross incomes	$62,000	$60,500		
	Operation expenses	26,000	25,500		
	Depreciation	2,650	2,650		
	Net returns	$33,350	$32,350	$ 1,000	$ 735

Total excess of the present worths of Plant E's net returns equals $36,147.

Plant D would probable earn \$14,000, July 1, 1913, to Dec. 31, 1913, with \$9,000 operation expenses (yearly rates \$28,000 and \$18,000, respectively). The Plant D curves of Fig. 12.1 were drawn through these points and made tangent to the corresponding Plant E curves, Dec. 31, 1915.

10. The data were assembled for final computations as in Table 12.1.

12.17. Estimates of Cost of Establishing the Business.—The United States Supreme Court decided in 1922[1] that going values cannot be based on a capitalization of past deficits. In lieu of this former practice, the estimated future "cost of establishing the business" came to be urged frequently thereafter as a basis for determining going value. Although cost is not value (Sec. 1.16), it may often be pertinent evidence bearing on value.

In *Fort Smith v. Southwestern Bell Telephone Co.* (Sec. 8.6), 1923, a United States District Court upheld the company's claim, based on the estimated "cost of establishing the business," for a going value of \$115,005 (15.54 percent of the claimed reproduction cost new); and District Judge Youmans said that " . . . The above method does not undertake to capitalize past losses." The district court's decree in this case was upheld by the United States Supreme Court, Jan. 25, 1926, in a memorandum decision which did not discuss the going value.

However, it soon began to be apparent that estimates of the "cost of establishing the business" often require, or at least use, so many assumptions as to development "programs," and so many surmises as to elaborately detailed estimated future data, as to justify their characterization by Mr. Chief Justice Hughes (Sec. 8.7), in the *Los Angeles Gas Case* decision, as "too conjectural" to approve.

In the *Southwestern Bell Telephone Co. v. Fort Worth Case*, instituted in 1924 but decided later than the *Fort Smith Case*, the special master and the district court rejected the Fort Smith "cost-of-establishing-the-business" method completely; they reduced the company's claimed \$807,394 going value to \$250,000 (6.4 percent of the other property).

In the recent (1933) *Los Angeles Gas Case*,[2] the company claimed \$9,228,667 going value; this was the cost of establishing the business, as estimated by Mr. Miller, one of its two witnesses on going value (the other was Mr. Luick). In the decision of the United States Supreme Court, Mr. Chief Justice Hughes said, in part, concerning the going value claimed:

An examination of the evidence offered by the company on this subject shows it to be of a highly speculative and uncertain character. . . . The other witness, Mr. Miller, took Mr. Luick's construction program in which the latter had figured the cost of reproduction, and had assumed that there would be turned over to the operating department "one-twentieth of the service mains during

[1] See the *Galveston Electric Co. Case*, Sec. 12.18, and case 33, Sec. 8.5.

[2] See Sec. 12.11, and case 66, Sec. 8.7.

each quarter of the second to sixth years inclusive." Estimating year by year the cost of the business during the construction period, the witness took the difference between 8 per cent interest on the property used and useful during the year and the net earnings estimated to have been received, and the total of these differences with interest, during the period assumed to be required, was taken to represent the cost of securing the present business of the company. This

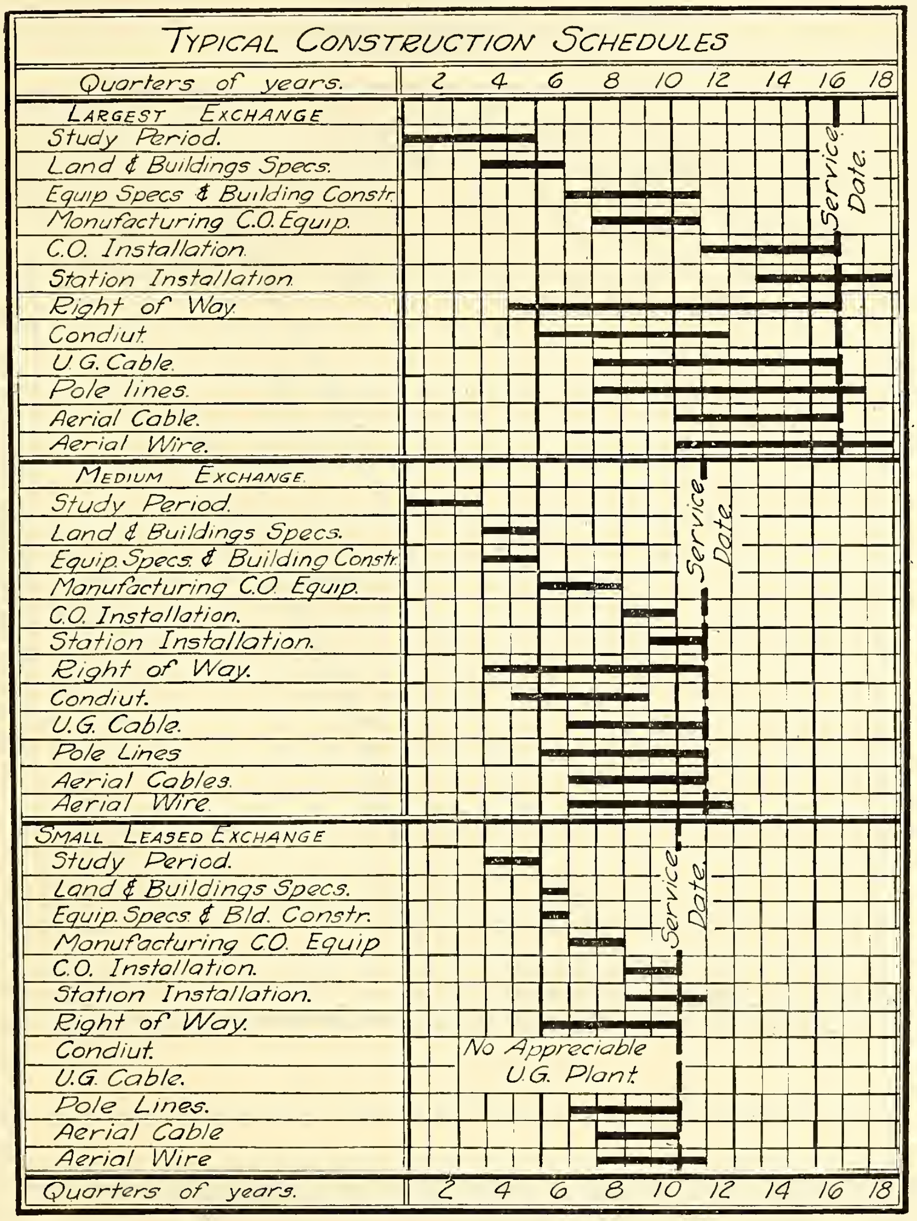


Fig. 12.2.—Construction program for Maine telephone exchanges.

was thus calculated to amount, from the second through the seventh year inclusive, to \$8,721,878. To this sum the witness added, as the estimated cost "of organizing property and personnel," \$506,789, thus reaching the total of \$9,228,667 which the company claims as going value. It is unnecessary to analyze the testimony of these witnesses, as it is obviously too conjectural to justify us in treating the failure to include their estimates as a sufficient basis for a finding of confiscation.

Elsewhere in the decision Mr. Chief Justice Hughes concluded that the California Railroad Commission, without avowedly making any

allowance whatsoever for going value, had really, in effect, allowed \$5,618,235 (about 11.08 percent of present physical value); and that with this allowance the rates questioned could not justly be held confiscatory.

A good illustration of the methods most favored for estimating the cost of establishing the business is furnished by the presentation[1] made to the Maine Public Utilities Commission by the New England Telephone & Telegraph Co., in 1925; *in re* its application for an increase in rates.

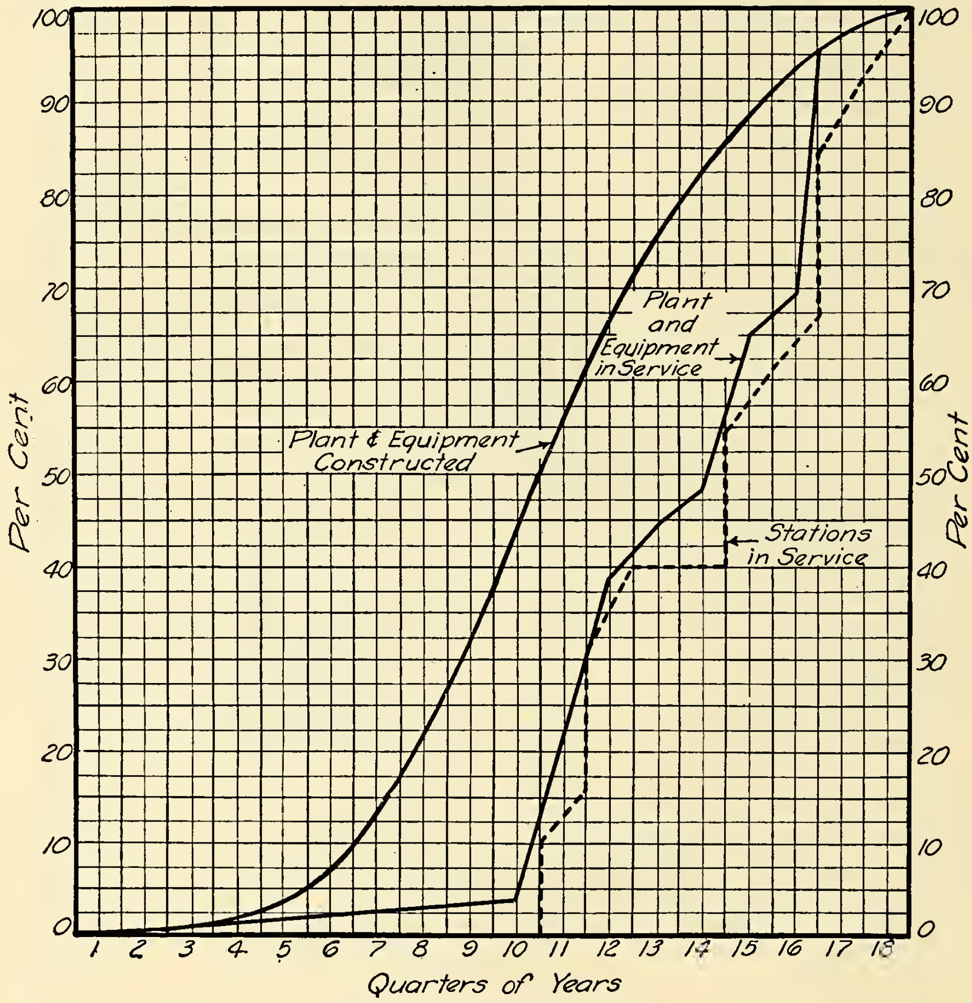


Fig. 12.3.—New England Telephone and Telegraph Company, reproduction program.

The methods and calculations used in this presentation are described briefly below. The table and the cuts are available through the courtesy of the company.

1. A program for constructing and putting into operation all the telephone exchanges owned in Maine was laid out as illustrated in Fig. 12.2. This program extended over a period of four and one-half years; which was divided into 18 "quarters."

2. The cost of the property which would be constructed each quarter was estimated, as also the cost of the property and the number of one-instrument "stations" put into service each quarter; the results are shown, on a percentage basis, in Fig. 12.3.

[1] See Maine Public Utilities Commission, *Re New England Telephone & Telegraph Co.*, P.U.R. 1926 B, p. 247.

TABLE 12.2.—CALCULATIONS OF COST OF ESTABLISHING THE BUSINESS
State of Maine—New England Telephone and Telegraph Co.

Quarters of years	Total average plant and general equipment in service	Working capital	Column I for preceding quarter	Total (A + B + C)	Return at 2% per quarter (0.02 × D)	Total operating expenses	Revenues	Return plus expenses minus revenues	Column H accumulated
	A	B	C	D	E	F	G	H	I
1	$ 44,284	$ 2,560	....	$ 46,844	$ 936	$ 3,839	....	$ 4,775	$ 4,775
2	123,947	5,121	$ 4,775	133,843	2,677	7,880	....	10,557	15,332
3	194,869	7,681	15,332	217,882	4,357	11,127	....	15,484	30,816
4	261,696	14,580	30,816	307,092	6,142	21,976	....	28,118	58,934
5	325,375	24,068	58,934	408,377	8,188	35,724	....	43,912	102,846
6	387,158	27,652	102,846	517,656	10,353	40,976	....	51,329	154,175
7	445,415	31,749	154,175	631,339	12,627	46,437	....	59,064	213,239
8	502,237	38,870	213,239	754,346	15,087	54,370	....	69,457	282,696
9	557,017	41,478	282,696	881,191	17,624	60,843	....	78,467	361,163
10	612,006	53,256	361,163	1,026,425	20,329	78,123	....	98,452	459,615
11	3,910,578	133,140	459,615	4,503,333	90,067	196,091	$ 207,622	78,526	538,141
12	7,318,641	208,415	538,141	8,065,197	161,304	306,634	324,473	143,465	681,606
13	8,399,428	227,874	681,606	9,308,908	186,178	335,560	359,630	162,108	843,714
14	9,124,523	245,285	843,714	10,213,522	204,270	361,947	419,125	147,092	990,806
15	12,271,253	325,169	990,806	13,587,228	271,745	478,092	581,449	168,388	1,159,194
16	13,141,655	370,744	1,159,194	14,671,593	293,432	545,512	708,584	130,360	1,289,554
17	18,371,020	483,913	1,289,554	20,144,487	402,890	711,607	961,328	153,169	1,442,723
18	18,729,568	512,077	1,442,723	20,684,368	413,687	753,867	1,071,992	95,562	1,538,285

The average plant and equipment in service each quarter is shown in column A of Table 12.2, apparently without any deductions for depreciation.

3. The working capital which would be in service each quarter, the total operating expenses, and the total revenues for each quarter were estimated in detail; the results are shown in columns B, F, and G, respectively, of Table 12.2.

The operating expenses, in column F, included $1,146,453 depreciation during the eleventh to eighteenth quarters, inclusive.

4. The cost of establishing the business was then calculated, to the ends of the successive quarters, to be the sums in column I of Table 12.2.

Study of Table 12.2 will show that the final sum in column I is the accumulation, at 8 percent interest compounded four times per year,

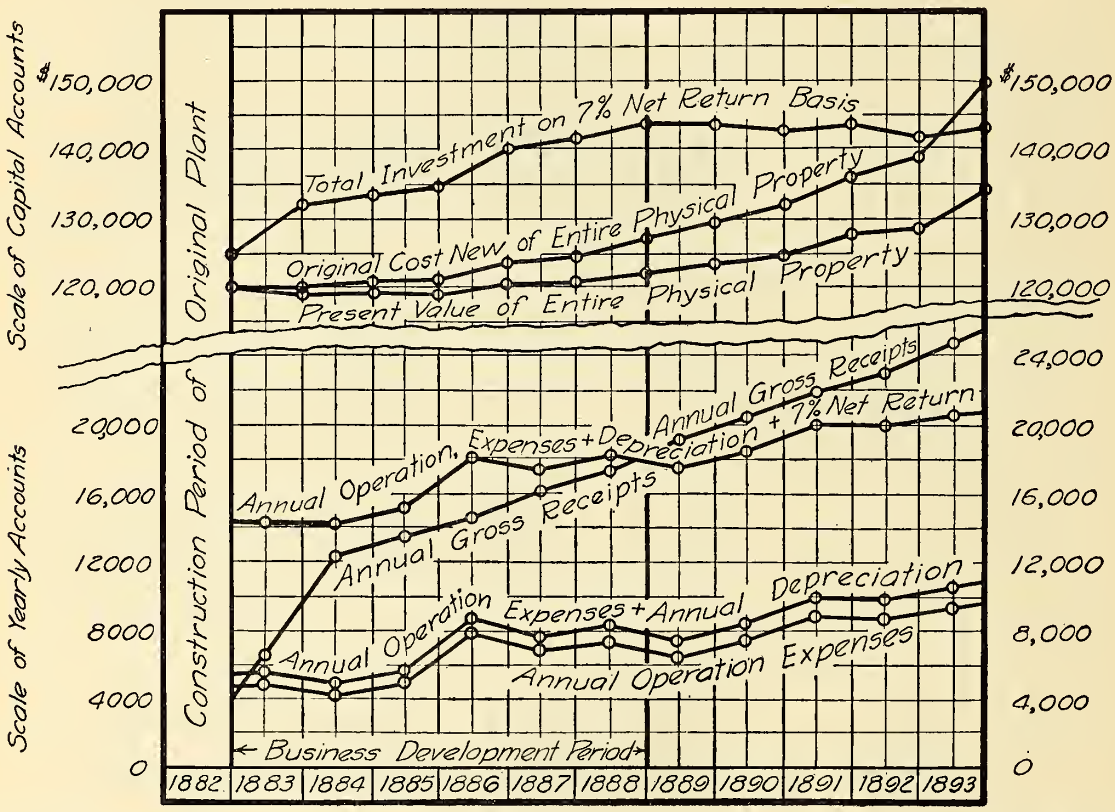


FIG. 12.4.—Illustrating calculation of deficits in the early operation of a waterworks plant.

to a date four and one-half years in the future as referred to the date of valuation, of the differences in the successive quarters between the estimated actual net returns (negative till the eleventh quarter) and 8 percent net return. Calculated at the same rate of interest (8 percent compounded quarterly), the present worth at the date of valuation of the $1,538,285 final sum in column I is only $1,076,882; in estimating the going value it would seem logical to reduce this still further, because depreciation was included in estimating operating expenses but excluded in estimating the values of property in service.

The Maine Public Utility Commission reduced the claimed $1,538,285 going value to $800,000 (5.13 percent of the present fixed-capital physical value).

Table 12.3.—Calculation of the Deficits in the Early Operation of a Waterworks Property

Value of entire physical property new, Jan. 1, 1883 = $120,000

Preliminary expense = $5,000

Fair rate of net return = 7%

Year	Annual operating expenses	Annual deprecia- tion	Gross annual receipts	Annual net return	Addition during year to physical property	Total original cost of physical property, Jan. 1	Present value of physical property, Jan. 1	Total* investment, Jan. 1	Fair return at 7%	Surplus (+) or deficit (−)	Accrued† deficit, Dec. 31
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1883	$4,776	$ 756	$ 6,507	$ 975	......	$120,000	$120,000	$125,000	$ 8,750	−$7,775	−$ 7,775
1884	4,163	786	12,324	7,375	568	120,000	119,244	132,019	9,241	− 1,866	− 9,641
1885	4,937	822	13,580	7,821	548	120,568	119,026	133,667	9,357	− 1,566	− 11,177
1886	7,897	860	14,676	5,919	2,701	121,116	118,752	134,929	9,445	− 3,526	− 14,703
1887	6,739	914	16,190	8,537	1,102	123,817	120,593	140,296	9,821	− 1,284	− 15,987
1888	7,320	958	17,465	9,187	2,415	124,919	120,781	141,768	9,924	− 737	− 16,724
1889	6,431	1,015	19,273	11,827	2,388	127,334	122,238	143,962	10,077	+ 1,750	− 14,974
1890	7,405	1,076	20,455	11,974	2,423	129,722	123,611	143,585	10,051	+ 1,923	− 13,051
1891	8,865	1,140	21,954	11,949	3,877	132,145	124,958	143,009	10,011	+ 1,938	− 11,113
1892	8,723	1,221	23,000	13,056	2,154	136,022	127,695	143,808	10,067	+ 2,989	− 8,124
1893	9,476	1,289	24,687	13,922	7,026	138,176	128,628	141,752	9,923	+ 3,999	− 4,125
Jan. 1, 1894		10,837				145,202	134,365	143,490			− 4,125

* Total investment equals present value physical property, Jan. 1, plus preliminary expense, plus accrued deficit, Jan. 1 (Dec. 31 of preceding year).

† Note that the largest accrued deficit was reached at the end of the sixth year of operation and amounted to 13.47 percent of the present value of the physical property at the beginning of that year.

12.18. Deficits below Fair Return.—In the *Galveston Electric Co. Case*[1] (1922), the United States Supreme Court ruled that

> The fact that a utility may reach financial success only in time or not at all . . . does not make past losses an element to be considered in deciding what the base value is and whether the rate is confiscatory. . . . It is doubtless true, as the master indicated, that a prospective purchaser of the Galveston system would be willing to pay more for it with a record of past losses overcome, than he would if the losses had continued. But would not the property be, at least, as valuable if the past had presented a record of continuous successes?

And in the *Los Angeles Gas Case* (1933) (Sec. 12.11) the same high court has said:

> Deficits in the past do not afford a legal basis for invalidating rates, otherwise compensatory, any more than past profits can be used to sustain confiscatory rates for the future.

Before 1922, it was quite customary for valuation engineers to justify their estimates of going values, in part, by calculations of the past deficits incurred during the original development of business period, immediately after the original plant started operations. The method used was known as the "Wisconsin Method," because originated, or at least much used, by the Wisconsin Railroad Commission; it is sufficiently illustrated by Table 12.3 and Fig. 12.4.

In considering the results of such facts and calculations as are illustrated in Fig. 12.4 and Table 12.3, it must be clearly understood: (1) that *past deficits cannot be capitalized;* (2) that the business developed during the early years of operation of the original plant is not the present business; (3) that the early conditions were not the present conditions; (4) that a comparatively small change in the rate of return used in the calculations will make a large difference in the calculated results (using 8 instead of 7 percent return in Table 12.3 would make the maximum accrued deficit $28,148 instead of $16,724).

ACTUAL ALLOWANCES FOR PRELIMINARY-EXPENSE AND GOING VALUES

The proper allowances for preliminary-expense and going values must be determined by sound judgment, not by formulas; their percentage relation to the other property values will vary with differences in the history and circumstances of different cases. A study of the actual allowances which have been made in valuation decisions will indicate the range within which correct allowances are most likely to vary.

12.19. Preliminary-expense and Going-value Allowances in Court Cases.—In Table 12.4, the authors present their conclusions as to the

[1] See case 33, Sec. 8.5.

actual allowances for preliminary expense and going values approved or implied in 45 court decisions studied and analyzed.

The allowances in Table 12.4 are stated both in amounts and in percentages of the present depreciated values of the fixed physical property; to this element of value, it seems obvious, such allowances are most likely to be roughly proportional.

Table 12.4 shows that separate allowances for preliminary-expense value were made in only 14 of the 45 court decisions analyzed. Apparently, in many cases organization and promotion costs have not been separated from overhead construction costs in the valuations; doubtless clear and complete records of the preliminary expenses actually incurred in starting industrial enterprises have often been lacking.

Correct going-value allowances must always be based on the values of the established incomes of properties, not upon their physical values. However, successful industrial enterprises must develop established incomes which will yield net returns equal to fair percentages of investments which are mainly represented by the present fixed physical property; for this reason, it is easy to see why the data in Table 12.4 show that 28 out of 43 going-value allowances were between 8 and 12 percent of the present depreciated fixed-property physical values of the properties.

Table 12.4.—Approximate Approved or Implied Actual Allowances Made for Preliminary-expense and Going Values in Valuation Litigations[1]

Dates of		Valuation court case	State	Court	Approximate depreciated physical value	Preliminary-expense value		Going value	
Valuation	Decision					Amount	Percent	Amount	Percent
	Jan. 4, 1909	Knoxville v. Knoxville Water Co.	Tenn.	U.S. Supreme Court	$ 461,000	$ 10,000	2.17	$ 60,000	13.02
	May 31, 1910	Omaha v. Omaha Water Co.	Neb.	U.S. Supreme Court	5,700,583	....	....	562,712	9.87
1912	June 14, 1915	Des Moines Gas Co. v. Des Moines	Iowa	U.S. Supreme Court	2,094,005	6,923*	0.33	....	.....
1914	Mar. 4, 1918	Denver v. Denver Union Water Co.	Colo.	U.S. Supreme Court	9,493,596	....	....	800,000	8.43
Dec. 1, 1919	May 21, 1923	S.W. Bell Tel. Co. v. P.S. Comm.	Mo.	U.S. Supreme Court	21,787,227	....	....	2,178,723	10.00
Jan. 1, 1924	June 11, 1923	Bluefield W.W. & Imp. Co. v. P.S. Comm.	W.Va.	U.S. Supreme Court	610,619	18,319	3.00	61,062	10.00
Jan. 1, 1924	June 11, 1923	Georgia Ry. & P. Co. v. Georgia R.R. Comm.	Ga.	U.S. Supreme Court	6,954,545	....	....	695,545	10.00
1923	Jan. 25, 1926	Fort Smith v. S.W. Bell Tel. Co.	Ark.	U.S. Supreme Court	....	....	....	115,005	15.54†
Jan. 1, 1924	Nov. 22, 1926	McCardle et al. v. Indianapolis Water Co.	Ind.	U.S. Supreme Court	17,056,366	....	....	1,620,355	9.50
Jan. 1, 1924	Jan. 2, 1929	United Fuel Gas Co. v. R.R. Comm.	Ky.	U.S. Supreme Court	26,288,644	....	....	3,000,000	11.41
Aug. 16, 1923	Dec. 1, 1930	Smith et al. v. Ill. Bell Tel. Co.	Ill.	U.S. Supreme Court	117,803,128	....	....	4,196,872	3.56
May 1, 1930	Jan. 9, 1933	Wabash Valley Elec. Co. } { Martinsville	Ind.	U.S. Supreme Court	84,589	....	....	8,500	10.05
May 1, 1930	Jan. 9, 1933	v. Young et al. } { entire property	Ind.	U.S. Supreme Court	2,569,395	....	....	250,000	9.73
Nov., 1930	May 8, 1933	Los Angeles Gas & Elec. Corp. v. R. R. Comm. et al.	Calif.	U.S. Supreme Court	50,707,639	429,126	0.85	5,618,235	11.08
1894	July 2, 1894	National W.W. v. Kansas City	Kans.	U.S. Cir. Ct. of App.	2,714,000	....	....	286,000	10.54
May 1, 1928	April 17, 1929	Vincennes Water Supply Co. v. P.S. Comm.	Ind.	U.S. Cir. Ct. of App.	929,422	....	....	81,642	8.78
June 30, 1926	Sept. 22, 1930	Bd. of P.U. Commrs. v. Elizabethtown Water Co.	N.J.	U.S. Cir. Ct. of App.	7,412,713	....	....	756,597	10.21
May 1, 1919	Apr. 21, 1921	Winona v. Wis.-Minn. Light & Power Co.	Minn.	U.S. Dist. Ct.	363,160	....	....	23,289	6.41
1913	June 6, 1921	Pacific Gas & Elec. Co. v. San Francisco	Calif.	U.S. Dist. Ct.	....	....	....	1,500,000	12.00
Sept. 1, 1921	June 4, 1923	Reno Power, Light & Water Co. v. P.S. Comm.	Nev.	U.S. Dist. Ct.	310,229	....	....	16,241	5.24
June 30, 1922	July 18, 1923	Arkansas Water Co. v. Little Rock	Ark.	U.S. Dist. Ct.	2,531,348	27,504	1.09	220,000	8.69
June 1, 1922	Jan. 4, 1924	Colorado Power Co. v. Halderman	Colo.	U.S. Dist. Ct.	8,515,818	25,000	0.29	300,000	3.52
Dec. 31, 1922	Dec. 13, 1924	Westinghouse Elec. & Mfg. Co. v. Denver Tramway Co.	Colo.	U.S. Dist. Ct.	20,275,375	....	....	2,900,000	14.30
Dec. 31, 1921	Dec. 27, 1924	Duluth St. Ry. v. R.R. & Warehouse Comm.	Minn.	U.S. Dist. Ct.	4,344,135	89,075	2.05	350,000	8.06
Dec. 31, 1923	Feb. 27, 1925	Chesapeake & Potomac Tel. Co. v. Whitman	Md.	U.S. Dist. Ct.	27,447,249	....	....	1,085,700	3.96
Dec. 31, 1923	Apr. 30, 1925	So. Bell Tel. & Telegr. Co. v. R.R. Comm.	S.C.	U.S. Dist. Ct.	6,185,000	....	....	815,000	13.18
Dec. 31, 1923	June 30, 1925	Kings Co. Lighting Co. v. Prendergast	N.Y.	U.S. Dist. Ct.	9,800,656	....	....	800,000	8.16
Dec. 31, 1922	Nov. 23, 1925	Springfield Gas & Elec. Co. v. P.S. Comm.	Mo.	U.S. Dist. Ct.	1,103,204	....	....	117,949	10.69

Table 12.4.—Approximate Approved or Implied Actual Allowances Made for Preliminary-expense and Going Values in Valuation Litigations.[1]—(*Concluded*)

Dates of		Valuation court case	State	Court	Approximate depreciated physical value	Preliminary-expense value		Going value	
Valuation	Decision					Amount	Percent	Amount	Percent
Oct. 31, 1924	Nov. 24, 1925	N.Y. & Richmond Gas Co. v. Prendergast	N.Y.	U.S. Dist. Ct.	4,729,683	....	....	644,000	13.62
Sept. 30, 1924	Jan. 7, 1926	Middlesex Water Co. v. Bd. of P.U. Commrs.	N.J.	U.S. Dist. Ct.	2,163,229	31,533	1.46	246,376	11.39
Jan. 1, 1923	Feb. 27, 1926	Monroe Gas, Light & Fuel Co. v. P.S. Comm.	Mich.	U.S. Dist. Ct.	359,000	....	....	36,000	10.00
Dec. 31, 1923	May 10, 1926	United Fuel & Gas Co. v. P.S. Comm.	W.Va.	U.S. Dist. Ct.	26,000,000	....	....	3,000,000	11.54
Aug. 31, 1925	Oct. 2, 1926	Brooklyn Borough Gas Co. v. Prendergast	N.Y.	U.S. Dist. Ct.	7,853,526	....	....	800,000	10.19
June 30, 1924	Apr. 20, 1927	Idaho Power Co. v. Thompson et al.	Idaho	U.S. Dist. Ct.	16,020,302	400,000	2.50	1,500,000	9.36
Jan. 1, 1927	Apr. 17, 1928	Plainfield-Union Water Co. v. Bd. of P.U. Commrs.	N.J.	U.S. Dist. Ct.	3,906,895	....	....	393,105	19.07
Dec. 31, 1927	Feb. 11, 1929	Worcester Elec. Light Co. v. Atwill et al.	Mass.	U.S. Dist. Ct.	12,774,093	119,446	0.94	1,494,461	11.70
Dec. 31, 1926	Sept. 28, 1929	Louisville Ry. Co. v. Louisville et al.	Ky.	U.S. Dist. Ct.	21,228,388	250,000	1.18	750,000	3.53
Mar. 31, 1927	Sept. 10, 1929	West Palm Beach Water Co. v. West Palm Beach	Fla.	U.S. Dist. Ct.	3,816,738	....	....	450,000	11.79
July 1, 1926	Nov. 11, 1929	New York Tel. Co. v. Prendergast	N.Y.	U.S. Dist. Ct.	375,353,760	2,961,792	0.79	8,575,000	2.30
Mar. 31, 1929	Nov. 25, 1930	Michigan Bell Tel. Co. v. Odell et al.	Mich.	U.S. Dist. Ct.	136,232,372	....	....	10,898,590	8.00
May 1, 1930	Oct. 20, 1932	Elko-Lamoille Power Co. v. P.S. Comm.	Nev.	U.S. Dist. Ct.	328,000	....	....	50,000	15.24
Sept. 1, 1931	Jan. 5, 1933	Wichita Gas Co. et al. v. P.S. Comm.	Kans.	U.S. Dist. Ct.	69,727,011	500,625	0.72	5,000,000	7.17
1899	July 19, 1901	Gloucester Water Supply Co. v. Gloucester	Mass.	Mass. Sup. Ct.	501,545	....	....	75,000	14.95
Jan. 1, 1925	June 4, 1927	Follett v. Seneca Water Co., Inc.	N.Y.	N.Y. Sup. Ct.	396,592	6,000	1.51	25,000	6.30
Aug. 31, 1926	Mar. 1, 1929	Erie v. P.S. Comm. et al.	Pa.	Pa. Sup. Ct.	3,417,425	....	....	300,000	8.88

[1] Data collected mainly by Prof. J. C. Hempstead, Iowa State College.

* Organization expense only.

† Percent of physical value new.

CHAPTER XIII

GOOD-WILL VALUE; THE VALUES OF OTHER INTANGIBLES

Preliminary-expense value and going value have been treated in Chap. XII; good-will value and the values of other intangibles complete the list of the intangible elements of the cost-values of properties.

There are two classes of good-will valuations: first, ordinary valuations, of the good wills of commercial and professional businesses, such as those of stores, hotels, restaurants, taxicabs, shops, lawyers, doctors; second, engineering valuations, of the good wills of factories, utilities, and other industrial enterprises.

ORDINARY VALUATIONS OF THE GOOD WILLS OF COMMERCIAL AND PROFESSIONAL BUSINESSES

Recognition of good-will values by the public and by the courts, developed first in connection with transfers of titles to established commercial businesses and to the offices and established practices of lawyers, doctors, and other professional men.

13.1. Legal Definitions and Concepts of Good Will.—Many treatises on valuation, and some court decisions, introduce the subject of good will by referring to Lord Eldon's[1] definition, which dates back to about 1810. It was: "Good will is the probability that the old customers will resort to the old place."[2]

The courts of England and of the United States have handed down a long line of decisions that include some reference to, and definition of, good will. It is perhaps sufficient to say that Lord Eldon's conception has gradually been expanded until good will is now understood to mean

> all that good disposition which customers entertain toward the house of business identified by the particular name or firm, and which may induce them to continue giving their custom to it. . . . Good will must mean every advantage . . . that has been acquired by the old firm in carrying on its business, whether connected with the premises in which the business was previously carried on, or with the name of the late firm, or with any other matter carrying with it the benefits of the business.[3]

[1] Lord Eldon was an eminent English jurist and one time Lord Chancellor.

[2] *Cruttwell v. Lye*, 17 *Ves.* 335, 346.

[3] Vice Chancellor Wood, in *Churton v. Douglas*, Johns. Eng. Ch. 174.

The New York Court of Appeals, in[1] *People ex rel. A. J. Johnson Co. v. Roberts, Comptroller*, April 25, 1899, stated that

In *Barber v. Insurance Co.*, 15 Fed. 312, it was said that "the good will of an established business, which is a common subject of contract, is nothing but the chance of being able to keep the business which has been established. . . . "

The United States Supreme Court (Sec. 12.11) has characterized good-will value as

That element of value which inheres in the fixed and favorable consideration of customers, arising from an established and well known and well conducted business.

When a valuation for sale is being made of a property that is of a type in which there is an element of good-will value, the further question arises as to whether the good will can be transferred with the property. The courts uniformly hold that it *may* be so transferred. If the business continues under the old name and with more or less the same personnel and policy the good will is *likely* to continue. But a sale may be made in such a way that good will *does not* follow the physical property. The firm name may be changed, new offices established, new personnel employed; even though the old line of goods is handled, good-will value might or might not be transferred; if transferred, its continuation *due to the existing good will* is bound to be limited to a comparatively short period of years; *indefinite* continuance must be due to the new owner's conduct of the enterprise.

13.2. The Legal Measure of the Good Wills of Ordinary Businesses. By a long series of decisions, the courts have established one "year's purchase" of good-will profits, in excess of reasonable "interest," as the *unit* of measure of the good-will values of ordinary commercial and professional businesses.

One year's purchase, of good-will excess profits, is the average (usually for three years) annual excess of the actual net profits of the business, above a reasonable rate of interest (usually 6 percent) on the fair value of the property—exclusive of good-will value and of the values of other property elements to which definite portions of the total net profits are attributed.

In its decision in *Von Au v. Magenheimer et al.*,[2] which is often cited in good-will-value litigations, the appellate division of the New York Supreme Court held (Oct. 12, 1906) that "our courts incline" to the following rule:

. . . The value of good will may be fairly arrived at by multiplying the average net profits [above fair interest] by a number of years, such number being suitable

[1] See case 13, Sec. 8.3.

[2] See case 17, Sec. 8.4.

and proper, having reference to the nature and character of the particular business under consideration, and the determination of such proper number of years should be submitted to and determined by the jury as a question of fact. . . .

The numbers of "year's purchase" actually allowed in several good-will litigations have been as follows:

Mellerish v. Kean, Eng. Cts., 1860	1	year's purchase
Page v. Ratliffe, Eng. Cts., 1896	3	year's purchase
Von Au v. Magenheimer et al., N.Y. Sup. Ct., 1908	6	year's purchase
In re Keahon's Estate, N.Y. Surrogate's Ct., 1908	3	year's purchase
In re Ball's Estate, N.Y. Sup. Ct., 1914	3	year's purchase
In re Demarest's Estate, N.Y. Surrogate's Ct., 1914	3	year's purchase
Yellow Taxicab Case, N.Y. Sup. Ct., 1915	3	year's purchase
In re McMullin's Estate, N.Y. Surrogate's Ct., 1915	1⅔	year's purchase

Note.—In all these cases, the so-called good-will values included whatever going values the businesses had established.

THE CORRECT MEASURE OF GOOD-WILL VALUE

Analyzing the concept of good will, and the various court decisions on good-will value, it is evident that the value of good will is due entirely to the probable future net profits attributable thereto, in excess of fair *net* returns.

13.3. The Correct Measure of Good-will Value.—The correct measure of good-will value is the present worth of the probable future net profits in excess, due to present existing good will, of fair net returns. It is customary to estimate the annual amount of such future excess profits at the three-years average of those actually being realized, in normal years.

The probable number of years' continuation, due to present existing good will, of the three-years average excess profits now being realized can only be estimated; in making the estimate, only reasonably good management can be presumed, for duration added by anything better is not due to existing good will; the estimate of probable future years continuation must be made by sound, conservative judgment, based on the history and circumstances of the particular enterprise; obviously, no high degree of accurate preciseness is possible.

Note.—The above "measure of good-will value" differs from the "legal measure of the good wills of ordinary businesses" (Sec. 13.2) in two particulars:

1. The present worth of the future annual excess profits is not equal to their sum, as assumed in the court decisions cited. If the number of "year's purchase" is small, the error may be less than the inevitable uncertainties of the estimates (Sec. 13.4).

2. Good-will excess profits should obviously be those above fair net returns for the particular property, not those above an arbitrary rate of interest.

13.4. The Present Worths of Future Excess Profits Due to Present Existing Good Will.—The probable future excess profits, due to the

present existing good will, should practically always be forecasted as uniform annual excess profits, for a conservatively estimated definite period, stated in years.

Owing to the uncertainties always unavoidable in making such forecasts, any higher degree of refinement of estimates of the future excess profits in different years would be merely speculative and fanciful.

The mathematics of calculating the present worths of future excess profits due to present good will are as follows:

Let f_n = the accumulation of an equal annual year-end payment of $1 continuing for n years at i compound-interest rate (Sec. 5.6).

pf_n = the present worth of f_n.

A = the forecasted uniform annual excess profits for n years, due to the present good will.

By formula (5–3),

$$Apf_n = A\frac{(1+i)^n - 1}{i(1+i)^n} = \text{the good-will value.}$$

NOTE.—The use of one year's purchase, multiplied by n, as the legal measure of the good-will value of ordinary businesses (Sec. 13.2), is equivalent to assuming that $pf_n = n$. How far this is in error may be seen in the following table.

TABLE 13.1.—CORRESPONDING VALUES OF n AND pf_n

All values i, n =	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
For i = 4%, pf_n =	0.96	1.89	2.78	3.63	4.45	5.24	6.00	6.73	7.44	8.11
For i = 6%, pf_n =	0.94	1.83	2.67	3.47	4.21	4.92	5.58	6.21	6.80	7.36
For i = 8%, pf_n =	0.93	1.78	2.58	3.31	3.99	4.62	5.21	5.75	6.25	6.71

ENGINEERING VALUATIONS OF THE GOOD WILLS OF INDUSTRIAL ENTERPRISES

The methods which have been approved by the courts for the valuation of the good wills of ordinary businesses by customary court juries (Sec. 13.2) are not satisfactory for application to highly technical industrial properties; for these, engineering valuations are required, of good will as well as of all other elements of value.

13.5. Ordinary and Engineering Valuations of Good Will.—In ordinary valuations of the good wills of businesses (Sec. 13.2), the juries customary in court procedures determine, as questions of fact, the *amount* of one year's purchase of good-will profits, in excess of customary interest (usually at 6 percent), and the *number* of year's purchase measuring the good-will value. No separate allowance for going value is customary.

In engineering valuations, going values are determined separately from good-will values; our highest court rules (Secs. 12.11, 12.12) that

"the going value thus recognized is not to be confused with good will." In engineering valuations, the rate of fair net return, commensurate with investment risks, is determined for each particular property; it is manifest that the difference between the rate of fair net return and the customary rate of interest is due entirely to investment risk, not at all to good will. In engineering valuations, the true present worth of the estimated future excess profits due to present existing good will is calculated; this eliminates the error (Sec. 13.4) in measuring good-will value by multiplying one year's purchase of good-will excess profits by the number of years a jury estimates they might continue.

13.6. The Good-will Values of Public Utilities.—The United States Supreme Court has repeatedly said (Sec. 12.11):

> That "element of value which inheres in the fixed and favorable consideration of customers, arising from an established and well known and well conducted business," . . . is not to be considered in determining whether rates fixed for public corporations are confiscatory.

In its decision *Willcox v. Consolidated Gas Co.*,[1] the United States Supreme Court gave the reason for excluding good-will value; it was "The complainant has a monopoly in fact, and a consumer . . . will resort to the 'old stand' because he cannot get gas anywhere else."

It undoubtedly is the settled rule that good-will values are not to be included in valuations of public-utility properties for rate-making purposes; the same rule would seem to apply to cases of condemnations for purchase for public ownership. It is possible that the rule might be questioned in cases where it can be shown conclusively that the public utility does not have a "monopoly in fact."

In the purchase of a privately owned public utility by another private owner, it would seem to be clear that a utility which has the active good will of its customers would be of greater value to its present and its prospective owner than if its customers were hostile, or even indifferent.

13.7. Making an Engineering Valuation of Good Will.—From what has been said already in this chapter, it is clear that this process should be about as follows:

1. Determine the total physical and intangible present fair cost-values of the elements of the property—exclusive of good-will value, and the values of other property elements (for example, contracts and patents) to which definite portions of the total annual net returns are attributed.

The cost-values to be thus determined are

a. The present fair cost-value of the fixed-capital physical property (Chap. XI).
b. The preliminary-expense value (Chap. XII).
c. The going value (Chap. XII).

[1] See case 20, Sec. 8.4.

d. The values of such other intangibles (for example, water rights and rights to reserves of natural resources) as cannot be credited with definite portions of the total net income (Chap. XIII).

e. The working capital (Chap. XIV).

f. The present value of other liquid funds, wisely tied up in the enterprise (Chap. XIV).

2. Determine the fair rate of net return, for this particular property; and the corresponding fair total annual net returns on the sum of the cost-values enumerated in 1, above.

3. Determine the average annual total returns which the enterprise is actually realizing; and forecast what these will probably average during a future period at least equal to the number of years during which the present existing good will is reasonably sure to continue. Three-years averages of present actual total net returns are often the most satisfactory. Usually, the average should be for normal years.

Note that the annual net return attributable to the present existing good will is the excess of the forecasted future total annual net returns over the sum of

a. The fair annual net returns, found in 2, above, on the cost-values enumerated in 1, above; plus

b. All annual net returns credited to specific other property elements (for example, contracts and patents).

4. Forecast the probable future net returns due to the present existing good will; they should be forecasted as equivalent to a uniform annual excess profit for a definite period of years (Sec. 13.4). The forecast must be by judgment, taking into account the history and circumstances of the particular property; only reasonably good management by the new owner should be assumed.

5. The good-will value of the enterprise will be the calculated present worth of the forecasted future excess profits, due to the present existing good will.

For the method of calculating the present worth, see Sec. 13.4. The interest rate to use in the computations should be the fair rate of net return for the particular property.

Note.—See Sec. 13.15 for a numerical example of the application to a particular property of the above process of determining good-will value.

THE VALUES OF OTHER INTANGIBLES

"Other intangibles" is a term used in this treatise to designate a miscellaneous group of intangibles, some of which may have to be valued, for any particular property, in addition to the preliminary-expense, going, and good-will intangible values, already discussed in Chaps. XII and XIII. Other intangibles include contracts, leases, options, easements, patent rights, trade secrets, copyrights, trade marks, trade names, water rights, rights to reserves of natural resources (gas, oil, coal, ores, timber, manufacturing materials, construction materials). "We do not say that there may not be others."

13.8. Contracts; Including Leases and Options.—Leases and options are particular kinds of contracts. Other contracts affecting the value of a property may be for power, raw materials, supplies, or other purchases, for a period of years; or for the sale of products or services for a period of years.

Each particular contract in which the contract prices for purchases, leases, or options are less than prevailing market prices has a value due to the future annual savings which it will effect in expenditures; similarly, contracts for sales, of products or services, at prices greater than prevailing market prices, have values due to the increases which they will effect in future incomes.

It is obvious that *unwise* contracts would have *negative* values.

The value of each particular contract is the sum of the present worths of the savings in future annual expenditures, or increases in future income, which it will effect before its date of expiration. The rate of interest used in calculating the present worths should be the fair rate of net return for the particular property.

Speculative or fanciful estimates of future savings or increases of income must not be made. When the future annual savings or increases of income are estimated to be uniform, their present worths may be computed by the good-will formula in Sec. 13.4.

Options.—Speculative technical calculations of probable future money returns from options are especially to be avoided. Options should usually be valued either on the basis of their actual cost or on well-established present market values for similar options.

13.9. Value of Easements.—Easements are legally established rights to make some definite, specified use, or uses, of property (usually lands) owned by others.

The value of an easement is best measured by the present cost at which a similar easement could be purchased; provided that this can be established conclusively; and provided, further, that this cost is not greater than the actual money worth of the easement to the enterprise.

Speculative estimates of the money worth to the enterprise must not be made. The actual original cost at which an easement was acquired is often entitled to weight in determining its present value.

Donated Easements.—In many cases, donated easements should be valued the same as if purchased. Usually, however, no value should be allowed, in public-utility valuations, for franchise easements granted for the use of streets and alleys.

Note.—In the *Baltimore Street Railway Case*,[1] the street easements granted the company were made taxable by a state law, and were valued at $5,000,000. This valuation, upheld by a state court, was not contested in the federal district court. For this reason, the United States Supreme Court refused to consider a later attempt to challenge the $5,000,000 valuation of the easements; the ruling on the challenge and objections was that " . . . if they ever possessed substance, they come too late."

[1] See case 55, Sec. 8.6.

13.10. Patents.—In factory valuation, it frequently becomes necessary to establish a value for each of the patents, or each group of patents, held by the company.

The value of a patent is arrived at by first estimating, if possible, the portion of the net returns of the enterprise that is attributable to the sale of the commodity affected by the patent. By a study of the sale records, and comparison with the price secured by competitors in the same field, the portion of these net returns that can be attributed to the protection afforded by the patent can sometimes be ascertained. An estimate is then made of the probable length of time in the future during which these returns may be expected to continue. The present worth of the estimated future returns due to the protection of the patent is the value established for the patent.

The computations are based on what is considered to be the probable average future rate of net return for the particular enterprise, as evidenced by its past record of profits.

In other instances no such estimates of earning are possible; such patents are carried at a value which is their actual original cost less depreciation; the depreciation is based on the age and legal life of the patent.

Of course, many patents are held by industries in the hope of some day using them; these may have little or no value, because they cover obsolete or commercially impracticable inventions.

13.11. Copyrights, Trade Marks, Trade Names.—The value of a copyright, trade mark, or trade name is like that of a patent and is estimated in the same manner.

13.12. Trade Secrets.—In many instances, companies, often by long-continued and expensive researches, develop special technical processes which they do not patent, or otherwise divulge; some of these may prove to be extremely valuable, others worthless. Trade secrets should be valued in the same manner as patents.

13.13. Water Rights.—Water rights are likely to be a factor in the valuation of a property in those cases where the water is available for the production of power; they are especially valuable in regions where water is scarce, though indispensable for domestic and municipal water supplies, for irrigation, and for furnishing supplies for steam power plants, including the necessary cooling water for condensers.

Titles to water rights are acquired in various ways in different states; those most common are the purchase of lands carrying riparian rights, the appropriation of the water itself, and the purchase of rights acquired by others through appropriations or through ownership of riparian rights. Water rights are usually transferable like other properties.

Once these rights are acquired, they may have value and in some instances do have considerable value.

In the *Denver Waterworks Case*[1] (1918), the United States Supreme Court did not question a valuation of $2,945,617 for water rights.

In its decision in *San Joaquin & Kings River, Canal & Irrigation Co. v. County of Stanislaus*[2] (1914), the United States Supreme Court reversed the decision of the federal circuit court because it had excluded the value of the water rights. But the Supreme Court said: "We are not called upon to decide what the rate shall be or even the principle by which it shall be measured."

The Valuation of Water Rights.—Three bases for valuing water rights have been suggested in various litigations:

1. *The Present Market Value of Similar Rights.*—This basis for valuing water rights was used in making the valuations accepted by the United States Supreme Court in the *Denver Waterworks Case*[1] (1918) and in the *Los Angeles & Salt Lake Railroad Case*[3] (1927).

In deciding the present market values of the railway-owned water rights in case 49, the Interstate Commerce Commission took into account the amount of water consumed by the carriers from each source, the total quantity available therefrom, and the next highest possible use of the water used by the carriers; the Commission ascertained the opinions of "many well-informed persons residing and engaged in business there."

If available to sufficient extent, data of actual recent prices paid for similar near-by water rights make the best criterion of present market value.

2. *The Actual Costs of Acquiring the Water Rights.*—If sufficiently recent, such costs may decide the value. However, this is only because they indicate the present market values.

3. *The Capitalized Values of the Savings in Annual Costs* (due to the use of the water rights in question, compared with the use of water or power from other sources). For example, water power may be compared with steam power. The estimates of such savings are pretty sure to be so speculative and fanciful as to be entitled to little weight. They must be based on assumptions and estimates subject to dispute. Widely different values may readily be calculated by varying the disputed fundamental data assumed.

Percolating waters are utilized in some localities by pumping for irrigating purposes, and in many localities these waters are the basis of municipal water-supply systems. In the valuation of irrigation systems or municipal waterworks, it may be necessary to take into account the value, if any, of percolating waters. It is the unanimous view of the courts and of experts in this field that percolating waters are a part of the land they underlie, and that their value, if any, is included in the value of the land. It is scarcely possible to estimate the value of percolating waters exclusive of the value of the land.

13.14. Rights to Reserves of Natural Resources.—Rights to reserves of natural resources are frequently an important part of the property of industrial enterprises; this is especially true of those enterprises whose service lives must terminate when they have used up their holdings of

[1] See case 31, Sec. 8.5.
[2] See case 26, Sec. 8.4.
[3] See case 49, Sec. 8.6.

gas, oil, coal, ores, timber, manufacturing materials, or construction materials.

When the reserves of such resources as those named can readily be measured and sampled by reliable means, they are valued by methods which are described in some detail in Chap. XVII. For example, the thicknesses and areas of horizontal coal strata may be ascertained by borings, and the quality of the coal by analyses.

In many cases, however, reliable detailed measurement and sampling are impracticable; this is likely to be the case when the rights have been acquired by purchases and leases, or the purchase of options therefor, covering specific tracts of lands, partly or wholly improved.

In such cases, the present market value of similar rights is the correct measure of value; the actual cost of acquisition may afford evidence worthy of material weight in determining the present market value.

Estimates, even by experts, of the future profits to be realized by the development, production, and sale of marketable products from undeveloped resources are sure to be speculative in large degree; hence, they are worthy of little weight. In *United Fuel Gas Company et al. v. Public Service Commission et al.*, for example, the company claimed a valuation of $36,449,176 for its gas lands, leases, and rights; the United States Supreme Court allowed only $6,732,920, saying:

"Appellants, as will more fully appear, reached their claimed value by an estimate by experts of the profits to be derived from the sale, in an unregulated market, of the quantities of gas estimated to underlie the proven and probable areas. The court below found that the value of appellant's gas field did not exceed its 'book cost,' which it took to be $6,732,920. . . . Such predictions [those of the company's experts] can only be made on the basis of data which are not and cannot be known, and most of which are highly speculative. Such a process of estimating value is without any known sanction."

A NUMERICAL EXAMPLE OF AN ENGINEERING VALUATION OF GOOD WILL AND OTHER INTANGIBLES

Since the value of good will must be ascribed solely to the probable future excess profits due to it alone, it is obvious that all other tangible and intangible cost-values must be determined before good will can be valued.

13.15. A Numerical Valuation of Good Will and Other Intangibles.— The application of the principles involved in estimating the value of good will, patents, and contracts will be illustrated by a typical problem, which has been set up to include these three intangibles.

The Problem: A corporation owns and operates a factory that manufactures certain specialties widely used in steam power plants. The product has been in use for 30 years, has been improved and modified from time to time, and is recognized as the leader in its field.

[1] See case 51, Sec. 8.6.

The several devices are covered by patents which, on the date of valuation, will afford protection for an additional 9 years on the key products. A careful study of the profits obtained for these key products, compared with those commanded by competing products, indicates that about $12,000 per year of the company's total net return is realized because of the protection afforded by the patents.

In its devices there is employed a certain mechanism made of sheet brass in a peculiar form difficult to fabricate. This was formerly made in its own factory but is now supplied by a factory devoted to the production of fabricated brass, on a price basis that effects a saving of $2,400 per year; the contract runs 6 years beyond the date of valuation.

The corporation has for many years maintained a service department to care for complaints and to provide for prompt inspection and repair or replacement of installations that fail to give satisfactory service; it maintains stocks of standard parts, and of items of supplies, at centers convenient for the users of the product. This service and the high reputation of its products have secured for the company the good will of a large clientele of desirable customers.

An engineering valuation of the factory has established the following fair present cost-values, exclusive of good-will value and the values of "other intangibles":

The depreciated, present fair cost-value of the fixed-capital physical property	$635,652
Working capital	22,200
Preliminary-expense value	9,535
Going value	64,000
Total	$731,387

A study of this and similar properties indicates that the fair rate of net return on investments in this type of industrial enterprise is 7 percent.

A study of the records of incomes and expenditures shows that, while the earnings of the corporation have fluctuated rather widely, the average net return over the 10 years prior to the date of valuation has been approximately $80,000 per year. A survey of the uses of the company's products, together with a forecast of the trend of power production, leads to the conclusion that this corporation may safely count on average annual net returns of at least $75,000 during the next 10 years.

The Valuation of the Other Intangibles

In this case, the "other intangibles" are (1) the patents, and (2) the contract for brass fittings.

Valuation of the Patents.—Since the patents are estimated to increase the net earnings of the enterprise about $12,000 per year, and have 9 years to run before expiration, the value[1] of the patents is

$$\$12{,}000\frac{(1.07)^9 - 1}{(1.07)^9(0.07)} = \$12{,}000\frac{0.838459}{(1.838459)(0.07)} = \$78{,}183.$$

Valuation of the Contract.—Since the contract for brass fittings will effect a saving of $2,400 per year for 6 years, the value[1] of the contract is

$$\$2{,}400\frac{(1.07)^6 - 1}{(1.07)^6(0.07)} = \$2{,}400\frac{0.50073}{(1.50073)(0.07)} = \$11{,}440.$$

[1] See the good-will-value formula in Sec. 13.4.

The Valuation of the Good Will

The process outlined in Sec. 13.7 will be used in valuing the good will of the enterprise.

1. The total physical and intangible present fair cost-values of the elements of the property—exclusive of good-will value and the values of "other intangibles" to which definite portions of the annual net returns are attributed.

As already stated, this has been found to be $731,387.

2. The fair rate of net return for this property is 7 percent. The fair annual net return on the fair present cost-value of the property elements enumerated in 1, above is $51,197.

3. The present average annual total net return is $80,000; and the forecasted future annual total net return is $75,000, both as already stated.

The annual return attributable to good will is $9,403. This is found by subtracting from $75,000, the sum of $51,197 fair net return on $731,387 (see 1 and 2 above), $12,000 for the net return due to the patents, and $2,400 for the net return due to the contract for brass fittings.

4. The probable future net returns, due to the present existing good will, are forecasted at $9,400 per year for 7 years.

Since this corporation has been highly regarded in the power-engineering field for more than a quarter of a century, and has constantly improved its product and its customer relationships, it is fair to assume that it will continue to enjoy the patronage of its old customers for many years in the future, if equally good management continues; however, it is not considered reasonable to assume that continuation beyond 7 years would be due to the present existing good will.

5. The present worth of the forecasted good-will net returns is the good-will value,[1] equal to

$$\$9{,}400\frac{(1.07)^7 - 1}{(1.07)^7(0.07)} = \$9{,}400\frac{0.60578}{(1.60578)(0.07)} = \$50{,}659.$$

The Total Fair Cost-value of the Factory

The total fair cost-value of this factory is now found to be as follows:

The depreciated present fair cost-value of the fixed-capital physical property	$635,652
Working capital	22,200
Preliminary-expense value	9,535
Going value	64,000
Patent rights	78,183
Contract for brass fittings	11,440
Good-will value	50,659
Total, depreciated present fair cost-value	$871,669

[1] See good-will-value formula in Sec. 13.4.

CHAPTER XIV

WORKING CAPITAL AND OTHER LIQUID FUNDS WISELY TIED UP IN THE ENTERPRISE: DETERMINATION OF THE FAIR COST-VALUE OF THE ENTIRE PROPERTY

The determination of the fair cost-value of the fixed-capital physical property of an industrial enterprise was explained in Chap. XI, and Chaps. XII and XIII were devoted to the intangible elements of value. Working capital and other liquid funds wisely tied up in the enterprise remain to be studied.

WORKING CAPITAL

"Working capital" is to be contrasted with "fixed capital." The two are defined in Secs. 2.16 and 2.17.

14.1. General Discussion of Working Capital.—The distinction between "fixed" and "working" capital is that the former is invested in the items of the comparatively permanent property; the latter in the temporary items which normally are consumed each year in operation.

Working capital is often defined in treatises on accountancy as the difference between current assets and current liabilities. This definition is not correct from the engineering valuation standpoint; such current assets as "accounts receivable" include profits, in addition to capital invested.

From the definition of working capital in Sec. 2.17, it appears that working capital, as defined in valuation engineering, includes two classes of property items:

1. Stocks of operation materials and supplies.
2. Operating cash resources; and temporary investments in paid-up operation expenses.

The amounts of working capital tied up in these two classes of items fluctuate from day to day; partly because of actual fluctuations in daily-use operation demands; partly because of differences in the reserves kept on hand to provide security against both temporarily high operation demands and the varying exigencies of procurement.

The correct valuation of working capital requires:

First, the correct determination of the average working capital actually used in the enterprise;

Second, a sound decision whether or not the actual average working capital used exceeded a wise working-capital provision.

14.2. Working-capital Stocks of Operation Materials and Supplies.—Only part of the stocks of materials and supplies kept on hand by industrial enterprises form a part of its working capital. All materials and supplies kept for use in making replacements of property items, or for improvements, or for additions to the property must be charged to construction costs instead of to working capital.

The amounts and present values of the stocks of operation materials and supplies on hand at any specific date are readily ascertainable by making an inventory of them, as of that date, and then deducting their accrued depreciations from the actual original costs new of the respective items. The difficulty in determining the working capital kept tied up in materials and supplies is that the average working capital may vary widely from its amount on any one date. Even monthly averages vary from month to month, and yearly averages from year to year. What must be sought in valuations for the determination of fair rates and/or fair prices is a fair average over a future period of three to five years.

The true average amounts and present values of the stocks of materials and supplies actually kept on hand in past years can be determined accurately only in the as yet rather infrequent cases where continuous inventories (usually of the card-index type) of all stores are maintained; and are kept up to date by entering all additions promptly, by allowing withdrawals only upon written work orders and by promptly crediting net withdrawals in the inventory.

In the absence of such continuous inventories, often the best that can be done in estimating average stocks of materials and supplies is to make a detailed study of past bills and accounts; and collect all available information of general character as to the varying demands for operation uses and the varying stocks kept on hand in different months and years.

In the case of long-drawn-out valuations, some helpful data may be secured by making the materials-and-supplies inventory early, and bringing it up to date at intervals afterwards.

After the average amounts and present values of the stocks of operation materials and supplies kept on hand have been estimated as accurately as practicable, a further inquiry should be made as to whether the actual averages do or do not exceed wise reserve requirements. Excessive reserves should not be included in valuations made for the determination of fair rates, and/or fair prices.

14.3. Working-capital Operating Cash Resources and Temporary Investments in Paid-up Operation Expenses. *Operating Cash Resources.* Both the daily and the average amounts of the company's total cash resources can usually be ascertained readily from its book records; the

TABLE 14.1.—APPROXIMATE APPROVED OR IMPLIED WORKING-CAPITAL ALLOWANCES MADE IN ACTUAL VALUATION DECISIONS[1]

Dates of		Valuation cases	State	Court	Approximate depreciated physical value	Working capital, %
Valuation	Decision					
1912	June 14, 1915	Des Moines Gas Co. v. Des Moines	Iowa	U.S. Supreme Court	$ 2,094,005	6.68
1914	Mar. 4, 1918	Denver v. Denver Union Water Co.	Colo.	U.S. Supreme Court	9,493,596	1.84
1919	May 29, 1922	Houston v. S.W. Bell Tel. Co.	Texas	U.S. Supreme Court	3,686,567	3.26
Dec. 1, 1919	May 21, 1923	S.W. Bell Tel. Co. v. P.S. Comm.	Mo.	U.S. Supreme Court	21,787,227	4.75
Jan. 1, 1924	June 11, 1923	Bluefield W.W. & Imp. Co. v. P.S. Comm.	W.Va.	U.S. Supreme Court	610,619	1.64
Jan. 1, 1924	June 11, 1923	Georgia Ry. & Power Co. v. R.R. Comm. of Georgia	Ga.	U.S. Supreme Court	6,954,545	5.03
Jan. 1, 1924	Nov. 22, 1926	McCardle et al. v. Indianapolis Water Co.	Ind.	U.S. Supreme Court	17,056,366	1.40
Aug. 16, 1923	Dec. 1, 1930	Smith et al. v. Ill. Bell Tel. Co.	Ill.	U.S. Supreme Court	117,803,128	2.55
May 1, 1930	Jan. 9, 1933	Wabash Valley Elec. Co. v. Young et al.	Ind.	U.S. Supreme Court	2,569,395	9.61
Nov. 24, 1930	May 8, 1933	Los Angeles Gas & Elec. Corp. v. R.R. Comm. et al.	Calif.	U.S. Supreme Court	50,707,639	2.16
May 1, 1928	Apr. 17, 1929	Vincennes Water Supply Co. v. P.S. Comm.	Ind.	U.S. Circ. Ct. App.	929,422	2.26
June 30, 1926	Sept. 22, 1930	Bd. of P.U. Commrs. v. Elizabethtown Water Co.	N.J.	U.S. Circ. Ct. App.	7,412,713	2.02
May 1, 1919	Apr. 21, 1921	Winona v. Wis.-Minn. Light & Power Co.	Minn.	U.S. Dist. Ct.	363,160	6.88
Sept. 1, 1921	June 4, 1923	Reno Power, Light & Water Co. v. P.S. Comm.	Nev.	U.S. Dist. Ct.	310,229	5.16
June 30, 1922	July 18, 1923	Arkansas Water Co. v. Little Rock	Ark.	U.S. Dist. Ct.	2,531,348	3.34
June 1, 1922	Jan. 4, 1924	Colorado Power Co. v. Halderman	Colo.	U.S. Dist. Ct.	8,515,818	2.64
Dec. 31, 1922	Dec. 13, 1924	Westinghouse Elec. & Mfg. Co. v. Denver Tramway Co.	Colo.	U.S. Dist. Ct.	20,275,375	1.18
Dec. 31, 1921	Dec. 27, 1924	Duluth St. Ry. Co. v. R.R. & Warehouse Comm.	Minn.	U.S. Dist. Ct.	4,344,135	3.11
Dec. 31, 1923	Feb. 27, 1925	Chesapeake & Potomac Tel. Co. v. Whitman	Md.	U.S. Dist. Ct.	27,447,249	3.55
Dec. 31, 1923	June 30, 1925	Kings Co. Lighting Co. v. Prendergast	N.Y.	U.S. Dist. Ct.	9,800,656	6.12
Dec. 31, 1922	Nov. 23, 1925	Springfield Gas & Elec. Co. v. P.S. Comm.	Mo.	U.S. Dist. Ct.	1,103,204	4.72
Oct. 31, 1924	Nov. 24, 1925	N.Y. & Richmond Gas Co. v. Prendergast	N.Y.	U.S. Dist. Ct.	4,278,663	8.04
Sept. 30, 1924	Jan. 7, 1926	Middlesex Water Co. v. Bd. of P.U. Commrs.	N.J.	U.S. Dist. Ct.	2,163,229	2.72
Jan. 1, 1923	Feb. 27, 1926	Monroe Gas Light & Fuel Co. v. P.S. Comm.	Mich.	U.S. Dist. Ct.	359,000	8.36
Dec. 31, 1923	May 10, 1926	United Fuel Gas Co. v. P.S. Comm.	W.Va.	U.S. Dist. Ct.	26,000,000	3.81

TABLE 14.1.—APPROXIMATE APPROVED OR IMPLIED WORKING-CAPITAL ALLOWANCES MADE IN ACTUAL VALUATION DECISIONS.[1]—(*Concluded*)

Dates of		Valuation cases	State	Court	Approximate depreciated physical value	Working capital, %
Valuation	Decision					
Aug. 31, 1925	Oct. 2, 1926	Brooklyn Borough Gas Co. v. Prendergast	N.Y.	U.S. Dist. Ct.	$ 7,188,526	6.96
June 30, 1924	Apr. 20, 1927	Idaho Power Co. v. Thompson et al.	Idaho	U.S. Dist. Ct.	16,020,302	4.06
Jan. 1, 1927	Apr. 17, 1928	Plainfield-Union Water Co. v. Bd. of P.U. Commrs.	N.J.	U.S. Dist. Ct.	3,906,895	2.56
Dec. 31, 1927	Feb. 11, 1929	Worcester Elec. Light Co. v. Atwill et al.	Mass.	U.S. Dist. Ct.	12,062,921	6.22
Mar. 31, 1927	Sept. 10, 1929	West Palm Beach Water Co. v. West Palm Beach	Fla.	U.S. Dist. Ct.	3,816,738	3.28
July 1, 1926	Nov. 11, 1929	New York Tel. Co. v. Prendergast	N.Y.	U.S. Dist. Ct.	375,353,760	3.64
Mar. 31, 1929	Nov. 25, 1930	Michigan Bell Tel. Co. v. Odell et al.	Mich.	U.S. Dist. Ct.	136,232,372	2.48
May 1, 1930	Oct. 20, 1932	Elko-Lamoille Power Co. v. P.S. Comm. et al.	Nev.	U.S. Dist. Ct.	328,000	6.71
Sept. 1, 1931	Jan. 5, 1933	Wichita Gas Co. et al. v. P.S. Comm.	Kans.	U.S. Dist. Ct.	69,727,011	2.04
Aug. 1, 1920	June 16, 1925	Cincinnati v. P.U. Comm.	Ohio	Ohio Sup. Ct.	16,783,545	2.68
Jan. 1, 1925	June 24, 1927	Follett v. Seneca Water Co.	N.Y.	N.Y. Sup. Ct.	396,592	1.77
Aug. 1, 1921	May 9, 1928	Hardin-Wyandot Light Co. v. P.U. Comm.	Ohio	Ohio Sup. Ct.	159,606	6.60
Dec. 31, 1925	July 19, 1928	Columbia R. Tel. Co. v. Dept. of Public Wks.	Wash.	Wash. Sup. Ct.	50,604	4.55
Aug. 31, 1926	Mar. 1, 1929	City of Erie v. P.S. Comm. et al.	Pa.	Pa. Sup. Ct.	3,417,425	5.56
1920	Apr. 7, 1931	Richmond, Fredericksburg & Potomac R.R. Co.	Va.	Interstate Com. Comm.	28,087,000	5.39
1921	Apr. 7, 1931	Richmond, Fredericksburg & Potomac R.R. Co.	Va.	Interstate Com. Comm.	27,880,000	6.53
1922	Apr. 7, 1931	Richmond, Fredericksburg & Potomac R.R. Co.	Va.	Interstate Com. Comm.	27,995,000	5.02
1923	Apr. 7, 1931	Richmond, Fredericksburg & Potomac R.R. Co.	Va.	Interstate Com. Comm.	28,805,000	4.50

[1] Data collected mainly by Prof. J. C. Hempstead, Iowa State College.

valuation engineer must deduct from the total cash resources whatever portion thereof should equitably be allotted to cash reserves needed for payments of construction costs rather than operation expenses.

Temporary Investments in Paid-up Operation Expenses.—The records of the payments of the various operation expenses and of the receipts of the various operation earnings must be carefully studied to determine:

1. The average monthly and yearly payments of operation expenses; and the monthly and yearly fluctuations thereof.
2. The average time elapsing between the payments of the various operation expenses and the receipts of the corresponding operation earnings.

14.4. The Valuation of the Total Working Capital of an Industrial Enterprise.—The purposes for which the valuation is made must be taken into account in valuing working capital.

1. *In Valuations for Determining Fair Rates and/or Fair Prices.*—The total working capital allowed should include
   a. The present (depreciated) value of either the *actual* or the *wise* (whichever is smaller) *average* stock of operation materials and supplies.
   b. An allowance for operating cash resources plus temporary investments in paid-up operating expenses; this should equal the product of the average operation payments during the average time elapsing between payment thereof and receipt of the corresponding operation earnings, multiplied by a fair fluctuation factor, to provide safely for temporarily high operation-expense demands from time to time.
2. *In Valuations for Sales or Condemnations.*—Include in the valuation only such parts of the total actual working capital at the date of sale as are transferred with the rest of the property. For example, title to materials and supplies on hand would usually be transferred to the new owners; title to "accounts receivable" might or might not be transferred.

14.5. Working-capital Allowances Approved or Implied in Actual Valuation Decisions.—As in the cases of overhead costs (Tables 11.9, 11.10), and preliminary-expense and going values (Table 12.4), the authors have studied the approved or implied allowances for working capital made in 39 court and 4 Interstate Commerce Commission valuation decisions. The conclusions reached by this study are presented in Table 14.1, to which the limitations explained on pages 330 and 181 apply.

As might be expected from the wide differences in the circumstances affecting the working capital required by different enterprises, the working-capital allowance percentages shown in Table 14.1 vary considerably, but, with three exceptions, only between 1.18 and 6.96 percent of the present (depreciated) cost-values of the fixed-capital physical properties of the various enterprises.

OTHER LIQUID FUNDS WISELY TIED UP IN THE ENTERPRISE

Besides fixed-capital physical property, intangible elements of property, and working capital, the assets of industrial enterprises often include a certain amount of notes, stocks, bonds, deposits, etc., which constitute liquid reserves; these are maintained for the purpose of readily securing funds to meet the demands of occasional exigencies in operation and management; including those due to fluctuations in business conditions.

The three main general classes of reserves are (1) the depreciation reserve; (2) reserves for surplus; (3) special reserves. In well-managed properties, the main part of the assets corresponding to the depreciation reserve should consist of fixed-capital physical-property items (Sec. 6.22); the same is apt to be the case with the assets corresponding to surplus.

14.6. Liquid Reserve Funds.—*In valuations for sales or condemnations*, all liquid reserves which are to be transferred to the new owner should be included in the valuation.

In valuations for determining fair rates and/or fair prices, the sources of the respective liquid reserve funds and their purposes must be considered in deciding whether or not they should be included in the valuation.

Actual depreciation sinking funds (Sec. 6.24) should be maintained only if really needed; in which case their funds should come out of annual depreciation appropriations, made before any net return is calculated. If wisely maintained, their present fair average values should be included in valuations for determining fair rates and/or fair prices.

Surplus investments in liquid securities are made with funds appropriated out of net-return income before distributing dividends. To the extent only of a wise provision to meet reasonably probable, legitimate exigencies, their present fair average values should be included in valuations for determining fair rates and/or fair prices.

Sinking funds for the payment of debts, including discounts, if maintained, should draw their funds from net-return income, appropriated for this purpose instead of being distributed as dividends. Such sinking funds *should not be included* in valuations made for determining fair rates and/or fair prices.

Other liquid reserves, if any. What to do with any other liquid reserve maintained must be decided by the purpose of each and the source from which its funds are drawn.

14.7. The Methods of Valuing Liquid Reserve Funds.—Although the actual costs of acquiring their securities should be ascertained and reported, liquid reserve funds should always be valued at their conservative, sound present fair values.

The securities to be valued may be separated into two classes:

1. *High-grade Securities.*—Examples are those which are listed as acceptable for investments, and/or as safe security for loans, by disinterested legally constituted authorities; these include the U.S. Reconstruction Finance Corporation authorities, state insurance laws and commissions, and the National Association of Insurance Commissioners.

In valuing high-grade securities, it seems reasonable to adopt the principle laid down by the United States Supreme Court,[1] for public-utility property in general, that: "An honest and intelligent forecast of future values, made upon a view of all the relevant circumstances, is essential." Valuations based on current average stock-exchange prices will be too high during speculative boom periods and too low during depressions.

For this reason, high-grade securities are often best valued at their true actual present worths; calculated upon the assumption that the principal and the interest (in the cases of bonds and mortgages) will all be paid when due and using a compound-interest rate in the present-worth computations equal to the average actual "yield" rates on similar securities, as indicated by actual market prices of similar securities over a long term of years.

Numerical Example: The present worth of a $1,000 high-grade 3 percent bond, maturing 25 years hence, with interest payable semiannually, is $957.43, if the conservative long-time average actual "yield" rate is 3¼ percent.

2. *Securities Not of High Grade* (as defined in 1, above).—Securities which are not of high-grade should not be regarded as safe investments. During speculative boom periods, they should not be valued at prevailing high stock-exchange prices. During depressions, their values should be greatly depreciated; often, many of them should be thrown entirely out of the valuation.

THE DETERMINATION OF THE TOTAL FAIR PRESENT COST-VALUE OF THE ENTIRE PROPERTY

The fair present cost-value of the entire property, including its fixed physical components, its intangible components, its working capital, and its wise liquid reserves, is an estimate of its total value made mainly from the bases of the actual original costs and the estimated reproduction costs of its various property elements. A tabulation of these elements and a diagram of their relations are presented in Sec. 14.8.

14.8. Table and Diagram of Cost-values.—The elements of the cost-value of an entire industrial property may be tabulated as follows:

Table 14.2.—The Elements of Cost-value

I. THE FAIR PRESENT COST-VALUE (DEPRECIATED) OF THE FIXED-CAPITAL PHYSICAL PROPERTY

1. The fair direct cost-values new of all fixed-capital physical-property units (giving due weights to their original costs and to their reproduction costs), plus

2. The fair overhead cost-values new of said physical units (giving due weights to their original costs and to their reproduction costs) equal

3. The fair cost-value new of the entire fixed-capital physical property; and 3 minus

[1] See *Southwestern Bell Telephone Case*, case 34, Sec. 8.5.

4. The accrued actual (cost-value) depreciations of said physical units, equals

5. The fair present cost-value (depreciated) of the entire fixed-capital physical property.

II. THE PRESENT INTANGIBLE VALUE OF THE PROPERTY

6. The fair present preliminary-expense value, plus

7. The fair present going value, plus

8. The fair present good-will value (if any), plus

9. The fair present values of other intangibles (if any), equals

10. The fair present intangible value of the property.

III. WORKING CAPITAL AND OTHER LIQUID FUNDS WISELY TIED UP IN THE ENTERPRISE

11. The present fair average working capital, plus

12. The fair present values of other liquid funds wisely tied up in the enterprise, equals

13. The present fair value of working capital and other liquid funds, wisely tied up in the enterprise.

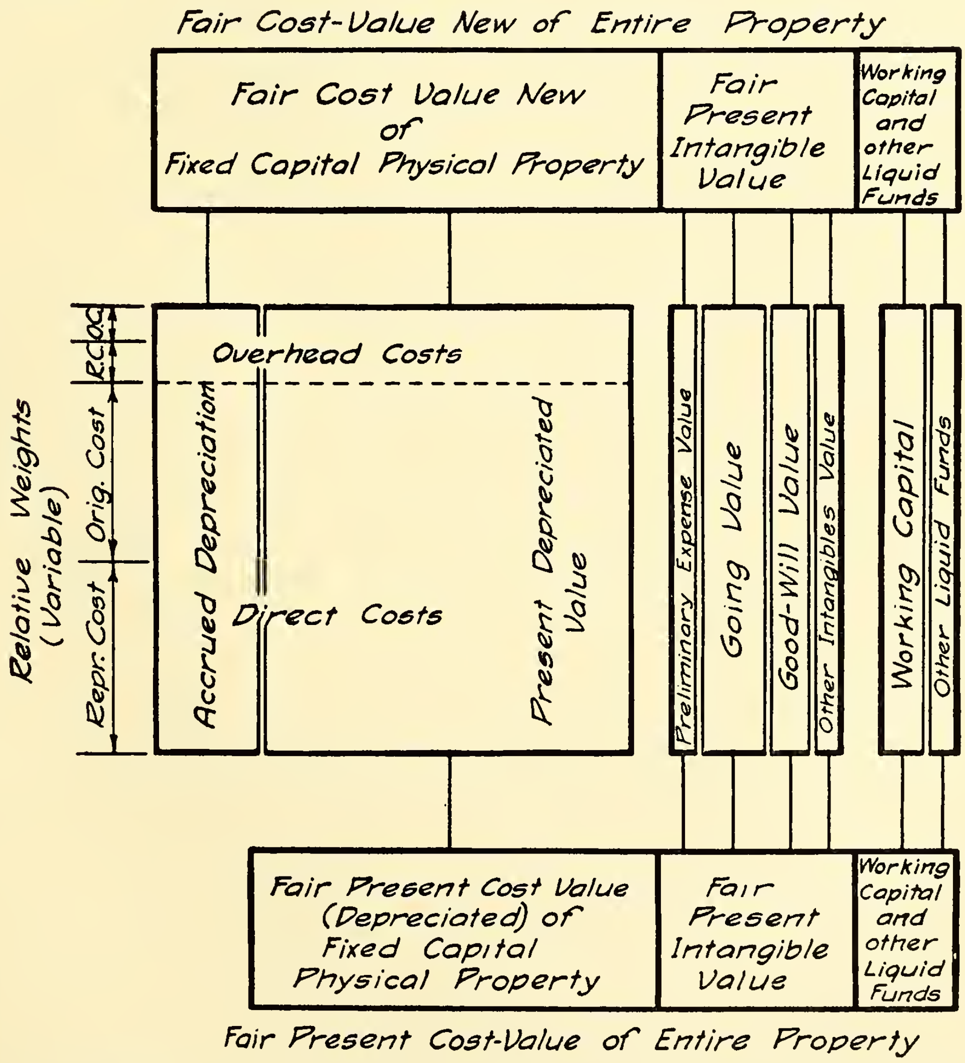


FIG. 14.1.—A diagram of the relations of the elements of the cost-value of an industrial property.

IV. THE FAIR PRESENT COST-VALUE OF THE ENTIRE PROPERTY

14. The fair present cost-value of the entire property is equal to the sum of 5, 10, and 13, above.

A diagram showing graphically the relations of the above 14 elements of cost-value is presented in Fig. 14.1, herewith.

14.9. Determination of Fair Cost-value.—As appears clearly from Table 14.2 and Fig. 14.1, the fair present cost-value of the entire property is determined by simply adding together:

I. The fair present cost-value of the fixed-capital physical property (Chap. XI).

II. The fair present intangible value of the property (Chaps. XII and XIII).

III. The fair present values of working capital and other liquid funds wisely tied up in the enterprise (Chap. XIV).

The fair present cost-value of the entire property is usually, but not always, entitled to great, sometimes dominant, weight in ultimately deciding what is the true fair value.

CHAPTER XV

EARNING VALUE; SERVICE-WORTH VALUE; STOCK-AND-BOND VALUE; FINAL DETERMINATION OF THE FAIR VALUE OF THE ENTIRE PROPERTY

The reason that, as stated at the end of Chap. XIV, fair present cost-value is usually entitled to great weight (Secs. 7.12, 7.13) in determining fair value is that, under usual circumstances, cost-value is a more reliable indicator of the probable future net earnings of a property than are either its actual recent and present earnings, or the prevailing prices of stocks and bonds. Present earnings vary with the temporary ups and downs of business conditions; prevalent stock-and-bond prices are subject to the changeability of the public's guesses at values.

THE EARNING VALUE OF THE ENTIRE PROPERTY

The present worth of the probable future net earnings of a property is the fundamental basis (Sec. 1.13) of its value; but the so-called "earning value" estimated in making an engineering valuation of the property is merely the calculated present worth of what its average future net earnings may be, prognosticated wholly on the basis of its actual recent and present expenses and earnings and the business outlook.

15.1. Earning Value and Its Determination.—For a definition of earning value see Sec. 1.25. The process of determining the earning value of a particular property may often be about as follows:

1. From the book records of the property, ascertain its actual annual net earnings each year during the last 3 to 10 years; and their averages over such periods as will give the most reliable indications of probable future net returns.
2. On the basis of these averages and his judgment, the valuator must decide the most probable future average annual net return which will be earned by the property.
3. Capitalize these probable future average annual net returns, at a rate of compound interest equal to what the valuator decides should be the probable future average fair rate of net return.

Numerical Example: A certain industrial property, which had been considerably improved and enlarged during 1925 to 1929 is to be valued as of date, Jan. 1, 1934.

1. The book records of the property show that its annual net earnings during 1925 to 1933 were as follows:

1925	$119,563	1928	$127,936	1931	$100,425
1926	121,870	1929	130,464	1932	85,639
1927	124,357	1930	125,740	1933	95,856
Average	$121,930	Average	$128,047	Average	$ 93,973

2. After consideration of actual past and present and probable future business conditions, the valuator concludes that the probable future average annual net returns from the property may be fairly estimated at $115,000.

3. The valuator also concludes that the fair rate of net return for this property was 9 percent during 1925 to 1929, is now 7 percent, and probably will average 8 percent in the future.

Hence,

$$\text{Present earning value} = \frac{\$93{,}973}{0.07} = \$1{,}342{,}471,$$

$$\text{Probable future earning value} = \frac{\$115{,}000}{0.08} = \$1{,}437{,}500.$$

It is clear that earning values determined in this manner depend very largely upon the *opinions* of the valuator, concerning the permanence of recent and present business conditions and the nature and effects of probable changes thereof. The opinions of different valuators on these subjects would be likely to vary materially; manifestly, by varying the estimates of future net earnings and of future fair rates of return, widely different earning values could readily be calculated.

The variability of values so largely based on variable opinions may be contrasted with the comparative reliability of fair values determined by giving great weight to cost-value. Giving great weight to cost-value is just a recognition of the general law (which, of course, does not govern under exceptional circumstances) that competition, in the case of private industrial properties, and regulation, in the case of public utilities, will operate to establish and maintain average prices and rates just sufficient, in the long run, to yield fair average net returns on fair value.

As indicated in Sec. 1.25, the valuator is seldom, if ever, justified in going so far into the realms of conjecture as to prognosticate definite future annual net earnings for specific future years; or even for periods of future years.

THE SERVICE-WORTH VALUE OF THE ENTIRE PROPERTY

In addition to the considerations discussed in Sec. 15.1 as affecting earning values, there is the added question of whether the actual recent earnings of industrial enterprises are or are not based on fair rates and/or fair prices.

In the case of *public-utility properties*, the United States Supreme Court has said:[1]

> What the company is entitled to ask is a fair return upon the fair value of that which it employs for the public convenience. On the other hand, what the public is entitled to demand is that no more be exacted from it for the use of a public highway [or other utility] than the services rendered by it are reasonably worth.

[1] See *Smyth v. Ames*, Sec. 8.3.

In the case of *private industrial properties*, more and more attention is being paid to the subject of fair prices; just sufficient to pay the average actual cost of production, in reasonably efficient plants, plus fair returns on their cost-values.

Thus the service-worth values of industrial properties must more and more frequently be substituted, in determining fair values, for their earning values, based on actual past and present earnings.

15.2. A Discussion of Service-worth Value.—Service-worth value is defined (Sec. 1.26) as what earning value would be if earnings were based on fair rates and/or fair prices.

The purpose of determining the service-worth value of an industrial property is to enable "such weight as may be just and right in each case" to be given to the principle that the public should not, and in the long run will not, pay more than their reasonable worths for the commodities produced by private industrial enterprises; and is "entitled to demand that no more be exacted from it" than the reasonable worths of the services rendered by public utilities.

In many cases, it is not necessary to make any elaborate, formal estimate of service-worth value; a simple general study of the facts often will show at once that the rates and/or prices, necessary for earning fair returns on fair cost-values, are not greater than the reasonable worths of the services rendered and/or products sold. The service-worth values should be determined in all cases in which it seems reasonably possible that rates and/or prices, high enough to earn fair returns on fair cost-value, may be materially higher than the reasonable-worth principle would justify.

15.3. The Determination of Service-worth Values.—The process of determining service-worth value may often be about as follows:

1. Determine the reasonable worths to customers of the services rendered and/or the commodities sold.
2. Determine what the actual annual net earnings over a sufficient number of recent years would have been if the rates and/or prices charged had been just equal to the reasonable worths to customers of the services rendered and/or the commodities sold.
3. By proceeding as described in Sec. 15.1, determine what the earning value would be on the basis of these modified net earnings.

15.4. Determination of the Reasonable Worths of Services Rendered and/or Commodities Produced.—There are two criteria for determining the reasonable worths to customers of the services rendered and/or commodities produced by industrial enterprises:

1. *Competitive Prices.*—Competitive prices can be relied upon to show the reasonable worths of services and/or commodities only when
   a. *Adequate free competition*, with unfair trade combinations and agreements barred, safeguards consumers against excessive rates and/or prices.

b. Absence of cut-throat competition safeguards producers against inadequate rates and/or prices.

Note.—Increasingly, as new methods are developed for rendering transportation and other indispensable utility services, public utilities are losing the status of "natural monopolies"; many are finding that the rates which it is feasible to charge are being determined by competition with other agencies. Street railways, for example, must now compete with buses, taxicabs, and privately owned automobiles.

2. *Fair Production Expense Plus Fair Return on Fair Cost-value.*—As our modern industrial civilization becomes more and more complex, the principle of fair rates and/or fair prices, which already for many years has been the law of the land for public utilities, is extending into the fields of private industrial enterprises.

Certain difficulties, some temporary and some permanent, must be overcome in applying this principle to specific enterprises.

a. The present lack of complete production-cost book records would not seem necessarily to be a permanent difficulty. Adequate and correct cost accounting, including continuous property-ledger inventories and correct current accountancy for all actual depreciation expense, should become the rule instead of the exception.

b. The wide variation in production costs between different plants, in the same or in different localities, is a permanent difficulty. As between plants in the same general locality, the average production cost should be estimated, by averaging all plants if possible; if this is not possible, average carefully selected representative plants. As between different localities, choice, depending on the marketing conditions in particular industries, must be made between (1) different average production costs (and different fair prices based thereon) in different regions, or (2) fair prices based on the same broad average production costs for all regions.

c. In the cases of natural monopolies (such as many public utilities), it may be necessary, in some instances, to use estimated costs of production, by a reasonably efficient substitute plant, designed but not built; however, it must always be remembered that production expenses so determined are mere estimates, not actual costs.

15.5. The Use of Substitute Plants in Estimating Service-worth Values.—As indicated in Sec. 15.4, the use of estimates of the costs of production by reasonably efficient substitute plants (designed but not built) may be necessary in some instances, in the absence of other available means, to determine the reasonable worths of the services rendered by natural-monopoly public utilities.

The Maine Supreme Court, in its instructions 13, 14, 15, issued *in re* the *Brunswick & Topsham Water District v. Maine* (1904) waterworks valuation, said (Sec. 8.4):

13. In estimating the value of a public service to the public or the customers, one of the elements necessary to be considered is the expense at which the public or customers, as a community, might serve themselves, were they free to do so, and were it not for the practically exclusive franchises of the supplying company. Water is to be regarded as a product, and the cost at which it can be produced or distributed is an important, though not the only, element of its worth.

14. The worth of a water service in such connection is the worth to the customers as individuals, but as individuals making up a community of water takers.

15. Communities are entitled to the benefit of existing natural advantages. If there is more than one source of supply, other things being equal, the community is entitled to have the least expensive one used, and the supplying company is not entitled to charge an enhanced rate, based in part at least, upon the cost of using a more expensive source.

The use of a substitute plant must be only for the purpose of estimating the reasonable worths to its customers of the services rendered by the existing plant; the object is to determine what the net earnings of the existing plant would be if only the reasonable worths of its services were exacted from the public.

In every valuation, it is the existing plant, not a substitute plant, which is to be valued. In the *Indianapolis Water Co. Case* (Sec. 8.6), the United States Supreme Court said:

There is to be ascertained the value of the plant used to give the service and not the estimated cost of a different plant. Save under exceptional circumstances, the court is not required to enter upon a comparison of different systems.

THE STOCK-AND-BOND VALUE OF THE ENTIRE PROPERTY

It is the law of the land, as laid down by the United States Supreme Court in *Smyth v. Ames* (Sec. 8.3) that " . . . the amount and market value of its bonds and stock . . . are to be given such weight as may be just and right in each case," in determining the fair values of public-utility properties.

15.6. A Discussion of Stock-and-bond Value.—For a definition of stock-and-bond value see Sec. 1.28. Since industrial properties are not bought and sold as entities, their stock-and-bond values are their only market values.

The purpose of determining stock-and-bond values is to enable such weight as is just and right in each case to be given them in determining fair values. As explained in Sec. 7.13, stock-and-bond value is entitled to little weight under usual circumstances, mainly because of its variable, speculative character; under exceptional circumstances, however, it may be entitled to great weight.

NOTE.—The stocks and bonds issued by an enterprise are in some instances based partly upon, and secured in part by, property "owned but not used," which it is not permissible to include in the valuation.

STOCKS AND BONDS AND THEIR HOLDERS

Only an extremely brief and elementary discussion of stocks and bonds and their holders will be given in this treatise. The student is urged to study good texts on corporation finance.

Stocks and Bonds.—The capital required for constructing an industrial property, including improvements and enlargements, is represented by its stocks and bonds. These are sold outright to investors; or are awarded in payment for the services of

individuals; or are issued to stockholders from time to time to cover the values added by property paid for by reservations from the net income of the property.

The *bond holders* are preferred creditors; they have loaned money for the enterprise upon a security for each bond issue which constitutes a lien upon the property prior to the rights of the stockholders. Bond interest is, therefore, usually materially lower than the fair rate of return on the property.

The *stockholders* own the property, manage it, and assume the risks of its possible failure. They are entitled to earn a net return on the fair value of the entire property (including that constructed with borrowed money and/or with money reserved from income), at a rate sufficiently higher than prevailing interest rates on good bonds to remunerate them for the risks of the undertaking. There are two general classes of stocks, common and preferred.

The *common stockholders* have no special safeguards against the risks of the enterprise. On the other hand, they are entitled to receive, wholly or in common with preferred stockholders, all excess of the total net returns over the sum of bond interest plus guaranteed preferred-stock dividends.

The *preferred stockholders* are given special guarantees (subject to the bond holders' liens); such as the right, after bond interest is paid, to receive dividends up to a guaranteed rate; and, in the case of "cumulative" preferred stock only, with the further guarantee that deficiencies in lean business years below the guaranteed rate shall be made up in later good years. The guaranteed rate of dividends on preferred stock will usually be higher than prevailing bond interest rates, but lower than the fair rate of net return on the entire property.

15.7. The Determination of Stock-and-bond Values.—As indicated by the definition in Sec. 1.28, the stock-and-bond value of an industrial property can be determined by proceeding about as follows:

1. Make a correct, complete list of all its various issues of bonds and stocks; including the total par value of each issue of bonds and the total par value, or the total number of shares, of each issue of stock.

2. From the various daily stock-exchange reports, determine the current market prices of the several issues; averaged over a period at least long enough to eliminate irregular temporary market fluctuations.

Note.—In the cases of properties whose securities are not listed on any stock exchange, it may be possible to establish roughly correct prices by investigations of private sales; and by the opinions of disinterested brokers. In many cases, especially of small properties, stock-and-bond values must be eliminated from consideration, because the values of their stocks and bonds are not ascertainable.

3. Calculate the total stock-and-bond value, using the total par values, and/or numbers of shares, listed for the several issues, and their respective current average market prices.

Numerical Example: By the above method, the stock-and-bond value of a certain industrial property was found to be as follows:

$35,250,000	first mortage 4½% 30-year gold bonds due Feb. 1, 1961, at 1.09⅝	$38,642,812
$ 6,000,000	first mortgage 4½% Series B, 30-year gold bonds due Jan. 1, 1957, at 1.08¼	6,495,000
40,000	shares cumulative 6% first preferred stock at 1.10	4,400,000
525,000	shares common stock at 0.50	26,250,000
	Total stock-and-bond value	$75,787,812

THE FINAL DETERMINATION OF THE FAIR VALUE OF THE ENTIRE PROPERTY

The culmination of the entire work of making an engineering valuation is the final determination of the fair value of the entire property; giving "such weights as may be just and right in each case" to every factor affecting value.

15.8. Final Determination of Fair Value.—The general method of making an engineering valuation, as outlined in Sec. 1.15 and described and explained in Chaps. X to XV, involves the making of a number of estimates of the value of the entire property; each from the standpoint of a different major factor affecting value.

From the standpoint of cost, the fair present cost-value of the entire property must be determined; giving due weights to original costs and to reproduction costs; all as described and explained in Chaps. XI to XIV.

From the standpoint of actual earnings, the earning value must be determined, as described and explained in Sec. 15.1.

From the standpoint of reasonable worths of services rendered and/or commodities produced, the service-worth value must be determined, whenever necessary, as described and explained in Secs. 15.2 to 15.5.

From the standpoint of the market values of its securities, the stock-and-bond value must be determined, whenever practicable, as described and explained in Secs. 15.6 and 15.7.

To make the final determination of the fair value of the entire property, these estimates of value, each made from the standpoint of a major factor affecting value, must each be given "such weight as may be just and right in each case"; in addition, all "other pertinent factors" affecting value must be given due weights, of a general character.

The process of doing this may often be about as follows:

First, compare the cost-value of the entire property with its earning value; or with its service-worth value, in the cases where that should be used instead of earning value; giving cost-value, and either earning value or service-worth value, the relative weights found to be "just and right" for the property being valued.

Second, determine whether any, and if any what, weight is due the stock-and-bond value (if feasible to ascertain it for the particular property).

Third, determine the fair value of the entire property; giving cost-value, earning value (or service-worth value), and stock-and-bond value such weights as are found to be just and right in this case; and also giving due weights, of a general character, to all other pertinent factors affecting value.

15.9. Weights Used in Final Determination of Fair Values.—The just and right weights to be used in determining fair values must be decided by sound, unbiased judgment; exercised by the valuator only after he has so carefully done the other work of making the valuation, and so thoroughly studied its results, that he has complete knowledge and understanding of the history, the circumstances, and the various

data of the enterprise and its property. The valuator must stick closely to facts and good common sense; shunning conjecture and purely theoretical reasoning. His work and study should be so thorough and sincere that, at the time of reaching his final conclusions, he will have become imbued with a strong conviction of their correctness and justice.

Discussions of the just and right weights which should be given, under different circumstances, to the different factors affecting fair values will be found in this treatise in the following references:

For the relative weights due original costs and reproduction costs, in determining the fair present cost-value (depreciated) of the fixed-capital physical property, see Secs. 7.11, 7.15, and 7.16.

For the relative weights due cost-value and earning value (or service-worth value), see Sec. 7.12.

For the weights due stock-and-bond value, see Sec. 7.13.

For the general weights due other pertinent factors affecting value, see Sec. 7.14.

PART III

THE ENGINEERING VALUATION OF DIFFERENT CLASSES OF INDUSTRIAL PROPERTY

CHAPTER XVI

VALUATION OF LAND

In engineering-valuation practice, the problem of valuing land arises in connection with the routine of factory, building, and utility valuation. While in many instances it is convenient and economical to employ specialists in land appraisal for the valuation of land, there are cases in which the engineer himself must make land valuations in connection with his appraisal of industrial property. For that reason a brief summary of the underlying principles of land valuation are presented in this treatise.

LAND OWNED BY PUBLIC UTILITIES

16.1. Legal Status of Utility Land Valuation.—The law with reference to the valuation of the land utilized by public utilities has been set forth unequivocally in several decisions of the United States Supreme Court. The basic principles enumerated in the *Consolidated Gas Case* (Secs. 8.4, 11.5) and reaffirmed and expanded in the *Minnesota Rate Cases* have been adhered to in all subsequent decisions of the Court. These decisions are inconclusive in that they do not point out what the Court believes to be the effect of existing facilities or buildings upon the value of the land when those facilities are of a character unsuited to the most productive use of the land. In all other respects the principles of land valuation seem to have been clearly enunciated.

16.2. The Value of Utility Lands.—The value of the land used for the facilities of public utilities has been defined by the courts as the *market value* as evidenced by the market value on the date of valuation of abutting and adjacent lands of a similar character. The courts specifically exclude an increment due to possible severance damages to adjacent property when the utility lands were purchased, and also exclude any increment of plottage value.

A little consideration will make it clear that in its rulings with reference to the value of utility lands, the United States Supreme Court has not departed from the general principles set out in *Smyth v. Ames* and adhered to rigidly since that time.

As set forth elsewhere in this treatise, there are certain factors recognized by the courts that must be taken into account in forming a judgment as to the value of an industrial property (it being understood,

of course, that in specific cases certain of these elements may have no significance). By applying that dictum to the valuation of land, it will be quite clear why the courts have established the market-value basis for the valuation of land.

In order to make this clear, each of the several factors ordinarily taken into account in valuing industrial property will be restated and its application to the valuation of land discussed.

1. *Original-cost value*, in the case of ordinary industrial property, is the actual cost of the existing units of property less the depreciation that has accrued to the date of valuation. The depreciation to the date of valuation is determined after a study of the mortality characteristics of the property.

In applying this principle to land there is often negative depreciation, that is, appreciation, to consider. And just as the comparison of a unit of ordinary industrial property with other units of industrial property is required to arrive at a decision as to its depreciation, a comparison of land with other units of land similarly situated and of the same general character is made to determine the amount of the appreciation or depreciation. The lands to be used as a basis of comparison obviously are those near by, and which are under such ownership that they can change hands on the open market according to the law of supply and demand. In a sense, then, determination of the market value of the land follows a process which also develops a figure closely having the same significance as the original-cost-value estimates employed for the physical property other than land.

2. *Reproduction-cost Value.*—When a unit of industrial property wears out it is generally possible to go on the market and purchase a like unit for replacement. In effect, then, an industrial property is being reproduced piecemeal during the time that it is in use. Such a situation cannot exist with reference to land. It does not wear out in the ordinary course of usage (farm land excepted), and, if it did wear out, it would be impossible to replace it in the sense in which other units of property are replaced. It seems clear, however, that the market value of comparable land is closely akin to the reproduction-cost value which is of significance in the valuation of other types of property.

3. *Earning value* is a factor that must not be given any considerable weight in a valuation which is to serve as a rate base, and, therefore, in the valuation of public-utility land this factor cannot be given any particular consideration.

4. *Service-worth value* is a factor that applies to properties as a whole and, like earning value, cannot enter into consideration when valuing the land owned by utilities.

5. *Stock-and-bond value*, as applied to industrial properties, cannot be applied to the land alone, since there is no means of separating the land from the remainder of the property in so far as its influence upon the value of the securities is concerned.

6. *Other factors* that may be taken into consideration in valuing land are the character of the improvements upon a specific piece of land and the extent to which the land is being utilized to its full productivity. These factors will usually be evidenced by the trend of land values in the vicinity of the tract that is being valued.

16.3. Severance Damages.—It has been claimed in some instances that, when the purchase of a tract of land for the purposes of a public utility would necessitate taking part of several parcels of privately owned land, there would be instances in which the value of the remaining parcels

would be reduced to the owners of those parcels. Consequently, it is claimed that there should be added, to the actual value of surrounding or adjacent lands, a factor for severance damages. The courts have uniformly held that at the time of valuation no parcel of land is being severed from any other, and consequently this factor cannot be taken into account in the valuation.

16.4. Plottage Values.—In securing lands for the right of way of a railway, an interurban railway line, or for a reservoir or dam, it is necessary to gather together a large number of parcels of land in order to provide an area of suitable size and shape for the purposes of the construction. It has sometimes been held that a tract of land made up in this way has a greater value than the sum of the individual values of the parcels thus gathered together, and this increased value is known as the "plottage value." The courts have uniformly held that there cannot be added to the valuation of the land of public utilities an increment for plottage value, because at the date of valuation no such gathering together is taking place and no increment of value is being added for that reason.

VALUATION OF LANDS OWNED BY PRIVATE INDUSTRIAL ENTERPRISES

16.5. Basis of Valuation.—The valuation of lands owned by private industrial enterprises is arrived at in precisely the same manner as for public utilities, except that, since the rate base is not a factor in the problem, the earning value of the property may be taken into account, and, if the earning value of the land itself can be established, that is a factor to be considered.

The earning value of a parcel of land may be estimated by assuming it to be developed to the highest usefulness possible in view of its location. The existing industrial enterprise may not be of a type permitting the development of the land in this way. Obviously at the beginning of the life of the enterprise the land was of such a character that it was possible for the enterprise to earn a return on the investment therein, but, as cities develop, the sites used by industrial enterprises may become of very great potential value because of a change in the character of the district. It is quite apparent, however, that a change in the potential value of the land does not in all cases increase its earning capacity to the enterprise of which it is the site. Sometimes the increase in value may be due to the development of a stream so that it becomes navigable past the site owned, permitting the establishment of docks which may be of considerable value to the existing enterprise, decreasing its costs and thereby increasing its earnings. The development of a railway adjacent to the property may have a similar effect. It not infrequently happens that the potential earning capacity of lands becomes so great as to warrant an industrial

enterprise moving its plant to another location, and selling the site for other and more productive types of development.

All of this will generally be reflected in the market price of adjacent and abutting lands, and consequently the most suitable basis and the one most uniformly applicable for determining the value of the lands of private industrial enterprises is precisely the same as that employed for the valuation of public-utility lands.

The appraisal of the land occupied by business buildings, such as stores, offices, hotels, and apartments, and that owned as a part of a homestead is not ordinarily a part of an engineering valuation, although in some instances such holdings are a part of an industrial property and their valuation becomes a part of the problem of the valuation of an industrial enterprise.

In most of the discussions of this subject, it will be found that great emphasis is placed upon the earning value of the land which is used for the types of development just mentioned. There apparently is a failure to recognize that the long-time earning value of land can be estimated accurately only by considering a number of factors common to all problems in engineering valuation. The actual future earnings will be governed largely by competition among the developments of a like character within a given area. It is quite possible that a modern hotel might show a very high rate of earnings for a period during which it was not subjected to any considerable competition. The very fact that its earnings are high encourages competition, and the usual experience is that there will be not only one competing hotel but perhaps two or three. As a result, the earnings of all will be on a basis that will indicate a lower value for the land than would have been estimated for the first hotel during the period in which it had no competition. For these reasons a correct determination of the value of a parcel of land can be reached only by an analysis of a number of factors of which the most important probably is the market value of the lands in the area adjacent to the parcel in question.

In estimating the market value of lands in the built-up portions of cities, it is important to take into account the relative price levels at the time of the appraisal. It may be that lands are being sold at speculative prices, as a natural consequence of over-optimism on the part of buyers in a period of business prosperity, and it is equally possible that the reverse may be the case. The long-time trend of land prices in an area may be analyzed by giving appropriate weight to fluctuations in the value of money as indicated by the fluctuations of the general price or commodity indexes, and with information of this type at hand it is possible to determine whether the prevailing market price is probably on a sound basis.

One of the factors to be taken into account is, of course, the present worth of the probable future earnings of the development of which the land is a part. If this estimate is carefully made and the estimates of future earnings are conservative, it affords a useful aid in arriving at land values. In this connection, however, it is important to make sure that the improvements upon a parcel of land are of such a character as to permit the maximum return possible to buildings and lands. If the land is occupied by old or obsolete types of buildings, then the present earnings are not a reasonable criterion by which to estimate the probable future earnings when the old buildings are replaced with modern structures.

Experience shows that the value of land reflects to a considerable extent its environment and its physical location and proximity to streets, parks, lines of transportation, and its geographical location with reference to the various zones in a city. No fixed rules can be laid down by which to estimate the effect of environment upon the future returns that may be earned by developing a parcel of land, but an analysis of the situation in any given instance may indicate the probable trend.

It has been the history of the cities that the main business district, which is devoted to wholesale and retail business, theaters, hotels, and the like, develops around some nucleus usually determined by the transportation situation. As the cities have grown, this main business district has gradually expanded and around the boundaries of the business district (which are never clearly defined) is a fringe in which the change is coming about gradually from residential to business use. In this area, land values are usually definitely on the increase. In the outer boundaries of this fringe the transformation to a business district is in the remote future while at the present it does not constitute a desirable residential district. As a result, land values in this zone are likely to be stationary or perhaps declining somewhat.

The value of lands occupied by a business property, as distinguished from industrial property, is likely to be affected by a large number of factors peculiar to the growth of the city, and any estimate of probable future earnings is, therefore, likely to be wide of the margin in many cases. Consequently, a correct value can be arrived at by taking into account not only the earning value but all other factors that are known to influence the value of lands of this type, and the most reliable index is the long-time trend of the market value of land in the adjacent area.

APPRAISAL OF URBAN LAND

Appraisal of urban land is required in connection with assessment for taxation, obtaining loans, the settlement of estates, and other similar purposes which do not always involve the sale of the land or the fixing

of rates. In all such cases the objective is an estimate of the fair market value of the land. A process of appraisal has been evolved for application in these cases which tends to make for uniformity and fairness where many parcels of land of varying shapes and dimensions are involved.

16.6. Calculated Value.—A number of empirical rules have been set up by real-estate boards and tax-assessment officers whereby a value can be calculated for a parcel of land in any city that has established standard unit values for lands. The inference might be that this calculated value is the real value of the land. Such is not always the case, however, and the calculated value must be modified according to judgment whenever it appears that modification is called for by the conditions.

There are two important reasons for scrutinizing carefully the calculated value before accepting it as the fair market value. First, the standard unit values are of necessity general and must apply to some considerable area. They cannot vary with each parcel of land for obvious reasons, and therefore individual parcels may be worth more or less than the average in the area. Second, land values change as the district develops or deteriorates, and these changes may be rather rapid. It is impossible to revise unit-value maps oftener than every five or six years because of the time and cost involved, and hence the calculated value may require adjustment to compensate for the general trend of land prices.

The calculated value is a suitable basis for valuation, however, because it insures the valuator starting with a figure that is in harmony with the general level of land values in the area.

16.7. Assessed Value.—The assessment of land for taxation presents the problem of arriving at fair comparative values for many parcels of land by methods that can be applied rapidly by assessors who are not necessarily appraisal experts. Those cities that have been progressive in such matters have systematized the work by various devices of which three are worthy of mention herein.

1. *Land-value maps* have been prepared for the entire city, upon which maps there is marked in every block and in every tract of undivided land the standard unit value that is to be used in making the assessment. These unit values are revised at intervals.
2. *Depth tables* are adopted for residential and business lots, and for rectangular and triangular lots, to aid in spreading the assessment to lots of various depths, or length perpendicular to the street. Special depth tables are sometimes furnished for corner lots also.
3. *Assessment rules*, setting forth the methods to be followed, are provided for the guidance of the assessors. These rules and illustrative examples of their application make the assessment almost a routine matter.

As a general rule the assessed value is a calculated value without adjustment, although most of the systems of assessment provide for the exercise of judgment under certain conditions. The nature of the

work and the requirement that favoritism be avoided preclude giving much latitude to individual assessors.

16.8. Standard Lots.—The appraisal of city lots is greatly facilitated by the establishment of a standard depth for lots in various categories, such as business, residential, manufacturing, and the like. The assessment bureaus of many cities have adopted standard depths, often after a study in which the local real estate board participated. While it is desirable that the standard depth correspond with the actual physical layout, if possible, the general experience has been that there is so much variation in the depths of lots in an area that some compromise depth must be employed. The standard lot depth most widely used is probably 100 feet for business districts and 120 feet for residential districts, but various other depths up to 150 feet have been adopted here and there.

16.9. Standard Unit Value.—The need of a common unit to which real-estate values could be reduced was recognized a good many years ago and led to the adoption of a *standard unit value,* in terms of a rate per front foot for a depth equal to that of the standard lot. In practice, the standard unit value is fixed on the basis of the prevailing market value of the land as evidenced by sales in the neighborhood, and after conferences with owners and many public hearings. In a typical case a unit value is fixed for each block in the city and for each undivided tract. These are indicated on the land-value maps and afford the starting place in the appraisal of a specific property.

16.10. Depth Factors.—The value of the portion of a parcel of land adjacent to a street is greater than the value of the portion remote from the street. It is generally agreed that the value gradually diminishes from front to rear, but not on a straight-line basis. Many theories have been advanced as to the law of this diminishing value, but none are universally accepted. The rule most widely applied is that the standard lot may be divided into four equal sections from front to rear and that 40 percent of the value lies in the front section, 30 percent in the second, 20 percent in the third, and 10 percent in the rear section. As a matter of fact the ratio of width to depth, the exact nature of the development of the area, the particular area of the city, and probably many other factors have a bearing on the depth rate proper in particular cases. A number of depth tables have been promulgated for use in determining the percentage of the value of a lot of standard depth to apply to a lot of any other depth. Several of the more commonly used depth tables are given in Table 16.1; others will be found in the literature of land appraisal.

16.11. Corner Influence.—It will be apparent that, if proximity to a street influences the value of a parcel of land, a corner lot, which has access to two streets, will usually be more valuable than an inside lot, which has access to but one street.

The exact influence of a corner location on value depends upon the size and shape of the lot and the relative standard unit values on the two streets. No generally accepted rule has been devised for computing the

Table 16.1.—Depth Tables for Assessment of City Lots

To assess a rectangular inside lot, multiply the standard unit value for the street upon which the lot fronts by the width of the lot and by the proper depth factor from this table. See text for special treatment of corner lots

Depth of lot, ft.	Somers' Cleveland rule	Baltimore standard	Milwaukee, Wis.		Newark rule	Florida, 130 ft., residential	Florida, 125 ft., business
			Residential	Business			
5	0.1435	0.09	0.06	0.17	....	0.0883	0.1285
10	0.2500	0.15	0.12	0.28	0.15	0.1678	0.2207
15	0.3322	0.21	0.17	0.37	....	0.2384	0.3032
20	0.4100	0.27	0.23	0.44	0.30	0.3002	0.3777
25	0.4790	0 33	0.28	0.49	....	0.3532	0.4417
30	0.5400	0.385	0.33	0.54	0.45	0.4018	0.4967
40	0.6400	0.49	0.42	0.63	0.55	0.4945	0.5877
50	0.7250	0.585	0.51	0.70	0.65	0.5784	0.6607
60	0.7950	0.67	0.59	0.77	0.74	0.6534	0.7247
70	0.8560	0 739	0.67	0.84	0.83	0.7196	0.7798
75	0.8830	0.769	0.71	0.87	....	0.7506	0.8053
80	0.9090	0.796	0.75	0.90	0.91	0.7788	0.8308
90	0.9560	0.842	0.82	0.95	0.96	0.8327	0.8763
100	1.0000	0.88	0.88	1.00	1.00	0.8830	0.9175
110	1.0400	0 911	0.95	1.03	1.04	0.9270	0.9545
120	1.0750	0.938	1.00	1.07	1.08	0.9660	0.9865
125	1.0905	0.95	1.02	1.09	....	0.9830	1.0000
130	......	.....	1.03	1.10	1.12	1.0000	1.0135
140	1.1300	0.982	1.07	1.13	1.16	1.0290	1.0385
150	1.1500	1.000	1.09	1.15	1.20	1.0530	1.0600
160	1.1680	.....	1.12	1.17	1.24	1.0750	1.0780
170	......	.....	1.14	1.18	1.27	1.0950	1.0940
175	1.1914	.....	1.15	1.19	....	1.1040	1.1010
180	1.1980	.....	1.16	1.20	1.29	1.1130	1.1080
200	1.2200	.....	1.19	1.22	1.33	1.1430	1.1300

value of corner lots so as to bring in corner influence on value, although several simple methods are in use. The judgment of the appraiser will be of necessity the main dependence in arriving at a fair value in these cases. However, a few principles may be set up to serve as a guide.

1. In Business Districts:
   a. Corner influence may be considered to extend 100 feet from a corner along each street side of the corner tract.
   b. Corner influence on lots adjacent to corner lots (within the 100-feet zone) is probably less for lots under separate ownership than when under the same ownership as the corner lot.
   c. Alley-corner influence is similar to street-corner influence, but somewhat less in magnitude.
2. In Residential Districts:
   a. Corner influence probably does not extend beyond the lot at the corner, except perhaps in apartment-hotel districts.
   b. Corner influence is a smaller factor in the value than for business property.
   c. Corner influence applies only when the corner lot is of a size and shape suitable for a residence (assuming the district is limited to residential property under a zoning ordinance or similar statute).

While the foregoing appear to apply in general, there will be many exceptions and these principles are at best no more than suggestive of the basic considerations to take into account in fixing the value of corner lots.

16.12. Appraisal of Inside Lots.—Inside lots are those outside the zone of corner influence. Their value is calculated by first determining from a land-value map the standard unit value for the street upon which they front. This will be in dollars per front foot for a standard depth. The standard unit value is then multiplied by a depth factor from the official depth table for the city which will be similar to those given in Table 16.1. This gives the calculated value per front foot, which is multiplied by the width to arrive at a value for the lot. This calculated value is then adjusted, as the judgment of the appraiser dictates, to take account of any special favorable or unfavorable factor applicable to the particular lot. Generally, the adopted value will not differ greatly from the calculated value.

If the lot is not rectangular, it may be divided into slices parallel to the front and each slice valued by obvious calculations, and the sum becomes the calculated value. Or the lot may be divided into rectangles and triangles, the rectangular part being valued as above and the triangular portion valued separately. Triangular lots are often valued by means of a special depth table, such as that given in Table 16.2.

16.13. Appraisal of Corner Business Lots.—Many methods for valuing corner lots are in use, and the following is one that is rather widely used. The standard unit value on each of the two streets is ascertained. The lot is then valued as an inside lot fronting on the street of higher unit value and then as an inside lot fronting on the other street, but excluding any portion more than 100 feet from the corner. The portion more than 100 feet from the corner is then valued as an inside lot fronting on the side street. The sum of the two values thus obtained for the corner, plus the value of any portion outside corner influence,

becomes the calculated value. This is then modified according to judgment, to compensate for favorable or unfavorable factors.

Some appraisers use a special depth table to distribute the corner influence of the secondary street. Others apply a special depth table based on the ratio of the standard unit values on the two streets. Still others appraise the lot as an inside lot fronting on the street of the higher

TABLE 16.2.—DEPTH FACTORS FOR TRIANGULAR CITY LOTS
Assessment Bureau of Milwaukee, Wis.

To assess an inside triangular lot, multiply the width at the base of the triangle by the standard unit value for that street and by the proper factor from this table. Special methods are needed for corner triangular lots

Business lots			Residence lots		
	Depth factors			Depth factors	
Depth of lot, ft.	When base of triangle lot is on street line	When apex of triangle lot is on street line	Depth of lot, ft.	When base of triangle lot is on street line	When apex of triangle lot is on street line
50	0.45	0.25	50	0.27	0.24
60	0.50	0.27	60	0.32	0.27
70	0.54	0.30	70	0.36	0.31
80	0.58	0.32	80	0.41	0.34
90	0.62	0.33	90	0.45	0.37
100	0.66	0.34	100	0.49	0.39
110	0.69	0.34	110	0.53	0.42
120	0.72	0.35	120	0.56	0.44
130	0.75	0.35	130	0.60	0.43
140	0.78	0.35	140	0.64	0.43
150	0.80	0.35	150	0.66	0.43
160	0.82	0.35	160	0.69	0.43
170	0.84	0.34	170	0.72	0.42
180	0.86	0.34	180	0.74	0.42
190	0.88	0.33	190	0.76	0.42
200	0.90	0.32	200	0.78	0.41

standard unit value and then add from 20 to 50 percent of the side-street value because of corner influence.

In all cases it is important to recognize that a value calculated by rule will be the true value in only a fraction of the cases dealt with. It must always remain the province of the valuator to use good judgment in fixing the value and to study each case with a view to arriving at a correct estimate in view of all the factors to be noted.

16.14. Appraisal of Residential Corner Lots.—It is conceded that the value of a corner lot in a residential district is generally greater than the

value of an inside lot of the same size. So many factors affect the value of residential corners that it is impossible to suggest any rule or adjustment method that is likely to have more than limited application. By inquiry as to the selling price of corner and inside lots in any neighborhood, a relationship can be worked out that will be applicable to that locality. The lot is appraised as an inside lot fronting the primary street and then a correction made for corner influence on the basis of conditions observed in the particular neighborhood and especially the way corner lots have sold.

ASSESSMENT OF LAND FOR TAXATION

The value placed on land for taxation purposes should bear some definite relation to its fair market value, and the effort of assessment bureaus is to bring about a situation in which the assessments will be made impartially and under a system that precludes injection of any personal bias of the assessor. While assessment work is not a part of engineering valuation, the values for land that are fixed in the valuation of a property must be consistent with the assessed value, or good reason shown for the departure from that value. Moreover, the base maps used in assessment work are invaluable aids in the appraisal of land.

16.15. Assessment of Inside Lots.—The assessment of an inside lot (one not affected by corner influence) of a rectangular form consists merely in multiplying the unit value by the proper depth factor and by the width. If a triangular lot is to be assessed, its base width is multiplied by the unit value and by the proper depth factor.

16.16. Assessment of Corner Lots.—In assessing corner lots for taxation, it is essential that uniformity be achieved and that a system be employed that can be handled by assistants of no particular training in appraisal, although experienced in assessment work. This situation has been met by providing that a corner residential lot shall be assessed just like an inside lot, except that its width shall be assumed to be increased a certain number of feet. For example the practice in Milwaukee, Wis., is to add 7 feet to the frontage for corner influence in the better residential districts with street improvements and 4 feet in less desirable districts with street improvements. In the same city, corner influence on business lots is obtained by assuming 7 feet to be added to the frontage and to this is added from one-sixth to one-third the value based on the side-street unit values. Corner influence on business lots is assumed to extend back 100 feet from the main street and 40 feet from the secondary street. If the lot is longer than 100 feet, the remainder is assessed as an inside lot fronting the side street. If the lot is less than 40 feet wide, the remainder of the corner influence is carried to the adjacent lot on the basis that 40 percent of value lies in the first 10 feet next to the side street, 30 percent

in the second 10 feet, 20 percent in the third 10 feet, and 10 percent in the fourth 10 feet. Alley-corner influence is handled in the same way.

Various other similar empirical rules are employed in several cities, but all of these have their limitations and are not always followed closely in appraisals for purposes other than taxation.

16.17. Assessment of Irregular-shaped Parcels.—Irregular-shaped parcels of land must be assessed by some less laborious method than the

Table 16.3.—Illustrating Determination of Calculated Value of Business Lots[1]

Lot	Primary street value				Secondary street value for depth subject to corner influence				Secondary street value part beyond corner influence				Total value
	Depth factor	Unit value	Frontage, ft.	Value	Depth factor	Unit value	Side-street frontage, ft.	Corner influence for depth 100 ft.	Depth factor	Unit value	Frontage, ft.	Value	
1*a*	0.28	$1,200	67.5	$22,680	0.820	$1,000	10	$ 8,200					
1*b*	0.16	1,200	62.5	12,000	0.785	1,000	10	7,850					
1*c*	0.10	1,200	57.5	6,900	0.755	1,000	10	7,550					
1*d*	0.09	1,200	52.5	5,670	0.720	1,000	10	7,200					
1*e*	0.07	1,200	47.5	3,990	0.685	1,000	10	6,850					
1*f*	0.07	1,200	42.5	3,570	0.650	1,000	10	6,500					
1*g*	0.07	1,200	37.5	3,150	0.610	1,000	10	6,100					
1*h*	0.06	1,200	32.5	2,340	0.565	1,000	10	5,650					
1*i*	0.05	1,200	27.5	1,650	0.515	1,000	10	5,150					
Total, lot 1..	....	......	....	61,950	.....	......	...	61,050	....	....	..	....	$123,000
3*A*	0.95	1,200	30	34,200									
3*B*	0.32	1,200	20	7,680									
Total, lot 3..	....	......	....	41,880	.....	......	...	....	....	....	..	....	41,880
6	1.05	1,200	30	37,800	0.05	800	100	4,000	....	....	..	....	41,800
7	1.05	1,200	40	50,400	0.25	800	100	20,000	....	....	..	....	70,400
8	1.13	1,200	50	67,800	0.70	800	100	56,000	0.70	$800	40	$22,400	146,200

[1] Based on layout of Fig. 16.1 and the Milwaukee depth table (Table 16.1) for business property, and following the method discussed in Secs. 16.10, 16.11, and 16.12.

one outlined in Sec. 16.12 for computing their value. The common procedure is to divide the lot into rectangular and triangular sections and value each by the method established for tracts of their respective shapes. In many cases it is sufficient to divide the lots into sections approximating the forms mentioned, when curved streets or alleys or stream banks make impossible the formation of exact rectangles or triangles.

16.18. Illustration of Calculation of Lot Values.—The plat given in Fig. 16.1 is used as the basis for calculating the value of certain lots, utilizing the methods discussed herein. The computations are shown in detail in Table 16.3. These calculated values would in many cases be

modified somewhat on the basis of the appraiser's judgment and study of the individual parcels involved. But the calculated values are usually the starting place in the equalization and often would be accepted as calculated.

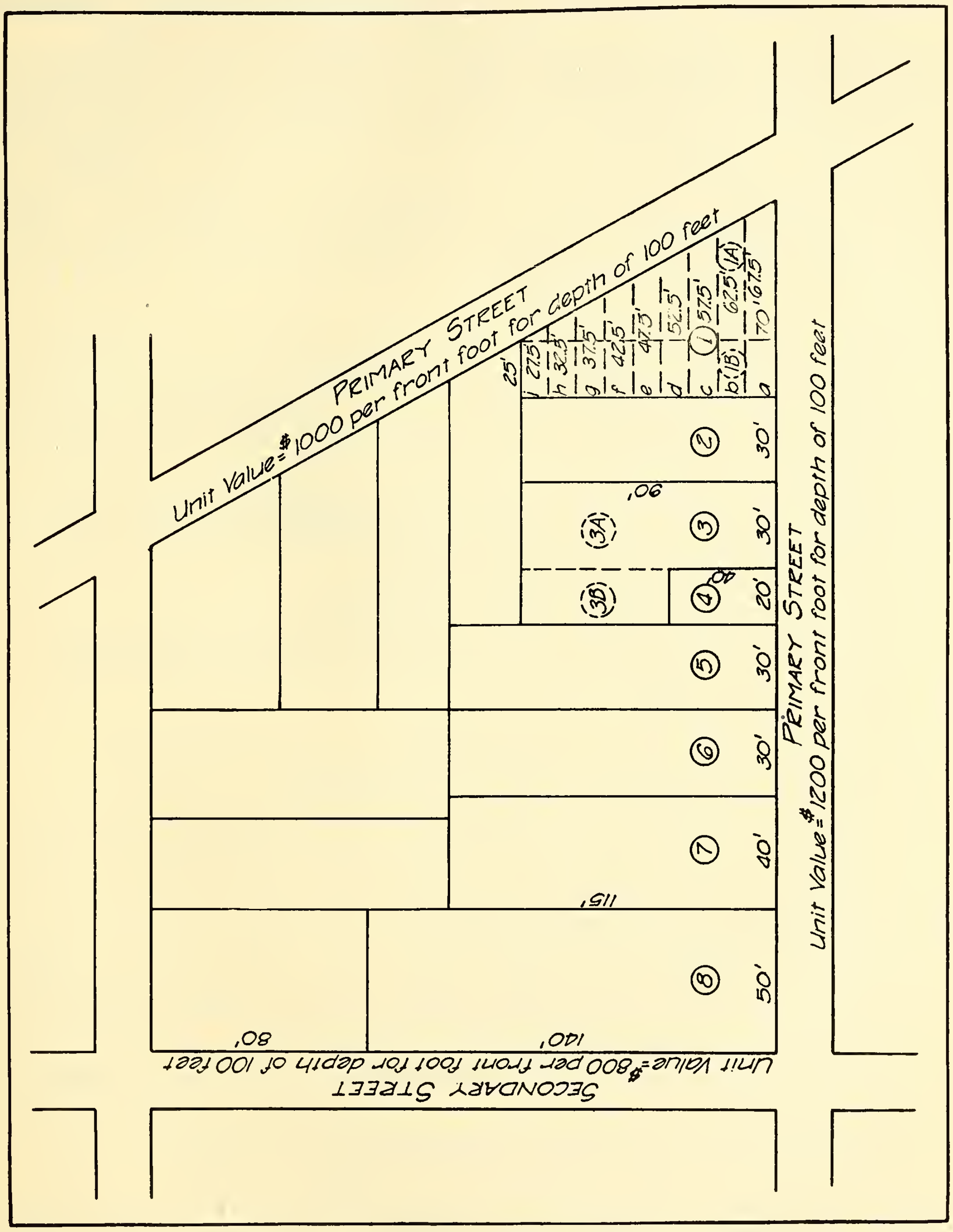


Fig. 16.1.—Illustrating use of standard unit values.

VALUATION OF FARM LAND

The farm ordinarily constitutes a combined production unit and a home. Its value is, therefore, compounded from factors that reflect the value from the standpoint of earnings and others that take into account the satisfactoriness of the particular farm as a home. The value of a farm

is arrived at by comparison with farms that have an established market price, because they have been sold sufficiently recently to make their sale price valuable for comparative purposes. It is in establishing the comparative value that the appraiser encounters the difficult problem of evaluating the many characteristics of the individual farm. In general, it is desirable to appraise the land and the buildings separately.

16.19. Factors Affecting the Value of a Farm.—It is comparatively easy to establish the general level of farm values in any area from the record of sales, except in abnormal years such as the period 1930 to 1935. The real problem confronting the appraiser is to apply these general values to a specific farm. The following factors, among others, will need to be considered in the usual case:

1. Lands in the Humid Regions:
   a. The kinds of crops that can be grown on the soil advantageously, in detail for each portion of the tract.
   b. The extent to which the vital elements of fertility in the soil have been depleted.
   c. The character of the road system serving the farm.
   d. The accessibility of markets for the several kinds of possible crops.
   e. The social character of the neighborhood.
   f. The educational facilities of the neighborhood.
   g. The racial and religious groups dominant in the neighborhood.
   h. The bonded indebtedness of the political unit in which the farm lies.
   i. The annual tax rate.
   j. The annual rainfall and its distribution.
   k. The adequacy of the natural and artificial drainage of the land.
   l. The adequacy and exact character of the fences.
   m. The fair cost-value of the buildings and other fixed improvements such as cribs, scales, mills, wells, and the like.
   n. The nature and abundance of water for stock and domestic use and its cost.
2. Lands in Semiarid and Irrigated Regions:
   The factors enumerated for humid regions and, in addition, the following:
   a. The source and cost of irrigation water.
   b. The nature of the control of the irrigation district, whether mutual, private company, U.S. Reclamation Service, or state.
   c. How long the land has been under irrigation and the effect thereof on the productivity.
   d. The need for expenditures for underdrainage in the future.
   e. The bonded or other obligations for the irrigation project.

16.20. The Basis of Farm-land Values.—Intrinsically the value of farm land rests upon the financial returns that may be secured by operation of the farm. As a practical matter it is found that the returns actually obtained depend very greatly upon the skill and industry of the owner, and the past history of a farm may give little clue to the probable future returns. In addition the farm is also a home, and the probable financial returns are often outweighed by factors that make the par-

ticular farm a desirable home. These are partly aesthetic (although the prospective owner might not recognize that word, he does sense the desirability of beautiful and healthful surroundings) and partly the social, religious, and educational opportunities of the neighborhood and accessibility to town over good roads.

In the main, all these things are reflected in the general level of sale prices in the neighborhood, and the appraiser must try to ascertain whether a given tract of land is worth more or less than the average farm of the region.

Note. A Production Method of Valuing Land. In this method a detailed soil survey is made of the farm by which it is divided into measured soil areas. The present and the potential yields of each of these areas are estimated, for an advisable system of rotation and farm management. The present values and "use values" of the buildings are also estimated from which they are credited with a fair rent per year. To this the values of customary landlords' shares of products are added to get gross return. Net return is found by deducting average annual expenditures for taxes, insurance, repairs, seed, lime, fertilizers, etc. The earning value of the farm is then found by capitalizing the net return thus estimated. See "A Production Method of Valuing Land," by W. G. Murray and H. R. Meldrum, *Bull.* 326, Iowa Agr. Exp. Sta., Ames, Iowa, 1935.

CHAPTER XVII

VALUATION OF MINES, MINERAL DEPOSITS, AND TIMBERLANDS

The value of timberlands, mines, and mineral and ore deposits is estimated upon the basis of the forecasted earnings of the plant developed to produce and market the material or a product produced therefrom. In certain respects the application of the principles of estimating earning value that have already been presented (Secs. 1.25, 15.1) involves special consideration when applied to the peculiar conditions that surround enterprises of this type, due to the gradual depletion during operation of the supply of material which is the principal asset of the enterprise.

For the purposes of this discussion, the term *mineral* is employed to include all materials belonging to the mineral kingdom that may be won and marketed by commercial enterprises. The term *mine* is used to designate the plant and workings employed in winning and preparing for market certain kinds of minerals. The terms *quarry*, *gravel plant*, and *clay pit* are used to designate certain types of mines.

While the discussion that follows deals specifically only with the valuation of timberlands and mining properties other than stone, gravel, or clay, the methods outlined are applicable to the valuation of quarries, gravel, or clay deposits and the like.

Enterprises like mines and lumbering must generally be considered as of the limited-life type. It is true that some mineral deposits are so enormous that many years of operation will be required to exhaust them, and no appreciable error will be incurred if they are considered to have unlimited life. In the great majority of cases, however, the life of the enterprise established to work a mineral deposit must be valued on the basis of a limited life. This necessitates the introduction of special amortization factors into the valuation computations, as will be discussed later.

17.1. Accuracy of Mine Valuations.—It is recognized that it is impossible to determine the true value of a mineral deposit in advance of its complete exhaustion. The best the valuator can do is to determine what he feels sure to be the minimum value that should be placed on the property. He may also hazard an estimate as to the probable maximum value, but this is likely to be more difficult than it is to deal with the minimum values; while these limitations are generally under-

stood by the promoters of enterprises of this type, investors probably do not recognize the questionable nature of the estimates of maximum value. While all valuations must be in the nature of forecasts of the future, those involving mineral properties are, in general, less substantial than most others.

The returns that may be secured from the operation of a mining property must usually be estimated on the basis of a highly competitive market in which to dispose of the product. For these reasons the beneficent atmosphere that often pervades the public-utility field, due to the assurance of a "fair rate of return on the fair value," is entirely absent in the field of mining. The earnings of the property, like those of other ordinary industrial enterprises, are wholly dependent upon what the product will fetch in the markets. On that basis the valuation of a mineral deposit involves the analysis of the production and marketing situation, as well as of the amount and richness of the ore or mineral.

The development of mines in particular, and to some extent other enterprises of this class, involves hazards that are elements of risk not attendant upon ordinary industrial enterprises; consequently, to attract capital, the prospective rate of return must be higher than is paid by ordinary enterprises.

Two distinct types of valuation problems are encountered in this field; one being the valuation of undeveloped deposits of raw materials and the other the valuation of mines that have become established and are operating as going concerns. These two types of valuation problems will be discussed in order.

VALUATION OF UNDEVELOPED DEPOSITS OF MINERALS

The valuation of undeveloped deposits of minerals requires two kinds of estimates that are but little related. The engineer must first ascertain the probable quantity and the quality of the material in the deposit; then he must estimate what it will cost to convert the material into a marketable product, and to market it, thus establishing a basis for estimating the probable annual operation returns during the life of the enterprise.

17.2. Estimating the Quantity of Material.—The methods employed in ascertaining the probable quantity of material in an ore body, or mineral deposit, are highly specialized and entirely too involved for presentation herein. Suffice it to say that the work calls for all of the resourcefulness of the experienced mining engineer, the geologist, and perhaps other specialists. There are practices extant that have gradually developed in each field of mining, such as coal mining, copper mining, silver mining, and the like, but in the final analysis each case is a special problem, and the valuator employs whatever methods he thinks will

aid him in forming a judgment as to the quantity of available material. Whatever his conclusions are, they can be no more than carefully considered estimates and it is the aim to arrive at a quantity that is the assured minimum. If development is warranted on the basis of the probable minimum quantity, then the deposit has a value that can be determined with some degree of assurance. Otherwise the project passes into the speculative field, which is outside the range of this discussion, although properties of this class are an important factor in the mining industry.

17.3. Estimating Quality of Material.—The determination of the quality of the material in a deposit is, of course, as important as is the estimate of quantity. By appropriate sampling, and testing or assaying, the quality is ascertained. In sampling ore deposits it is necessary to follow highly specialized methods; not only in taking the samples, but also in evaluating the results of assays. The sampling of rock, clay, and gravel deposits is also a specialized procedure, but one much less complicated and trying than mine sampling. In all cases an attempt is made to ascertain the quality and quantity of marketable material in the deposit. In the case of ores or other minerals, it is a question of tons of iron, copper, lead, or other metal; in the case of minerals like asbestos, it is a question of tons of marketable mineral of the degree of purity required by industry.

17.4. Estimating Market Value of Product.—Metals and minerals are generally sold to industries that utilize them for the production of commodities. Markets have been definitely established for many minerals and metals, and the prices paid for these are matters of record. The valuator ascertains the prices paid over a period of years by a study of price trends, tries to look into the future to gain an impression of the possibilities of changes in market conditions, and then sets a price that he thinks is assured during the period of productivity of the property under consideration. This price is converted into a price per ton at the site of the proposed development by taking account of transportation, possible tariff imposts, taxes on tonnage, and all similar factors. Obviously, all of this involves an intimate knowledge of the business and a thorough analysis of all economic and engineering factors involved. At best, the result is no more than an estimate; but, if it were necessary to have a more exact basis for securing money for development, mining would never have become the tremendous industry it is today.

There are instances in which a deposit of mineral, clay, stone, or gravel is to be developed and converted into a finished product as one enterprise. A mineral may be used in a manufacturing process, clay converted into ceramic ware, stone finished for use in buildings, or gravel made into pipe or building blocks. In these instances, the

valuator is interested primarily in the suitability of the material for the intended product and its value as a raw material; therefore, the basic problem is to estimate the probable value of the deposit for these purposes and differs only in method from that employed in the valuation of ore deposits.

17.5. Estimating Cost of Production.—The next step in the valuation of this type of property is to estimate the cost of production, which will then serve as a basis for computing the annual operation return to be expected from the enterprise. This involves the development on paper of a method of winning the material, the general design of a plant with all accessories, including transportation, and an estimate of operation costs. In many cases the methods employed are so completely standardized that general cost estimates may be employed with some slight modification to compensate for any especially favorable or unfavorable conditions that surround the particular project. In this connection it is necessary to estimate the time required for development, the rate of production that can be counted upon, the capacity of the markets to absorb the output, and many similar factors. Out of this study the valuator finally reaches a decision on three things:

1. The annual operation return to be anticipated from the development of the deposit, or possibly a schedule showing the probable operation returns from year to year, if it is expected that the returns will fluctuate widely.
2. The number of years during which the returns may be expected to continue.
3. The amount of risk attending the enterprise and the effect of that risk upon the rate of net return that will be required to attract investors.

In general, the enterprise will pass through the stages common to all industrial enterprises and already made familiar by previous discussion in this treatise (Sec. 2.7):

1. The promotion stage, during which the project is being organized and financed.
2. The construction stage, during which the plants are built.
3. The development of production stage, during which the plant will be in partial operation with production gradually increasing to the maximum contemplated by the promoters of the enterprise.
4. Production at the capacity rate, subject, however, to periodic reduction or cessation because of market conditions. At the average rate achieved during this stage, the operation returns should be approximately those upon which the project was predicated, if the enterprise is well managed and soundly developed.
5. In addition, many enterprises of the character under consideration experience a final stage during which production gradually diminishes until the returns shrink to the point where it no longer pays to continue operation.

17.6. Basis of Estimating Value.—The value of mineral, or other deposits of the kind being considered, must be predicated upon the fact that the enterprise is one of wasting assets. Consequently, the investor must be assured of the return of his investment during the life

of the property and, in addition, a suitable return each year on the portion of his investment remaining tied up in the property. The determination of the present worth of the anticipated annual operation returns involves some special considerations. Since the life of the enterprise is limited to that required to exhaust the supply of material, the following principles apply:

1. The owners are entitled to recoup their investment during the life of the enterprise.

2. The owners are entitled to a rate of net return on such capital as remains invested in the business in any year[1] that is commensurate with the risk involved in developing and operating the property.

17.7. Rate of Net Return.—It is generally stated that an investor in an enterprise of this type will demand, and should receive, a rate of net return greater than the rate he is assured in risk-free investments. The magnitude of the risk factor obviously varies with the hazards of a particular enterprise. He may be assured of a 4 or 5 percent rate of return on funds invested in risk-free securities, such as state bonds. If so, he will exact a higher rate if he is to invest in a mining enterprise, and the factor by which the assured rate must be multiplied to give a suitable rate of return for a mining property is called the "risk factor." In general, the risk factor varies from 2 to 5, but it may be higher in exceptional cases. In the computation of the value of properties of this class, the risk factor is fixed on a basis of the hazards of the particular enterprise under consideration.

17.8. Computation of Value.—In the computation of the value of an ore or mineral deposit, certain data are available from the examination of the property as has been set forth. These may be summarized as follows:

1. The estimated probable average operation returns from the sale of the product over a period of years in the future.

2. The estimated investment required to bring the enterprise to steady production at a profitable rate.

3. The rate of net return that will probably be necessary to attract capital to the development of the enterprise.

With the foregoing as a background it is customary to assume that the annual operation returns should be sufficient to

1. Pay a return to the investor on sums actually invested in the enterprise at the rate estimated as proper in view of the risks.

[1] It is the common practice to assume that the owner of a mining property is entitled to a satisfactory rate of return each year on the *total* capital he has invested, although he has perhaps had most of it returned to him through extra dividends that provide the annual amortization payments set up in accordance with 1. In order to compare mine values with the values of other types of properties, however, the assumption of 2 is necessary.

2. Provide an equal-annual-year-end-contribution to a fictitious sinking fund, which will extinguish the investment during the estimated life of the enterprise. It is customary to assume that the annual sinking-fund contribution will immediately be reinvested at the assured rate of interest, and hence the interest rate to use in the sinking-fund computation is usually 3 to 4 percent. Actually this investment is made by the recipient of dividends in whatever type of property he desires to own.

Let A = the estimated annual operation return in dollars, to continue at a uniform rate for n years. This is equal to the annual gross receipts from the sale of the product less the annual operation cost, since no depreciation reserves are set up (but operating costs include replacements of worn-out equipment).

n = the number of years during which the property is expected to produce an annual operation return of A dollars per year. This period begins on the date when the property reaches full productivity.

i = the rate of interest assured in risk-free investments, such as government bonds.

s = the rate of return required to attract capital for the development of the specific property; = i times a proper risk factor.

d = the number of years required to develop the property to full production.

V_p = the value of the specific property under consideration on the date when development begins.

The value of the property may be computed from the following formula:

$$V_p = A\frac{(1+i)^n - 1}{i + s[(1+i)^{d+n} - 1]}. \qquad (17\text{–}1)^1$$

[1] Formula (17–1) is known as the O'Donahue modification of the Hoskold annuity formula, and its derivation is as follows:

The accumulation of A in n years is

$$A\frac{(1+i)^n - 1}{i}. \quad \text{(See Formula 5–2).}$$

At the end of $d + n$ years, the investor must be in possession of his original investment V_p, plus the accumulation at compound interest at rate i of the annual return sV_p.

$$\text{Accumulation of } sV_p = sV_p\frac{(1+i)^{n+d} - 1}{i}.$$

$$\text{Total accumulation} = V_p + sV_p\frac{(1+i)^{n+d} - 1}{i}.$$

Hence,

$$A\frac{(1+i)^n - 1}{i} = V_p + sV_p\frac{(1+i)^{n+d} - 1}{i};$$

$$A[(1+i)^n - 1] = V_p\{i + s[(1+i)^{n+d} - 1]\};$$

$$V_p = A\frac{(1+i)^n - 1}{i + s[(1+i)^{n+d} - 1]}.$$

NOTE 1.—If $s = i$, the above formula becomes $V_p = A\frac{(1+i)^n - 1}{i(1+i)^{n+d}}$.

NOTE 2.—The "fictitious" sinking-fund conception is employed in developing the O'Donohue formula and consequently the annual return to the investors is computed on the "value new" of the property instead of the "present value" (Secs. 5.6, 5.25).

17.9. Example of the Valuation of an Ore Reserve.—The present value of a certain iron-ore reserve is estimated on the following basis: The period of active production required to exhaust the reserve is assumed at 20 years. The period required for development is 2 years. The accumulative or sinking-fund rate of interest is taken at 3 percent and the risk factor 2⅓, which gives a rate of return of 7 percent on the investment.

1. *Reserve Tonnage.*—The tonnage in the ore reserves was determined by core drilling, and the quantities were computed according to the usual practice. Since conditions in part of the area were favorable to open-pit mining, and part would have to be mined by underground methods, the reserves of each type had to be ascertained because of the differences in cost. The estimated reserves are as follows:

	Tons
Available for open-pit mining	5,102,100
Available for underground mining	1,025,000
Total	6,127,100

Annual production, on the basis of 20 years for exhaustion:	
Open pit	255,100
Underground	51,250
Total	306,350

2. *Selling Price.*—The selling price was fixed by determining the average Lake Erie selling price for the five-year period immediately preceding the valuation for ore of basic grade. The average Lake Erie value was then computed upon the basis of the analyses of core-drill samples as follows:

	Tons	Lake Erie value	Total
Bessemer ore	3,490,500	$4.763	$16,625,251
Non-Bessemer ore	2,217,000	4.189	9,287,013
Paint Rock A	355,800	3.178	1,130,732
Paint Rock B	63,800	3.274	208,881
Total	6,127,100	......	$27,251,877

Average Lake Erie value per ton = $27,251,877/6,127,100 = $4.448.
Annual gross receipts = (306,350)($4.448) = $1,362,644.

3. *Mining Costs.*—The mining costs were determined from the records of operating units in the area, supplemented by an analysis of the operating conditions expected to prevail when this particular ore reserve is developed. The unit costs upon which these estimates are made do not include future development costs or plant investment, these being estimated separately.

Method	Tons handled	Cost per ton	Total cost
Power-shovel mining	3,334,200	$0.25	$ 833,550
Power-shovel mining, lean ore	119,200	0.30	35,760
Milling and scram	1,767,900	0.35	618,765
Milling and scram, lean ore	49,800	0.40	19,920
Total	....	.....	$1,507,995

Tons of open-pit ore realized = 5,102,100.
Average cost of open-pit mining = $1,507,995/5,102,100 = $0.296 per ton.
Cost of underground mining = $1.30 per ton.

4. *Transportation.*—The cost of transportation by rail and lake, including insurance, is calculated from published tariffs and standard insurance rates and for this property is $1.741 per ton. Annual cost of transportation and insurance is

$$306{,}350 \times \$1.741 = \$533{,}355.$$

5. *Miscellaneous.*—Miscellaneous costs include administration, legal, fire insurance, medical and hospital expenses, workmen's compensation, contingencies, crushing and screening, cost adjustments, stockpile loading, and taxes on stockpile and equipment. For this case these costs were determined from average experience and are

For open-pit mining	(5,102,100)(0.121) =	$617,354
For underground mining	(1,025,000)(0.201) =	206,025
	Total	$823,379

Miscellaneous cost per year, $823,379/20 = $41,170

6. *Selling Commission.*—The selling commission is fixed for this class of ore at 5 cents per ton.

Selling cost per year, (306,350)($0.05) = $15,317.

7. *Development.*—The development consists in stripping the reserves that are to be operated by the open-pit method of mining and sinking a shaft with the accompanying underground development in those parts of the reserves to be operated by underground mining methods.

Costs of open-pit stripping:

23,500 cu. yd. of soil overburden at $0.35 =	$ 8,225
779,800 cu. yd. of rock overburden at $0.75 =	584,850
Total cost of open-pit development	$593,075

Underground plant, direct estimate, $29,725.

It is considered that 75 percent of the cost of stripping and the total cost of shaft development will be incurred during the period of deferment. The cost of underground plant will be incurred over the period of production and 25 percent of the cost of stripping will likewise be spread over the period of production.

$$\text{Annual cost of development} = \frac{\$593{,}075}{(4)(20)} + \frac{\$29{,}725}{20} = \$8{,}900$$

8. *Plant.*—The cost of plant was estimated from experience of the properties in the area and the records and data in the files of the valuator.

Open-pit plant	$510,210
Shaft and underground development	25,625
	$535,835

It is expected that the cost of plant will be incurred during the period of deferment.

9. *Taxes.*—Taxes were estimated on the basis of rates of taxation including federal income taxes prevailing at the date of valuation. The actual taxes then being paid on active properties were of three kinds as follows:

	Per Ton
An active tax on production amounting to	$0.376
An occupational tax averaging	0.111
Federal income tax	0.118
Taxes during production period	$0.605

Annual taxes = (306,350)($0.605) = $185,340.

During the deferment period an "inactive" annual tax of $0.0346 per ton will be paid, or $106,000 each 6 months.

10. *Working Capital.*—The amount of working capital required was determined by estimating the average monthly production during the life of the property by each mining method, which gives a monthly production of 42,516 tons by the open-pit mines working 6 months each year and 4,271 tons per month by underground mines.

The capital required for open-pit operations is three-fourths of 2 months' mining, miscellaneous, and transportation costs plus 6 months' taxes (taxes paid twice a year)

	Per Ton
Mining costs	$0.296
Miscellaneous costs	$0.121
Transportation	$1.741
Total	$2.158

Mining costs, miscellaneous, and transportation = ($2.158)(42,516) = $91,750 per month.

Active tax = ($0.376)(42,516)(6) = $95,914 for 6 months.

Open-pit production capital = $\frac{3}{4}\{[(\$91,750)(2)] + \$95,914\}$ = $209,560.

Open-pit supplies for 1 month = $11,000.

Total open-pit capital required $209,560 + $11,000 = $220,560.

The working capital for underground mining is taken as the cost for 6 months of mining, miscellaneous, and taxes, plus 2 months' cost of transportation.

	Per Ton
Mining	$1.300
Miscellaneous	0.201
Taxes	0.376
Total	$1.877
Transportation	$1.741

Required production capital =

$$[(6)(4,271)(\$1.877)] + [(2)(\$1.741)(4,271)] = \$62,970.$$

Underground supplies for one month = $7,000.

Working capital required for underground mining = $69,970.

Total working capital required, open-pit plus underground = $290,530.

11. *Expenses during Period of Deferment.*—During the deferment period (2 years) the expenditures will be as follows:

Stripping	$444,806
Plant and underground development	535,835
	$980,641, or $490,320 per year

Inactive taxes, each 6 months, $105,990.

12. *Present Worth at Date of Appraisal of Expenses during Deferment.*—It may be assumed that these expenditures are spread over a two-year period and, therefore, may be considered to be concentrated at the middle of each year. That is, by the end of 6 months half of the expenditure will have been made, and by the end of 18 months the entire expenditure will have been completed. The value of these future expenditures at the beginning of the deferment period is as follows:

Value of development expense at beginning of deferment, or date of appraisal =

$$\frac{\$490{,}320}{1.03^{1/2}} + \frac{\$490{,}320}{1.03^{3/2}} = \$952{,}179$$

The investment, I_t, at the date of valuation to provide the funds for taxes during the deferment period is as follows:

$$I_t = \frac{\$105{,}990}{1.03^{1/2}} + \frac{\$105{,}990}{1.03^{1}} + \frac{\$105{,}990}{1.03^{1\frac{1}{2}}} + \frac{\$105{,}990}{1.03^{2}} = \$408{,}640$$

The investment required at the date of valuation to pay inactive taxes during the two-year period of deferment and provide funds for development costs during the deferment period, assuming the investment to accumulate until needed at 3 percent per annum, will be $408,640 + $952,180 = $1,360,820.

The annual operation return during the active life of the property required to amortize the investment in development and pay 7 percent thereon during the active period plus the period of deferment (using 3 percent and 7 percent as sinking-fund rate and rate of return, respectively) is A in the following:

$$\$1{,}360{,}820 = \frac{A[(1.03)^{20} - 1]}{0.03 + 0.07[(1.03)^{22} - 1]} = 8.564A$$

$$A = \$158{,}899$$

13. *Computation of Appraised Value.*—The various items detailed above are assembled and summarized as follows:

(*a*) Annual gross receipts	$1,362,645
(*b*) Annual operating expenses:	
Mining, open pit	$ 75,510
Mining, underground	66,630
Transportation and insurance	533,355
Miscellaneous	41,170
Selling commission	15,320
Development costs	8,900
Taxes	185,340
Interest on working capital at 7%	20,340
	$ 946,565
(*c*) Annual operation return required to amortize development costs and taxes incurred during deferment period	$ 158,900
Total (*b*) + (*c*) =	$1,105,465

(*d*) Annual operation return (available for "return" to investors and the amortization of the initial investment) is

$$(a) - [(b) + (c)] = \$257{,}180.$$

The value, V_p, of the ore reserve is determined by use of the O'Donohue modification of the Hoskold formula, with $r = 0.03$, $s = 0.07$, $n = 20$, $d = 2$ and $A = 257{,}180$.

$$V_p = \$257{,}180 \frac{(1.03)^{20} - 1}{0.03 + 0.07[(1.03)^{22} - 1]} = \$2{,}202{,}500$$

17.10. Summary.—It will be apparent from the foregoing that many elusive and none-too-clearly defined factors appear in the estimates needed in the valuation of mines and mineral deposits. Such is the nature of the problem, and the engineer must perforce assign to each quantity a sum that represents his best judgment with reference thereto. Certainly, the estimates that result and the final value assigned are no more than an expression of the engineer's best judgment, but this is always the case in valuation, and, by following out an orderly and theoretically correct analysis, he is more likely to arrive at a reasonable result than if he follows short-cut methods. In many cases, it is true, the data upon which computations must rest are so tenuous that any refinement in computation would be foolish, and an estimate based on a consideration of the present worth of estimated future annual net returns without considering amortization of investment would indicate a value as closely as the conditions would warrant.

With reference to the assumed annuity included in the formula for the purpose of extinguishing the investment, it might be said that no such sinking fund is actually set up. The sinking fund is purely fictitious, for the reason that the investors prefer to have their investment returned in the form of dividends so they can reinvest it themselves. But only by assuming such a sinking fund can results be obtained that are comparable with investments in ordinary industrial enterprises that do not involve the mining risk.

VALUATION OF GOING MINES

The valuation of mines that have reached the production stage presents a much more definite problem than that of the undeveloped mineral deposit. In part, the field of speculation is left behind and the solid ground of a tangible operating enterprise is reached. There still lingers the elusive problem of determining the available ore or mineral in the holdings of the company or available to be added to those holdings. This can be determined with much greater certainty in an operating mine than it can in an undeveloped property, because the workings give access to the lower portions of the deposit and permit drill exploration to greater depths than is feasible by drilling from the surface. The

extension in the depth of the deposit always remains a somewhat doubtful factor in the estimate of the quantity of material remaining in an ore body or mineral deposit. The situation is analogous to that presented in re-estimates of expectancy of units of physical property, and by frequent re-estimates of ore reserves the value of the property can be kept in line with the actual facts as they develop.

17.11. Basis of Value of an Operating Mine.—The value of a mine is the present worth of the future annual operation returns estimated to be obtainable after setting aside an annuity sufficient to extinguish the investment required to acquire the property. Earning value is the dominating consideration in the valuation of properties of this class as has been set forth in the preceding pages. The problem of mine valuation is, therefore, that of estimating the probable annual operation returns during the remaining life of the enterprise. When a mineral deposit is exhausted, the plant is generally of little or no value. Some parts may be salvaged, but the sum realized for them is usually so small as to be negligible.

17.12. Procedure in Valuation of Mines.—The first step in the valuation of a mine is to determine the adequacy of the existing plant and the additional investment that will be required, if any, to permit continued operation at the desired rate.

The second step is to determine the remaining quantity and quality of ore or mineral in the deposit. This is accomplished by examination of the property, sampling and testing the materials, core drilling to explore the unworked portions of the holdings, and a study of the geological formations in the area in, and adjacent to, the holdings. From the data thus obtained, and the past history of the property, an estimate is made of the probable future annual production and the remaining years of operation to be anticipated.

The third step is to estimate the annual gross receipts during the remaining life of the enterprise. This involves a study of the market situation for the product and the probable price that may be obtained during the remaining life of the enterprise. It involves a study of prices during the preceding 10 or 15 years and a consideration of the trend of general industrial conditions and their relation to the use of the particular product. The factor is influenced by contracts held for the delivery of the product, and agreements as to prices for processing the product, such as contracts with smelters for the reduction of the ore from a mine.

The fourth step is to estimate the operating costs during the remaining life of the property. The past history of the enterprise is available to be used as a guide in estimating production costs, and a decision is reached as to whether these costs will continue on the same basis or will increase or decrease. In many instances, advances in methods of mining or

smelting may justify a modification of past costs in estimates for the future.

The final step is to determine the rate of net return to employ in computations of the value. The market price of the securities affords a basis for this estimate, but due account must be taken of abnormal conditions in the securities market. The hazards of the particular enterprise will be a factor, but these are largely known at the time of valuation and can be compensated for. The one most likely to influence the rate of net return is uncertainty with reference to the remaining quantity in the deposit of material.

17.13. Computations.—The data outlined in the preceding paragraphs are assembled, and the annual operation returns during the remaining life of the property are estimated. Using the nomenclature of the preceding example, the value of the property is computed from Hoskold's annuity formula.

$$V_p = A\frac{[(1+i)^n - 1]}{i + s[(1+i)^n - 1]} \qquad (17\text{–}2)^1$$

17.14. Example of the Valuation of an Active Mine.—The present value of an active mine is estimated on the following basis: The period of active production required to exhaust the ore reserves is calculated to be 8 years on the basis of the average production during the past history of the property. The accumulative or sinking-fund rate of interest is taken at 2½ percent, and the risk factor is established as 3 on the basis of the past history of the property. The shaft and surface plant and the sidings are adequate for the present production, but it is

[1] Hoskold's annuity formula, using the nomenclature of Sec. 17.8, is derived as follows:

sV_p = the annual return to be paid each year for n years.

$A - sV_p$ = the annual sum available for the sinking-fund investment.

$(A - sV_p)\left[\frac{(1+i)^n - 1}{i}\right]$ = the accumulation of $A - sV_p$ in n years at i rate, which must equal V_p. (See Formula 5–2).

$$V_p = (A - sV_p)\left[\frac{(1+i)^n - 1}{i}\right];$$

$$V_p = \frac{A[(1+i)^n - 1] - sV_p[(1+i)^n - 1]}{i};$$

$$iV_p + sV_p[(1+i)^n - 1] = A[(1+i)^n - 1];$$

$$V_p\{i + s[(1+i)^n - 1]\} = A[(1+i)^n - 1];$$

$$V_p = A\frac{(1+i)^n - 1}{i + s[(1+i)^n - 1]}.$$

NOTE.—If $s = i$, the above formula becomes

$$V_p = A\frac{(1+i)^n - 1}{i(1+i)^n}.$$

estimated that \$50,000 must be expended immediately for underground plant, principally pumping equipment. It is estimated that upon exhaustion of the ore reserves now in sight, the mining plant will have no net salvage value.

1. *Reserve Tonnage.*—The reserve tonnage has been rather accurately determined to be 2,100,000 tons.

Annual production, 2,100,000/8 = 262,500 tons per year.

2. *Selling Price.*—The average selling price during the past 3 years has been \$6.48 per ton.

Annual gross receipts, (262,500)(\$6.48) = \$1,701,000.

3. *Mining Costs.*—The mining costs during the past 5 years have averaged \$3.20 per ton, but it is evident these costs have been increasing in recent years and they are therefore taken at \$3.75 per ton.

Annual mining costs, (262,500)(\$3.75) = \$984,375.

4. *Transportation.*—The cost of transportation to smelter is fixed by standard tariffs at \$1.35 per ton.

Annual cost of transportation, (262,500)(\$1.35) = \$354,375.

5. *Miscellaneous Costs.*—Miscellaneous costs include administration, insurance, legal, hospital, workmen's compensation, and contingencies, averaging in the past \$0.16 per ton.

Annual miscellaneous costs, (262,500)(\$0.16) = \$42,000.

6. *Taxes.*—The taxes paid during the past 5 years, including federal income taxes, have been \$0.52 per ton.

Annual taxes, (262,500)(\$0.52) = \$136,500.

7. *Working Capital.*—The amount of working capital required has been found in the past to be equal to the mining costs for 3 months.

Working capital, $\frac{(262{,}500)(\$3.75)}{4} = \$246{,}100.$

Working capital can be secured for 5 percent, and the annual cost of working capital is \$12,305.

8. *Computation of Appraised Value.*—The items enumerated above are assembled, and the value calculated as follows:

(*a*) Annual gross receipts		\$1,701,000
(*b*) Annual operation expenses:		
Mining	\$984,375	
Transportation	354,375	
Miscellaneous	42,000	
Taxes	136,500	
Interest on working capital	12,305	\$1,529,555
(*c*) Operation return for "returns" and amortization		\$ 171,445

The present value of the future earnings of the property V_p may be calculated from the Hoskold formula, with $i = 2\frac{1}{2}$ percent and $s = 7\frac{1}{2}$ percent.

$$V_p = 171{,}445\frac{(1.025)^8 - 1}{0.025 + 0.075[(1.025)^8 - 1]} = 171{,}445\frac{0.2184}{0.04138} = (\$171{,}445)(5.277) = \$904{,}770$$

From the computed value of the future earnings there is deducted \$50,000, required immediately for plant betterment.

The present value is therefore \$854,770.

VALUATION OF LEASES ON MINERAL LANDS

17.15. Valuation of Mining Leases.—While mineral rights are often purchased outright, it is quite the custom to secure a lease on the property under the terms of which the owner is compensated on the basis of a cash bonus plus a royalty paid on each ton (or other unit) of product marketed during the life of the enterprise. Generally, the owner is paid a certain sum for an option to lease on or before some fixed date in the future, thus giving time to prospect the property and establish a value. If the property proves to have real value, the owner will be paid the bonus for the lease and the purchaser may then develop the property himself, form a company, and sell its securities; or sell the lease to an operating company. Even if he develops the property himself, he is likely to need to borrow capital or sell stock. Hence, in nearly all instances, it is necessary to establish a value for the lease as a basis for borrowing capital, for the sale of securities, or for the sale of the lease.

The value of such a lease is the difference between the value of the mineral deposit and the cost of development, with proper account being taken of the dates when payments must be made. Since the development is fraught with all of the hazards of mining, the investor should receive a risk rate of return upon his investment during the period of deferment as well as during the period of production. Hence the O'Donohue formula may be applied in estimating the present value of the future operation returns.

In some cases a mining company adds to its holdings by leasing adjacent proven lands, which it will work at a future date when the present workings are exhausted. In the meanwhile no mining risk is involved in carrying the leased land, and the investors are not entitled to risk interest during the period. Suppose the period of deferment to be d years and the period of active production n years. The value of the lease at the beginning of production would then be determined by applying Hoskold's formula for the period of n years, using the two appropriate interest rates. The present value of the lease would then be computed for the period of d years on the basis of a risk-free investment rate of interest.

17.16. Leases Held by Public Utilities.[1]—Public utilities often hold leases on lands in gas-, oil-, or coal-producing areas. Litigation over the valuation of leases of this type have been frequent in the state courts and in general have been bitterly contested and long drawn out. The cases have been too diverse to permit drawing any very positive general conclusions as to the views of the courts on this subject.

[1] See Sec. 13.14 and case 51, Sec. 8.6.

A review of the legal decisions germane to the subject indicates that the courts generally take the position that the values of leases of this type often depend primarily upon the original cost, since that is the one substantial fact that can generally be determined. Where market value can be established, as has been possible in a few instances involving leases on gas-producing lands, that is strong evidence of value. Ordinarily the stock-and-bond value of an enterprise is assumed to be an indication of its market value, but in the case of enterprises organized to develop oil or gas there are so many other facilities necessary that the portion of the indebtedness that rests upon the lease itself is hard to segregate. Oil-land leases and gas-land leases are sometimes the object of barter, and thus a market value is established. Generally this is not the case, as the companies that are in a position to bid for such leases are not in competition with each other, and while the willing seller element is present, the willing buyer is not.

In no case would the value of mineral leases owned by a utility be included in the rate base, unless the mineral resources were necessary in the future production of the services covered by the rates in question.

17.17. Numerical Problem Involving a Lease.—A prospecting organization purchases an option to lease a mineral property on or before a date two years in the future. After a period of prospecting and development, they establish that the property is commercially valuable and proceed to negotiate for the sale of the lease. Let it be assumed that the lease carries the provision that a royalty of 25 cents per ton shall be paid on the marketable product. Suppose the deposit in question be the one considered in Sec. 17.9. Since the operating company does not purchase the mineral rights but pays a royalty instead, it must determine what it can afford to pay for the lease.

The first step would be to estimate the present value of the mineral deposit as before (Sec. 17.9), except that the annual operation return is reduced by 25 cents per ton, the agreed royalty rate, and the annual return available for return and amortization of the investment is

$$\$257,180 - \$76,590 = \$180,590.$$

The value of the lease is determined by use of the O'Donohue modification of the Hoskold formula with $i = 0.03$, $s = 0.07$, $n = 20$, $d = 2$, and $A = \$180,590$. In practice, the computed value of the lease would be reduced by a factor that provides against errors in the estimates of quantities upon which the computation rests, and which varies with the nature of the property. In the illustrative problem, the lease would be purchased on the basis of a negotiated price, which should not exceed about $1,546,000.

VALUATION OF TIMBERLANDS

The valuation of timberlands follows quite closely the method outlined for the valuation of mineral deposits. The value is based on the estimated earning value or present worth of the operation returns that may in future years be derived from the exploitation of the stand of timber. If the timber, only, is purchased from the owner of the land, obviously no account is taken of the value of the land after the timber has been removed. If the land is purchased in order to get the timber, an estimate is made of the value of the land after the timber has been cut, and this is in the nature of salvage value.

The distinctive feature of timberland valuation is the method of sampling (called "cruising") employed to determine the quantity of marketable timber. This, like mine sampling, is a highly specialized operation and can be accomplished acceptably only by trained timber cruisers.

17.18. Timber Cruising.—Two methods of sampling are employed, the choice being based on the nature of the stand of timber. These are the *plot method*, in which systematically selected areas of from one to five acres each are sampled, and the *strip method*, by which strips extending entirely across the stand are sampled. The proportion of the total area to be sampled varies from 1 to 10 percent, but occasionally a higher percentage is sampled. The size of the plots or the width and distribution of the strips selected for sampling depend upon the kind and character of the growth and the nature of the topography. These are matters that can be determined by experienced cruisers. As a result of the cruising, an estimate is made of the amount of merchantable timber in the area.

17.19. Estimating Logging and Sawmill Costs.—The next step in the valuation is to estimate the cost of converting the timber into marketable lumber and getting it to a market. This involves estimating the cost of the sawing and the cost of loading and shipping to a market. There is generally an established market at some pulp mill for pulp logs, so that estimates of the market price of pulp wood can be made with reasonable certainty. There is no established general wholesale market for lumber, and estimates of the market price of lumber must be based on selling in competition with other producers of like material. Generally the valuator knows in which market the product will be sold, since transportation costs are a factor of considerable relative magnitude in getting the lumber to market.

On the basis of the foregoing process, an estimate is set up showing the probable annual operation returns to be expected during the period required to cut and market the available timber. From this stage of

the valuation, the procedure is as outlined for the valuation of ore and mineral deposits.

Since the quantity of timber available can be estimated rather closely, and the hazards of production are lower than in mining, the risk factor is generally lower than is ordinarily required in mining valuation.

The value of the stumpage on timberlands is generally expressed in dollars per acre, and ranges from $1.50 per acre to $15 per acre.

The value of the land after the timber is removed may be predicated on its use as agricultural land, or it may be based on natural reforestation or replanting. In general, cutover lands have very little value.

CHAPTER XVIII

VALUATION OF PRIVATE INDUSTRIAL PROPERTY

This chapter deals with the valuation of manufacturing enterprises and properties other than public utilities that are of such a character that engineering considerations enter into the appraisal of the physical property. Engineering personnel will be employed in such appraisals in connection with the determination of the probable service lives of the units of property, the unit prices to employ in the reproduction-cost-new estimates, and in checking the original costs new of buildings, equipment, and other facilities.

SOUND VALUE CONTRASTED WITH FAIR VALUE

18.1. Sound Value.—The term *sound value* came into use in connection with appraisal work where the value of the physical property was ordinarily fixed at the cost of reproduction new less accrued depreciation, calculated according to the condition of the *existing* property. It was considered that whatever arguments might be advanced for some other basis of fixing value, a value established on the basis of the actual cost of reproduction was certainly sound. Consequently, it is found that in many appraisals the sound value is set up as the reproduction-cost value of the physical property. The term sound value is also used to designate the value established by appraisals on some basis other than the reproduction-cost value of the physical property. As a matter of fact the term may be used with several meanings as follows:

Sound value may be

1. The reproduction-cost value of the physical property.
2. The original-cost value of the physical property.
3. The fair cost-value of the physical property, giving due relative weights to original cost and reproduction cost.
4. The fair cost-value of the entire property (including intangibles and cash).

The term *fair value* applies alike to private industrial properties and public utilities. There are many instances, however, in which appraisals of the sound value of private industrial property on a basis similar to one of those listed above will suffice.

The process of making a valuation of a public utility has gradually evolved as the courts, valuation experts, and regulatory commissions have expressed their views as to what constitutes the law of the land

in regard to the value of properties of this character. It has been pointed out that certain elements of value have come to be generally recognized in public-utility valuation. These have been set up under the scrutiny of the courts and commissions over a period of 30 or 40 years, and constitute a guide as to the data that are evidential of value.

In the valuation of private industrial property, there has been no such thorough discussion of the elements of value, nor has the process of valuation been subject to the intense legal scrutiny and expert study that have attended the development of the valuation of public utilities. However, it has repeatedly been stated in this treatise that value is after all a matter of judgment, and that the value of any kind of industrial property is correctly arrived at by bringing to bear the judgment of experts who have gathered together the various data that are essential to forming a correct judgment. Furthermore, it is believed that the general process for making an engineering valuation that has grown out of the experience of those who have been valuing public utilities affords a correct basis for gathering together the information necessary for forming a judgment as to the value of any other type of industrial property.

When valuations of ordinary industrial properties are undertaken, there may be a variety of objectives. For example, in many cases the purposes of the valuation are best served if there is established the present cost-value of the physical property. In other instances, the necessities require a valuation which will assure the establishment of the fair value of the entire enterprise and when such is the case it will be found expedient to follow the process outlined herein for the formal valuation of industrial properties.

THE PURPOSES OF VALUATION OF PRIVATE INDUSTRIAL PROPERTY

The valuation of a private industrial property may be undertaken for any one of a variety of purposes, and, to some extent, the degree of detail employed and the items to be included in the valuation will depend upon the purpose. The fact that a correct value for at least certain parts of the property is desired should not be overlooked, however, and short-cut methods that leave the final conclusion open to criticism are to be avoided.

Some of the major purposes of these valuations will be discussed briefly to indicate the various phases of the problem.

18.2. Administrative Purposes.—The management of an industrial enterprise obviously ought to know the value, and especially the annual depreciation, of the property employed in the fabrication of the product. Without this information, it is impossible to fix a selling price that will insure a real profit. This is particularly true with those types of products

that must be marketed in the face of vigorous competition. The danger in such instances is that the selling price will be based on assumptions with reference to costs in which an incorrect allowance for plant depreciation is included. A valuation that is made for the purpose of administrative control should furnish the management with an estimate of the fair value of the entire property on the date of valuation, which includes the value of both physical and intangible property. In such an instance, an appraisal of the value of the actual physical property would not be sufficient.

It has already been pointed out that one of the elements having a very great influence upon actual profits is annual depreciation. It is the practice in many industrial enterprises to account for depreciation of the units of physical property upon the straight-line theoretical depreciation basis in accordance with estimates of service life that are seldom even approximately correct. As a result, the management really has no adequate conception of the extent to which the plant is being consumed in the production process, and, if the element of plant overhead is a large factor in production costs in a given case, an error in the estimates of depreciation is likely to be exceedingly serious. In such instances the management would be entirely justified in incurring the expense necessary to secure an accurate estimate of accrued depreciation at rather frequent intervals.

18.3. Sale or Transfer.—When a business is sold or transferred in connection with a consolidation, or when it changes hands because of retirements or for other reasons, an appraisal may be made for the purpose of fixing a basis upon which to negotiate for sale or transfer. The appraisal that is made for this purpose should take into account every element of value that is recognized in law. As a matter of fact, appraisals of this kind not infrequently find their way into the courts. The fair value of the property can most reliably be established by following the formal process that has already been described in this treatise (Sec. 1.15 and Chaps. X to XV). In applying the formal valuation process to private industrial properties, it is not only necessary to take into account each factor affecting the value but also to give to the estimates for each of these elements the proper weight in view of the fact that the property under valuation is of a private character and subjected to open competition. The rate of net return the property can earn in the face of competition is indicated by what it has been able to earn in the past.

18.4. Assessment for Taxation.—The laws relating to the taxation of industrial properties vary widely among the states, and the provisions with reference to the transfer tax which will be assessed in case the property changes hands are growing in complexity. It is undoubtedly the part of wisdom for the management to have appraisal data available

covering the elements of the plant that are subject to taxation, to be used in connection with the adjustment of assessments against the property. If a complete valuation has been made, then, of course, those parts that are pertinent to tax assessment will be available. The data on actual annual depreciation are now required to be filed with income-tax returns when deductions for actual depreciation are claimed.

18.5. Insurance Adjustments.—In determining the amount of insurance to be carried on a plant, and for the appraisal of the damages that may have been suffered through fire or other disaster, there must be available complete valuation data on the physical property. Insurance companies employ short-cut methods for checking the value of a property before they assume a risk thereon, but these methods are not satisfactory for the adjustment of losses. Here the detailed inventory with the actual value correctly determined affords the most dependable basis for the adjustment.

18.6. Financing.—The necessity frequently arises for refinancing an industrial enterprise by the issuance of bonds or stock, or to meet temporary stringencies by borrowing money from the banks. In all such instances (except when obtaining limited loans for working capital), it is required that the value of the property be established to the satisfaction of the banking group who are to underwrite the financing. In such a case the most significant element of value will be that of the physical property. Nevertheless, no banking group would ordinarily finance an enterprise wholly on the basis of the established value of the physical property. Therefore, it is highly desirable that there be available an appraisal which gives the fair value of the enterprise, including all of the intangible elements, to show that it is a real going concern.

18.7. Condemnation.—Private property is sometimes taken over for public use through exercise of the right of *eminent domain* and the owner compensated in accordance with the findings of a special jury or board set up in accordance with law. Except in simple cases it is necessary to present evidence of value to these boards, and the owner may find it expedient to have an appraisal made for the purpose. The value sought in such cases is a sound value equivalent to the fair value of the property, exclusive of cash, or its equivalent, that is employed in operation of the property but does not go with the property if it changes hands.

ELEMENTS OF VALUE OF PRIVATE INDUSTRIAL PROPERTY

It will perhaps serve a useful purpose to review the elements of physical and intangible value that may be encountered in connection with appraisals of private industrial properties, and to point out the

extent to which the particular item is likely to be an important factor in the appraisal.

18.8. Factors Affecting Judgments of the Value of Private Industrial Properties.—The factors to be taken into account in the valuation of private industrial properties are substantially those already discussed in connection with the valuation of public utilities. They are repeated here with slight modifications.

1. The present value based on the original cost new of the units of physical property now in use, with proper deductions for the accrued depreciation thereon. This has been termed the original-cost value of the physical property.
2. The cost of reproducing the physical property new on the date of valuation, with deductions for depreciation as determined in 1. This has been termed the reproduction-cost value of the physical property.
3. The value of all intangible elements, such as preliminary expenses, going value, good will, contracts, patents, and trade marks.
4. The market value of the outstanding stocks and bonds, if that can be ascertained.
5. The capitalized value of the estimated future earnings at the rate of net return believed proper in view of the past history of the property and a forecast of its future possibilities.
6. The amount of working capital required, the stability of the market for the product, and any other special factors that may be pertinent.

The exact weight to be given to each of these estimates will vary from time to time and in various locations, but earning value and original- or reproduction-cost value will frequently be the dominating factors.

18.9. Physical Property.—In the types of private industrial property that are likely to be subjected to engineering valuation, there will be a considerable amount of physical property in the form of land, buildings, and equipment. The value of these will, therefore, be an important element in the total value of the property. An intelligent estimate of the present value of such a property must generally take into account both the original-cost value and the reproduction-cost value. The original-cost value indicates the capital costs that will have to be carried by competitors whose plants were built in the period contemporary with that of the plant under valuation, while the reproduction-cost value will indicate the capital costs that will have to be carried by competitors whose plants are newly completed or who contemplate erecting plants in the immediate future. In addition, the reproduction-cost value gives the proprietor some idea of the trend of his investment as he replaces (reproduces) his equipment from time to time as the old units wear out.

18.10. Earning Value.—There are types of industrial properties in which the investment in plant is relatively small and the product of which is marketed through a selling organization the support of which represents the major operation cost. In this case the value of the

physical property is relatively unimportant, and the value of the enterprise will rest largely on other factors, the dominating one among them being the earning value. Engineering valuations will seldom be employed for such properties. But, even in the case of a property of relatively high physical value, the element of capitalized earning value is also highly important and significant. It will seldom be the dominating factor in a valuation, however, because of the impossibility of forecasting future earnings with the degree of accuracy necessary if that were to be the controlling element of the valuation of the property. The valuation of the physical property gives ar indication of the investment necessary for the particular kind of enterprise and, consequently, some intimation of the price basis upon which competitors obviously must operate. This, in turn, will aid in estimating what the future history of the earnings of the enterprise is likely to be.

18.11. Going Value.—Going value is an element to be considered in most industrial enterprises of the character under consideration. It requires time and substantial expenditures of money to develop the clientele necessary to a profitable business, and, especially in those properties where the amount of physical property is relatively small compared with the value of the business as a whole, erroneous estimates of value will result, if going value is not taken into account.

18.12. Preliminary Expenses.—The item of preliminary expense may reach a rather substantial sum when a factory is being developed to market a new line of products. It is not uncommon, however, to find that preliminary expenses, good will, and going value are lumped together in estimates of value, and for many purposes that may be permissible. However, in the long run, the accuracy and dependability of a valuation will be enhanced if these items are estimated separately, even though the clients do not require them to be separated in the final valuation.

18.13. Good-will Value.—The item of good-will value is not given weight in the valuation of public utilities as a general rule, but in many instances it is a very important element of the value of an industrial property. This was discussed quite fully in Chap. XIII, and it will suffice at this time to call attention again to the fact that this element of value should be appraised if possible when it is desired to secure a correct estimate of the fair value of an industrial property.

18.14. Patents.—Many industrial enterprises are founded almost entirely on the protection afforded by patents covering the products that are manufactured. In other cases a factory manufactures a product which is trade-marked, and the protection and advertising value of the trade mark are an important asset. Various policies are pursued with reference to carrying the value of patents in industrial-property account-

ancy, but, when a valuation is being made of an enterprise in which patents are a factor, they should be evaluated just as carefully as if they were a part of the physical property of the enterprise, subject to the provision that patents, trade marks, and copyrights are not included in valuation for taxation.

18.15. Patterns.—Industrial enterprises engaged in the fabrication of metal ordinarily have a stock of patterns for use in the foundries; templates for use in their sheet-metal shops, and jigs and dies for use in the machine shops. The number and variety of these will vary greatly from time to time, and the future usefulness of any stock of this sort of material is always subject to considerable speculation because of the possibility of changes in the product that will necessitate replacing the old patterns or dies. Likewise garment factories, glove factories, shoe factories, metal-container factories, and box and carton plants, all make use of a great many dies and patterns the value of which may reach substantial figures. Blueprints and tracings may be classified in the same category as patterns. All of these may be valued according to the principles employed in the valuation of other physical property.

18.16. Goods in Process.—The determination of the amount of working capital actually tied up in the business may be arrived at by a study of the record books, supplemented by an appraisal of the quantity of raw materials carried in storage, the investment in goods in process, and completed product in storage and in transit. The value of the raw materials in storage may be obtained from the inventory of the quantity of the materials and the delivered price including any handling charges that have been incurred. The value of goods in process can be checked from the inventory of the quantity of each class of product en route through the plant and the value as determined by shop cost sheets. Finished products in storage, ready for shipment to customers, and in transit may be valued on a basis of the actual cost plus the carrying charges. The cost of insurance is likely to be an important factor in the cost of these items.

18.17. Working Capital.—The working capital is in part required to carry the cost of raw materials and products in warehouse, in part for meeting bills for labor and other operating expenses, and in part for carrying the customers' accounts until collections can be made for goods delivered. With many enterprises the amount of working capital becomes a very significant element of the cost of doing business. The amount of working capital to include in a valuation is determined as set forth in Chap. XIV.

18.18. Contracts.—Contracts for the disposal of a product, or contracts for power or raw materials, may be favorable or unfavorable and consequently need to be taken into account in estimating the value of

an industrial property. It often happens that the premises are leased instead of being owned by the business that occupies them. The lease, which is a form of contract, may be favorable or unfavorable. In any case, the existing contracts of all kinds must be taken into account in arriving at a fair estimate of the value of the property. This was discussed in Chap. XIII.

18.19. Stock-and-bond Value.—Corporations whose securities are regularly traded in will have established market quotations that permit an estimate of the market value of its stocks and bonds. In other cases there may be no established market for the securities. In any case the stock-and-bond value is a useful check on the other estimates of value, but stock-and-bond-value estimates may be useless in the case of speculative types of enterprises, and those whose securities fluctuate widely in value because of inevitable fluctuations in the annual earnings of the business.

18.20. Fair Return—Actual Return.—The term *fair rate of return* is employed in connection with the valuation of public utilities and has been given a legal status by the courts. In theory, there is a fair rate of return applicable to industrial enterprises of a private character, but it is perhaps unwise to use that term since the question of confiscation through enforced rates that are too low cannot enter into the valuation of these properties. The term *actual rate of net return* is used in connection with valuations of private industrial property to denote the return which the property has been able to earn on the average over a period of years, and the term *estimated future rate of net return* to denote the future return which is forecast on the basis of the past history of the property. In computations of earning value and in selecting depreciation condition-percent tables for depreciation computations, the estimated future rate of net return should be employed.

METHODS OF APPRAISAL OF PRIVATE INDUSTRIAL PROPERTY

The usual appraisal of private industrial property is directed toward the determination of a *sound value* for the property. Aside from any consideration of the exact terms employed, it is found that the valuations of these industrial enterprises are frequently confined to the actual physical property employed in the business. Depending upon the particular purpose of the appraisal, the sound value may be fixed on the basis of the original cost or of reproduction cost, whichever is the more appropriate.

18.21. Depreciation Estimates.—Valuations of private industrial properties are generally based on appropriate depreciation estimates set up in the manner described in Chap. V. That is, the basis of the estimate for any unit of property is the age and expectancy of the unit,

the expectancy being estimated separately for each unit of the property. There is a tendency for appraisers to adhere rather closely to expectancies for *average* units, obtained by subtracting the service age from the average life given for that type of property in some average-life table. However, when the unit of property is obviously in better or poorer condition than the average, adjustments in the expectancy will be made according to the best judgment of the appraiser.

Depreciation is computed on the straight-line basis in most instances, and, if depreciation condition-percent tables are available, the percentages can be taken directly from the table using the zero percent rate of return. While it is customary to use the straight-line method, it is equally convenient to follow the present-worth method, since the depreciation condition-percentages may be taken directly from the tables for a rate of return equal to the estimated future return on the property.

As has already been pointed out, depreciation estimates based on the actual present-worth theory are to be preferred to those based on the straight-line method, and this theory is gradually gaining recognition and acceptance although it is not as yet widely employed.

18.22. Cost and Price Data.—In many cases it will be found that the records kept by a factory will not give the valuator the correct cost new of the units of physical property. Quite frequently the records will show no more than the cost of the unit at the factory where it was produced. The freight, drayage, and the cost of installation must be added to the f.o.b. price in order to arrive at the cost new, ready to operate.

It is quite the common practice for a factory to utilize an organized force of mechanics for the maintenance of the plant equipment. These maintenance gangs are not infrequently also employed for installing new equipment that is secured to replace retired items. Unless the accounting system has been especially set up to develop the true facts with reference to maintenance, it will be found that the cost of installing replacement units of equipment has been charged against the maintenance of the factory. Whatever may have been the practice in this respect, the valuator wishes to know the true cost of the units of physical property installed ready to operate, and, if the costs cannot be secured from a study of the records of the business, the valuator will be compelled to estimate them on the basis of his experience and such records as he may find available, including recourse to historical data on prices and construction costs.

18.23. Surplus Equipment.—In determining what units of physical property to include in the valuation, the engineer will encounter the problem of surplus equipment. In many factories there has been a

tendency to install equipment in excess of the production requirements as one means of avoiding the accumulation of abnormally high surpluses. Some of this machinery is needed as stand-by equipment, but in other instances the surplus equipment represents an investment dictated by over-optimism and not warranted by the needs of the business. The valuator must determine the extent to which this surplus equipment really enters into the value.

The various "values" discussed in connection with the appraisal of ordinary industrial property should not be thought of as indicating that a property has several actual values. On the contrary, the various estimates of value are made up by including certain elements of the property and deliberately excluding others as not pertinent in certain cases. If fair value is desired, all elements of value are included; but, if sound value is the objective, certain elements may be omitted.

CHAPTER XIX

EXAMPLES OF THE VALUATION OF PRIVATE INDUSTRIAL PROPERTY

The application of the principles of valuation to private industrial properties will be illustrated in this chapter by a typical appraisal of a factory and an example of the valuation of an office building. The appraisal method actually employed for determining sound value is presented as well as that required for the estimation of the fair value.

ESTIMATE OF SOUND VALUE OF A FACTORY

19.1. Book Accounts.—An examination of the books of the corporation for which this appraisal was made showed that the following practices had prevailed in carrying the book value of the physical property:

1. Each unit of property had been entered on the books at its actual cost new, including cost of installation but without an addition for overhead.

2. Depreciation deductions were made in the book-value accounts annually, according to the following schedule:

Buildings of all kinds, 2 percent per year.
Machinery of all kinds, 10 percent per year.
Purchased patents, one-seventeenth of the cost per year.
Patterns of all kinds, 20 percent per year.

3. Each plant belonging to the property maintains a property ledger of the card type for the purpose of recording the costs new, installation costs, repair costs, and age of each unit of equipment.

4. The units of property are grouped in the accounts in accordance with the primary accounts of the business. There are some twenty-odd such accounts, corresponding to an arbitrary grouping of the several lines of products manufactured.

5. Retirement accountancy is employed, but no retirement reserve is maintained. Replacements are financed out of the annual receipts in much the same way that maintenance costs are paid.

19.2. Plan of the Appraisal.—The first step in the field work of appraisal was to make a complete inventory of the property. The inventory procedure was organized by buildings, and the equipment in the buildings was grouped in the inventory according to floors and bays with a notation as to the department to which each unit is assigned.

Equipment in the yards was listed by type, there being an inventory of railway sidings, roads, sewers, pipe lines, walks, conduits, and the like. These were finally grouped and their value listed under the heading "General." Each unit of property of each kind was examined

as it was inventoried and its condition noted. Its service age was taken from the record on the property-ledger cards in the plant office.

When the inventory was completed, the cost of reproduction new was estimated for each unit and from that cost the estimated depreciation of the actual unit was deducted on the straight-line basis, due weight being given to the present condition in estimating expectancy (straight-line actual depreciation).

19.3. Typical Inventory Units.—The inventory units and groups of units for one building and its contents are listed to indicate the practice followed by the appraisal company that valued this property. The inventory of the buildings themselves was in the form shown in Table 19.1. The inventory units and groups of units that were adopted for the inventory of this property were as follows:

1. Excavation
2. Foundation Walls and Piers:
   - *a.* Walls
   - *b.* Wall piers
   - *c.* Interior piers
   - *d.* Column piers
   - *e.* Floor piers
3. Walls
4. Framing:
   - *a.* Constructed wall columns
   - *b.* Constructed trusses
5. Roof
6. Stairs
7. Partitions
8. Ladders
9. Platforms
10. Vestibules
11. Canopies
12. Outside Drains
13. Plumbing:
    - *a.* First floor
    - *b.* Second floor
    - *c.* Mezzanine
    - *d.* Underground pipe
    - *e.* Pipe above ground
    - *f.* Soil and vent pipe
14. Heating System:
    - *a.* First-floor radiators and pipe fittings
    - *b.* Hot-blast heating
    - *c.* Hot-blast piping
    - *d.* Heating system, mezzanine floor
    - *e.* Heating system, second floor
15. Electric Lighting System:
    - *a.* First floor
    - *b.* Second floor
    - *c.* Mezzanine floor
    - *d.* Outside
16. Sprinkler System
17. Foundations for Building Fixtures
18. Foundations for Machinery and Equipment
19. Equipment:
    - *a.* Motors
    - *b.* Machinery
    - *c.* Extra machinery
    - *d.* Power plant
    - *e.* Power piping
    - *f.* Power feed wiring
    - *g.* Forges and furnaces
    - *h.* Shafting
    - *i.* Pulleys
    - *j.* Belting
    - *k.* Tanks
    - *l.* Manufacturing piping
    - *m.* Factory furniture and fixtures
    - *n.* Service equipment
    - *o.* Scales
    - *p.* Trucks
    - *q.* Fire-protection equipment
    - *r.* Perishable equipment
    - *s.* Tools
    - *t.* Perishable equipment, fixtures
    - *u.* Perishable equipment, office
    - *v.* Jigs and fixtures
    - *w.* Office furniture and fixtures

19.4. Depreciation Rates Employed.—The depreciation rates employed for units in average condition are given in Table 19.2. Due account was taken of the nature of the past and of the probable future conditions of service in estimating expectancy.

19.5. Inventory.—The details of a small part of the inventory are shown by Table 19.1, which also shows the estimate of the cost of reproduction. The estimates of cost of reproduction new are not made during the inventory process, of course, but the items in the table are assembled from the detailed inventory sheets, which were similar to, but not identical with, the form given in Fig. 10.1. The table is introduced to show the detail with which the inventory is taken and how it may be employed as the basis for estimating the cost of reproduction new. The original cost is, in general, a matter of record, and from those records the total original cost new of the building and of each unit of equipment can be abstracted and assembled in whatever form is most suitable for study.

19.6. Summary Tables.—The sound value for part of one building and its contents is tabulated in Table 19.3. It will be noted that the details that comprise an item in Table 19.3 are given in Table 19.1 in such manner as to permit convenient calculations to be made, as may be seen by comparing the subtotals marked "A" in the "Cost of reproduction" columns of the two tables. This summary table covers one building and its contents, in part. Each building on the property, the equipment in the yards, and the land were summarized in this manner.

19.7. Sound-value Summary.—The sound value for the entire property is set forth in Table 19.4, which is prepared by adding the appropriate items from the summary sheets for individual inventory sections, which in this case were buildings. It will be noted that sound value was in this case taken as the cost of reproduction new less depreciation of the physical property, including appropriate overhead costs, and the value of the land, a total of $488,137.

19.8. Valuation-report Form.—The valuation report was submitted to the owners in bound volumes, the first of which contained a brief outline of the method of valuation employed and the summary table (Table 19.4). The second volume included a discussion of the history of the property, the depreciation-accountancy methods and allowances, and like general data. It also included a valuation of the property assigned to each division of the business according to the primary accounts. The detailed inventory for each inventory section (a building in this case) was bound in a separate volume. There was also a volume giving the valuation for insurance purposes, which was obtained by eliminating from sound value those parts of the property that it was deemed unnecessary to cover with insurance.

ESTIMATE OF FAIR VALUE OF A FACTORY

The application of engineering-valuation methods to the determination of the fair value of the property for which a sound-value appraisal had been made as illustrated in the preceding sections will now be presented in summarized form to afford a basis for comparing fair value with sound value and to illustrate the difference in method.

19.9. Historical.—The enterprise in question was originally established about 1890 for the manufacture of certain kinds of mining machinery. Its growth was at first slow, but it was well managed and prospered from the beginning. Gradually other lines of products were added, competing plants were purchased and consolidated with the original property, and, while the enterprise has never gained great size, it has continued to do a good business and its products are recognized as the leaders in its field. It has continued to protect its equipment by patents, as is the custom in well-managed enterprises of this character. The records show that the net returns over a number of years have averaged $77,500 per year, and that the average value of raw materials, goods in process, and complete equipment and spare parts held for shipment has been $100,000. The cash required to carry on the business has been, on the average, $115,000. The property is being valued as of Jan. 1, 1931.

19.10. Original-cost Value.—Except for one of the older buildings and a few pieces of old equipment, the book records were adequate to the determination of the original cost new of the physical property. This is the identical property, item by item, that is included in the inventory, a part of which is given in Tables 19.1 and 19.4. The present value of each unit of the property on the basis of its cost new and its probable service life was determined by the present-worth method, using an 8 percent rate of return, and the sum of these was found to be $508,575, to which $2,125 is added for land, making a total of $510,700.

19.11. Reproduction-cost Value.—The cost of reproduction new of the physical property is given in Table 19.3. The present value on the reproduction-cost-new basis was computed by applying the respective condition-percents determined for the several units in computing original-cost value to the estimated cost of reproduction new of the units, and the reproduction-cost value of the physical property was found to be $587,875, to which $2,125 is added for land, making a total of $590,000. This differs from the figure of $488,137 given in Table 19.4 because the present-worth method of computing the actual accrued depreciation was employed, instead of the straight-line method. Overhead construction cost was included at 7½ percent, which is the average overhead cost of recent enlargements and improvements of the property.

Table 19.1.—Combined Inventory and Estimate Sheet Used in Factory Valuation

Units	Cost of reproduction	Sound value
Below Ground		
Excavation:		
1—4′ × 42′ trench area, 2′1″ deep	$ 15.96	
1—4′ × 36′ trench area, 1′10″ deep	12.24	
1—3′6″ × 17′ trench area, 1′6″ deep	4.20	
Foundation Walls:		
148—8′6″ sides		
4—9′3″ sides		
2—96′9″ ends		
1—41′6″ side, toilet and washroom		
3—15′3″ ends and cross walls, toilet and washroom		
1—62′ side, hospital and washroom		
3—13′6″ ends, toilet and washroom		
6′ deep trench excavation		
2′ × 1′ rubble-sandstone footing		
18″ × 5′ rubble-sandstone wall, lime and cement mortar	9,566.03	
150 wall piers		
6′ deep excavation		
3′ × 3′4″ × 1′10″ concrete footing		
2′6″ × 2′10″ to 1′6″ × 1′10″ × 4′ concrete shaft, including forms 4 sides		
1—1′ × 1′4″ × 1′ sandstone cap	3,579.00	
30 interior piers		
6′ deep excavation		
4′ × 4′6″ × 1′ concrete footing		
3′6″ × 4′ to 2′6″ × 3′ × 4′ concrete shaft, including forms 4 sides		
1—2′ × 2′6″ × 1′ sandstone cap	1,825.00	
44 interior piers		
6′ deep excavation		
6′ × 6′6″ × 1′ concrete footing		
5′6″ × 6′ to 3′ × 3′6″ × 4′ concrete shaft, including forms 4 sides		
1—2′6″ × 3′ × 1′ sandstone cap	4,725.16	
18 mezzanine-floor, crane, and second-floor column piers		
6′ deep excavation		
3′8″ × 3′8″ × 1′ concrete footing		
3′2″ × 3′2″ to 2′2″ × 2′2″ × 4′ concrete shaft, including forms 4 sides		
1—1′8″ × 1′8″ × 1′ sandstone cap	645.84	
3-2-floor piers		
6′ deep excavation		
4′ × 4′ × 1′ concrete footings		
3′6″ × 3′6″ to 2′6″ × 2′6″ × 4′ concrete shaft, including forms 4 sides		
1—2′ × 2′ × 1′ sandstone cap	157.83	
Total	$20,531.26 (A)	
Depreciation at 33 %, deducted	....	$13,755.94

TABLE 19.1.—COMBINED INVENTORY AND ESTIMATE SHEET USED IN FACTORY VALUATION.—(*Concluded*)

Units	Cost of reproduction	Sound value
Walls:		
2—675′7″ sides		
2—52′4″ ends		
First-story walls 14′6″ high; 13″ common brick, lime and cement mortar, struck joints 2 sides; 2-coat lead and oil painting one side all doors and all windows, all walls, 2-coat lead and oil painting, 2 sides		
1 pair doors 2′8″ × 9′8″; 1″ boards; 1″ cleats and brace; 3″ × 8″ plank frame; 1 fixed transom sash; 10 lts., 12″ × 13″, 1¾″ thick; 3-rowlock arch lintel		
1 pair doors 2′8″ × 7′; 1″ × 6″ D. & M.; 1″ stiles and rails one side; 1 sash, 6 lights, 10″ × 12″, 1⅜ thick; 3″ × 8″ plank frame, two-hinged transom sash, 10 lights, 12″ × 13″, 1¾″ thick; 3-rowlock arch lintel		
1—60′ × 14′6″ area, 2-coat sand-finished plaster including finish on brick walls of washroom, 2 corner guards of 1—5″ × 5″ × ½″ × 4′ long angle		
NOTE.—In like manner the estimate continues and is then summarized at	$56,245.00 (A)	
Depreciation at 37%, deducted	....	35,434.35
Hot-air Heating System:		
2 American Blower Corp.		
Sirocco unit heaters, size 3, type 30F, No. A3255, with 1 Westinghouse 3-h.p. punch press motor, type CS, frame 250F, style No. 535215, 3-phase, 60 cycles, 220 volts, with 1 Noark safety switch, T/P, 30 amp., 250 volts, No. 8133, including wiring		
Installed	1,150.00	
Depreciation at 3%, deducted	....	1,115.50
Etc., etc		
Electric Lighting System—First Floor:		
Wiring in ½″ concealed conduit		
Type B construction, average 15′ on centers		
2—150-watt Ivanhoe-Trojan suspension fixtures, with 14″ diameter reflector	34.50	
6—200-watt Ivanhoe-Trojan suspension fixtures, with 16″ diameter reflector	114.00	
20—200-watt Ivanhoe-Trojan suspension fixtures, with 16″ diameter reflector, ceiling pull switch	420.00	
Etc		

19.12. Earning Value.—The net returns have averaged $77,500 per year for a number of years, which is approximately 8 percent of the book value, but conditions on the date of valuation indicate somewhat lower earnings in the immediate future, and the earning value will, therefore, be estimated on the basis of net earnings of $70,000 per year, and a rate of return of 7½ percent. The capitalized earning value is, therefore, $70,000/0.075 or $933,000.

19.13. Stock-and-bond Value.—The stock in this corporation is not traded on the exchanges, and such sales as have occurred have been handled through a trust company. The sales have been too scattered to permit an estimate of market value. The company has no outstanding bonds.

19.14. Value of Patents.—The enterprise holds 27 live patents, of which eight are important factors in the profits of the business because they permit the manufacture of a line of equipment that is distinctive in its field and not subject to direct competition. In comparing the profit made on this special equipment with the competing equipment most nearly equivalent, it is apparent that some \$8,000 of the annual net returns can be attributed directly to the protection afforded by this group of eight patents. The most recent patent of the group is 5 years old and consequently will afford protection for 12 years. While the other patents will expire at intervals between the date of valuation and 1942, the product is really protected until 1942. Therefore the value (Sec. 13.10) of this group of patents is as follows:

$$\text{Value of patents} = \frac{\$8{,}000\,[(1.075)^{12} - 1]}{(0.075)\,(1.075)^{12}} = \$61{,}882$$

The remaining 19 patents cover features of the competitive lines of products and are valued on the books at \$500 each, at a total of \$9,500, which will be allowed.

The total value of the patents is therefore \$71,400.

19.15. Preliminary Expenses and Going Value.—The records of the early history of the company do not include enough data on the organization costs and the costs of establishing the business to permit accurate estimates of these factors. It appears that the preliminary expenses did not exceed \$20,000, and that figure will be used.

The going value determined by the comparative method (Secs. 12.15, 12.16) is \$55,000.

19.16. Good-will Value.—The fair value of the property, exclusive of good will, is \$812,125 (Sec. 19.17). The average rate of return to competitive businesses in the same field is 6½ percent. The annual net return on \$812,125 at 6½ percent would be \$52,790. The future annual net return for this property is estimated at \$70,000, of which \$8,000 is attributed to the patents, leaving \$9,210 annual net return that may be attributed to good will. In view of the history of this property it seems likely that the present good will may be effective for at least 6 years, during which the rate of net return on the property is expected to average 7½ percent. The good-will value is therefore

$$\text{Good-will value} = \frac{(\$9{,}210)\,[(1.075)^{6} - 1]}{(0.075)\,(1.075)^{6}} = \$43{,}230$$

TABLE 19.2.—RATES OF DEPRECIATION USED FOR UNITS IN AVERAGE CONDITION IN THE EXAMPLE OF FACTORY VALUATION
Date of Valuation, Jan. 1, 1931

Unit of property	Depreciation per year, %	Description of type
Machine shop, type A	2½	Ironclad steel frame
Machine shop, type B	2½	Brick with tar and gravel roof
Storehouse	2	Ironclad steel frame
Machine shop, type C	2¼	Brick with tar and slag roof
Shop, type D	1¾	Brick with tile roof
Shop, type E	2	Brick with tile roof
Shop, type F	2½	Brick with slate roof
Shop, type G	1¾	Brick with tile roof
Shop, type H	2¼	Ironclad steel frame
Retaining walls	3	Concrete
Sand and coal bins	2½	Concrete
Scrap shed	2	Iron-covered steel frame
Boiler house A	1¾	Brick, tar, and slag roof
Boiler house B	3¾	Brick, tar, and slag roof
Reservoir	2½	
Stacks	5½	Radial brick
Transformer house	2½	Concrete
Warehouses	2	Brick, tar, and slag roof
Office building	2¼	Brick with slate roof
Storage building	2	Ironclad steel frame
Garage	2	Tile with tile roof
Gasoline storage	2	Brick-concrete roof
Sheds	3½	Open frame
Concrete walls and runways	3	
Conduit, piping—housings	2	
Conduit, piping—equipment	4	
Sewer system	2	
Track system, railroad sidings	4	
Track scales	3	
Water system	3¾	Pumps and piping
Drafting-room equipment	5	
Woven-wire fences	5	
Office equipment	5½	
Photographic equipment	5½	
Air compressors	4½	
Motors for air compressors	5	
Printing-shop fixtures	4½	
Auxiliary business machines	4½	
Compressor equipment	3½	
Compressor-equipment buildings	6	
Power-machine buildings	2½	
Power machinery	3½	
Printing machines	3¾	
Power tools	6	
Compressed-air tools	7½	

Table 19.3.—Extract from the Value Summary of a Factory Building and Its Contents

Date of Valuation, Jan. 1, 1931

(For the detailed inventory of the shop building, see Table 19.1)

PART I. BUILDING AND BUILDING ACCESSORIES

Items	Cost of reproduction new	Depreciation, %	Sound value
Building:			
Below Ground:			
Excavation	$ 32.40		
Foundation	20,498.86		
Total foundations	$ 20,531.26 (A)	33	$ 13,755.94
Above Ground:			
Walls	$ 56,245.68 (A)		
Framing	41,922.92		
Floors	27,221.85		
Roof	50,714.37		
Stairs	1,405.13		
Partitions	5,710.97		
Ladders	76.00		
Platforms	73.61		
Vestibules	102.92		
Canopies	36.10		
Outside drains	1,620.32		
Total superstructure	$185,129.87	37	$116,631.82
Plumbing:			
Total, first floor	$ 4,520.00	20	$ 3,616.00
Total, mezzanine floor	1,239.00	20	991.20
Total, second floor	1,939.00	20	1,551.20
Supply pipe, underground	42.76	40	25.66
Supply pipe, above ground	570.35	27	416.36
Soil and vent pipe	162.77	20	130.22
Total plumbing	$ 8,473.88		$ 6,730.64
Heating System:			
1st-floor radiators and pipe fittings	$ 1,078.00	25	$ 808.50
Hot-blast heating	3,415.00	50	1,707.50
Hot-blast heating, con'd	4,459.00	55	2,006.55
Hot-blast piping	3,331.00	55	1,498.95
Mezzanine floor	339.00	20	271.20
2d floor	249.00	20	199.20
2d floor, hot blast	3,415.00	50	1,707.50
Total heating	$ 16,286.00		$ 8,199.40
Electric Lighting System:			
1st floor	$ 4,684.00	28	$ 3,372.48
2d floor	101.95	20	81.56
Mezzanine floor	44.60	20	35.68
Outside	60.00	20	48.00
Total electric lighting	$ 4,890.55		$ 3,537.72

Table 19.3.—Extract from the Value Summary of a Factory Building and Its Contents.—(*Continued*)

Items	Cost of reproduction new	Depreciation, %	Sound value
Sprinkler system (complete with all pipe and heads).....	$5,934.00	45	$ 3,263.70
Foundations for Building Fixtures:			
Machine 11	$ 106.14	50	$ 53.07
12	127.02	50	63.51
13	337.35	45	185.54
Total foundations for fixtures	$ 570.51		$ 302.12
Total building	....	..	$152,421.34

PART II. EQUIPMENT FOR PRODUCTION PROCESSES

Items	Reproduction cost new	Total cost of reproduction	Depreciation, %	Sound value
Motors:				
No. 24, Factory cost	$ 263.54			
Freight, cartage, wiring and installation	40.00	$ 303.54	30	$ 212.48
No. 25, Factory cost	$ 570.24			
F.c.w. and i	189.00	759.24	30	531.47
No. 26, Factory cost	$ 375.48			
F.c.w. and i	93.00	468.48	30	327.94
No. 27, Factory cost	$ 176.74			
F.c.w. and i	44.00	220.74	50	110.37
No. 28, Factory cost	$ 375.48			
F.c.w. and i	103.00	478.48	30	334.94
No. 29, Factory cost	$ 375.48			
F.c.w. and i	125.00	500.48	30	350.34
No. 30, Factory cost	$ 375.48			
F.c.w. and i	103.00	478.48	30	334.94
No. 31, Factory cost	$ 580.24			
F.c.w. and i	163.00	743.24	30	520.27
No. 32, Factory cost	$ 375.48			
F.c.w. and i	97.00	472.48	24	359.08

TABLE 19.3.—EXTRACT FROM THE VALUE SUMMARY OF A FACTORY BUILDING AND ITS CONTENTS.—(*Concluded*)

Items	Reproduction cost new	Total cost of reproduction	Depreciation, %	Sound value
Motors (*Continued*):				
No. 33, Factory cost	$ 585.24			
F.c.w. and i	224.00	$ 809.24	30	$ 566.47
Machinery:				
No. 13, Wheel press	$2,800.00			
Installation	95.00			
Countershaft	32.00	2,927.00	45	1,609.85
No. 80, 4 automatic chucking machines, each	$3,650.00			
No. 81, Installation	140.00			
No. 82, Belt guard	24.00			
No. 83, Operator's platform	7.00	15,284.00	45	8,406.20
	$3,821.00			
No. 91, 1 automatic chucking machine	$1,880.00			
Installation	83.00			
Belt guard	20.00	1,983.00	42	1,150.14
	$1,983.00			
No. 92, No. 93, 2 (like 91)	....	3,966.00	45	2,181.30
No. 94, 2 chucking machines, each	$3,650.00			
Installation	140.00			
Belt guard	24.00			
Operator's platform	8.00	7,644.00	35	4,968.60
	$3,822.00			
No. 21, 1 I-beam trolley, 9″	....	60.00	25	45.00
No. 22, 10 I-beam trolley	....	62.00	25	46.50
No. 23, 9 I-beam trolley	....	44.00	25	33.00
No. 24, 12 I-beam trolley	....	47.00	25	35.25
No. 25, Swing-jib crane	....	50.00	25	37.50
No. 26, 9 Swing-jib crane	....	53.00	25	39.75
No. 27, 2 Theald grinders, each	$2,250.00			
Installation	53.00			
Guards	30.00	4,666.00	45	2,566.30
	$2,333.00			
No. 29, Norton grinder	$6,060.00			
	205.00			
	30.00	$ 6,295.00	55	$2,832.75

NOTE.—The inventory continues in like manner to include several hundred items similar to those listed above.

19.17. Summary of the Valuation Data.—The data obtained from the estimates that have been discussed in the foregoing sections may be summarized as follows:

Land	$ 2,125
Original-cost value of physical property, except land	508,575
Reproduction-cost value of physical property, except land	587,875
Earning value	933,000
Stock-and-bond value not ascertainable.	
Preliminary expenses	20,000
Going value	55,000
Value of patents	71,400
Good-will value	26,755

In view of the downward trend of prices at the time of this valuation and the likelihood of lower prices in the future, it seems unwise to give too much weight to the reproduction-cost value, and the present fair value of the physical property was accordingly fixed at $520,000. The present fair value of the property on the basis of the estimates for the several elements is as follows:

Land	$ 2,125
Physical property, except land	520,000
Going value	55,000
Preliminary expenses	20,000
Working capital	215,000
Total, except patent and good-will values	$812,125
Good-will value	43,230
Patents	71,400
Total fair value	$926,755

In view of the estimated earning value of $933,000, based on a forecast of the future earnings of the property and the influence of the rate of net return (estimated at 7½ percent) on the computed value of patents, earning value, and good-will value, it is apparent that the actual value of this property lies between $920,000 and $940,000. If one were optimistic about the future 10 or 15 years following the valuation, the fair value might be fixed at the higher figure. However, the exact figure to adopt will usually be apparent in an actual case, since many impressions will be gained by the study of the property during valuation that can scarcely be brought into an illustrative problem such as the foregoing.

A comparison of a fair value of, say, $925,000 with the sound value of $488,137 will serve to illustrate the fundamental difference between

the two processes. The sound value as set up in this case was a conservative estimate of the present value of the physical property only.

TABLE 19.4.—SUMMARY OF COMPUTED SOUND VALUE OF A FACTORY
Date of valuation, Jan. 1, 1931
(This summary is prepared by adding together the appropriate items from the inventory sheets for the entire property, not all of which are reproduced herein)

Items	Cost of reproduction new	Sound value
Land	$ 2,125.00	$ 2,125.00
Railroad sidings	11,444.00	6,272.25
Buildings, Construction, and Fixtures:		
Construction of buildings	179,696.99	127,770.89
Stairs	32.78	22.95
Flagpole	275.00	233.75
Fences	1,275.86	718.11
Paving and walks	812.93	651.45
Roads	633.00	506.40
Retaining walls	1,026.43	615.86
Reservoir	543.88	407.91
Culvert	544.57	462.88
Pits	314.00	281.28
Pipe tunnels	945.77	851.20
Wells	2,132.50	1,567.50
Plumbing	5,402.26	4,273.66
Sewerage	3,293.83	2,799.75
Heating system	16,580.18	10,297.92
Electric light system	3,249.92	2,299.82
Sprinkler system	5,371.36	3,416.55
Water lines	6,976.73	3,768.26
Building elevators	556.25	422.75
Foundations for building fixtures	127.02	63.51
Engineer's fees	16,016.48	11,259.15
	$245,807.74	$172,691.55
Machinery and Equipment:		
Foundation for machinery and equipment	$ 6,159.53	$ 4,482.21
Power plant	38,541.62	22,964.77
Power piping	10,905.26	7,052.16
Motors	5,219.56	4,006.81
Power-feed wiring	3,850.72	3,237.95
Machinery	251,237.21	155,521.09
Extra machinery	32,274.23	12,331.43
Forges and furnaces	1,977.37	899.91
Shafting	4,531.34	2,393.55
Pulleys	1,933.69	1,199.81
Belting	3,727.03	2,683.63
Tanks	1,787.75	1,322.90
Manufacturing piping	2,895.83	1,853.79
Factory furniture and fixtures	16,256.59	12,662.19
Service equipment	689.00	512.35
Scales	1,596.00	1,060.86
Trucks	564.25	351.77
Fire-protection equipment	1,001.74	752.55
Perishable equipment—tools	53,591.31	31,571.85
Tools	12,667.43	10,494.93
Perishable equipment—fixtures	1,140.87	605.41
Perishable equipment—office fixtures	778.35	529.45
Jigs and fixtures	55,774.50	22,522.46
	$509,101.18	$301,013.83
Office furniture and fixtures	8,553.22	6,034.97
Total	$777,031.14	$488,137.60
Sound value, $488,137		

The fair value is an estimate of the market value of the enterprise as a going business.

ESTIMATE OF THE FAIR VALUE OF AN OFFICE BUILDING

The appraisal of buildings and land for many purposes is a special field for real-estate experts and building-appraisal companies;[1] they have developed highly specialized and, in some instances, short-cut methods for this work. The purposes of these valuations are frequently not comparable with those served by valuations employed for establishing fair value or sound value. It is particularly the function of these special methods of appraisal to establish values to be used as the basis for assessment for taxation, determination of inheritance and transfer taxes, real-estate loans, insurance, and many like purposes.

In making a valuation of a private industrial property, it is necessary to include a valuation of the buildings that are a part of the property. Moreover, when the property is a public utility it is imperative that the fair value of the buildings and land be established in conformity with the principles that have been developed for public-utility valuation. Sometimes buildings that are of themselves independent industrial properties are valued by methods calculated to establish the fair value to be presented to the courts in connection with litigation. The methods of formal valuation, however, are too laborious and costly to be applied in detail where it is only necessary to secure approximately equivalent values on many properties, as is the case in assessments for taxation. The same situation prevails in many other instances.

Sections 19.18 to 19.33 present an illustration of the application of engineering-valuation principles to the determination of the fair value of a building, followed by the determination of the sound value of the same building.

19.18. Inventory Methods.—A wide diversity of practice exists with reference to the inventory methods employed in listing the items in the construction schedule for a building. For purposes of estimating the present value on the basis of the original cost, it is necessary to consider as units (Sec. 2.4) only those parts of the building that depreciate as units and deliver service as units.

The following list suggests the degree of detail ordinarily necessary in itemizing the parts of a building for the purposes of estimating the present value based on the original cost. Some such subdivision is necessary in order to permit appropriate computations of accrued depreciation.

1. The building proper.
2. Plumbing and plumbing fixtures.
3. Boilers and accessory equipment.

[1] For an excellent presentation of this subject see "Appraisers and Assessors Manual" by Prouty, Collins, and Prouty, McGraw-Hill Book Company, Inc., New York, 1930.

4. Piping and radiators.
5. Electrical wiring and fixtures.
6. Ventilation equipment.
7. Elevators.
8. Miscellaneous, under which would be listed any special equipment that is essentially a part of the building and not properly classed as equipment that may or may not be permanently installed in the building.

While the foregoing list is suggestive of the method employed for assembling the elements of the building into groups for computation purposes, the actual grouping of like items, and determination of the cost of reproduction of each of the groups of items, can be accomplished only after making an examination and detailed inventory of each of the individual parts that are finally assembled into the various groups. This is most readily accomplished by following some systematic method in examining the property and listing the items. The most common and perhaps most satisfactory method is to employ what might be called a geographical system in preparing the detailed inventory, as illustrated in Table 19.1. In this method the inventory begins with the portion of the foundation and other parts that are below ground, then follows with the portion of the foundation and other appurtenances below the level of the actual building walls but lying above ground where they can be seen. Then, finally, the inventory of the portion of the building above ground will be made up, including outside and partition walls, columns, floors and ceilings, roof trusses, and roof proper. This is followed by a detailed inventory of the appurtenances in outside and partition walls, such as doors, windows, louvers, ducts, gratings, gates, platforms, and all similar items.

If detailed plans are available, a preliminary inventory can be made up by taking off the quantities from the plans, these to be checked later by examination of the building to determine the condition of each of the elements of the structure.

For each size of window, the size of the sash, the size and type of frames, the kind of lintel, the number of lights, the kind of glass, and the condition of the paint are noted; likewise for doors, walls, floors, steel work, and each of the other items that comprise the structure.

The dimensions and the character of the materials in each of these elements are ascertained, and the cost of reproduction is estimated on the basis of the prices established as outlined in Chaps. IX and XI. From these detailed estimates the cost of the various groups of units selected for computing depreciation are assembled. There is given in Table 19.1 a group of items selected from the inventory and reproduction-cost analysis prepared by an appraisal company in connection with the actual appraisal of one of the buildings in an industrial property. This

will illustrate in detail the method employed in that particular case which is typical of present-day practice.

From the data obtained in the analysis just described, the original cost and reproduction cost of the building are fixed. Depreciation is then deducted according to the condition of the several parts of the building and thus original-cost and reproduction-cost values are set up.

The intangible values are then determined, completing the data required for estimating the fair cost-value of the building. By considering earning value and other factors, the fair value may then be estimated.

The method of estimating the fair value of a building is illustrated by the valuation of an office and store building that had been completed late in 1925 and valued as of Jan. 1, 1933. The building is located advantageously in a city where the development has proceeded far enough to define clearly the areas suitable for a building of this type.

19.19. General Conditions.—The preliminary expenses were incurred in organizing a company to promote the project, selling bonds to finance the construction, incorporation and lawyers' fees, preliminary reports on the selection of location, and brokerage fees, totaling $45,000. The actual cost of the building new, ready for occupancy, is shown in Table 19.5, which is an abstract of the contractor's bidding schedule.

A drug company leased a part of the ground floor for a term of 20 years, beginning Jan. 1, 1927, at an annual rental that gives a net return of 8 percent on the estimated value ($135,000) of the space occupied.

19.20. Market Value of Land.—A survey of the trend of land prices in the area near this building developed the fact that few sales have occurred during the past two years. Some plots are available at distress prices, but most lots are held at prices near those of 1926 to 1928. It seems apparent that to secure enough land for a building of this size the price would be about what was originally paid, and therefore the reproduction-cost value of the land was considered to be identical with the original-cost value.

19.21. Estimating Original-cost Value.—The owner company's records included a tabulation of the original cost as given herein in Table 19.5, and detailed information as to the earnings to date. The financing was on a basis that indicates 6 percent to be a satisfactory rate of return, and accordingly all computations are based on that rate. The original-cost-value computations for the building are given in summarized form in Table 19.6.

19.22. Estimate of Reproduction-cost Value.—The first step in the estimation of reproduction-cost value is to determine the cost of constructing the building and installing equipment at the prices current

TABLE 19.5.—ORIGINAL-COST AND REPRODUCTION-COST ESTIMATES OF AN OFFICE BUILDING

These costs include contractor's overhead and profit prorated for each item, except that the cost of the performance bond is included as a separate item

Quantity, cu. yd.	Item	Actual original cost new, 1925		Estimated cost of reproduction new, 1933	
		Unit price	Amount	Unit price	Amount
	Group I. Building Proper				
	Cellar and subcellar excavation; piling and pile caps	......	$ 65,000	......	$ 47,000
200	Trench excavation	$ 1.75	350	$ 1.50	300
3,000	Backfilling	1.50	4,500	1.30	3,900
1,220	Foundation concrete walls in place, incl. forms, steel, etc	20.00	24,400	16.00	19,520
5,200	Concrete fireproofing, floors, columns, etc., incl. forms, wire, tile, etc	30.00	156,000	25.00	130,000
	Performance bond, etc	......	13,646	......	11,023
	Total contract cost	......	923,408	......	748,943*
	Engineering and architect's fees	......	51,500	......	41,750
	Owner's overhead costs	......	17,500	......	14,200
	Interest lost during construction	......	52,300	......	42,500
	Total cost new, ready for occupancy	......	$1,044,708	......	$847,393
	Group II. Heating and Ventilation				
	Performance bond	......	$ 1,564	......	$ 1,217
	High-pressure boiler	......	1,020	......	900
	Smoke breeching	......	1,000	......	850
	Unit heaters	......	7,280	......	6,100
	Coal-handling equipment	......	2,370	......	1,580
	Boilers and stokers	......	25,000	......	18,700
	Thermostats	......	3,000	......	2,280
	Extras, Etc	......	4,795	......	4,200
	Total contract cost	......	130,851	......	101,067
	Engineering and architect's fees	......	6,300	......	4,900
	Owner's overhead costs	......	2,100	......	1,625

Interest lost during construction	......	5,950	......	4,600
Total cost new, ready for occupancy	......	$ 145,201	......	$113,192
Group III. Plumbing				
Soil, waste, drainage and vent piping	......	$ 8,585	......	$ 6,500
Exterior sanitary and storm sewer	......	200	......	180
Water piping	......	4,900	......	3,800
Ejectors	......	1,330	......	1,210
Performance bond	......	587	......	469
Extras, Etc	......	1,827	......	1,550
Total contract cost	......	39,714	......	31,814
Engineering and architect's fees	......	2,380	......	1,700
Owner's overhead costs	......	800	......	626
Interest lost during construction	......	2,190	......	1,950
Total cost new, ready for occupancy	......	$ 45,084	......	$ 36,090
Group IV. Wiring				
Performance bond	......	$ 872	......	$ 618
Conduit and boxes	......	14,235	......	12,100
Fiber ducts and floor boxes	......	10,724	......	9,300
Wire	......	4,213	......	3,150
Light fixtures, installed complete, Etc	......	14,700	......	11,300
Total contract cost	......	65,870	......	53,138
Engineering and architect's fees	......	3,550	......	2,900
Owner's overhead costs	......	1,180	......	950
Interest lost during construction	......	3,320	......	2,700
Total cost new, ready for occupancy	......	$ 73,920	......	$ 59,688

* Reproduction overheads at the same rate as for actual original cost.

TABLE 19.6.—COMPUTATION OF ORIGINAL-COST VALUE OF AN OFFICE BUILDING
Based on 6% rate of net return
(Data for columns (1) and (2) taken from Table 19.5)

Item (1)	Actual original cost new (2)	Estimated salvage value (3)	Depreciable value new, (2) − (3) (4)	Age, years (5)	Probable service life, years (6)	Condition-percent / 100 (7)	Present depreciable value, (4) × (7) (8)	Original-cost value, (8) + (3) (9)
Building only	$1,044,710	$ −20,000	$1,064,710	7	50	0.9711	$1,033,928	$1,013,930
Plumbing and fixtures	45,085	2,000	43,085	7	25	0.8470	36,493	38,490
Heating and ventilation	145,200	9,000	136,200	7	20	0.7718	105,122	114,120
Electrical wiring and fixtures	73,920	1,000	72,920	7	15	0.6394	46,623	47,625
Elevators	87,600	4,000	83,600	7	17	0.7025	58,727	62,730
Total	$1,396,515	....	....	..	..	......	....	$1,276,895
Land	73,000	....	....	..	..	......	....	73,000
Grand total	$1,469,515	....	....	..	..	......	....	$1,349,895

TABLE 19.7.—COMPUTATION OF REPRODUCTION-COST VALUE OF AN OFFICE BUILDING

Based on 6% rate of net return

(Data for columns (1), (2), and (5) taken from Table 19.5)

Item	Contract cost of reproduction new	Estimated salvage value	Reproduction depreciable value new, (2) − (3)	Condition-percent / 100	Present reproduction depreciable value, (4) × (5)	Contract reproduction-cost value, (6) + (3)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Building only	$ 847,393	$ −20,000	$867,393	0.9711	$842,325	$ 822,325
Plumbing and fixtures	36,090	2,000	34,090	0.8470	28,875	30,875
Heating and ventilation	113,192	9,000	104,192	0.7718	80,415	89,415
Electrical wiring and fixtures	59,688	1,000	58,688	0.6394	37,525	38,525
Elevators	61,320	4,000	57,320	0.7025	40,265	44,265
Total	$1,117,683	....	....	......	....	$1,025,405

in 1932 and likely to hold for the period of construction. If plans for the building are available, the work involved is identical with that of preparing a proposal for the construction of the building by contract. In general, the usual procedure is to have quantity surveys made from the plans, in so far as quantity surveys are applicable, and to price the various items and thus establish a basis for computing the estimate. There is given in Table 19.5 the original construction-cost schedule for this building, and an additional column showing the estimated cost of reproduction new in 1933 on the basis of average prices prevailing in 1932.

In the very usual case in which the plans for the building are not available, the reproduction-cost estimates are made up from the detailed inventory sheets as illustrated in Table 19.1 (which is an inventory of a factory building, not the building for which a valuation is being presented in this section), by applying appropriate prices to each of the inventory items. This method is very widely used in appraisal work. The reproduction-cost value was computed in this illustrative example from the data given in Table 19.5, and the detailed computations are shown in Table 19.7.

19.23. Estimate of Value of Lease.—The rate of net return required to secure funds for the building was 6 percent. The lease to the drug company is on a basis that gives a rate of net return of 8 percent, or 2 percent in excess of the rate of fair return. This produces an excess net return of \$2,700 yearly, which will continue for 14 years after the date of valuation. The value of the lease is, therefore, the present worth of an annual year-end payment of \$2,700 continuing for 14 years at 6 percent interest. Hence,

$$\text{Present value of lease} = \frac{\$2{,}700[(1 + 0.06)^{14} - 1]}{(1 + 0.06)^{14}(0.06)} = \$25{,}100.$$

19.24. Going Value.—The going value was estimated by the comparative method at \$94,200. The method is described in detail in Chap. XII.

19.25. Estimate of Good-will Value.—The average of the annual net returns for the seven years prior to date of valuation has been a little more than \$95,000. It is forecast that during the 10 years following the date of valuation the average annual net returns will be at least \$80,000. The fair value of the building and land exclusive of good will is \$1,262,000 (Sec. 19.31). The rate of net return obtainable on similar investments in the same city has averaged 6 percent. The annual return required to pay 6 percent on the fair value of this property is \$75,720 and the annual net return attributable to a favorable lease is \$2,700, making a total of \$78,420. By deducting this total from the forecasted annual

net return of $80,000, there is left a balance of $1,580 which may be attributed to good will. It is apparent that the good-will value is so small as to be negligible.

19.26. Earning Value.—The earning value is estimated on the basis of an average annual net return of $65,000 for the life of the building. Earning value is therefore $65,000/0.06 = $1,084,000.

19.27. Stock-and-bond Value.—While the construction of this building was financed in part through a bond issue, conditions in the securities market during 1932 precluded any trading in bonds of this type and consequently there were no market quotations to indicate how the securities were regarded. The underwriters advised that very few of the bonds had been offered for sale, and the few that were offered sold at a discount of about 25 percent. It is, therefore, impossible to ascertain bond value, but the evidence indicates that the securities are highly regarded. The common stock is held by a real-estate trust, and none has been sold since the building was erected.

19.28. Working Capital.—The average monthly balance carried to meet the running expenses of the building has been about $40,000. It appears, however, that, since the rentals are mostly paid monthly, the working capital needed is not in excess of $25,000 and that sum will be allowed.

19.29. Depreciation Reserve.—It has been the practice of the owners to pay current replacements from the annual depreciation appropriation and to employ the balance in retiring the bonds, which are of the serial type. At the date of valuation, $110,000 of the depreciation reserve had been so employed and had, therefore, really been added to the net return. The actual depreciation sinking fund amounted to approximately $10,000, held on deposit for emergencies and earning 3 percent. The interest actually earned had been credited to gross receipts in accordance with accepted practice.

19.30. Other Pertinent Factors.—A survey of the availability of office space in the city in which the building is located indicated a considerable surplus in space during 1932 but that prior to 1930 there had been no considerable excess. No additional office buildings of a competitive type are known to be planned for construction during the near future. The city has shown a steady growth for a number of years and has a well-diversified industrial development which indicates a rather steady demand for office space. This building is advantageously located in an area that is accessible and convenient to hotels and transportation lines. Zoning ordinances and established through-street systems make improbable any marked shift in the business district. The building itself is of a conservative type with a high ratio of rentable floor space to cubic contents, yet sufficiently modern in its appointments to insure a high type of tenant.

19.31. Determination of Fair Value.—The several estimates of value may be summarized as follows:

Land	$ 73,000
Original-cost value of building, incl. all overheads	1,276,900
Reproduction-cost value of buildings, incl. all overheads	1,025,405
Going value	94,200
Preliminary expenses	35,000
Rental contract	25,100
Working capital	25,000
Earning value	1,084,000
Stock-and-bond value not determinable.	
Depreciation sinking fund	10,000

On the basis of the foregoing and the best forecast of the future that was possible at the beginning of 1933, the following values were adopted.

Land	$ 73,000
Building	1,000,000
Going value	94,000
Preliminary expenses	35,000
Rental contract	25,000
Working capital	25,000
Depreciation sinking fund	10,000
Fair value	$1,262,000

On the basis of the foregoing analysis, and the nature of the times when the property is valued, it appears that reproduction-cost value and earning value should be given dominant weight; consequently the fair value of this building, including land, was fixed at $1,262,000, including working capital and depreciation sinking fund.

SOUND-VALUE APPRAISAL OF AN OFFICE BUILDING

19.32. Sound-value Appraisals.—A very common building-appraisal problem is the one of establishing the sound value of a building and the land upon which it stands. The almost universal practice in such cases is to consider sound value as being analogous to what was called the reproduction-cost value of the building (exclusive of intangibles) in the earlier part of this chapter. In some cases where approximate results are acceptable, this value is estimated by applying a cost-per-cubic-foot factor and in others by applying a cost-per-square-foot-of-floor-area factor. If the appraisal is to be employed as a basis of sale or purchase, or is required in connection with litigation, more precise methods of estimating cost of reproduction are employed. In general, the method illustrated in Table 19.1 will of necessity have to be followed for buildings of any considerable age, but the one illustrated in Table 19.5 is also applicable if plans are available so that a quantity survey may be taken

off. Having estimated the reproduction cost, depreciation is deducted, usually on the fixed-percentage basis, which gives what is commonly called the sound value of the building. The value of the land is next determined by one of the methods described in Chap. XVI.

The application of the ordinary appraisal method to the building described in this chapter will serve to illustrate a common method of determining the sound value of a building.

19.33. Estimating Reproduction Cost and Sound Value.—It is assumed that the cost of reproduction of this building is determined as outlined in Table 19.5, and, if it is, the resulting estimates must be identical with those given in that table; that is, the basic data are identical whether the building is valued by ordinary appraisal methods or by the methods of engineering valuation.

The second step is to determine the depreciation deductions to bring the reproduction cost to the basis of the actual building in its depreciated condition. In all cases where appraisals are made by reliable appraisal companies, the rate of depreciation is determined from a knowledge of the mortality characteristics of various kinds of buildings and their equipment and after a careful inspection of the building under appraisal. The straight-line method of computation is usually employed for the computation of depreciation. Here, again, the process is identical with the methods employed in estimating fair value. In the example under consideration, the probable service life of each of the several elements of the building is given in column (6), Table 19.6. This information permits the calculation of the depreciation to apply to the reproduction cost in order to arrive at sound value. The computations are shown in Table 19.8.

TABLE 19.8.—COMPUTATION OF SOUND VALUE OF AN OFFICE BUILDING

Item	Cost of reproduction new	Total depreciation, %	Depreciation	Sound value
Building only	$847,390	14	$118,640	$728,750
Plumbing and fixtures	36,090	28	10,100	25,990
Heating and ventilation	113,190	35	39,620	73,570
Electrical wiring and fixtures	59,690	47	28,050	31,640
Elevators	57,600	41	23,620	33,980
			Sound value	$893,930

The foregoing determination of sound value for the building may serve as the basis for estimates of value that include a value for the

land, and such of the intangibles as may be pertinent to the purposes of the valuations.

19.34. Other Applications of Sound Value.—It was stated that in appraisal work the usual practice is to estimate the sound value on the basis of the cost of reproduction. In some cases it is desired to ascertain the sound value on the basis of the original cost, in which case the procedure is exactly as outlined in Sec. 19.33 except that the basic figures are those representing the original cost new. Here, again, the value of certain of the intangibles will be added if the problem in hand requires.

CHAPTER XX

VALUATION OF PUBLIC UTILITIES

At many places in this treatise references have been made to the valuation of public utilities, and the basic principles have been discussed at length. This chapter will present some of the general aspects of utility valuation and current practice with reference thereto.

GENERAL CONSIDERATIONS

20.1. Nature of Public Utilities.—In the literature of jurisprudence, references to businesses endowed with a public interest are to be found in cases arising as early as A.D. 1400. One of the earliest of such cases had to do with a common carrier, which, of course, was a stagecoach. Of the businesses now recognized as endowed with a public interest, one of the most stupendous is that of the modern common carrier, the railroads.

In the *Munn Case* (Sec. 8.3) decided in 1876, Mr. Justice Waite makes reference to the oft quoted saying of Lord Hale that "when private property is affected with a public interest, it ceases to be *juris private* [within the scope of private right] only." One of the earliest references to this doctrine will be found in *Allnutt v. Inglis,* King's Bench, 1810, in a case involving the rates to be charged for storage in a bonded warehouse which, due to governmental regulations, had a monopoly on the handling of certain imports. Thus it will be noted that, from the beginning, various phases of transportation and related activities have been considered to be vested with a public interest.

The term *public utility* (Sec. 2.1) came gradually into use as a general term to apply to certain classes of business that were recognized as being endowed with a public interest, and in the jurisprudence of today any business so endued is considered to be a public utility. The layman generally thinks of such things as light, water, gas, and railroad properties, street-car lines, and bus lines as being the public utilities without sensing that many other kinds of industrial enterprises are really public utilities.

20.2. Catalogue of Kinds of Public Utilities.—The list of businesses that have definitely been labeled as public utilities by the courts is not long and appears in Sec. 2.1.

The regulation of rents has been recognized as constitutional under emergency conditions such as those prevailing in Washington, D.C.,

during the World War. The status of bus, truck, and air transportation will doubtless be considered at an early date. Likewise radio broadcasting.

It will be apparent from the foregoing that the amount of business transacted annually under government scrutiny in the United States reaches a staggering total.[1] It will also be noted that not all are monopolistic in character.

20.3. Valuation and Regulation.—The valuation engineer is concerned with those developments because government regulation inevitably sooner or later involves rates of compensation. Rates of compensation can seldom be established with justice to all concerned without going into the question of value. The conditions in the public-utility field in 1936 point to increasing need for systematic valuation of utility properties to provide a suitable basis for determining fair rates for the services offered.

From the valuation standpoint, the most significant statement that has appeared in judicial decisions is: "What the company is entitled to ask is a fair return on the fair value of that which it employs for the public convenience." (*Smyth v. Ames*, Sec. 8.3.)

The "fair-value" basis for rate regulation has been discussed and criticized adversely from the very beginning (Sec. 8.7), and, in recent years especially, a good many writers on utility regulation seem unable to see anything logical or reasonable in the fair-value basis. Some of the regulatory commissions, notably that of Massachusetts, have wholly disregarded fair value in fixing rates. Nevertheless the cryptic statement of Mr. Justice Harlan quoted above is still the law of the land so far as public utilities are concerned.

20.4. Significance of the Fair-value Rate Base.—It seems probable that the attitude toward the fair-value basis of rate regulation is due to a lack of understanding of the real significance of the rate base thus established. The impression seems to prevail in certain quarters that the purpose of a valuation, and the establishment of a rate base as a result thereof, is to fix rates that are as near the "confiscation" level as is possible without danger of having the courts set aside the rates thus established. Such is not the case in the practice of the efficient regulatory commissions.

Impartial students of utility regulation recognize the necessity for providing the well-managed utility with an income sufficient to keep the

[1] As this is being written in 1934, practically all business has gone under a form of government supervision through a system of codes designed to take care of a national emergency arising out of the economic upheaval that began in 1929. Probably some of the regulatory features of these codes, perhaps many of them, will become a fixed part of future control of industry under the guise of public interest.

property in good repair and to maintain the credit of the corporation so that it can operate efficiently and render satisfactory service. The rates required to insure that condition must be materially above the confiscation level. Just what constitutes a *fair* rate of return has never been defined precisely, but it must be at a level that will permit the corporation to finance itself; judgment must be exercised in fixing the *rate of return*, just as it must be exercised in fixing the *rate base*. It seems perfectly clear that an intelligent and reasonable solution of this admittedly difficult problem is impossible without a knowledge of the value of the property employed in the public service. It is entirely feasible, if all concerned collaborate in the effort, to set up for rate-making purposes a rate base that takes proper account of all factors that should be considered in a given case. The rate base would be closely analogous to market value, although market value as such cannot usually be established for public utilities. The fair value thus established will permit a proper study of the effect of various rates for services on the net returns of the enterprise.

Probably the unfavorable criticism of the fair-value basis for rate regulation has also been due in part to the long-drawn-out litigation that has been common in valuation cases. While the number of such cases has been small in comparison with the total number of valuations that have been made, they have attracted wide attention. Valuation as a means of establishing the rate base, however, has not been responsible for the law's delay, although valuation seems to have borne the onus.

It is perhaps not out of place to remark that lack of confidence in fair value as a rate base is also due to certain indefensible practices that have been all too common in rate cases. Frequently the technical experts representing opposing sides in a rate controversy have filed separate valuations which differ by so large a sum that it is not surprising that reviewing boards and courts have had little confidence in the work of experts. The testimony of such technical experts as to the methods they employed in arriving at their figures has not been very convincing because obviously the figures were in error by a wide margin. The valuation of an extensive property is a tedious matter and costs a large sum of money. It often happens that clients are unwilling to finance an adequate formal valuation and consequently make it necessary for their valuation experts to testify on the basis of inadequate data. All of these things have contributed to much litigation and confusion in the field of public-utility regulation, and particularly have greatly delayed the decision which is so important to the users of the services.

20.5. Substitute Rate Bases.—It may be of passing interest to observe that no rate base that has been proposed as a substitute for the fair-value base has the support of any considerable group of qualified persons, much

less the sanction of the courts. The substitute method perhaps most widely discussed at this time (1936) is what is known as the prudent-investment formula (Sec. 7.10). A careful appraisal of the discussion of this subject will convince one that there are as many loose ends to be gathered together in setting up such a rate base as there are in the fair-value method, and just as great diversity of opinion in regard to the procedure that should be followed.

Some of the questions that would need to be answered authoritatively before a prudent-investment base could be established are these:

1. Who shall determine what expenditures are "prudent" and by what criteria shall the expenditures be judged? Is "prudent" to be interpreted to mean "honest," or is it to mean that good "business judgment" was used in the expenditures constituting the investment? (Is it any easier to check judgment as to investment than to check judgment as to value?)

2. If securities sell at a premium, is the prudent investment the par value of the securities or the actual market value? If the actual market value, what shall be done if the securities sell below par?

3. What shall be done about improvements made by "plowing back" a part of the earnings? Shall these be considered a part of the prudent investment? Suppose the plowing back is accomplished (as it has been in some cases) by curtailing dividends, is it then a prudent investment? Perhaps a part of the money reinvested in the business is in the "retirement reserve," and, if so, does that become a part of the prudent investment?

4. How shall depreciation be treated in determining the prudent investment? Shall it be ignored as is done in Massachusetts regulation (which is not strictly on the prudent-investment basis)?

If depreciation is to be brought into the rate base, by what process shall it be determined?

Of course the United States Supreme Court has not as yet approved "prudent investment" in any form as a suitable rate base. One would assume that it might be considered among "other factors" to be taken into account, but there is no basis for judging how much weight might be given this factor without meeting the adverse criticism of the courts.

Various modifications of the prudent-investment formula have been proposed and in fact applied in rate regulation to a limited extent, although in no case have they appeared except in the practice of the state commissions. All are intended to base the rates on the investment and to insure a suitable return on the investment regardless of the condition of the property. These methods have been most widely favored in those states where for many years the commissions have had authority over the issue of securities and have scrutinized closely each issue of stocks or bonds and have prescribed the purposes for which securities could be issued, or at least have approved the purposes for which the new capital was to be employed.

The cost of reproduction new, less depreciation, has also been suggested as a substitute for the fair-value rate base, but the advocates of this system are vociferous only when reproduction costs are high. When these costs are low, one hears nothing about the reproduction-cost rate base. Since the advocates of this system do not appear to maintain a consistent position with reference to this rate base, it need not be taken very seriously. The cost of reproduction new, less depreciation, is one factor to take into account in establishing a rate base and that is all.

20.6. Property Accounts in Rate Regulation.—A careful appraisal of all of the factors involved in this intricate problem seems to indicate rather conclusively that some simplification of the process of establishing the rate base for public-utility regulation would be of inestimable benefit to the public and to the utilities. But such simplification must reckon with the attitude of the courts toward any departure from the fair-value rate base. It has been pointed out in a previous chapter that the task of valuation would be greatly simplified if each utility were required to keep as careful a record of property (Sec. 6.15) as it does of money. The up-to-date property ledger would eliminate a large part of the cost of a valuation inventory and would permit rapid calculation of the cost of reproduction new. The cost of keeping the property ledger would be part of the recognized operation expenses. If such a system were established on the basis of an inventory of a property and the correct determination of the accrued depreciation, and subsequently maintained under the supervision of a utility commission, the determination of fair value to serve as a rate base would be a relatively short and clear-cut problem. The cost of keeping the property ledger would be saved many times over through elimination of the cost of making inventories and avoidance of some of the litigation that frequently follows attempts to fix a rate base.

20.7. Status of Rate Regulation.—It is becoming increasingly apparent that many public utilities occupy a favored position due to having a monopoly of the business in an area. This position is established by public grants and carries the obligation that the utility furnish its services at fair rates. Fair rates are those that are just sufficient to meet the operation costs, care for depreciation, and pay a fair return on the fair value of the property. Thus, while many factors enter into the determination of fair rates, one of them must always be the value of the property employed in giving the service.

The valuation of a property of the nature of a public utility obviously requires the systematic assembling of many data evidential of value to enable the valuator to form a correct judgment as to value. Here the formal valuation reaches its most elaborate and comprehensive form.

Since there are many factors to appraise in a utility property of considerable size, there are opportunities for differences of opinion, even among experts, with reference to the present value of elements of the physical property. This is particularly true of the estimates of the cost of reproduction new, and yet it is surprising to find that even in properties in which the cost of reproduction new runs into many millions of dollars the separate estimates made by two or more independent experts are often in relatively close agreement. The wide differences in value claimed by the two parties in a valuation controversy are generally attributable to the failure to agree on the basic principles applicable to one or more of the following factors in the problem:

1. The overhead construction costs (Chap. XI).
2. The depreciation of the physical property (Chap. IV)
3. The preliminary-expense value (Chap. XII).
4. The going value (Chap. XII).

The estimation of these factors has been discussed at length in other places in this treatise and the approved method of attack in each case has been set forth, and these will not be discussed further here.

It is evident that there must be some tribunal to determine which of the estimates of value set up by the parties in controversy shall be accepted as the rate base. In states having public service commissions, these commissions constitute the first resort, but either party may appeal to the courts and the United States Supreme Court is the final resort.

In rate cases, the usual procedure is for each party to the controversy to establish what it contends is the fair value of the property. This is accomplished by causing a valuation to be made by qualified experts who follow closely the process outlined in this treatise. Quite frequently the opposing parties will find it desirable to confer with reference to the inventory because of the questions of fact involved. Obviously, the existence of certain elements of the property is not a matter of judgment but rather a question of fact. Aside from such conferences as may be held with reference to the inventory, the valuators work independently, each adopting the unit prices, the depreciation rates, and the mortality data he believes to be correct.

Sometimes the valuation is made by a board constituted to represent both parties to the controversy, which is furnished an inventory of the property that is acceptable to both parties. The board then proceeds to apply unit prices, depreciation rates, and other factors as required to determine the original- and the reproduction-cost values of the inventoried property. At each step in the computations, agreement is reached, if possible, with reference to the several factors involved. The estimates

of the various intangible values are worked out in the same manner. Where such a board is made up of technical experts who are familiar with the type of property involved, it is usually possible to reach an agreement on all vital factors and the fair value reached is likely to be acceptable to all concerned. For small or moderate-sized properties, especially, this method has many things to commend it and usually will be less costly than the method whereby each party has an independent valuation made, which must later be referred to the courts for a decision as to whether there is confiscation under the rates established on the basis of one or the other of the valuations.

20.8. Difficulties Inherent in Utility Valuation.—One of the difficulties inherent in the problem of rate regulation is that the officers of governmental units and commissions are rarely experienced beforehand in dealing with the intricate technical problems involved. In these bodies there is frequent change of personnel, which adds to the confusion and causes inexcusable delays in formulating decisions.

Another of the unsatisfactory phases of rate regulation arises out of the slow progress of these cases when they must be reviewed by the courts. The time that must elapse is rarely less than two years, and it may run to five years or more. Quite frequently the final decision turns on some question of valuation procedure, and in many such cases the outcome might have been expedited had the valuations been made in accordance with the well-known views of the courts. Instead, there have been many cases in which a fruitless effort has been made to introduce into rate cases theories of value that are repugnant to the courts, with the result that the courts reject the resulting rate base and the procedure must begin over again.

A third difficulty involved in public-utility rate cases is that of securing suitable valuation experts to represent the public. Such experts have found their most remunerative employment to be with corporations and many are under retainer of some form and consequently are not available to represent the public. In some of the very important cases, it has required six months or more for the city to find experts to handle the city's valuation work. The solution to this problem would appear to be in the hands of the state utility commissions who should furnish experts from their own staffs to make valuations for municipalities.

20.9. Example of Rate-case Procedure.—A typical proceeding before a state public service commission will be illustrated by an abstract of a case involving the determination of the fair value of a waterworks property which was being sold to a city by a corporation that had constructed the plant and operated it for a number of years. Upon receiving notice that the city officials had been authorized to purchase the property, the Public Service Commission had sent members of its staff to make an

TABLE 20.1.—COMPARISON OF APPRAISALS OF OCONTO WATERWORKS PROPERTY—COST OF REPRODUCTION NEW
(As of Jan. 1, 1928)

Item	Appraisal as filed with the commission			Appraisal adjusted to a common date of Jan. 1, 1928, and corrected for errors and machine trenching			Cost of reproduction new adopted by commission
	Company's appraisal as of Dec. 1, 1926	City's appraisal as of Nov. 1, 1927	Commission's appraisal as of Jan. 1, 1927	Company's appraisal	City's appraisal	Commission's appraisal	
Land	$ 4,000	$ 2,750	$ 2,750	$ 4,000	$ 2,750	$ 2,750	$ 2,250
Cast-iron mains	127,253	80,036	95,719	112,802	80,036	81,268	81,268
Galvanized mains	21,112	8,243	10,566	21,112	8,243	10,566	10,566
Hydrants and connections	8,151	10,027	12,061	8,151	10,027	12,061	12,061
Meters and connections	6,774	6,507	5,390	5,774	6,507	6,104	6,104
Meter boxes	5,083	2,759	2,563	5,083	2,759	2,563	2,563
Suction main	1,790	1,770	1,800	1,790	1,770	1,800	1,800
Reservoir	16,883	11,956	15,930	16,883	11,956	15,930	15,930
Wells	20,334	21,920	19,903	20,334	21,920	19,903	19,903
Other structures	18,868	16,530	17,333	18,868	16,530	17,333	17,333
Plant equipment	17,552	19,155	17,456	17,552	19,155	17,456	17,456
General equipment	3,632	2,324	2,337	3,632	2,324	2,337	2,337
Total	$251,432	$183,977	$203,808	$235,981	$183,977	$190,071	$189,571
Overhead costs at 15% (on above items)	37,730	27,597	30,481	35,397	27,596	28,510	28,435
Total	$289,162	$211,574	$234,289	$271,378	$211,573	$218,581	$218,006
Deducted for machine trenching[1]	....	....	....	10,120	10,120	10,120	10,000
				$261,258	$201,453	$208,461	$208,006
Paving, including 15% for overhead	11,820	Not appraised	....	11,820	....	....	Not allowed
Total	$300,982	$211,574	$234,289				
Nonoperating property	2,421	Not appraised	....	2,421	....	....	Not allowed
Total cost of reproduction new adopted by the commission for this case, as of Jan. 1, 1928	....	....	....	....	....	....	$210,000

[1] This deduction was decided upon during the hearings on this case.

appraisal of the property. The value fixed by the commission's engineers is given in Table 20.1 in the column headed "Commission appraisal." The city employed an appraisal company to make a valuation which is given in this table under the heading "City's appraisal." The water company also had an appraisal company prepare a valuation which appears in this table under the heading "Company's appraisal." There was introduced in evidence also an old appraisal which appeared to have no direct bearing on the questions raised and will not be discussed herein. The value was to be established as of Dec. 31, 1928.

The commission held hearings on the valuations that were before it and took the testimony of the several valuation engineers and others as to the basis of the several estimates of value and how they were arrived at. It then proceeded to reconcile so far as possible the several reports that were before it. The major questions and the decisions reached were as follows:

1. *Land.*—Two of the appraisers valued the land at $2,750 and the third at $4,000 (Table 20.1). The Commission stated that none of the engineers presented first-hand evidence of the value of the land. There was before the commission a deposition of the assessor for the city to the effect that he had been assessor for 28 years and was familiar with the value of the land in question and thought it was worth $2,250. This value was accepted by the commission.

2. *Mains.*—It will be noted that the greatest differences in the cost of reproduction as estimated by the three appraisers were in the items of cast-iron mains and galvanized-iron mains (Table 20.1). The commission questioned each of the appraisers to learn why the differences existed. While two of the appraisers were fairly close together on this item, the third was much higher than the other two. The testimony of the appraisers brought out the fact that one had used a unit price for laying that was about 50 cents a foot higher than the unit price used by the others. The valuation engineer for the commission presented data on the unit costs for laying in other nearby cities where labor costs and digging conditions were similar to those in the city where the plant was located. The others testified as to average costs or general costs. The commission accordingly accepted the appraisal of its own engineer, which is somewhat higher than the figures presented by the appraiser employed by the city and much lower than the figures of the appraiser employed by the company.

3. *Other Physical Property.*—The differences among the three appraisers as to the cost of reproduction new of the other items of physical property were not very large, and, after some discussion of such differences as did exist, the commission adopted the figures of its own appraisal engineer for each of the items as shown in Table 20.1.

4. *Nonoperating Property.*—The commission rejected an allowance for nonoperating property that appeared in the appraisal made for the company.

5. *Machine Trenching.*—The commission discovered that the estimates of cost of reproduction new of the cast-iron and galvanized-iron mains were based on hand digging. It concluded that the estimates should be based on machine digging in so far as machine digging was practicable. In certain parts of the city the ground was built up over old sawmill refuse which would preclude machine digging, and at railroad crossings hand digging was required. Accordingly, the commission requested the valuation engineers to estimate how much the cost of laying would be reduced by machine digging. One appraiser did so and his figures were accepted.

6. *Cost of Reproduction New Finally Adopted.*—The final column of Table 20.1 gives the figures finally adopted by the commission as the "Cost of reproduction new." Apparently they decided that some question remained as to certain items aggregating a total of about $2,000, which they finally added without clear explanation in the record of the case.

7. *Overhead Costs.*—The allowance of 15 percent for overhead costs was adopted in accordance with the established practice of the commission. There is no statement as to what this covers, but apparently it includes all the allowance that is made for preliminary expenses and construction overhead. The percentage was applied to all items including land. (It is now authoritatively established that the overhead should not be applied to land.)

8. *Depreciation.*—The Commission next devoted itself to a consideration of the actual depreciation of the property. There was considerably more variation in the

Items	Present condition-percent			
	Company's valuation (1)	City's valuation (2)	Commission's valuation (3)	Accepted by the commission (4)
Distribution system, complete	89	78	81	66 ±
Galvanized mains	47	43	38	43
Gate valves and boxes	97	75	85	85
Specials, crossings, fittings on galvanized mains, and discharge piping	....	..	79	80
Hydrants and connections	90	66	79	79
Meters	78	64	70	75
Meter boxes	95	90	85	90
Suction main	96	85	85	85
Pump house	80	82	80	80
Stack	80	46	80	80
Reservoir	83	57	80	57
Frame coalshed	30	5	4	4
Main office building	73	60	60	60
Small office building	85	90	90	46.2 (Small office building, Storage sheds, Well house)
Storage sheds	63	50	50	
Well house	28	40	25	
Sidewalk curbing and fencing	80	80	80	80
Wells	95½	77	79	85
Steam pump 1	54	36	33	39
Steam pumps 2 and 3	77	26	20	56
Air compressor	80	35	30	50
Air-compressor tank	80	50	80	50
Boilers	50	18	10	30
Boiler feed pump	50	18	10	30
Feed-water heater	50	18	25	30
Plant piping and covering	60	59	25	30
General and office-equipment	75	65	60	65

figures adopted by the several appraisers for depreciation than in the figures for cost of reproduction new. The more important items were as shown in the table on page 464.

The commission considered in detail the reasons advanced by the appraisers for each estimate of condition-percent and adopted the figures given in column (4) above as a result of careful consideration and much discussion. Unfortunately, the data in the record of this case do not permit direct computations from the foregoing table because the tabulation of reproduction cost new in the report does not separate the items as above. However, in the report the cost of reproduction new less depreciation, on the straight-line basis, using the foregoing established condition-percents, is given as in Table 20.2. It will be noted that the allowance for overhead has also been depreciated on the basis of an over-all condition-percent of about 70.

9. *Historical Cost* (*Original Cost*).—The commission had made a valuation of this property in 1911 and had as of record the historical cost as determined by that valuation. It was brought down to date by deducting retirements and adding the cost of additions subsequent to 1911.

Value on original-cost basis before depreciation, as of June 30, 1911	$125,847
Net additions, June 30, 1911, to Dec. 31, 1928	26,614
Original cost, Dec. 31, 1928	$152,461

The above was then depreciated on the same basis as was the cost of reproduction new, by using a computed composite over-all condition-percent which gives the original-cost value as $113,408.

10. *Investment.*—The reports that had been filed with the Commission showed that the investment to Dec. 31, 1928, had been $160,083, which was also depreciated by an over-all condition-percent as above and showed the present investment to be $120,529.

11. *Fixing Fair Value of Physical Property.*—The several analyses of value as of Dec. 31, 1928, which have been presented above, may be summarized as follows:

Original-cost value of the physical property	$113,408
Reproduction-cost value of the physical property	$157,009
"Investment" value (prudent investment)	$120,529
From the foregoing the commission fixed the fair value of physical property at	$155,000

12. *Intangible Values.*—The commission rejected an allowance for going value because of the depressed condition of the city. In its decision the Commission stated: "Certainly nothing is in sight which would justify a conclusion that because of future prospects the property of the Oconto City Water Supply Company has a material going concern value."

13. *Materials and Supplies.*—Materials and supplies are excluded from the foregoing figures but were to be included at the time the city took over the property at a value to be agreed upon at the time.

14. *Working Capital.*—Apparently the working capital, if any, did not go with the property when it was transferred, as no mention is made of working capital in the report on the case.

Table 20.2.—Cost of Reproduction New Less Depreciation—Oconto Waterworks Property

(As of Jan. 1, 1928, based on inventory of Jan. 1, 1927)

Land	$ 2,250
Cast-iron mains and galvanized main	60,834
Gate valves and boxes	2,294
Specials, crossings, fittings on galvanized mains, and discharge piping	4,034
Hydrants and connections	9,528
Meters connections and fittings	4,508
Meter boxes	2,307
Suction main	1,530
Pumping-station building and stack	9,975
Reservoir and cover	9,080
Miscellaneous buildings	2,247
Wells	16,408
Pump, air compressor, and tank	3,968
Boilers and accessories	2,591
General and utility equipment	1,519
	$133,076
Allowance for construction overhead at 15%	19,961
Arbitrary addition by commission on account of uncertainty in certain items	1,714
	$154,751
Depreciated value of additions during 1927 and 1928 not included in inventory of Jan. 1, 1927	2,258
Total cost of reproduction new less depreciation	$157,009

VALUATION OF THE RAILROADS OF THE UNITED STATES

The most extensive valuation project in history, perhaps, is that of determining the value of the railroads of the United States; a task imposed upon the Interstate Commerce Commission by Congress in 1913. The methods devised by the Commission to accomplish its task comprise an almost complete treatise on valuation technique and are entirely too voluminous for presentation here except in barest outline. Such an outline will, however, serve to indicate the general plan of the work, which plan with some modifications might be applied advantageously to many other types of utility valuation.[1]

20.10. Valuation Sections.—Each railroad was arbitrarily divided into valuation sections for inventory purposes, the section of railroad between selected termini being considered an entity and all property of every kind on that section being inventoried separately from the property

[1] The methods employed by the Interstate Commerce Commission in railroad valuation are set forth in detail in Appendix 3, of the valuation report on the Texas Midland Railroad, *I.C.C. Repts.*, July, 1918–July, 1923, **75**, 103. The report is obtainable from the Superintendent of Documents, Government Printing Office, Washington, D.C.

of another section. Original ownership, present use, character of construction, records of original cost, and similar factors determined the division into sections, which vary greatly in length in the different properties that have been valued. In no case did valuation sections extend across state lines, as it was mandatory that the value of each property in each state be determined as well as the total value of the entire property.

20.11. Inventory Methods.—A complete and detailed inventory of the physical property constituted the first phase of the valuation. Inventory parties consisted of two engineers, one employed by the commission and one by the carrier. Additional personnel as required was provided by the Commission. The two engineers worked together, checking each other as the data were assembled. These parties did not fix prices or compute costs. They confined themselves strictly to the task of securing an accurate inventory. In order to insure personnel with the requisite experience, the inventory groups were set up according to the type of property as follows:

1. Land (not a part of the engineering inventory).
2. Road and track.
3. Bridges.
4. Buildings.
5. Mechanical equipment.

20.12. Inventory Units.—The Commission, after considerable study, adopted the following as inventory units, among many others required in special cases:

Ties, each.
Rails, tons.
Bridges, each.
Signal stands, each.
Excavation and filling, cubic yards.
Track fastenings and plates, tons.
Cars and locomotives, each.
Buildings, each; with foundations, platforms, heating, and lighting as items in many cases.

20.13. Unit Prices.—Perhaps the most difficult problem confronting the Commission was the determination of the proper unit prices to apply to the inventory quantities in estimating the cost of reproduction new. After much discussion the Commission decided to ascertain what prices had been paid in the period just prior to the time when these computations were being made. Accordingly they requested the roads to furnish attested reports of the actual costs incurred for various types of construction and for various materials at intervals over a period of five years and sometimes ten. From these a unit price was determined

which was suitable for the conditions in each area and for each kind of work. These unit prices were comparable with period prices, as those terms are used in this treatise, and were what the Commission considered to be normal prices on June 30, 1914. Perhaps no part of the valuation of the railroads received more careful study than that of unit prices, and in no case was a figure adopted until there had been exhaustive study of all available data and the proposed figures had been submitted to the carriers for comment.

20.14. Depreciation.—The practice relative to depreciation is set forth in the following extract from *Interstate Commerce Commission Engineering Board Memorandum* 226, 1915:[1]

The board is of the opinion that the principles contained in the following definitions should govern in determining the depreciation of physical property used for common-carrier purposes:

(1) Depreciation is the lessening in worth of physical property due to use or other causes.

(2) Labor depreciates with the items of property of which it is a part.

(3) Service-condition percent is the ratio between remaining capacity for service and total capacity for service in each cycle of use.

(4) Depreciation shall be determined by a consideration of observations of actual conditions of the property and mortality statistics of similar property in like use applied where practicable on the straight-line basis.

(5) Salvage is the net value which an article possesses for carrier purposes other than that for which the article was originally used.

(6) Scrap is the net market value which an article possesses after its service for carrier purposes has been exhausted.

(7) Normal life shall be the estimated average total service life under ordinary conditions.

20.15. Reproduction-new Cost Estimates.—In estimating the cost of reproduction new the following principles were adopted:

1. *Appreciation of Roadbed.*—After exhaustive study the Commission decided not to include an item for appreciation of roadbed, mainly because it seemed impossible to find a suitable basis for estimating this factor.

2. *Contingencies.*—In estimating the cost of reproduction, no allowance for contingencies (Sec. 11.23) was included. The reason given is that with a complete inventory and full information as to the nature of the work to be done the contingency of the original construction is now a known factor and is adequately covered in the unit prices and inventory quantities.

3. *Engineering Costs.*—Engineering costs were allowed in amounts ranging between 2 and 5 percent of the road accounts (all items except land and equipment).

[1] It will be noted that the methods of handling depreciation in railway valuation depart in many respects from those advocated in this treatise, and from the principles that have become established through the decisions of our highest courts. The practice established by the Commission in 1915 has been in use without change since that date.

4. *General Expenditures.*—An allowance of 1.5 percent (Sec. 11.12) of the road accounts was allowed for general expenditures exclusive of interest lost during construction. The items covered are:

a. Organization expenses.	*b.* General officers and clerks.
c. Law.	*d.* Stationery and printing.
e. Taxes.	*f.* Other general expenditures.

5. *Interest during Construction.*—Interest during construction (Sec. 11.24) is computed on the road accounts (except land) and upon general expenditures for one-half the period of construction plus three months and upon equipment for a period of three months. The rate applied was 6 percent, which was assumed to include commissions.

20.16. Determinations of Original Costs New.—At first the Commission reported it had found it impossible to ascertain the original costs new of railroad properties because of imperfect book records and lack of sufficient reliable data to permit supplying reasonable estimates for items not of record.

With the lapse of time, records of cost of replacements of old property and the accumulation of data on costs have made it feasible to set up reasonable estimates of original cost new for some roads. In the valuation of the Richmond, Fredericksburg and Potomac, the Commission reported both the original costs and the reproduction costs.[1]

20.17. Comment.—As an example of engineering valuation the inventory and the estimates of the cost of reproduction new of the railroads are classics, and a study of any of the volumes giving the details of these computations and the discussions at the hearings on the tentative valuations will repay any one interested in the subject of engineering valuation. These valuations have been widely used in connection with many types of railway problems. Unfortunately, dissension in the Commission relative to theories of depreciation and the basis for establishing fair value have resulted in adverse decisions of the courts in the cases that have been litigated.[2]

[1] I.C.C., Finance Docket No. 3898, 170 I.C.C. 451.

[2] See *St. Louis and O'Fallon Case*, case 53, Chap. VIII.

CHAPTER XXI

AN EXAMPLE OF THE VALUATION OF AN ELECTRIC UTILITY

There is presented in this chapter a somewhat detailed report of an actual valuation of the city of Ames electric utility, which furnishes light and power to a city of about 10,000 population. This valuation was made to establish the fair value of the property for the city manager to use in connection with the administration of the property, and the valuation has not been reviewed by a state utility commission or by the courts. It is presented to indicate a procedure which may be employed in making such valuations.

The preliminary examination and study of the property to be valued showed that it consisted of a steam-generating station, located on a railway siding and provided with the necessary cooling towers and ponds for condenser water, since there was no stream available to furnish water for that purpose. Adjacent to the generating station were storerooms for supplies to be used in the maintenance of the property, and garages to house the equipment.

The distribution system consists of four feeder circuits, each of which supplies a definite section of the city with light and power. In addition, there are nine feeder circuits supplying light and power to areas adjacent to the city but outside the corporate limits. The distribution system is described in detail in a later section.

An examination of the book records of the utility disclosed that the utility had fairly complete records of its property, including the costs and in most cases installation costs. However, these records have not been kept in accordance with any particular standard form.

After consideration of the nature of the property, the personnel for the valuation was determined upon, inventory sections delineated, and a system of grouping the property according to a standard system of accounts was set up. With these preliminaries out of the way, the active work of the valuation was undertaken.

The city manager desired a separate valuation of the equipment used for each type of electrical service provided by the utility, namely, street lighting of the ordinary type, street lighting by electroliers, and commercial service which includes domestic lighting and power, industrial lighting and power, and rural service.

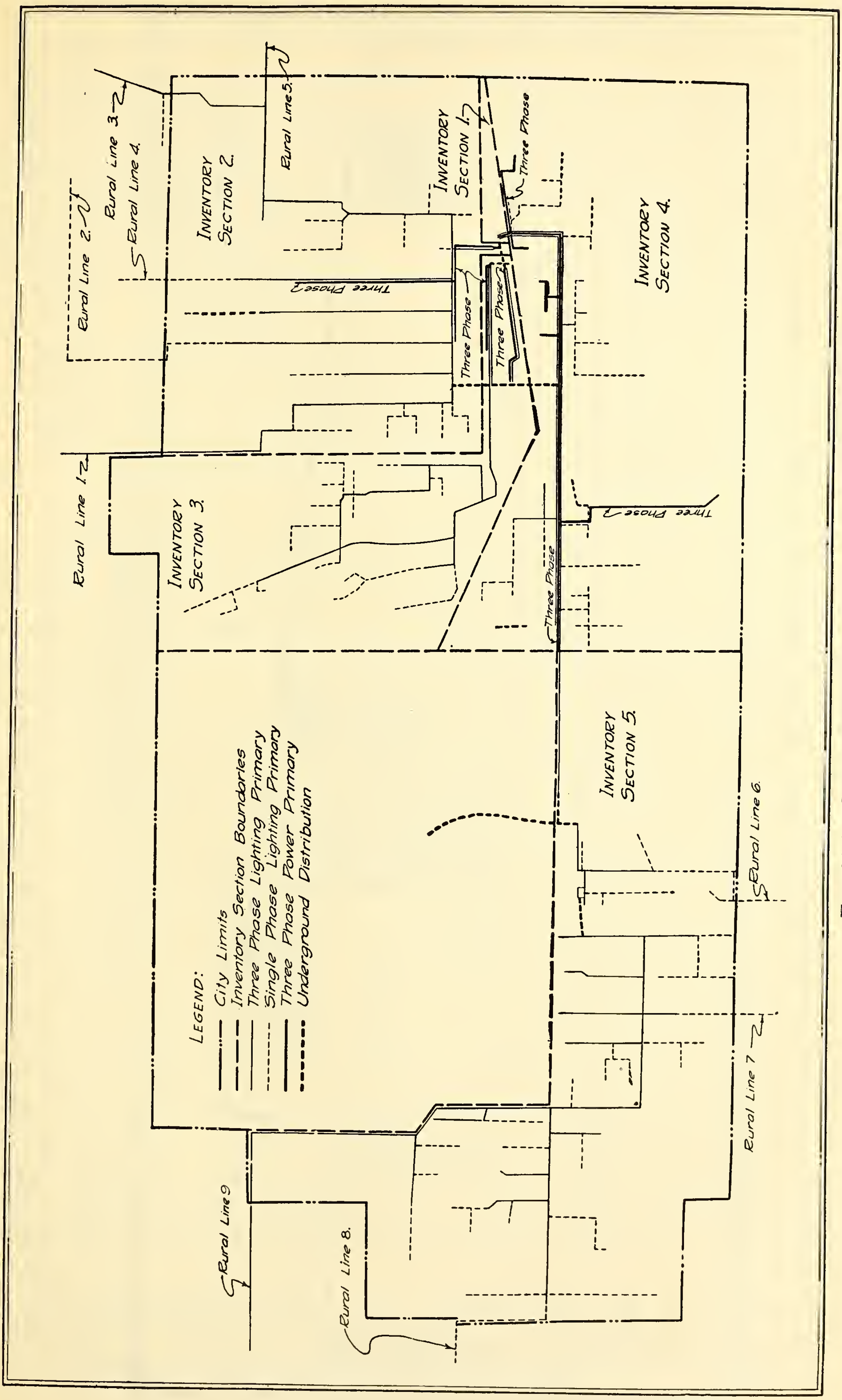


FIG. 21.1.—Map showing inventory sections.

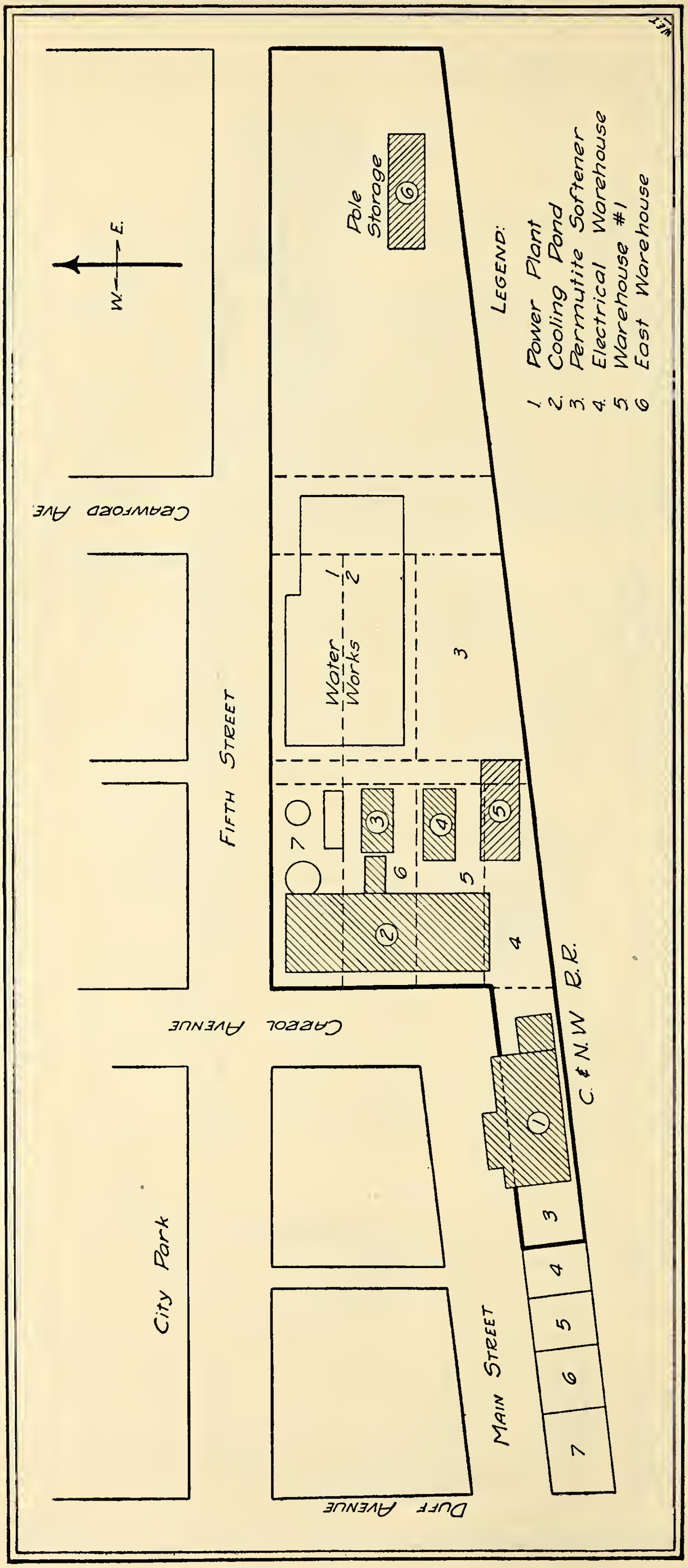


Fig. 21.2.—Plat showing inventory section 1 in detail.

PLANNING AND MAKING THE INVENTORY

The utility did not possess an inventory of its property that was suitable for valuation purposes, but the property records of the utility were of considerable assistance in formulating and checking the inventory, which was made as the first step in the valuation.

21.1. Inventory Sections.—The property was divided into six inventory sections, as follows:

Section 1. The power plant with all its accessories and the storerooms.
Section 2. The Sixth Street light and power feeders and all services connected therewith.
Section 3. The Duff Street feeders and all services connected therewith.
Section 4. The South side feeders and all services connected therewith.
Section 5. The West side feeders and all services connected therewith.
Section 6. All of the rural lines being served by the utility, including only those portions outside the corporate limits. There were nine such lines to be inventoried.

These inventory sections are shown on the map in Fig. 21.1. Inventory Section 1 is shown in detail in Fig. 21.2.

21.2. Property Groups.—In order to make the valuation report of the widest possible value, the property items of the utility were grouped according to certain of the standard accounts recommended in "Uniform Classification of Accounts for Electrical Utilities," issued by the National Association of Railway and Utilities Commissioners.[1] The classification was set up in accordance with the schedule in Table 21.1.

21.3. Inventory Parties.—The inventory parties consisted of two men each. One party made the inventory and computations for valuation section 1, and a second party made the inventories and computations for all of the remaining valuation sections. As rapidly as the field inventory progressed, the data on the field-inventory sheets were summarized by the inventory party according to property groups and were transferred to property-group classification summary sheets, conveniently arranged for making the final computations of depreciation and of value.

21.4. Original-cost Unit Prices.—The unit prices used in the computation of original costs were obtained by a study of the actual prices paid by the utility for the various existing items of property as shown by the records. In a few instances omissions in the records had to be supplied on the basis of price-trend curves and indexes, and of cost data obtained from various other sources. The method was that described in detail in Chaps. IX and XI (see Tables 11.1, 11.2, and 11.3).

21.5. Reproduction-cost Unit Prices.—Unit prices employed in estimating the cost of reproduction new were those obtained by averaging

[1] The State Law Reporting Company, New York, 1933.

TABLE 21.1.—CLASSIFICATION OF ACCOUNTS IN THE VALUATION OF THE AMES ELECTRIC UTILITY

Standard account	Character of property	Pamphlet reference,[1] pages
	Land	
311a, 311g-1.	Land	28, 36
	Power Plant	
312a	Structures	28, 37
313	Boiler-plant equipment	28, 39
315	Turbogenerator units	28, 40
316	Electric plant (switchboard)	28, 40
317	Miscellaneous power plant	28, 40
	Distribution System	
328	Substation equipment	29, 43
330	Underground conduits	29, 44
331	Poles, towers, and fixtures	29, 45
332	Overhead conductors	29, 45
333	Underground conductors	29, 45
335	Services	29, 46
336	Transformers	29, 46
338	Consumer meters	29, 47
342	Street-lighting equipment	29, 48
	General Structures and Equipment	
312g-1.	Structures	28, 37
344	General equipment	29, 48
	Working Capital[2]	
116	Materials and supplies	2, 11
336 stock	Transformers in stock	29, 46
338 stock	Meters in stock	29, 47

[1] Uniform Classification of Accounts for Electrical Utilities, issued by the National Association of Railway and Utilities Commissioners, the State Law Reporting Company, New York, 1933.

[2] Working capital also includes fuel and current supplies and the average working cash balance necessary.

the prices actually paid by the utility for various kinds of property during the year 1932, supplemented in a number of cases by quotations from manufacturers and by other pertinent cost data available to the valuators. Extreme care was exercised to make sure that all data employed in estimating the cost of reproduction new were fairly representative of the average prices for the year 1932. In this instance, average prices for 1932 were used in preference to unit prices based on the averages for the last three to five years, because of the fluctuation in construction costs during 1930, 1931, and 1932 and the uncertainties as to the future trend of prices. It is believed that the situation in 1932 was such that average prices for the past three to five years would have

had little significance and would be no indication as to the prices likely to prevail during the next few years. It is also problematical as to whether 1932 prices are likely to hold for very long in the future. Nevertheless, they afforded a definite basis for a valuation and a definite starting point for future revisions in the value, when that becomes necessary. The method employed for selecting these unit prices is discussed in much greater detail in Chaps. IX and XI.

INVENTORY OF LAND, POWER PLANT, MISCELLANEOUS PROPERTY, AND STORES

The power-plant building is of steel-frame and brick construction, having a total volume of 482,000 cubic feet. In addition there are a coal storage pit, a cooling tower and pond, and a one-story brick building housing a water softener. The installed capacity of the power plant is 5,750 kilowatts made up of three turbogenerating units of a capacity of 3,000-, 2,000-, and 750-kilowatt capacity, respectively. These machines operate at 170 pounds per square inch of steam pressure. The condenser cooling water is recooled by a natural-draft cooling tower, having a capacity of 2,000 gallons per minute. A water softener, having a capacity of 200 gallons per minute, furnishes the softened water for the boilers and condensers. There are three boilers having a total of 1,042 boiler horsepower, equipped with chain grate stokers, superheaters, and soot blowers. Three one-story mill-type buildings are used for shop and storage purposes.

The general equipment inventory listed the tools, furniture, and fixtures used in the generating station, and office equipment in the business office and the engineering office. In addition, there are several cars and trucks employed. The stock of materials and supplies is warehoused in bins at various places and includes a stock of spare parts and supplies for the generating station and for line work.

21.6. Field Inventory, Inventory Section 1.—In making the inventory of section 1, each unit of property at the generating station was carefully examined and, so far as possible and practical, the following facts were ascertained for each unit: its age, present physical condition for its age, its past and probable future service conditions, and other data that would have a bearing on its expected future service life and its ability to give satisfactory and economical service.

All the buildings were measured in detail and the volume of each material computed and recorded. To facilitate the work of making and checking the inventory, the buildings, particularly the power-plant building, were divided into small units. This was done by numbering the columns and measuring each column and the wall between pairs of columns. The masonry and concrete were computed in cubic yards, floors and roof in square yards or square feet, and steel in pounds.

Advantage was taken of all office records of equipment purchases to make up a list of the units purchased in recent years. This list was then checked in the field, which served the dual purpose of completing the field inventory and also of checking it. This method also provided a way of obtaining an accurate description of each piece of equipment. The possibility of overlooking any concealed items was thus eliminated.

The equipment that was not of record consisted mostly of small units which were inventoried in the usual way; by counting and recording the quantity and description of each item in a systematic geographical order. Figure 21.3 shows the form used in making the field inventory.

Form ________ Valuation of ________ Sheet *41*

By ________ *Check* *1✓ 2✓ 3 4* Of *✓*

Inventory Field Notes for Inventory Section *No. 1* *315 H.P. Boiler*

(1) Physical Condition-E, Excellent, G, Good, A, Average, P, Poor, B, Bad—For the Age
(2) Service Conditions (Past and Future)—M, Mild, F, Favorable, A, Average, U, Unfavorable, S. Severe.

Mfgr's No. and Plant No.	Quantities	Location 315 H.P.	Description	Age	Cond. (1) Phys.	(2) P.Ser.	(2) F.Ser.	Remarks
✓	2	✓	*315 H.P. Murray Boilers, Type A*	*10/*	*G*	*A*	*F*	*$9074 –*
			Extra Labor, Estimated (Ball)	*/17*				*250 –*
								Abt. Jan. 1, 1923
✓	2	✓	*Foster Superheaters*	*10*	*G*	*A*	*F*	*$2400–*

FIG. 21.3.—Inventory form for section 1.

21.7. Collection and Tabulation of the Inventory Data.—The data recorded on the field-inventory sheets were collected into the smallest possible number of groups of units. One group of units can include only property of the same age, kind and size.

The grouped units were then recorded on classified summary sheets, like those shown in Fig. 10.3, and were ready for pricing.

21.8. Unit-cost Data.—The original costs of almost all units of equipment were obtained from office files. Sometimes this figure included freight and installation costs, while in other instances only one or perhaps neither of these charges were included in the recorded contract price. Missing data were estimated or computed with the aid of information obtained from those directly connected with the work when the equipment was installed.

Reproduction unit costs were, where possible, obtained from current purchases. Current quotations on like or similar equipment were obtained, for most of the larger power-plant units, from the manufacturers. Curves that were used in checking the original costs and present

reproduction costs are shown in Fig. 11.2. Standard published indexes and price data were also used in checking a number of the prices. The Constructograph index was used in checking the adopted reproduction unit costs for the small buildings (Fig. 9.16).

The unit prices to employ in determining the reproduction cost of the power-plant building were obtained from the actual cost of an addition to the building that was erected the same year (1932) the valuation was made. Some of the original costs of the building items were obtained from the records, while others had to be estimated from a mass of available data.

Table 21.2.—Probable Service Lives Adopted for Power-plant Units and Structures in the Valuation of the Ames Electric Utility

Unit of property	Expectancy	Probable service life, years
Structures:		
Power-plant building	20	37
Coal pit	20	33
Cooling pond (north)	15	33
Cooling pond (south)	17	25
Water-softener building	22	25
Warehouse 1	25	28
Warehouse 2	23	25
Warehouse 3	17	25
Power-plant Equipment:		
315-hp. boilers and equipment	7	17
412-hp. boilers and equipment	15	19
Feed-water heater	10	18
Surge tank (steel)	15	18
Boiler feed pumps	10	13
Stack, steel, brick lined	20	30
Breeching (original)	15	25
Breeching (addition)	15	19
Coal-handling equipment	8	13
Ash-handling equipment	10	12
2,000-kw. turbine	15	18
750-kw. turbine	7	14
Permutit water softener	5	8
Cooling tower (original)	10	13
Cooling tower (addition)	10	12
Switchboard equipment	15	15–33
Piping—pipe and covering	Same expectancy as the equipment served	

21.9. Determination of Probable Service Lives.—A reasonable probable service life was determined for each item in the following manner: The valuators determined the physical condition of the units

of property, considered the normal range of probable service lives used in valuations of this type, as presented in Appendix A, studied the past performance of similar units in service in the plant, considered the past growth in load and improvement in equipment and the probable growth in load and increase in efficiency of power-plant equipment, consulted with officials having knowledge of the equipment, and then fixed probable service lives for all major items of equipment as seemed reasonable in view of all the facts. These figures were checked in a general way by using mortality type curve S_3, (Appendix B). The probable service lives adopted for the main units are listed in Table 21.2.

21.10. Annual Depreciation.—The depreciation of the property for the years 1932 and 1933 was computed by determining the depreciation for each unit shown in the valuation for each of the two years, including the depreciation of the additions to the property in 1932 and 1933 which were not inventoried in detail. Considerable property was retired during 1932, prior to the date of the inventory, and consequently was not included in the inventory, but these units are included in the year's depreciation. Since, at the beginning of the year, this property had considerable value, the difference between that value and the net salvage value was the loss in value due to the retirement of these units. This loss in value plus the computed depreciation on the inventoried items gave the total amount of depreciation for 1932. The explanation of the fact that the depreciation for 1932 is larger than that estimated for 1933 is that no major retirements were anticipated in 1933. Moreover, it is probable that the annual depreciation during each of the years 1932 and 1933 is somewhat higher than the normal depreciation of this property because in those years several sizable units of equipment were retired because of obsolescence or inadequacy, and such a condition may not recur for some years.

INVENTORY OF THE DISTRIBUTION SYSTEM

The distribution system is mostly of wooden pole-line construction, carrying 2,300 volts on the primary circuits and 115 or 230 volts on the secondary. The general arrangement of the distribution system is shown in Fig. 21.1. The utility owns and maintains nine rural lines, serving customers outside the corporate limits. The utility owns 92 customer meters on these circuits, and there are 32 transformers on the rural circuit that are owned by the utility, and 7 that are owned by the customers.

21.11. Field Inventory.—The party engaged in the task of taking the inventory of the transmission system was composed of two men, one of whom recorded all equipment on the utility-owned poles, the other listed all utility-owned equipment on Bell System poles. Each

CITY OF AMES, ELECTRICAL DEPARTMENT

Distribution Sheet *4*

Inventory Section No. *2* Map No. *2,* By *R & H* Date *6-28-32* Of *17*

Pole Number	Poles-W.R. Cedar: Length	Top Diam.	Set In	Age	Treatment	Phys. Cond.	P.S.C.	F.S.C.	Cross Arms. 2-P Single	2-P Double	4-P Single	4-P Double	6-P Single	6-P Double	Alley Arm Eqpt.	Pins: Locust	Steel Screw	Steel Forged	Clamp
76	30	6	11	11	B	G	M												
77	30	6	1	9	B	A	M												
78			10	10												10			
79	30	6	9	9	B	A	F				1					3			

Pole Number	Insulators: Side Groove	Top Groove	Loop	Dead End	L.M. Dead End	Thread Thru	Rks. Small	Rks. Large	Brackets: Wood	Single Trans.	Double Trans.	St. Lt. Eqpt.	Guys Lt. Down	Lt. Span	Med. Down	Med. Span	Hy. Down	Hy. Span
76							1	1					1					
77							1	1										
78	12			4				1	2									
79	3														1			

Pole Number	Grd. Equipment	Grd. Guard	Lightning Arrstr.	Cutouts	F's 60 Amp.	F's 100 Amp.	Transformers	Ser's Two Wire	Three Wire	Special	Misc. Angle Irons	Hub Guards
76					1EP4565	2P4665		7				
77								1				
78								4				
79												

CITY OF AMES, ELECTRICAL DEPARTMENT

Wire Inventory

Sheet *1*

Type of Service *Distribution-Primary* Inv. Sec. No. *2* By *H & R* Date *7/8/32* Of ____

Map No.	Line Description	Age	Cond.	Wire: Type	Insul.	Size	No. of Wires	Meas. Length (feet)	% Allow.	No. Feet Allow.	Total Est. Len. (feet)
21	Plant to No. 1			Cable	4/0	- temporary					
	No. 1-943	3		Solid	W.P.	2	3	51		3.5	164
21.1	No. 1-No. 7, North Side Primary	3		Str.	W.P.	0000	3	549		29.5 *4-35.0	1736 35
	, Power Primary	3		Solid	W.P.	2	3	549		27	1728

Fig. 21.4.—Distribution-equipment and wire-inventory sheets.

man checked the quantities counted by the other. These data were entered on special "Distribution equipment inventory sheets," Fig. 21.4, a form devised to fit this particular job (similar to Fig. 10.2). In studying the sheet it can be seen that some of the units include a number of smaller items or parts which go to make up the larger units. This is true of a six-pin single crossarm, for example. It always consisted of a six-pin crossarm, one through bolt, two machine bolts, two crossarm braces, and one lag screw.

This grouping greatly reduced the possibility of errors in taking the inventory. Similar grouping was made of all crossarms, alley arms, guys, racks, street-lighting brackets, special insulators, ground equipment, line devices, and numerous other items.

The inventory sheets were carried in two pads, one for each of the following groups of property:

1. Utility-owned poles and equipment:
   a. Jointly used poles and equipment owned by the city.
   b. Commercial distribution only.
   c. Street lighting only.
2. Utility equipment on Bell System poles:
   a. Jointly used equipment.
   b. Commercial distribution only.
   c. Street lighting only.

In taking the inventory, one line on the inventory sheet was used for each pole location. Since the poles were numbered consecutively, it was thought best to take the inventory in the order the poles were numbered. The condition of each pole was noted, examination being made by removing dirt from around the base of each pole and for a distance of several inches above and below the ground line testing the pole as to soundness. Data were also recorded as to the size, age, treatment, and service condition of the pole. The data were completed from, and checked with, the utility pole records which had been compiled during the fall and winter of 1931 to 1932. All equipment was considered to be the same age as that of the pole, unless otherwise known.

The wire inventory was taken from the utility's official distribution maps. These maps gave the scaled lengths (which by actual measure were found to check within 2 percent) and the various wire sizes. A member of the line crew of the utility aided in checking the service age of wire and its size and condition. The wire-inventory sheet is shown in Fig. 21.4. The allowances for connections, corners, dead-ending, transformer settings, and sags were estimated.

All underground equipment, substation equipment, and the electrolier system, being comparatively new, were inventoried and checked with various records of the utility and listed on the form shown in Fig. 21.3.

Inventory Section No. 2 — CITY OF AMES, ELECTRICAL DEPARTMENT — Sheet 1

Type of Service Distribution — Collection Sheet — Of

Name / Descpt. / Age	Poles 25'-6" U	25'-6" B	25'-6" P	30'-7" B	30'-7" P	30'-6" U	30'-6" B	30'-6" P	35'-9" P	35'-8" P	35'-7" B	35'-7" P	35'-6" P	40'-8" P	40'-9" P
0												1			
1					1			1				5	1		
2					9			1				16		4	1
3										1					
4					4			2				1			

Inventory Section No. 2 — CITY OF AMES, ELECTRICAL DEPARTMENT — Sheet 5

Type of Service Distribution — Collection Sheet — Of

Name / Descpt. / Age	Fuses Small Exp. EP23C3	Med. Exp. EP45C6	Throttle Fuses P45C5	Throttle Fuses P66C1	Large Exp. EP75C1	Transformer Number	Services 2-Wire Ave.	2-Wire Long	3-Wire Ave.	3-Wire Long	Special 3-Wire	Special 2-Wire	3φ-3 Wire Com.	3φ-3 Wire Special	Angle Irons	Hub Guards
0		2				17 45 257 165	16 29 29 2		5 9 5 2		2-#151 #224 #355		4	#74 #181		
1						312 67 60 264	38 49 21 53		3 21 17 7		2}-354 377			2}-#182		
2		3				279 81 343 289	32 19 30 36		1 11 4 11						4	
3		8	2		6	82 84 79 307	50 27 41 14		15 7 7							

FIG. 21.5.—Collection sheets.

The distribution inventory sheets gave the location and serial number of each transformer. This record was completed in detail from the transformer record, which showed the make, size, age, and location of each transformer.

The method of taking the inventory of meters varied considerably from that used in taking the other property in that no field inventory was made. All meters are numbered, and a card file is maintained which provides that, when a meter is installed, a card is made out for it. In cases where the data on the cards were inadequate, the meter readers obtained the necessary data. The meters actually in service were ascertained by checking the meter readers' books against the card index. All meters whose numbers appeared in the meter readers' books were assumed to be in service or in serviceable condition. The meters were grouped according to size, age, and make. Meters on rural lines were separately inventoried.

The property making up the various rural lines was inventoried in the same manner as the distribution system within the corporation limits. The wire, however, was measured by a revolution counter, mounted on the wheel of an automobile.

21.12. Collection and Tabulation of Data.—After the entire inventory had been taken, a collection sheet like that shown in Fig. 21.5 was used to collect the items of like kind, class, size, and age before transferring the data to the final classified summary sheets. The data thus assembled on the collection sheets were then transferred to the classified summary sheets, shown by Fig. 10.3. On these sheets were recorded the name and general description of the equipment or property, the quantity, the age, the probable service life, and the condition-percent as taken from the condition-percent tables (Appendix C). The table for 6 percent rate of return was used in this valuation. The original unit cost f.o.b., additions for handling and installation, and reproduction costs were entered in their proper columns. The total cost new was determined on the original-cost basis and estimated on the 1932 reproduction-cost basis; and the present values were calculated by the use of the condition-percent tables as explained in Chap. V.

21.13. Mortality Data.—With information obtained from many sources, supplemented by the experiences and observation of those making the inventory and a study of the policies of the utility with respect to retirements, probable average service lives for the various items in the distribution system were set up as shown in Table 21.3.

A mortality type curve upon which to base the estimated probable service lives of the various items was next selected from information such as that discussed in Chap. III and Appendix B. It was decided to use the S_3 mortality type curve because it appeared to represent

fairly the life characteristics of this property. When this curve is used for the average life shown at the head of each column in Table 21.4, the probable service of the average survivor life will be that shown opposite the service age in the first column.

Table 21.3.—Average Service Lives Adopted in Valuation of the Distribution System—the Ames Electric Utility

Equipment	Average service life, years
Poles:	
Untreated	10
Brush-treated	12
Penetration-treated	17
Wire:	
Bare	15
Triple-braid, weatherproof (T.B.W.P.)	10
Tree	8
Services	10
Meters	20
Transformers	20
Voltage regulator	12
Underground:	
Cable, lead, in duct	18
Duct in concrete	25
Steel conduit	18
Cable, nonmetallic	12
Parkway cable	15
Crossarms, wood	12
Galvanized steel	17
Insulators, line and strain	15
Hardware, pole	12
Guy equipment	to 1930, 10; 1930 on, 12
Ground equipment	12
Racks, primary, lead insulated, dead-ends	12
Pole-top switches	12
Lightning arresters	8
Transformer cutouts	6
Disconnect fuses	6
Street-light brackets	8
Electroliers	15
Wood brackets	12

21.14. Designation of Accounts.—After the inventoried property was placed on the classified summary sheets, the various accounts were totaled. All accounts made up of property used for commercial distribution were designated by D placed before the account number, street-lighting property by S, jointly used property by J, and rural property by R. The various accounts are totaled on the "Certified valuation sheets."

Table 21.4.—Probable Service Lives for Various Ages and Average Lives—the Ames Electric Utility
(Based on mortality type curve S_3)

Service age, years	Average life, years									
	30	25	20	18	17	15	12	10	8	6
0	30	25	20	18	17	15	12	10	8	6
1	30	25	20	18	17	15	12	10	8	6
2	30	25	20	18	17	15	12	10	8	6
3	30	25	20	18	17	15	12	10	8	6
4	30	25	20	18	17	15	12	10	8	6
5	30	25	20	18	17	15	12	10	8	7
6	30	25	20	18	17	15	12	10	9	7
7	30	25	20	18	17	15	12	11	9	8
8	30	25	20	18	17	15	13	11	10	9
9	30	25	20	18	17	15	13	11	10	9
10	30	25	20	18	17	16	13	12	11	10
11	30	25	20	19	18	16	14	13	12	11
12	30	25	21	19	18	16	14	13	13	
13	30	25	21	19	18	17	15	14	14	
14	30	25	21	20	19	17	16	15	14	
15	30	26	21	20	19	18	16	16	15	
16	31	26	22	20	20	19	17	17		
17	31	26	22	21	20	19	18	18		
18	31	27	23	22	21	20	19	18		
19	31	27	23	22	22	21	20	19		
20	31	27	24	23	22	21	21			
21	32	28	25	24	23	22	22			
22	32	28	25	24	24	23	22			
23	32	29	26	25	25	24	23			
24	33	29	27	26	26	25				
25	33	30	27	27	26	26				
26	34	31	28	28	27	27				
27	34	31	29	28	28	28				
28	35	32	30	29	29	28				
29	36	33	31	30	30					
30	36	33	32	31	31					
31	37	34	33	32	32					
32	37	35	33	33	32					
33	38	36	34	34						
34	38	37	35	34						
35	39	37	36							
36	40	38	37							
37	42	39	38							
38	42	40	38							
39	42	41								
40	43	42								
41	44	42								
42	45	43								
43	46	44								
44	47	45								
45	47	46								
46	48	47								

21.15. Jointly Used Equipment.—The value of jointly used equipment was apportioned to the several types of service on the basis of the relative weights of wire or cable used for each class of service; similarly for jointly used poles, towers, and underground conduit.

The Determination of the True Fair Value and the Accrued and Annual Depreciation of the Property

The law of the land on the determination of a fair value of public-utility properties is that *every* factor affecting value must be taken into account and given such weight as may be just and right in each case. In particular, the United States Supreme Court has enumerated original cost, present (reproduction) cost, earnings, and stock-and-bond value as among the factors requiring just consideration.

In the case of public utilities, however, earning value is not entitled to any weight, when, as is true of this property, the rates required to yield a fair return on the fair value are not higher than the services are reasonably worth.

This property has no stocks or bonds outstanding, hence it does not have a stock-and-bond value.

The fair value of this utility must be determined mainly by consideration of its original cost and its reproduction cost.

21.16. Overhead Costs.—In determining the fair value of the physical property, $12\frac{1}{2}$ percent for overhead costs has been added to both the original and the reproduction costs new of all fixed-capital physical property except land.

The manager's annual financial statements for the fiscal years ending March 31, 1929, 1930, 1931, and 1932, respectively, show that the recorded overhead costs of the property have averaged 9.8 percent for the four years and 9.7 percent for the years 1931 and 1932 (of costs exclusive of overheads). (See Table 11.5.)

In addition, there should be an overhead allowance for the interest lost during an average period of about five months between the dates on which construction costs were paid and the later dates when earnings by the property constructed were received.

A very small overhead allowance for *contingencies* has also been made, which in this case is an allowance for omissions. The smallness of this allowance is justified by the painstaking care exercised in making the inventory and in determining the original and reproduction costs.

Note.—Overhead cost allowances of 15 percent or more are common in valuing similar utility properties.

21.17. Determining the Actual Accrued Depreciations.—In this valuation, the true actual depreciation of each unit has been deter-

mined: first, by making an examination and noting its age, its present physical condition for its age, its past and probable future service conditions, and, when necessary, its probable future operation-return ratio; second, by the aid of these data and other information and, with the aid of type mortality curves, carefully estimating the unit's expectancy, which, added to its age, gives its probable service life; third, by the use of condition-percent tables (Appendix C), determining the fair present value of the unit, subtracting which from its value new gives its accrued depreciation. The 6 percent condition-percent tables were used because, for this property, they give the present values in accordance with the actual-present-worth-of-probable-future-operation-returns method.

21.18. Fair Cost-value of the Fixed-capital Physical Property.—In determining the "present market values" of the respective tracts of land used by the utility the principles of Chap. XVI were applied with the assistance of competent realtors who are familiar with land values in the area of the city in which this property lies.

In estimating the reproduction cost of physical properties, it has been quite customary in the past to use unit prices equal to the average prices during the preceding 3, 5, or even 10 past years. The valuators are of the opinion that such averages are not entitled to much weight at the valuation date (1932) in making "an honest and intelligent forecast of future values." This is because of the violent drop in prices since 1929, and the then uncertainty as to the prices of the next future three to five years. For this reason, the reproduction-cost unit prices used in this valuation are based on the average prices prevailing in 1932. The resulting reproduction costs are designated "1932 reproduction costs" and must not be confused with the materially higher reproduction costs which would have resulted from using the average prices of the three to five years just preceding.

The differences between the "original-cost values" and the "1932 reproduction-cost values" of the various property groups of this utility's depreciable physical property are relatively very small; so small as to be less than the unavoidable margin of uncertainty in all valuations. With the exception of the power-plant building (much of which was built before the World War), the 1932 reproduction costs are practically all a little lower than the original costs actually paid by the utility for the various property units. However, an average of prices for the three to five immediately preceding years would have given costs higher than the original cost of most, if not all, of the several property groups.

It is impossible at date of valuation (Dec. 31, 1932) to be certain what the trend of construction costs will be during the next three to five years. Judging from the history of similar business depressions in the past, the existing severe business depression will be succeeded

ultimately by a material up-trend, but unfortunately this is not assured; and it is even possible that any future rise may be preceded by a drop below the present prices. Moreover, present construction costs are still nearly 60 percent above the pre-war level, although average commodity prices are about 18 percent below pre-war prices; and farm products prices are about 50 percent below.

Taking all these considerations into account, the present fair physical value of the fixed-capital depreciable physical property of this utility was fixed at a sum equal to

1. The 1932 reproduction cost, less accrued actual depreciation, of the power-plant building; plus
2. The original cost, less accrued depreciation, of all other fixed-capital depreciable physical property.

21.19. Working Capital.—The working capital of this utility consists of three items, as follows:

1. *Stock, Materials, and Stored Supplies.*—These are mostly located in three storage buildings, but limited amounts are kept stored in other plant buildings. All stock, materials, and stored supplies, except fuel and certain current supplies which fluctuate from day to day, have been carefully inventoried and valued at their actual costs to the city, less the accrued depreciation on the transformers and meters kept in stock.

The manager stated that the amounts of stock, materials, and stored supplies on hand on the dates when inventoried correctly represent the average amounts kept on hand during the year. Accordingly, the stock, materials, and stored supplies were valued at $25,151.

2. *Fuel and Current Supplies.*—The average value of fuel and current supplies kept on hand was stated to be $1,000.

3. *Average Working Cash Balance Necessary.*—Partly in order to secure all cash discounts, the utility is accustomed to paying promptly for all equipment, materials, repair parts, supplies, and fuel required, either for operation or for renewals; and the labor rolls were paid twice monthly. On the other hand, customers were not billed until the end of the month and had until the tenth of the following month to secure their discounts, of which discounts apparently most of them take advantage.

The average monthly total operation cost expenditures had been $10,541, $10,666, $9,506, $8,927, respectively, during the fiscal years ending March 31, for the years 1929, 1930, 1931, 1932. Taking into account monthly fluctuations and possible contingencies, it was deemed wise for the utility to maintain an average working cash balance about equal to two months' average operation expenditures. Accordingly, in the valuation of the working capital, $20,000 was included for the working cash balance necessary.

21.20. Preliminary-expense Value.—The property of this utility has been built up gradually, over a period of 36 years, during all of which time the utility had been incurring some general expenses not represented by actual physical property. Some of this expense was incurred preliminary to the construction of the original property in 1896, and some of it was made necessary by the yearly enlargements since that date. The amount of such expenses is not of record.

It is not uncommon for valuators to allow, and courts to approve, preliminary expense values of 2 percent to 2½ percent of the present value of the physical property in valuations of public-utility properties. In this case the preliminary expense was fixed at $13,000, on the basis of the considered judgment of the valuators.

21.21. Going Value.—This utility also has a going value of considerable amount, owing to the fact that it has a well-established and profitable income (Chap. XII). It seems reasonably certain that the prospective income earned by this property during the few years subsequent to the valuation will be sufficient to yield an annual net return of 8 percent (about $64,000) on the fair value of the property, in addition to paying the annual operation costs (at the date of valuation, about $106,000) and the annual depreciation (at the date of valuation, about $51,000).

If this income had not already been developed, the utility would have been fortunate indeed to be able to secure enough business the first year following the date of valuation to pay merely the operation costs and depreciation during the year, leaving nothing at all for net return. Hence, the going value of the established income to the end of the first year following the valuation would be the $64,000 net return it would yield above operation costs and depreciation. The present worth of $64,000 receivable in one year is $59,259 (at 8 percent fair rate of return).

The utility would have been fortunate to be able to secure enough additional business during the second year after the date of valuation to pay the operation costs and depreciation during the year and, in addition, yield a net return of $50,000; $14,000 short of the $64,000 net return yielded by the actual income at the date of valuation. Hence the going value of the established income during the second year after the date of valuation would be $14,000 at the end of the second year; the present worth of which at 8 percent, compounded for two years, is $12,003.

Hence, even if the utility could secure enough business the third year after the date of valuation to yield the full $64,000 net return it was earning when valued, the going value of the established income would be $59,259 + $12,003 = $71,262.

The going value of this utility property was fixed at $70,000.

NOTE.—Many public-utility valuations have been approved by the courts in which going values of about 10 percent of the present physical value of the fixed-capital physical property have been allowed. In the case of this property, this rule would give $68,000 going value.

21.22. Summary.—The fair value determined for this property is summarized in Table 21.5.

Table 21.5.—Valuation of the Ames Electric Utility—Grand Summary of the Value of the Entire Property
Inventory sections 1, 2, 3, 4, 5, 6

Valuation group	Total cost new			Present value, Oct. 1, 1932			Depreciation	
	Original cost	1932 reproduction cost	Fair value	Original cost	1932 reproduction cost	Fair value	1932	1933
I. Land	$ 16,650	$ 19,500	$ 19,500	$ 16,650	$ 19,500	$ 19,500		
II. Power plant	495,980	500,714	520,386	400,336	404,548	419,104	$31,641	$27,051
III. Distribution system	311,291	282,185	311,291	215,729	197,095	215,729	18,925	20,395
IV. General structures and equipment	38,017	33,380	38,017	28,052	25,358	28,052	3,272	3,083
Total physical value	$861,938	$835,779	$889,194	$660,767	$646,501	$682,385	$53,838	$50,529
V. Working capital						46,151	207	270
VI. Preliminary-expense value						13,000		
VII. Going-concern value						70,000		
Total present fair value						$811,536	$54,045	$50,799

Table 21.6.—Valuation of the Ames Electric Utility—Grand Summary of the Value of the Physical Property by Property Groups

Nat'l Assoc. Railway and Utility Commissioners' standard accounts	Condition-percent	Total cost new			Present value, Oct. 1, 1932			Depreciation	
		Original cost	1932 reproduction cost	Fair value	Original cost	1932 reproduction cost	Fair value	1932	1933
No. 311 Land	100.0	$ 16,650	$ 19,500	$ 19,500	$ 16,650	$ 19,500	$ 19,500		
No. 312 Structures	83.8	118,701	137,696	143,107	101,096	115,512	119,864	$ 7,804	$ 4,112
No. 313 Boiler-plant equipment	73.2	131,750	122,300	131,750	96,401	89,474	96,401	7,274	7,126
No. 315 Turbogenerator units	83.9	213,139	207,198	213,139	178,810	174,864	178,810	13,633	14,456
No. 316 Electric plant	78.7	45,638	44,650	45,638	35,899	35,054	35,899	3,345	1,797
No. 317 Miscellaneous power-plant equipment	65.4	720	671	720	471	440	471	50	53
No. 328 Substation equipment	94.1	6,662	6,640	6,662	6,269	6,248	6,269	395	419
No. 330 Underground conduit	98.2	11,379	10,887	11,379	11,170	10,699	11,170	144	265
No. 331 Poles, towers, and fixtures	69.9	62,648	57,800	62,648	43,803	40,558	43,803	3,774	4,066
No. 332 Overhead conductors	63.1	73,577	61,324	73,577	46,427	38,823	46,427	6,477	6,789
No. 333 Underground conductors	97.7	10,992	10,444	10,992	10,739	10,225	10,739	170	252
No. 335 Services	64.8	23,307	20,458	23,307	15,113	13,287	15,113	2,195	2,237
No. 336 Transformers, excl. of stock	68.7	39,533	36,787	39,533	27,177	25,617	27,177	1,721	1,999
No. 338 Consumer meters, excl. of stock	61.0	53,314	50,649	53,314	32,518	31,314	32,518	2,340	2,560
No. 342 Street-lighting equipment	75.3	29,879	27,196	29,879	22,513	20,324	22,513	1,709	1,808
No. 344 General equipment	65.3	24,049	21,579	24,049	15,711	14,562	15,711	2,807	2,590
Total physical value	76.7	$861,938	$835,779	$889,194	$660,767	$646,501	$682,385	$53,838	$50,529

Table 21.7.—Valuation of the Ames Electric Utility—Summary of the Value of the Power Plant
Inventory section 1

Nat'l Assoc. Railway and Utility Commissioners' standard accounts		Total cost new			Present value, Oct. 1, 1932			Depreciation	
		Original cost	1932 depreciation cost	Fair value	Original cost	1932 depreciation cost	Fair value	1932	1933
No. 312a	Power-plant structures	$104,733	$125,895	$129,139	$ 88,755	$104,716	$107,523	$ 7,339	$ 3,619
No. 313	Boiler-plant equipment	131,750	122,300	131,750	96,401	89,474	96,401	7,274	7,126
No. 315	Turbogenerator units	213,139	207,198	213,139	178,810	174,864	178,810	13,633	14,456
No. 316	Electric plant (switchboard)	45,638	44,650	45,638	35,899	35,054	35,899	3,345	1,797
No. 317	Miscellaneous power-plant equipment	720	671	720	471	440	471	50	53
Total power plant		$495,980	$500,714	$520,386	$400,336	$404,548	$419,104	$31,641	$27,051

TABLE 21.8.—VALUATION OF THE AMES ELECTRIC UTILITY—VALUE OF THE LAND
Inventory section 1

Nat'l Assoc. Railway and Utility Commissioners' standard account 311	Year bought	Cost	Value, 1932	Percent		Cost based on		1932 value based on	
				Owned	Used	Ownership	Use	Ownership	Use
Standard account 311a, Power-plant land:									
Lots 1 and 2, Block 40									
50′ × 114.5′ and 50′2″ × 106′	1894	$ 250	$ 7,000	100.0	100.0	$ 250	$ 250	$ 7,000	$ 7,000
Lot 3, Block 40									
50′ × 50′	1929	4,500	3,000	100.0	100.0	4,500	4,500	3,000	3,000
Lots 4 and 5, Block 39									
128′ × 180′ and 105.5′ × 181.4′	1915*	5,400	4,000	100.0	79.8	5,400	4,309	4,000	3,192
Lot 6, Block 39									
60′ × 180′	1913†	2,000	2,000	100.0	96.7	2,000	1,934	2,000	1,934
Lot 7, Block 39									
60′ × 180′	1903‡	350	2,500	0.0	46.5	....	163	....	1,163
Total power-plant land	.....	$12,500	$18,500	.....	.....	$12,150	$11,156	$16,000	$16,289
Standard Account 311g-1 Storage land:									
Lots 1 and 2, Block 39									
180′ × 60′, each	1924¶	$ 4,500	$ 3,500	100.0	0.0	$ 4,500		$ 3,500	
Unplatted lot, east of Block 39	1927*‡	9,500	9,500	0.0	35.0		$ 3,325		$ 3,325
Total storage land	.....	$14,000	$13,000	.....	.....	$ 4,500	$ 3,325	$ 3,500	$ 3,325
Total land	.....	$26,500	$31,500	.....	.....	$16,650	$14,481	$19,500	$19,614

* Used by all departments.
† Partially used by water department.
‡ Owned by water department.
¶ Used exclusively by water department.

TABLE 21.9.—VALUATION OF THE AMES ELECTRIC UTILITY—SUMMARY OF THE VALUE OF THE ENTIRE DISTRIBUTION SYSTEM
Inventory sections 2, 3, 4, 5, 6

Subvaluation groups	Total cost new			Present value, Oct. 1, 1932			Depreciation	
	Original cost	1932 reproduction cost	Fair value	Original cost	1932 reproduction cost	Fair value	1932	1933
Commercial distribution	$241,735	$219,474	$241,735	$166,149	$152,239	$166,149	$14,667	$15,766
Street lighting	52,137	46,669	52,137	37,563	33,792	37,563	3,238	3,514
Total lines in city	$293,872	$266,142	$293,872	$203,712	$186,031	$203,712	$17,905	$19,280
Rural lines	17,419	16,042	17,419	12,017	11,064	12,017	1,020	1,115
Total distribution system	$311,291	$282,185	$311,291	$215,729	$197,095	$215,729	$18,925	$20,395

Summary of Entire Commercial Distribution System

Nat'l. Assoc. Railway and Utility Commissioners' standard accounts:								
No. D328 Substation equipment	$ 6,662	$ 6,640	$ 6,662	$ 6,269	$ 6,248	$ 6,269	$ 395	$ 419
No. D330 Underground conduits	7,089	6,608	7,089	6,886	6,426	6,886	138	181
No. J330 Jointly used underground conduits	3,544	3,544	3,544	3,544	3,544	3,544	......	67
No. J331 Jointly used poles, towers, and fixtures	18,129	16,302	18,129	13,157	11,758	13,157	1,121	1,137
No. D331 Poles, towers, and fixtures	26,174	24,726	26,174	18,534	17,592	18,534	1,562	1,700
No. D332 Overhead conductors	60,049	50,009	60,049	37,515	31,317	31,515	5,314	5,560
No. D333 Underground conduits	10,061	9,525	10,061	9,818	9,316	9,818	155	209
No. D335 Services	22,566	19,811	22,566	14,553	12,799	14,553	2,126	2,167
No. D336 Transformers	35,459	32,898	35,459	24,231	22,763	24,231	1,572	1,828
No. D338 City consumer meters	52,002	49,411	52,002	31,642	30,476	31,642	2,284	2,498
Total commercial distribution	$241,735	$219,474	$241,735	$166,149	$152,239	$166,149	$14,667	$15,766

TABLE 21.10.—VALUATION OF THE AMES ELECTRIC UTILITY—SUMMARY OF THE VALUE OF THE PHYSICAL PROPERTY IN DISTRIBUTION SYSTEM BY PROPERTY GROUPS

Nat'l. Assoc. Railway and Utility Commissioners' standard accounts		Condition-percent	Total cost new			Present value, Oct. 1, 1932			Depreciation	
			Original cost	1932 reproduction cost	Fair value	Original cost	1932 reproduction cost	Fair value	1932	1933
No. 328	Substation equipment	94.1	$ 6,662	$ 6,640	$ 6,662	$ 6,269	$ 6,248	$ 6,269	$ 395	$ 419
No. 330	Underground conduits	98.2	11,379	10,887	11,379	11,170	10,699	11,170	144	265
No. 331	Poles, towers, and fixtures	69.9	62,648	57,800	62,648	43,803	40,558	43,803	3,774	4,066
No. 332	Overhead conductors	63.1	73,577	61,324	73,577	46,427	38,823	46,427	6,477	6,789
No. 333	Underground conductors	97.7	10,992	10,444	10,992	10,739	10,225	10,739	170	252
No. 335	Services	64.8	23,307	20,458	23,307	15,113	13,287	15,113	2,195	2,237
No. 336	Transformers, excl. of stock	68.7	39,533	36,787	39,533	27,177	25,617	27,177	1,721	1,999
No. 338	Consumer meters, excl. of stock	61.0	53,314	50,649	53,314	32,518	31,314	32,518	2,340	2,560
No. 342	Street-lighting equipment	75.3	29,879	27,196	29,879	22,513	20,324	22,513	1,709	1,808
Total physical value, distribution system		69.3	$311,291	$282,185	$311,291	$215,729	$197,095	$215,729	$18,925	$20,395

The values determined for the physical property are shown by valuation groups in Table 21.6; along with the values of the depreciations estimated for 1932 and 1933.

The valuation summary for the power plant is shown in Table 21.7 and for land in Table 21.8.

The valuation summary for the distribution system by valuation groups and of the commercial distribution system by accounts is shown in Table 21.9.

The valuation summary for the entire distribution system by accounts is shown in Table 21.10.

APPENDIX A

TABLE A.1.—AVERAGE SERVICE LIFE OF INDUSTRIAL-PROPERTY UNITS

This table has been compiled from data obtained from many sources and represents the practice of valuation engineers, regulatory commissions, and similar authorities, rather than the actual average service life as determined from mortality data on the kinds of property listed. Doubtless, the average life given is in many cases based on actual mortality data, although that fact cannot be ascertained. In a few cases the average life for a kind of property is known to be based on actual mortality data, and the unit is marked with an asterisk. The average life is given in years.

The arrangement of the table is as follows:

When several values for average life are found to have been used, and there is no indication that any one particular value has been used more frequently than another, the lowest and the highest figures are given thus: 25–35.

When several values have been used, but some one value has been used more generally than the others, the highest and the lowest are shown in parentheses and the most widely used value, or values, are given in the average-life column.

A study of published average-life tables indicates that they have been copied one from another to a considerable extent and in many cases the actual origin of the estimate of average life is impossible to ascertain. This table has been compiled with care, and no average-life data have been included that appeared to be applicable only to units of property subjected to peculiar or abnormal service. Nevertheless, the average lives reported herein should be considered as no more than aids to judgment in cases where actual mortality data on the specific property cannot be obtained.

Useful average-life tables will be found in the following publications, which have been consulted in preparing this manuscript.

WINFREY, ROBLEY, and EDWIN B. KURTZ: Life Characteristics of Physical Property, *Iowa Eng. Exp. Sta., Bull.* 103, Ames, Ia.

DAVIDSON, J. B.: Life, Service, and Cost of Service of Farm Machinery, *Iowa Eng. Exp. Sta.*, Ames, Ia.

Depreciation Studies, *Prelim. Rept. Bur. Internal Revenue,* U.S. Treasury, Washington, D.C., 1931.

PROUTY, COLLINS, and PROUTY: "Appraisers and Assessors Manual," McGraw-Hill Book Company, Inc., New York.

GRUNSKY, CARL EWALD: "Valuation, Depreciation and the Rate Base," John Wiley & Sons, Inc., New York.

McKAY, CHARLES W.: "Valuing Industrial Properties," Industrial Extension Institute, Inc., New York.

"Valuation of Elevated Railroads," Chicago Subway Commission, Chicago, Ill., 1916.

"Appraisal, Rate, and Service Investigation of the Suburban Water Company." Woodland, Pierce Co., Washington, 1931.

TURNEAURE, F. E.: "Valuation of the Davenport, Ia., Water Company."

Valuation of Street and Interurban Railway Lines in the City of Los Angeles, Calif., 1914.

MOULTON, J. S.: Report on Estimated Service Lives and Corresponding Sinking Fund Annuities for the Electric Property of the San Joaquin Light and Power Corporation, 1932.

NOTE. Additional average-life data will be found in Appendix B, especially numbers 66 to 176, Table B-1.

CODE OF REFERENCES

The following symbols have been used in the table to indicate the source of the average-life data, or the person to whom the data are usually credited.

* indicates that this figure for average life is known to have been secured from actual mortality data.

† indicates values of average life based on mortality studies made by the San Joaquin Light and Power Corporation and reported in 1931.

b	Pacific Gas and Electric Co. data used in rate hearings in 1913 to 1914
c	St. Louis Public Service Commission
d	Wisconsin Railroad Commission data from various published reports
e	Data from "Appraisers and Assessors Manual" by Prouty, Collins, and Prouty. Used by permission of the publishers
f	Data from Depreciation Studies, *Prelim. Rept. Bur. Internal Revenue*, U.S. Treasury, Washington, D.C.
g	John W. Alvord
h	California Railroad Commission
i	Chicago Telephone Company
k	Chicago Union Traction Company
l	Chicago Traction Commission
m	Henry Floy
n	H. A. Foster
o	H. P. Gillette
p	Robert Hammond
q	W. Kiersted
r	Leonard Metcalf
s	Traction Valuation Commission data employed in the case of the Chicago Consolidated Traction Co.
t	Stone and Webster
u	Telephone data
v	San Joaquin Light and Power Corporation data used in rate hearing 1915. Items indicated + are from mortality studies reported in 1931
w	T. C. Parsons
x	Sir William Preece
y	W. H. Rosecrans
z	Milton G. Starrett
aa	Board of Arbitration, Atlanta, Ga., rate case, 1899
ab	Data summarized by authors from various reliable sources
ac	Bion J. Arnold
ad	T. R. Agg. Data from personal files
ae	William H. Bryan
af	Charles B. Burdick
ah	B. L. Dodge
ai	Milwaukee Electric Railway and Light Co. data, presented in the 3-cent fare case in 1912
aj	W. G. Wilgus data from appraisal of Lehigh Valley Railroad, 1914
ak	Benzette Williams
al	Iowa State College. Data from *Iowa Eng. Exp. Sta., Bull.* 103
am	American Appraisal Company. Data from various appraisals
an	Data from Chicago and Northwestern Railway construction reports
ao	George Craven
ap	William Arthur
aq	R. E. Belt
ar	Prentis-Hall
as	F. E. Turneaure. Data from valuation, Davenport, Ia., Waterworks
at	Mortimer E. Cooley
au	J. B. Davidson. Data from *Iowa Eng. Exp. Sta., Bull.* 92
av	Nicholson & Rohrbach
aw	Portland Cement Industry
ax	E. G. Connette
ay	Associated General Contractors, Equipment Ownership Expense, 1930
az	American Foundrymen's Association
bb	C. E. Grunsky
cc	C. W. McKay
dd	*Coal Age*
ee	W. B. Read
gg	Montgomery
hh	Bay State Rate Case
kk	Bureau of Census
ll	Charles Piez

Item	Average service life	References	Item	Average service life	References
Accumulators, hydraulic	15	*p,w,x*	Barrows, cupola charging	5	*f*
hydraulic	15–20	*e*	Batchbox, steel	5	*ay*
Adding machines	10	*f,d*	wood	4	*ay*
Addressing and mailing machines, publishers	15	*f*	Batcher plants, all steel	5	*ay*
Addressographs	10	*f,d*	wood bin	4	*ay*
Agitator pipes, connections, and air-cooling coils	15	*f,d*	Batchers, see kinds	4	*ay*
Agitators, ice manufacturing, metal products	15	*f,d*	Batteries, storage	10	*ai,d*
wooden, dairy	12	*f,d*	storage	12½	*b*
Air brakes	20	*f,d*	storage	15	*d,k*
Air compressors (25–12½)	20	*b,s*	storage	20	*t,c,m*
Air hammers	12	*e*	storage, airplane and automobile	2	*f*
Airplanes	5	*e*	storage, plant installations	10–20	*ao*
Ammeters, recording	15	*f,d*	Belting, in general	8	*b*
Ammonia concentrators and tanks	15	*s,d*	in general	10–20	*d*
Annealing boxes	8	*f*	canvas, rubber	8–20	*e*
Apartments and flats, building only, concrete	40	*f*	leather	18–25	*e*
Apartments, frame	25	*f*	leather	20–25	*n*
masonry with frame interior	30	*f*	Benches, gas plant	25	*d*
steel and stucco (fireproof) masonry, slow burning, with or without steel frame	35	*f*	power plant, shop, etc.	20–25	*f*
Arc lamps (15–6⅔)	15	*d*	rolls, for metals	15–22	*e*
Ash- and coal-handling equipment	13–20	*f,d*	shop	10	*f*
Ash-pipe vacuum system	3	*e*	Benders, bar reinforcing	5	*f,ay*
Asphalt plant accessories, asphalt pump	4	*ay*	Bending machines, angles, pipe, etc.	10–15	*f*
dust collector	4	*ay*	Billing machines, office	10–20	*f*
asphalt and oil storage tanks	7	*ay*	Binders, corn	12	*f*
heaters for tools	6	*ay*	* grain	12–16	*au*
jacketed steam pipe	3	*ay*	* grain	14	*al*
kettles, heating	6	*ay*	loose-leaf	20	*f*
superheaters	5	*ay*	Bituminous plants, see asphalt plants		
Asphalt plants, railroad type	10	*ay*	Bituminous distributors with or without truck	4	*ay*
portable	7	*ay*	Blacksmith shop equipment	15	*f*
Aspirators, food manufacturing	15	*f,d*	portable	4	*f*
Augurs, clay-working	14	*f*	Blowers and fans in general power-plant use	15	*f*
Autoclaves, clay-working	10	*f*	Blowers, gas plant	15	*d*
rubber manufacturing	10	*f*	gas plant	20	*b*
Automobiles (10–4)	5	*ab*	gas plant	18	*e*
* (retired in 1926)	8	*al*	mine, textile mill, quarry	15	*f*
light	2	*ay*	soot	65	*e*
medium	3	*ay*	ventilating	30	*e*
heavy	4	*ay*	Blueprinting machines	15	*f*
Awnings, duck	5	*e*	Boiler, steam, construction equipment upright	13–14	*f,ay*
Backfillers, power	3–6	*e*	steam, farm	15	*f*
power	4	*ay*	steam, fire tube (30–10)	15	*ab*
Balers, paper, rags, leather	10–20	*e*	steam, fittings only	18	*f*
Ball mills	12	*e*	steam, locomotive type, portable	10	*f,ay*
Barrels, tumbling	13–15	*f*	steam, oil and gas drill rigs, bad water	2	*f*
			steam, oil and gas drill rigs, good water	5	*f*

Item	Average service life	References
Boiler, steam, plant equipment	20–25	*v*
steam, water tube (33–20)	20	*ab*
Boilers, cast iron, heating	35	*e*
hot water	35	*e*
in general (30,10,15)	20	*ab*
waste heat	20	*e*
water tube	25–30	*as*
* waterworks	14	*al*
Bookkeeping machines	6	*f*
Boosters, gas line	18	*e*
Boring mills	30	*e*
Bottle machines	10	*f*
Breeching and flues	22	*f*
Brick-making machines	12	*f*
Bridges, concrete or masonry	50–75	*f*
skip, foundry	20	*f*
steel (75–40)	50	*ab*
wood	10–28	*ap*
Buckets, cableway, clam shell, orange peel	6	*ay*
concrete	5	*ay*
Buggies, brick yard	12	*f*
Buildings, apartments, factories, foundries, private garages, grain elevators, loft buildings, power stations, stores	25	*f*
Buildings, all kinds	40–50	*o,r*
apartments, frame	25–50	*ar*
brick	30	*ah*
brick	30–50	*b*
brick	35	*af*
brick	40–50	*ar*
brick	60	*p*
brick	67	*s*
brick	80	*x*
gas retort	30	*d*
standard steel construction outfits for	3	*ay*
with slate roof	27–40	*am*
with tar and gravel roof	40–75	*am*
with tile roof	50–75	*am*
car barns and shops, dwellings, foundries, private garages, grain elevators, power stations, roundhouses, stores	40	*f*
car barns and shops, dwellings, grain elevators, power stations	33	*f*
car barns and shops, roundhouses, stores	28	*f*
car barns, shops, warehouses	50	*f*
concrete	50–66	*d*
transformer building	18	*am*
dwellings	35	*d*
Buildings, 1-family	33–40	*f*
1-family	50	*f*
2-, 3-, or 4-family	30	*f*
factories	40	*f*
factories, machine shops	28	*f*
factories, frame	20	*f*
fireproof	40	*i*
foundations only	100	*x*
foundries	50	*f*
garages, public or private	50	*f*
† general office	40	*v*
† general shops	40	*v*
† main substation	24–30	*v*
grain elevators	50	*f*
hotel, brick and stone, wood interior	30–40	*ar*
hotel, fireproof throughout	70	*ar*
hotel and apartments	35	*f*
hotel and elevator apartments	25	*f*
hotel and elevator apartments	30	*f*
in general	25–50	*d*
in general	50	*c,ax*
ironclad steel frame	40–50	*am*
iron covered	20–36	*b*
machine shops	33	*f*
machine shops	40	*f*
masonry, brick, concrete, brick and steel, steel and stucco (fireproof) apartments	40	*f*
masonry, slow burning, with or without steel frame, apartments, factories, public garages, lofts, mill type, theaters	35	*f*
masonry with frame interior, apartments, factories, public garages, mill-type machine shops, office, store, and apartment	30	*f*
mill type	40	*f*
office	40	*f*
power stations	50	*f*
public garages, mill-type buildings, theaters	20	*f*
lofts	45	*f*
railroad	33	*o*
railroad station	10	*h*
reinforced concrete	50	*b*
reinforced concrete	50–100	*ar*
residence of brick veneer, brick and stucco on wood	50–60	*ar*
residences, good frame construction	45–55	*ar*

Item	Average service life	References	Item	Average service life	References
Buildings, residences of brick, stone, or stucco	50–70	*ar*	Cables, mains, solid	30	*p*
residences, poor frame construction	30–40	*ar*	telephone	33	*e*
round houses	50	*f*	Cableways	9	*aw*
second-class shops, etc.	50	*d*	Can washers, milk	15	*e*
sheds, open frame	30	*am*	Cars, all kinds, wood or steel	20	*f*
shop and car houses, railroad stations, concrete, steel, power houses, substations, all of brick	66	*l*	batch for ceramic work	12	*f*
solid pressed brick	35–70	*am*	batch for concrete construction	5	*ay*
stone and brick, first class	75	*d*	* box	28	*al*
stores	35–50	*f*	box	30	*e*
street railway	35–75	*d*	* coal	19	*al*
street railway	50	*k,z,at,o*	concrete hopper	8	*ay*
street railway (75–33)	50	*ab*	composite, motor or trailer	45	*l*
street-railway power plants	20	*k*	dump, steel	8	*ay*
street-railway power plants	25	*at*	dump, wood	6	*ay*
street-railway power plants	50	*t*	electric street railway	12	*at*
street-railway power plants	60	*d*	electric street railway	15–20	*d*
stucco on frame	30–55	*ar*	electric street railway (30–12)	20–30	*ab*
stucco on tile	35–70	*ar*	electric street railway	17	*z*
substations	50	*x,h*	electric street railway	30	*ac*
telephone and telegraph	30–50	*ab*	* flat	26	*al*
theaters	20	*ab*	flat construction, service, n.g. and s.g.	6	*ay*
theaters	22	*f*	freight	30–50	*aj*
theaters	25	*f*	* freight, all kinds	20	*al*
theaters	33	*f*	freight, steel (50–18)	30	*ab*
tile with tile roof	50	*f*	gondola	20	*e*
2-, 3-, or 4-family res.	45	*f*	hopper	20	*e*
warehouses	35	*f*	mine, steel	10	*e*
warehouses	40–50	*f*	mine, wood	4–5	*e*
warehouses	45	*f*	* passenger	33	*al*
wood frame (50–20)	50	*ab*	passenger	35–40	*aj*
Bulldozer	5	*ay*	passenger, dining, sleeping	25–30	*ax*
			Pullman	25	*e*
*Cables	10	*al*	refrigerator	15	*e*
* aerial	15	*al*	* Rodgers ballast	20	*al*
aerial (15–10)	15	*d,n*	spreader, n.g. and s.g.	10	*ay*
aerial exchange	12–15	*u*	steel, motor or trailer	50	*l*
aerial, lead covered (15–10)	10	*n*	* stock	19	*al*
aerial, terminal (12–10)	12	*c,u*	street railway, bodies	15	*d*
and feeders	15–25	*d*	street railway, bodies and trucks	13	*ai*
and feeders	20	*ai*	street railway, bodies and trucks	20	*k,t*
armored	25	*p*	street railway, closed bodies	20	*s*
armored	33	*e*	street railway, open bodies	25	*s*
armored	35	*x*	street railway, trucks for	15–20	*d*
cableway service	4	*ay*	street railway, trucks for	30	*s,ac*
copper	60	*e*	tank	20	*e*
feeder	25	*e*	wooden, motor or trailer	40	*l*
lead covered, underground (25–10)	20	*ab*	Carts	8	*f*
main, lead covered	20	*b*	steel for concrete	3	*ay*
mains	20	*i*	Cash registers	12	*e*
mains	30	*w*	Cast-iron pipe specials	75	*as*
			Cast-iron water mains	100	*as*
			Chains	20	*e*

Item	Average service life	References
Charging boxes, cinders	3	*f*
steel works	5	*f*
Charging machines, cement mill	10	*f*
foundry	15	*f*
Check writers	8	*f*
Chimneys, brick	14–50	*o*
brick	33	*s,m*
brick	50	*as*
brick, concrete	50	*f*
concrete	75	*e*
Custodis	60	*as*
steel	12	*b*
steel	14	*m*
steel (25–10)	10	*o*
steel lines, self-supporting	28	*f*
unlined, self-supporting	22	*f*
Classifiers, chemical industry	12	*e*
Cleaners, electric vacuum	6	*f*
Clocks, recording	7	*e*
time	15	*f*
time stamping	10	*f*
wall	20	*f*
watchman's	8	*e*
Clutches, jaw	20	*e*
†Commercial lamps, etc	10	*v*
†Communication system equipment	24	*v*
Compressors, air, stationary	20	*b*
air, stationary	20–25	*s*
portable, air	13	*f*
portable, air construction service	6	*ay*
station, natural gas	10	*h*
stationary, all types (28–20)	20	*ab*
stationary, gas	25	*e*
motor-truck unit	5	*ay*
Comptometers	10	*f*
Concrete mixers (8–2)	6.5	*ab*
Concrete spouting accessories, buckets	5	*ay*
chutes	2	*ay*
hoppers, towers	5	*ay*
towers	5	*ay*
Concrete spouting plant, complete	4	*ay*
Condensers, barometric	30	*e*
† distribution	20	*v*
gas plant	30	*f*
gas plant	30	*e*
ice manufacturing	12	*e*
in general	10	*aa*
in general	15	*c*
in general	20	*d,m,ac,b*
in general	25	*v*
in general (25–10)	20	*ab*

Item	Average service life	References
Condensers, steam, atmospheric	25	*f*
surface	20	*f*
surface	25	*e*
Conductors, overhead, distribution system	24	*v*
† transmission line	35	*v*
† transmission overhead	30	*v*
Conduits, cast iron	100	*e*
clay in concrete	50	*l*
concrete and vitrified	50	*f*
duct system	30	*p*
ducts	30	*p*
fiber duct	30	*e,b*
fiber in concrete	20	*u*
fiber or steel	30	*f*
in general (100–50)	50	*ab*
iron	20	*u*
main, concrete	55	*u*
main, fiber or creosoted wood	20	*u*
main, vitrified clay	50	*u*
mains	30	*w*
mains	40	*x*
manholes and paving, service holes	40	*b*
masonry	75	*e*
paper	25	*b*
solid system	30	*p*
solid system in wood	40	*x*
steel, large	50	*e*
steel, small	30	*e*
under tracks	40	*b*
vitreous clay tile	50	*n,b*
vitreous clay tile and fiber	40	*b*
wood stave	33	*e*
wooden	25	*b*
wooden (25–20)	20	*n*
wrought-iron pipe	20	*b*
Controllers, electric	15	*f*
Converters, rotary	18	*f*
rotary	20	*p*
Conveyors and elevators	11	*f*
ash and coal	10	*d,ai*
ash and coal	14	*s,ac*
ash and coal	15	*d,k*
ash and coal	20	*t,m*
ash and coal (20–10)	15–20	*ab*
belt, portable	3	*f,ay*
belt, stationary	6	*f,ay*
bucket, cable, drag	6	*f*
cable monorail	15	*f*
chain, portable	5	*f*
general all types	10	*p*
general all types (26–10)	20	*ab*
in general	20	*e*
scraper	6	*f*
Cooling towers	15	*e*

Item	Average service life	References	Item	Average service life	References
*Corn binder	14	au	Cutters and folders, textile	28	f
*Corn planters	15	au	and sizers, confectioners	15	f
*Corn planters	11	al	bar, power	5	f,ay
*Corn sheller	18	au	bolt	15	f
Cranes, bridge and cantilever	10	f	coal	6	e
crawler, electric, 2½ to 5 tons	5	f	corrugated iron, hand	10	f
crawler, electric, 10 to 15 tons	7	f	for metal, all kinds	17	am
crawler, electric, 20 tons up	9	f	gear	30	e
crawler, gas, 5 tons	4	ay	lens	7	f
crawler, gas, 10 tons	5	ay	meat	12½	f
crawler, gas, 15 tons	6	ay	paper	20	e
crawler, gas, 20 tons	8	ay	paper board	20	f
crawler, gas, 10 to 15 tons	6	f	paper trimmers for print shops	15	f
crawler, gas, 20 tons up	8	f	pipe	25	e
crawler, steam, 5 tons	5	ay	Dams, concrete	50	v
crawler, steam, 10 tons	6	ay	concrete	100	e,h
crawler, steam, 15 tons	8	ay	concrete, diverting	100	h
crawler, steam, 20 tons	9	ay	crib	25	f
electric hoist	18	e	earth	100	h,e
electric traveling	17	f	earth and loose rock	50	v
gantry, jib	15	f	earth and loose rock	75	e
hand hoist	30	e	earth or concrete masonry	75	f
jib	15	f	loose rock	60	f
jib and locomotive	15	f	steel	40	f
locomotive	15	e	Derricks, boat	10	f,ay
locomotive	18	f	circle swing, hand	8	f
locomotive, constructors (15–10)	8–10	ab,ay	crab, hand	16	f,ay
overhead traveling	20	f	crab, power	10	f,ay
power plant, steel in general (50–15)	50	ab	guy steel	12	f,ay
steam or gas, 2½ to 5 tons	4	f	guy wood	8	f,ay
traveling	30	e	pumping, oil well, steel	11	f
wooden	15	b	pumping, oil well, wood	8	f
Crossarms, electric transmission (14–8)	12	ab	scrap breaker	20	f
Crusher plants, portable, without power	5	ay	steel	13	ay
Crushers, rock or stone quarry	20	f	steel	18	e
stone, jaw, general	12	e	steel	20	f
stone, jaw, portable construction equipment	6	f,ay	stiff-leg steel	12	f,ay
stone, jaw, stationary	8	f	stiff-leg wood	8	f,ay
*Cultivators, all types	15	au	wood	6½	f
* corn, one row	13	al	wood	12	e
horse drawn	15	e	Dictographs	15	e
power drawn	10	e	and dictaphones	6	f
Culverts, cast iron	16	h	Dies, metal stamping	4	e
concrete	100–60	ab	metal stamping	6	f
log and timber	17	o	Distribution system, public service	33	e
steel	40–20	f	street railway	12	at
vitrified pipe	38	e	street railway	14	z
wood	18–14	f	street railway	22	e
Cupolas	16	e	street railway including transformers	30	d
Cupolas	20	f	Distributors, see Bituminous distributors		
			Ditches	50	v
			concrete lining	20	v

Item	Average service life	References	Item	Average service life	References
Drag line, crawler, electric	5–8	*ay*	Engines, fire	30	*e*
crawler, gas	4–8	*ay*	gas	10	*f,ay*
Drills, air, tripod	7	*ay*	gas	15	*e,d*
air, drifter or tunnel	5	*ay*	gas	16	*o*
blast hole, all types	7	*ay*	gas	10–15	*n*
Drill boat	12	*ay*	gas and gasoline	14–17	*f*
sharpeners, pneumatic, all			gas or gasoline, well drilling	6–8	*f*
sizes	8	*ay*	* gasoline, stationary	15	*au*
Dredge, clamshell	12	*ay*	hoisting	18	*e*
dipper	8	*ay*	marine	10	*f*
hydraulic	10	*ay*	oil	8	*f*
Dryers, ceramic	11	*f*	oil, well drilling	10	*f*
ceramic, rotary	15	*f*	pumping	14	*f*
†Dwellings, garages, etc.	20–30	*v*	* pumping	31	*al*
Dynamos and alternators	25	*p*	steam	11	*f*
and alternators	30	*x*	steam, Corliss	30	*e*
			steam, high speed	15	*d*
Economizers	10–20	*m*	steam, high speed	15–20	*m*
Economizers	15	*k*	steam, high speed	16	*f*
Economizers	20	*t*	steam, low speed	25	*f*
Economizers (20–10)	15–20	*f*	steam, slide valve	28	*e*
Edgers, planing mill	18	*f*	steam, uniflow	30	*e*
Electric hand tools all kinds	3	*ay*	steam, well drilling	8–10	*f*
Elevators and conveyors, for			turbine, steam (30–20)	20	*ab*
food products	15	*f*	*Ensilage cutter	10	*au*
bucket, chain type	6	*f,ay*	†Equipment, boiler plant	20–25	*v*
bucket for food product	20	*f*	† electrical, steam plant	20–25	*v*
cage, steel tower	5	*f*	† electrical, water power plant	30	*v*
electric, building	20	*e*	† miscellaneous power plant	20	*v*
flour, endless-chain type	15	*f*	Evaporators for tank products,		
freight	25	*f*	meat packing	15	*f*
grain	33	*o*	vacuum	25	*f*
grain, concrete	50	*e*	Excavators, cableway	8	*ay*
hydraulic, building	25	*e*	gasoline, 7 ft. depth	3	*f*
passenger	20	*f*	gasoline, 12 ft. depth	5	*f*
towers, cages and sheaves	5	*ay*	gasoline, 18 ft. depth	7	*f*
wood	35	*e*	in general	10	*e*
Engines, steam	15	*c,k*	steam, 7 ft. depth	5	*f*
steam	15–25	*r*	steam, 12 ft. depth	6	*f*
steam	20	*ai,t,b,m,*	steam, 18 ft. depth	8	*f*
		d,aa	trench	6⅛	*ay*
steam	25	*p*	trench vertical boom	5	*f*
(25–20)	20	*ab*	wheel or ladder type	5	*f*
and machinery, power plant	25	*x*	Exchangers, heat, oil refining	15	*f*
and machinery, power plant	27	*w*	Exciters	20	*f*
and machinery, power plant	10–33	*l*	Exhaust systems	11	*f*
and machinery, power plant	13–33	*ac*	Exhausters, gas	17	*f*
and machinery, power plant	15–20	*d*	gas plant	25	*d*
and machinery, power plant	20–33	*s*	Extinguishers, fire	3	*f*
blowing or haulage, indus-			Extractors, centrifugal	25	*f*
trial plants	12½	*f*	tar	33	*f*
chemical	20	*e*			
Corliss, slow speed	25–30	*n*	Fans, electric desk	10	*f*
Diesel	20	*e*	exhaust	15	*f*
Diesel and semi-Diesel	20	*f*	forced draft	25	*e*
fire	7	*f*	induced draft	15	*e*

Item	Average service life	References	Item	Average service life	References
Fans, mine ventilation	20	*e*	Forms, concrete construction		
ventilating	12	*e*	metal pans	5	*f*
ventilating	13	*az*	steel, for pavements, tunnels	4	*f,ay*
*Feed grinder	15	*au*	steel, for pipe	3	*f*
Feeders, for crushers or conveyors	12	*f*	steel, for walls	5	*f*
Fence machines, wire	10	*f*	Foundations, same as life of article supported	......	*s,m*
Fences, masonry	25	*f*	Frogs and switches	14–15	*f*
railroad	14	*o*	†Fuel gas lines	25	*v*
railroad	15	*h*	†Fuel gas piping in plants	20–25	*v*
snow	12	*e*	Fuel oil equipment	25	*l*
steel posts and wire	15	*f*	Fuel oil handling apparatus	25	*l*
wire	12	*av,b*	†Fuel oil piping, tanks, etc.	20–25	*v*
wire	13	*aq*	Furnaces, annealing	18	*f*
wire	15	*e*	annealing, tunnel type	17	*f*
wood	10	*aq,av*	blast	20	*f*
wood	12	*e,b*	blast	25	*e*
wood and wire	12	*f,b*	continuous heating	17	*f*
Filing machines for wood saws	17	*f*	electric melting	18	*f*
Filter beds, sewage disposition	35	*e*	hardening, drawing, electric	18	*f*
water purification	15–20	*ae*	heating, large	15	*e*
waterworks	30–50	*n*	heating, small	10	*e*
Filters, oil	20	*f*	melting	15	*e*
pressure or vacuum	15	*f*	metal melting, coal fired	10	*f*
presses	40	*e*	metal melting, gas or oil fired	7	*f*
Finishing machines, highway	4	*ay*	metal melting, electric	12½	*f*
Fire alarm and fire prevention equipment for hotels and apartments	20	*f*	open hearth	18	*e*
			open hearth	25	*f*
			open hearth, electric	15	*e*
Fire alarm system for apartments	25	*f*	puddling, reheating, welding	17	*f*
			Furniture and fixtures	12½	*at*
Fire alarms	18	*e*	Furniture and fixtures	20	*z,ai,d*
Fire extinguishers, hand	10	*e*	Furniture and fixtures	25	*e*
Fire hydrants	40–50	*r*	Furniture, dining and guest room, hotel service	12	*f*
Fire hydrants	50	*as*			
Fire hydrants (50–40)	40	*ab*	lobby, hotel service	8	*f*
Fire protection apparatus	12	*b*			
Fixtures, light, hotel and rest.	8	*f*	Gages, pressure	20	*e*
Flotation machines, mines	10	*f*	recording	15	*f*
Flumes, concrete	50	*e,h*	Garage equipment	15	*v*
concrete or masonry	75	*f*	Gas holders	40–50	*r*
steel	22	*e*	Gas holders	50	*d*
steel	25	*h,v*	Gas holders, steel-tank type	45	*e*
steel	40	*f*	Gas producers, blue gas	25	*e*
wood	30	*f,h,e*	carburetted water gas	25	*e*
wooden (50–25)	30	*ab*	coal gas, brick	28	*e*
Folders, rotary, paper	20	*f*	steel plant	20	*e*
†Forebays, penstocks and tailraces	50	*v*	Gates and valves, waterworks	40	*ak*
			waterworks	40–50	*r*
Forges, blacksmiths	20	*f*	Gates, head, on ditches	30	*v*
blacksmiths, portable	10	*f*	†Generating units, turbine	35	*v*
gas or oil burning	10	*f*	Generators, acetylene	12	*e*
Forging and upsetting machines	18	*f*	acetylene gas	14	*f*
			and dynamos, large units	28	*f*
Forming machines, sheet metal	20	*f*	and dynamos, medium units	25	*f*
Forms, adjustable supports	4	*f*	and dynamos, small units	14–25	*f*

Item	Average service life	References	Item	Average service life	References
Generators, electrical	10–20	*bb*	Hemming machines, metal working	15	*f*
electrical	13–33	*s*	Hobbing machines	18	*f*
electrical	16	*aw*	Hoists, air	20	*e*
electrical	20	*v*	air, electric, steam	8	*f*
electrical, belted	10–20	*l*	chain	6	*f*
electrical, direct connected	20	*l*	chain	10	*e*
electric, engine drive	33	*e*	electric	15	*e*
electric, hydraulic drive	40	*e*	electric, monorail	5	*f*
electrical, modern types	20	*d*	gasoline	6	*f*
electrical, obsolete types	15	*d*	gasoline or steam	10	*e,ay*
electrical, steam-turbine drive	30	*e*	hand power	30	*e*
telephone plant	20	*d*	hand power	8	*f,ay*
Gin machinery, cotton	15	*f*	slew, electric	8	*f,ay*
Governors, consumers	25	*d*	slew, steam	12	*f*
engine	25	*e*	steam, in general	15	*e*
gas plant	50	*d*	Holders-on, pneumatic	4	*f*
pump	20	*e*	Hose, air	8	*e*
waterwheel	12	*e*	fire	5	*e*
Graders, road	6	*e*	steam	2	*e*
road, blade	3–6	*f,ay*	Hospital equipment	15	*f*
road, elevating	5	*f,ay*	Hydrants, fire	40	*ak*
rooter, wheel	5	*f*	fire	40–50	*r*
subgrade planes	4	*f,ay*	less than 6 in.	33	*f*
*Grain binder	16	*au*	6 in. and over	50	*f*
*Grain drill	18	*au*	Hydrators	15	*f*
*Grinders, grain and feed	15	*au,f*			
swing, for castings	15	*e*	Instruments, surveying, levels	10	*ay*
stove	30	*e*	†Installations, lighting and power, on customer's premises	10	*v*
Hammers, air	10	*e*			
air	15	*f*			
drop	22	*e*	Jacks, hand	25	*e*
electric portable and pneumatic riveting	3	*f*	hydraulic	8	*f,ay*
			hydraulic	20	*e*
steam	28	*e*	rail	25	*f*
Harness	7	*f*	ratchet	8	*f*
Harrows	14	*f*	screw, or lever	5	*f,ay*
* disc	13	*al*	Jibs, steam	17	*f*
* disc	15	*au*	Jigs	3–10	*cc*
* smoothing	20	*au*	Jigs	10	*f*
* spring booth	8	*au*	Jigs	10	*e*
*Hay loader	20	*au*	Jointers, bench, electric, steam or gas	5	*f*
Head frames, mine, steel	30	*f*			
mine, wood	15	*f*	Jointer, wood	20	*f*
Heaters, asphalt, kettles, etc.	4	*f*	wood	30	*e*
feed water	15–30	*d*			
feed water	16–25	*s*	Key seater	30	*e*
feed water	20	*b*	Kilns, brick manufacturing	20	*e*
feed water	22–25	*f*	cement	18	*e*
feed water	25	*e*	ceramic	15	*f*
feed water	30	*k*	dry, for wood	18–20	*f*
feed water	33	*ac*	foundry	25	*f*
ladle or pit, oil fired	10	*f*	lime	15	*f*
Heating systems, boilers and furnaces	20	*f*	lime	25	*e*
			rotary	15	*f*

Item	Average service life	References
Laboratory equipment, metallurgical	13	*f*
† utility	15	*v*
Ladles, hot metal	7	*f*
hot metal	10	*e*
hot metal	11–12	*f*
Lamps, arc	7	*aa*
arc	12	*x,c*
arc	15	*b,d*
Nernst	8–10	*n*
Lapping machines	18	*f*
Lathes, automatic	14	*f*
bench	15	*f*
engine	16	*f*
metal working	15	*f*
metal working	25	*e*
roll	20	*f*
spinning	18	*f*
steel	10	*aq*
turret	16	*f*
wood working	17	*f*
wood working	20	*f*
wood working	30	*e*
Launches, gasoline	7	*f*
Lifting device for oven doors	15	*f*
Lighting protection	10	*b*
Lighting protection	15	*c*
Lighting protection	15–20	*n*
Lighting system, conduit, fitting, wiring	25	*f*
electric, for home installation	14	*f*
fixtures only	15	*f*
incandescent, street	15	*b*
incandescent, street	20	*h*
Lightning arresters, station type	20	*f*
Loaders, bucket, crawler, portable	5	*f,ay*
bucket, crawler, stationary	6	*f*
coal	20	*f*
rail	15	*f*
Loading machines, brick	10	*f*
Lockers, office, metal	20	*f,e,az*
Locomotives, contractors	10	*e*
contractors, standard gage	11	*f*
electric	16	*av*
electric	20	*e,f*
electric, passenger	25	*f,e*
gas, over 20 tons	10	*f,ay*
gas, 10 to 20 tons	7	*f,ay*
gas, up to 10 tons	4	*f,ay*
gasoline	6	*e*
industrial, electric	9	*f*
mine	10	*dd,ee*
mine	13	*aw*
oil and gasoline driven	17	*f*

Item	Average service life	References
Locomotives, steam	20	*m,d*
steam	20–35	*aj*
steam	25	*ap*
steam (30–10)	25	*ab*
steam or electric, all gages, for industrial service	15	*f*
steam, over 20 tons	10	*f*
steam, 30 to 70 tons	11	*ay*
* steam, railroad main-line service	25	*al*
steam, saddle tank	6–10	*ay*
steam, 10 to 20 tons	8	*f*
steam up to 10 tons	6	*f*
Machine-shop equipment, small	20	*f*
Machinery, in general	27	*w*
in general, including engines	25	*x*
Machines, adding	12	*e*
air compressors	22	*am*
all kinds in general	28	*am*
automatic screw	15	*e*
auxiliary business, in general	22	*am*
bandsaw, wood	30	*e*
belt lacing	20	*e*
bending	15	*e*
bending rolls	30	*e*
boring	30	*e*
broaching	30	*e*
buffing	30	*e*
calculating	10	*e*
cement mixing	8	*e*
clay mixing	20	*e*
cold saw	20	*e*
compressed-air tools	13	*am*
drilling	30	*e*
drying	25	*e*
engraving	20	*e*
flanging	20	*e*
flotation	12	*e*
gear cutting	30	*e*
key seating	30	*e*
lacing, belt	15	*e*
lathes, metal working	25	*e*
lathes, wood working	30	*e*
mechanical hacksaw	25	*e*
milking	15	*e*
pipe cutting	25	*e*
pipe threading	25	*e*
planer, metal	32	*e*
power-driven tools	17	*am*
printing	15	*e.*
printing machinery, in general	27	*am*
refrigerating	20	*e*
screw	20	*e*

Item	Average service life	References	Item	Average service life	References
Machines, sewing	20	*e*	Mills, boring	15	*f*
shapers, metal	25	*e*	boring	30	*e*
shears, metal	20	*e*	ceramic	15	*f*
slotting	25	*e*	tube	12	*e*
thread cutting	18	*e*	Mixers, concrete	10	*f*
welding	12	*e*	concrete, building, electric	5	*ay*
wood saws	30	*e*	concrete, building, gasoline	2–4	*ay*
wood working, in general	30	*e*	concrete, paving	4	*ay*
Magnets, lifting	15	*f*	concrete, agitator	3–4	*ay*
lifting	20	*e*	concrete, truck-mix type	3–4	*ay*
Mains, gas collecting, steel mills	20	*f*	hot metal	20	*f*
Mangles	12	*f*	Molding, machines	12–15	*f*
*Manure spreaders	10	*al*	Molds, ingot	5	*f*
Manure spreaders	12	*e*	Mortiser, wood	30	*e*
*Manure spreaders	14	*au*	Motors, A.C. and D.C. (construction service) large	12	*f*
†Meters, consumers	30	*v*	A.C. and D.C. (construction service) medium	10	*f*
customers, electric	25	*e*	A.C. and D.C. (construction service) medium	8	*f*
customers, gas	25	*d*	hydraulic and pneumatic	5	*f*
customers, gas	30	*e*	electric	10	*p*
customers, water	20	*ak*	electric	28	*e*
customers, water	20–30	*r*	electric, in general	10	*aa,ay*
customers, water	50	*h*	electric, in general	14	*aw*
electric	12	*x,w,c*	electric, in general	20	*n,p,b*
electric	12–20	*m*	electric, in general	25	*w,x*
electric	15	*b*	electric, in general (25–8)	20	*ab*
electric	24	*h*	electric, in general, obsolete type	15	*n*
electric, service	15	*d*	electric railway	20	*n,d*
electric, switchboard	20	*d*	electric railway	30	*s*
gasoline	25	*o,d*	*Mower, farm	11	*al*
oil	10	*gg*	* farm	15	*au*
oil	13	*az*	Nail-making machines	10	*f*
orifice plate, recording	12	*e*	Nailing, machines	15	*f*
pitot tube, recording	10	*e*	Nut and bolt machines	15	*f*
venturi, recording	25	*e*	†Office equipment	15	*v*
water	20	*o*	Oil separators	20	*e*
water	20	*ak*	Ore bridges	20	*f*
water service	25	*as*	Ovens, bakery	12	*e*
Mill machinery, billet, blowing merchant bar, plate, puddle, rail, rod and wire, sheet, strip, structural, tube	20	*f*	brick drying	12	*e*
Milling machines	15	*f*	backstaves, structural, steel	12	*f*
automatic	13	*f*	coke	20	*f*
Milling machinery, die trimming and sinking	15	*f*	core	12	*e*
hand	14	*f*	Pallets and trays, ceramic	8	*f*
horizontal	20	*f*	Pans, dry, ceramic	15	*f*
Milling, multiple spindle	15	*f*	Pavements, asphalt	10	*k,t*
planer type	20	*f*	asphalt	12	*d*
thread	18	*f*	asphalt	14	*m*
universal	20	*f*	asphalt	18	*ad*
vertical	20	*f*	asphalt	20	*e*
†Miscellaneous power-plant equipment	20	*v*	brick	12	*d*
Mills, ball	12	*e*			
ball	13	*f*			

Item	Average service life	References	Item	Average service life	References
Pavements, brick	22	*o*	Pipe lines, hydraulic	22	*f*
brick	25	*e*	lines, steel, over 2½	30	*e*
brick	25	*ad*	lines, steel, under 2½	25	*e*
concrete	15–25	*ad*	lines, wood stave	35	*e*
concrete	30	*e*	Piping, air	20	*az*
granite	16	*k,t*	air, gas, liquor, oil, steam, tar, and water	20	*f*
granite	16–21	*ax*	blast, bessemer, gas, hot and cold air	25	*f*
granite	21	*d*	cast-iron 4 in. and under	50–75	*r*
granite	25	*ad*	cast-iron mains 3–4 in	50	*d*
macadam, bituminous	7	*ad*	cast-iron mains 6 in. and over	75	*d*
macadam, bituminous	8–18	*hh*	cast iron, small sizes	20–40	*r*
wood block	3–20	*hh*	cast iron, small sizes	20–40	*r*
wood block	10–15	*ad*	cast-iron water pipe	75	*q*
wood block	12	*d*	cast-iron water pipe	80	*ak*
wood block	30	*e*	cast-iron water pipe	100	*ab,ah,g,h*
Paving, in general	10	*at,ai*	distributing systems	16½	*h*
in general	10–12½	*d*	for pumping engines, water service	40	*as*
in general	10–25	*k*	fuel oil	10–20	*az*
in general	10–26	*t*	† fuel oil and tank	20–25	*v*
in general	12½	*z*	galvanized wrought iron including fittings	30–50	*n*
Pile drivers	12	*e*	gas, wrought iron or steel, 3 in. and under	20	*d*
large	8	*f*	gas, wrought iron or steel, over 3 in	30	*d*
steam	10	*ay*	pipe and covering	15	*k*
steam on skids	7	*f*	pipe and covering	20	*t,d,m*
track	13	*ay*	pipe and covering	22–25	*k*
track	10	*f*	pipe and covering	28	*ac*
steam, 200–400 ft.-lb	3	*ay*	power plant	20	*ah*
steam, 600–2,500 ft.-lb	4	*ay*	power plant penstocks	30	*v*
steam, 4,000 ft.-lb	5	*ay*	power stations	20	*b*
steam, 8,000 ft.-lb	6	*ay*	services	20	*ah,d*
steam, 15,000–22,000 ft.-lb	7	*ay*	steam	20	*kk*
Pile hammers, steam or air, heavy	7	*f*	steel	25–50	*r*
steam or air, light	3	*f*	underground, wrought iron	30	*ak*
steam or air, wood and steel	6	*f*	wood stave	20–30	*r*
Piling, untreated, cedar	20	*an*	wood stave	25	*kk*
untreated, rock elm	5	*an*	wood stave	30	*v*
untreated, southern oak	5	*an*	wrought iron	20	*q,ah*
untreated, tamarack	10	*an*	wrought iron, services	15–30	*r*
untreated, white oak	15	*an*	Pits, casting, concrete and steel	25	*f*
Pipe, cast iron, 8 in. up	75	*f*	soaking, steel mill	18	*f*
cast iron, less than 8 in	50	*f*	Planers and joiners, wood	20	*f*
concrete or masonry	50	*f*	metal	18	*f*
fuel oil	20	*f*	metal	32	*e*
galvanized iron	30	*f*	wood	30	*e*
galvanized iron, construction service	4	*ay*	Platforms, concrete	25	*f*
gas, ammonia	20	*f*	ladders, stairways, railings, foot bridges (structural steel)	18	*f*
wrought iron and steel, 6 in. and larger	33	*f*	wood	10	*f*
wrought iron and steel, less than 6 in	22	*f*			
valves and fittings	15	*f*			
cutting and threading machines	18	*f*			
line and fittings, dredge	6	*f*			

Item	Average service life	References	Item	Average service life	References
*Plows, farm, all kinds	12	*al*	Poles, telephone	12–15	*d*
furrow, construction service	3	*ay*	telephone, average exchange	10	*u*
* gang	15	*au*	telephone, average toll line	15	*u*
gang	18	*e*	telephone, crossarms	8–12	*d*
rooter	6	*ay*	trolley poles, wooden	20	*l*
snow, highway, rotary	10–12	*ad*	untreated cedar in earth	14	*e*
snow, railroad	20	*e*	untreated cedar in concrete	18	*e*
* sulky	16	*au*	wood, creosoted	22	*f*
* tractor type	9	*au*	wood, dipped butt	20	*f*
snow, V-pusher type	6–8	*ad*	wood in concrete	10–20	*n*
* walking	14	*au*	wood in concrete	20	*d*
walking single	20	*e*	wood in earth	10	*aa,l*
Plumbing fixtures	25	*f*	wood in earth	10–18	*n*
Pneumatic tools for construction, backfill tampers, wood borers, caulking and chip hammers, clay digger, drills, holder-on, pavement breaker, riveter, and saw	3	*ay*	wood in earth	12–18	*d*
			wood in earth	13	*ai*
			wood in earth	15	*f*
			wooden transmission line, including fixtures	20	*l*
			Polishing machines	10	*f*
Pointers, rotary for wire	12	*f*	for wire	15	*f*
†Poles and fixtures, transmission	24	*v*	Pontoons	10	*f*
			†Power plant buildings, main	25–40	*v*
Poles and towers, concrete	50	*f*	Power plant equipment	12½	*at*
steel	33	*f*	Power plant equipment	13	*aq*
Poles, cedar	10	*b*	Power plant equipment	20	*z*
cedar	10–20	*u*	Power plant equipment	21	*d*
cedar, in concrete	12–20	*n*	Power plant equipment	25–32	*l*
cedar, in earth	10–18	*n*	Presses, arbor, drop forge, hydraulic	20	*f*
cedar, with crossarms	12	*d*			
chestnut	8–15	*u*	bench	18	*f*
concrete reinforced	50	*e*	drill	12½	*f*
† distribution, and fixtures	17	*v*	drill	18	*f*
* electric transportation lines	10–11	*al*	drill (22–10)	10	*aq*
iron	20	*k*	hydraulic	20	*f*
iron	40	*d,ac*	hydraulic forging	14–26	*gg*
iron and steel	33	*f*	multiple plunger	17	*f*
iron or steel in concrete	15–29	*n*	punch	22	*ll*
pine, creosoted	20	*u,e*	scrap bundling	22	*f*
redwood	16	*b*	Pressure filters, water purification	50	*as*
steel	40	*l*			
steel	50	*m,e*	†Prime movers and auxiliaries	20–25	*v*
* telegraph (German and Prussian telegraph system) copper sulphate treated	10	*al*	†Print-shop equipment	15	*v*
			Producers, gas	15	*f*
			Pullers, car	20	*f*
			Pulleys and clutches	20	*e*
* telegraph (German and Prussian telegraph system) zinc chloride treated	10	*al*	Pulverizers	15	*f*
			Pulverizers	10	*e*
			Pump stations, gas and oil transportation, gas	25	*f*
* telegraph (German and Prussian telegraph system) coal-tar treated	11	*al*	gas and oil transportation, gathering	16	*f*
* telegraph (German and Prussian telegraph system) mercuric chloride treated	9	*al*	gas and oil transportation, main line	25	*f*
* telephone	8–10	*al*	Pumping engine, steam, compound condensing, for water-supply service	30–40	*as*

Item	Average service life	References	Item	Average service life	References
*Pumping stations	13½	*al*	Punching machines for metal,		
Pumping units, electric, centrifugal, diaphragm, piston	6	*f,ay*	hand or power	15	*f*
gasoline, centrifugal and			Punches, hydraulic	20	*f*
diaphragm	5	*f*	power for metals	20	*e*
highway contractors	4	*f*	Purification equipment, feed		
piston	5	*f*	water	20	*f,e*
Pumps, air lift	10	*f*	Purifiers, gas plant	30	*f*
and condensers	15–25	*d*	gas plant	50	*e,d*
and auxiliary equipment	20–30	*r*	Pushers and levelers, coke		
boiler feed (40–15)	20	*n*	plants	20	*f*
boiler feed and slush, oil-well			Pushers for mill furnaces	17	*f*
rigs	4	*f*	Pushers, track	25	*f*
centrifugal	16	*v*	Quenching equipment (coke)	20	*f*
centrifugal, no power included	10	*ay*	Radiators, for heating systems	25	*f*
centrifugal	25	*e*	Rails, railroad	25	*h*
centrifugal and rotary	20	*f*	railroad track	10–30	*hh*
centrifugal for sand and			railroad track	12½	*o*
gravel	5	*f,ay*	railroad track (30–12½)	15–25	*ax*
diaphragm	5–6	*ay*	railroad with fasteners	40	*o*
direct acting	25	*f*	railroad with fasteners and		
filling station, retail	10	*f*	joints	20	*f*
† for wells	10	*v*	steel	10	*f,ay*
gas	20	*f*	*Railway stations	23	*al*
gear driven	22	*f*	*Railway stations, buildings	23	*al*
gear driven	20–30	*n*	Rakes, hay	15	*e*
hydraulic	15	*f*	* hay dump	20	*au*
in general	15	*c*	* side delivery	16	*au*
in general	16	*b*	Rammers	10	*f*
in general	20	*m,s,ac,*	Reamers, electric, pneumatic	3	*f*
		aa	Reapers	12	*e*
in general	21	*g*	*Reapers	16	*au*
in general	22	*y*	Recorders, clock operated	10	*e*
in general	25	*p,x*	Refrigerators, electric	15	*f*
in general	25–30	*ak*	Regenerators, gas	30	*f*
in general	30	*q,af*	Regulators, feed water	20	*e*
in general	40	*ah*	gas or voltage	12	*e*
mine	10	*e*	Reheater, gas	15	*f*
oil	4	*f*	Reservoirs	50–100	*r*
oil	16	*b*	Reservoirs	100	*af*
portable piston, electric	6	*ay*	Reservoirs, connected with		
portable piston, gas	5	*ay*	boiler plant	50	*f*
portable, road construction	4	*ay*	† dams and intakes	50	*v*
plunger	30	*e*	earthen	100	*d*
power-plant auxiliary	18	*f*	Retaining walls, concrete	45	*e*
small steam	15	*d*	Retorts, gas manufacturing	28	*e*
stationary, entrifugal	6	*ay*	house floors	15–30	*d*
stationary, piston, electric,			power plant	25	*f*
high or low pressure	6	*ay*	Riddles, gyratory	10	*f*
stationary, piston, steam,			Rivet and bolt clippers	15	*f*
high or low pressures	10	*ay*	Rivet-making machines	18	*f*
vacuum	25	*e*	Riveters, hydraulic	20	*e*
waterworks, duplex	20–25	*d*	hydraulic, pneumatic	14–20	*gg*
* waterworks, general	21	*al*	pneumatic	3	*f*
waterworks, triplex	20–30	*d*	pneumatic	2	*e*

Item	Average service life	References	Item	Average service life	References
Riveters, steam	15	*e*	*Seeder, broadcast	16	*au*
Roadmaking machinery, fresno	2	*e*	* end gate	16	*au*
grader	5	*ad*	Sewers	50	*f*
grader	8	*e*	Shapers	18	*f*
slip	2	*e*	Shears, electric, steam	15	*f*
wheel scraper	3	*ad*	hydraulic	20	*f*
wheel scraper	4	*e*	rotary	11	*f*
†Roads and trails used for production or transmission	40	*v*	†Shop equipment, general	15	*v*
Roll trains, steel mills	18	*e*	Shovel, air tunnel	4	*ay*
Rolls, bending and straightening	30	*e*	electric mucker	4	*ay*
*Roller, agricultural	16	*au*	crawler, gas	4–6	*ay*
Rollers, concrete finishing, steel	5	*f,ay*	crawler, steam	7–10	*ay*
road, gasoline, tandem type	7	*f,ay*	attachment, all types	4	*ay*
road, steam, tandem type	9	*f,ay*	Signals, automatic	12	*f*
3-wheel, gas	7	*ay*	interlocking	16	*f*
3-wheel, steam	9	*ay*	railway, and gates	18	*f*
Rolling machines, cold rolling, forming, shaping	20	*f*	Sintering plants	20	*f*
Rooter grader, wheel, 4 ft.	5	*ay*	Slotters for metals	18	*f*
Rotary converters, electric	28	*e*	Snowsheds, railroad	20	*f*
			railroad	25	*o*
Sand-blast and shot-blast equipment	10	*f*	Spike machines	15	*f*
Saw-filing, settling, and sharpening machinery	17	*f*	Spooling machines, wire	15	*f*
Saws, cold and hot	17	*f*	Spreader, stone	5	*ay*
metal working	15	*f*	Sprinklers, city street	20	*e*
wood working	20	*f*	fire-protection system	35	*e*
wood working	10	*ay*	Stampers billet	12	*f*
Scales, automatic, counter, or crane	10	*f*	Standpipe, concrete	50	*f*
platform and car track or fixed platform, general	20	*f*	steel	25	*f*
platform, movable	15	*f*	steel	25	*y*
railroad	20	*f*	steel	30	*q*
Scarifier attachment to road grader	4	*ay*	steel	25–40	*r*
steerable block type, heavy	5	*ay*	steel	40	*ak*
Scraper, fresno, marmon, or rotary	3	*ay*	wood	10	*f*
Macey or wheel	4	*ay*	Steam vessels, on Great Lakes	40	*aj*
slip	2	*ay*	on tidewater	32	*aj*
wheel	4	*ay*	Stills, ammonia	15	*f*
Screens, bar for coke	20	*f*	Stokers, furnace	17	*f*
revolving or vibrating	10	*f*	Stokers, fixed parts	20	*e,s*
revolving or vibrating	5	*ay*	moving parts	5	*e,s*
screening plant, portable	5	*ay*	†Stores, department equipment	15	*v*
Screw machines	15	*f*	Street-lighting equipment	24	*v*
Scrubbers and condensers, gas plant	30	*d*	Street-railway electric equipment	10	*ai*
Scrubbers, gas	15	*f*	Street-railway electric equipment	10–20	*d*
Sedimentation basin, concrete including cast-iron piping and fittings	60	*as*	Street-railway electric equipment	12–15	*k,t*
			Strippers, ingot	18	*f*
			Structural steel electric-railway structure	100	*l*
			Structural steel-truss bridges	100	*l*
			Squeezers, puddle mill	15	*f*
			Sumps and wells, gas plant	30	*b*
			Sumps, tar and ammonia	5	*d*
			Superheaters	20	*d*

Item	Average service life	References
Superheaters	30	e
Surfacers, wood working	30	e
Surge tanks, concrete	30	v
Substation equipment including batteries	20	v
Substation equipment including batteries	25	x
Substation equipment including batteries	33	l
Substation equipment including batteries	50	d
Switchboards and wiring	35	e
Switchboards, distribution	15	v
P.B.X.	8	i
P.B.X.	10	u,d
power plant	15	k
power plant	15–20	v,d
power plant	17	ac
power plant	20	t,ai,m,b,p
power plant	33	s
power plant	50	ac,l,d
* telephone	5	al
telephone central	8	i
telephone central	8–10	u
telephone central	12	d
† transmission	17	v
Tanks and standpipes, concrete	50	f
steel	30	f
general	20	f
Tanks, oil	25	e
steel, for air	25	e
steel, for gas	45	e
steel, for ice and brine	20	e
steel, for water	30	e,d
wood, for acids	12	e
wood, for water	12	d
wood, for water	15	e
Tapping machines	18	f
†Telephone instruments and equipment	15	v
Telephone systems	20	e
Telephones	10	e
Testing instruments, electric	10	h
Thread-cutting machines	18	f
*Threshing machine	15	au
*Ties, beechwood, treated with zinc chloride	9–11	al
including bonding and rails	13	e
cedar	12	f
* cedar, untreated	13–17	al
* Douglas fir and hemlock, zinc chloride treated	9–11	al
fir, pine, tamarack	8	f
hemlock and mixed oak, untreated	7	f

Item	Average service life	References
Ties, hewn oak, cedar, etc.	8–12	l
sawed pine	12–24	l
steel	11	f
treated	14	f
white oak	11	f
* white oak and tamarack, untreated	8	al
Tipples, coal, steel construction	25	e
coal, wood construction	18	e
Tools and shop machinery	5–25	d
Tools and shop machinery	13	ai
Tools and shop machinery	20	k,t
Tools and shop machinery	14	z
†Towers and fixtures, transmission	35	v
Track, frogs and switches, street railway	25	h
portable	6	ay
portable switches	4	ay
special work	10	ac
special work	12	d
special work	13	k
special work	14	t
special work, street railway	8.3	ai
straight	18	d
street railway, ties and bonding	12	d,ak,z
street railway, ties and bonding	13	k
street railway, ties and bonding	13	ai
street railway, ties and bonding	14	t
*Tractor	8	au
Tractor	6	e
Tractor, crawler, 10 hp.	2	ay
crawler, 15 hp.	3	ay
crawler, 20–30 hp.	4	ay
crawler, 60 hp.	5	ay
wheel, small	2	ay
wheel, 20 hp.	3	ay
Trailer, steel, bottom dump or side dump	5	ay
Transformers	15	p,c,d
Transformers, distribution	20	b
† distribution	25	v,e
line	20	h
power	20	b
static	15	p
station service	20	d
station service	25	v
†Transmission lightning arresters	15	v
Transmission lines	40	e
†Transmission, miscellaneous equipment	20	v

Item	Average service life	References	Item	Average service life	References
†Transmission, overhead conductors	30	*v*	*Wagon, farm	24	*au*
† poles and fixtures	24	*v*	dump, contractor's	4	*ay*
† towers and fixtures	35	*v*	tank, contractor's	5	*ay*
† towers, line conductors	35	*v*	Washer, gravel	5	*ay*
Traps, steam	20	*e*	Water softeners	15	*e*
Trenching machine, chain bucket, 7 ft	3	*ay*	Water towers, factory	30	*e*
chain bucket, 12 ft	4	*ay*	†Water wells and pumps	20–25	*v*
chain bucket, 18 ft	6	*ay*	Waterwheels	40	*e*
1 rotary scoop, 7 ft	3	*ay*	*Waterworks sources	17	*al*
Trestles and viaducts, steel	30	*f*	Welding equipment, all kinds	10	*f*
timber	14	*f*	all kinds	4	*ay*
Trestles, concrete	75	*e*	Wells, driven or drilled	50–75	*n*
steel	50	*e*	gas	10	*h*
wood	15	*e*	large open masonry	75–100	*n*
Trucks, annealing furnace	10	*f*	(water)	30	*v*
charging	10	*f*	Wire, aerial	20	*c*
core oven	11	*f*	aerial copper	15	*l*
motor, stake body	2½–5	*ay*	bare	25	*b*
motor, dump body	3–5	*ay*	bare aluminum	25	*l*
Tube mills	12	*e*	bonds for track	5–15	*l*
Tunnels and subways	75	*e*	copper, weatherproof	10–15	*n*
†Turbines and wheels, water	35	*v*	copper, weatherproof	13	*aa*
Turbines, hydraulic	50	*e*	copper, weatherproof	16	*d*
steam	15	*c*	4-kv. distribution	15	*v*
steam	20	*t,ai,d,p*	insulated	20	*b*
steam	30	*e,b*	rubber- and lead-covered copper	30	*l*
steam (30–5)	20	*ab*	telephone, copper	20	*d*
Turbo-blowers	15	*f*	telephone, distribution	10	*d*
Turn tables, railroad	15	*e*	telephone exchange, bare copper	10–15	*u*
Tuyeres	10	*f*	telephone exchange, bare iron	8–10	*u*
Typewriters	10	*e*	telephone exchange, insulated copper	10	*u*
Typewriters	4	*ad*	telephone, iron	8–15	*d*
Underground conduits, tile or concrete	50	*l*	telephone, iron	10	*d*
Valves and valve boxes	75	*as*	telephone, bare iron	15	*u*
Valves, back pressure, safety and general	20	*e*	telephone toll, bare copper	40	*u*
reducing	18	*e*	telephone, weatherproof iron	15	*d*
water gate	40	*e*	10-kv. distribution	20	*v*
Ventilating systems	25	*e*	2-kv. distribution	15	*v*

APPENDIX B

INDUSTRIAL-PROPERTY MORTALITY TYPE CURVES

Appendix B presents the mortality type curves which have been developed by the Iowa Engineering Experiment Station of Iowa State College, Ames, Ia., during some 15 years of research. It is believed that the 18 curves already completed (possibly increased later to 20) cover practically the entire range of the mortality characteristics of the thousands of different types of industrial property. For explanations and discussions of these type curves, see Secs. 3.13 to 3.18.

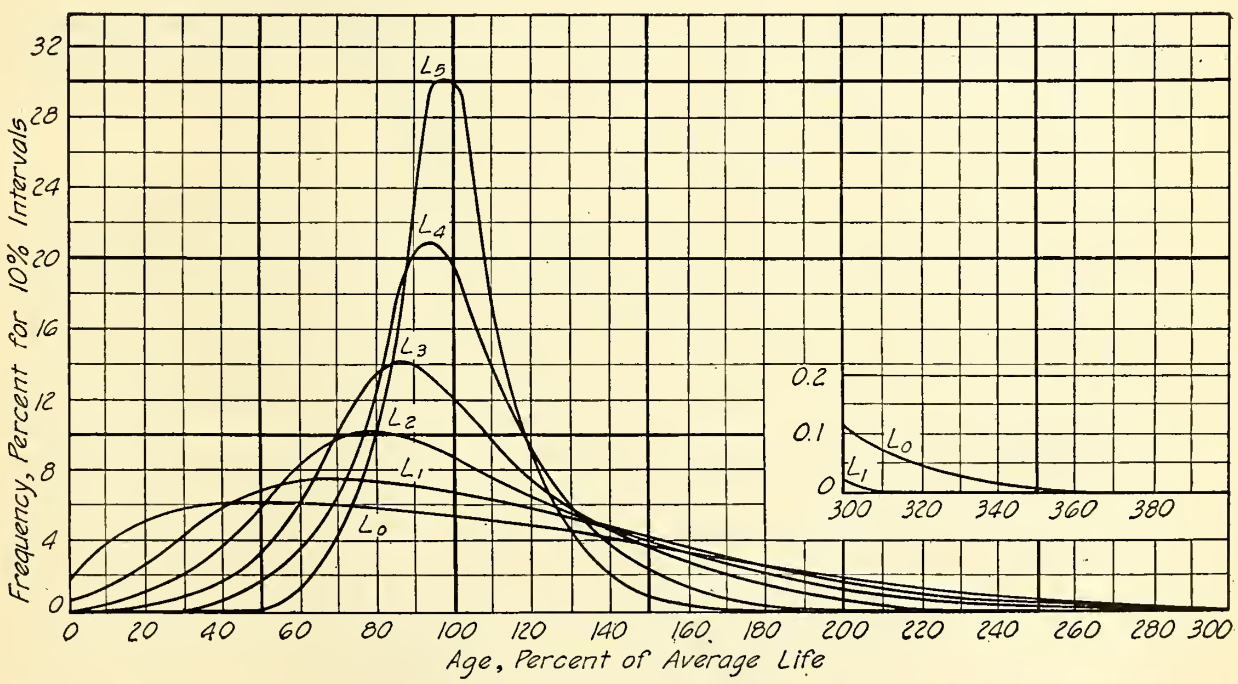


FIG. B.1.—Frequency curves of the left mode type group.

Table B.1, below, shows which of these type curves best fit the respective mortality characteristics of each of the 176 different groups of industrial-property units of different kinds of industrial property for which the Experiment Station has thus far collected mortality data.

Table B.2 is a summary of the mortality characteristics of the 18 mortality type curves already completed.

Figures B.1 to B.3, inclusive, show the 18 type frequency curves already completed.

Figures B.4 to B.6, inclusive, show the corresponding 18 type survivor curves and their resulting probable-life curves.

Figure B.7 illustrates the method of using the survivor and probable-life type curves in forecasting the probable actual service life of a particular property unit (or average survivor of an age-group of like units).

Figure B.8 shows three examples of the 18 renewals type curves.

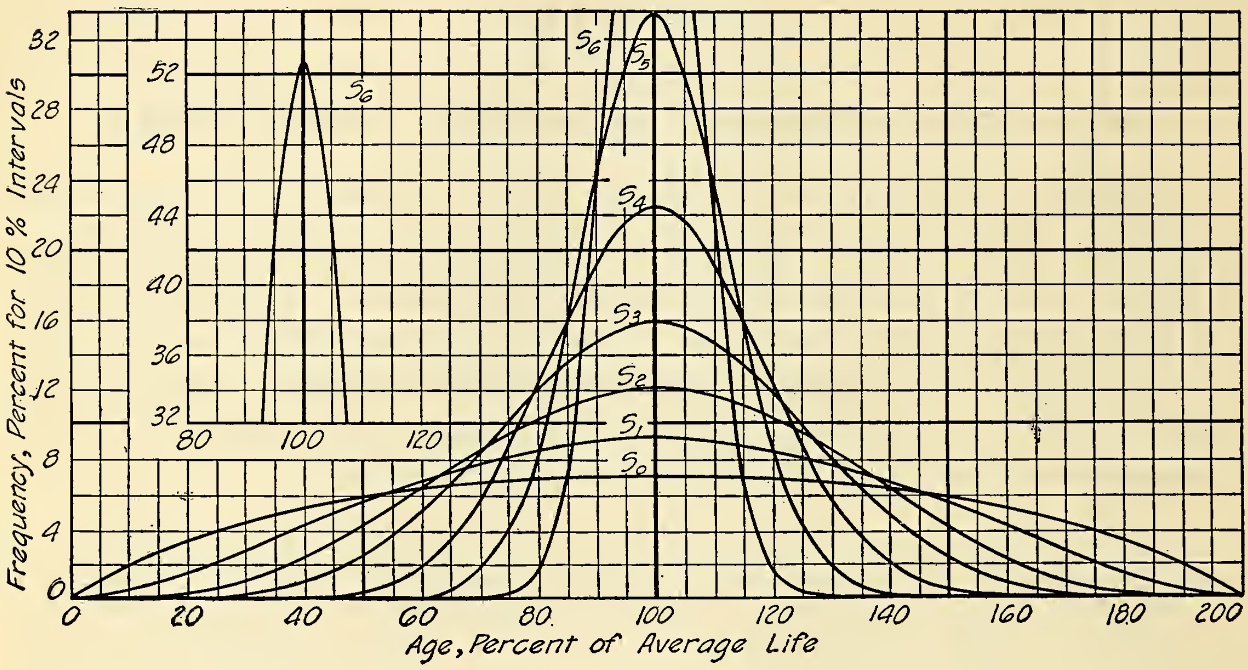


Fig. B.2.—Frequency curves of the symmetrical type group.

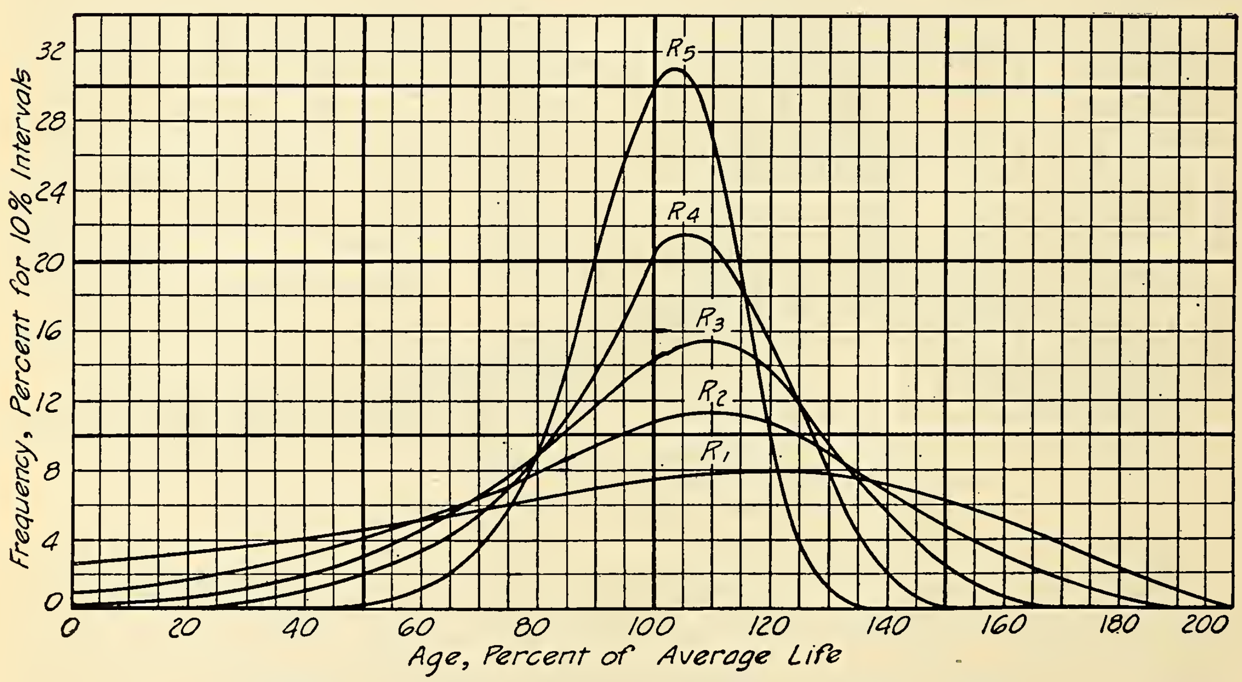


Fig. B.3.—Frequency curves of the right mode type group.

Table B.3 gives the results of the renewals calculations for the entire 18 type curves extended through the future time periods found theoretically to be necessary to bring about uniform "normal" annual renewals.

THE METHOD OF USING THE MORTALITY TYPE CURVES

A summary is given below of the method of using the mortality type curves to assist in estimating the probable life, and probable annual

renewals, of any particular industrial-property unit, or age-group of like units (Secs. 3.13 to 3.18 and 3.21).

1. *Select the type curve* which best fits the mortality characteristics of the particular kind of industrial property under consideration:
   a. Whenever possible, by direct comparison with an actual mortality curve of like units under like service conditions.

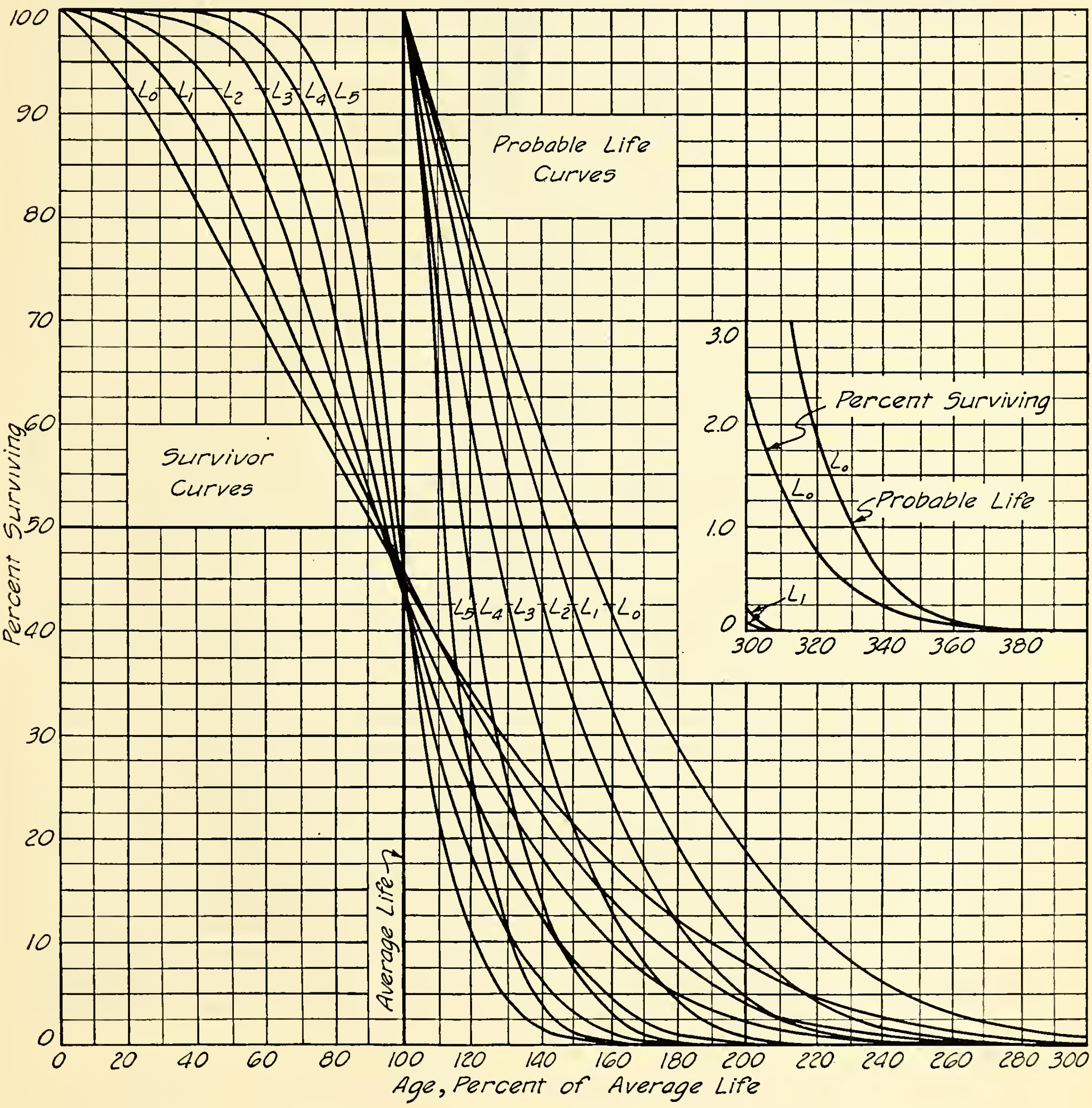


FIG. B.4.—Survivor and probable-life curves of the left mode type group.

   b. When data for method *a* are not available, use Table B.1 for all types of property listed therein.
   c. When neither method *a* nor method *b* is practicable, use two or more tentative type curves (and, if necessary, two, or more, tentative average lives), judged necessary to forecast retirement dates within the range of probable uncertainty which they indicate.
2. *Determine the average service life* of like property units under like conditions:
   a. Whenever possible, by computation from an actual mortality curve of like units under like service conditions.

b. When data for method *a* are not available, refer to the average life table in Appendix A, supplemented by any other available pertinent mortality data; and, when considered necessary, try two or three average lives, judged to cover the range of uncertainty.

3. *Determine the probable life of the average survivor unit* at the age (in percent of average life) of the property unit, or age-group of like units, under consideration.

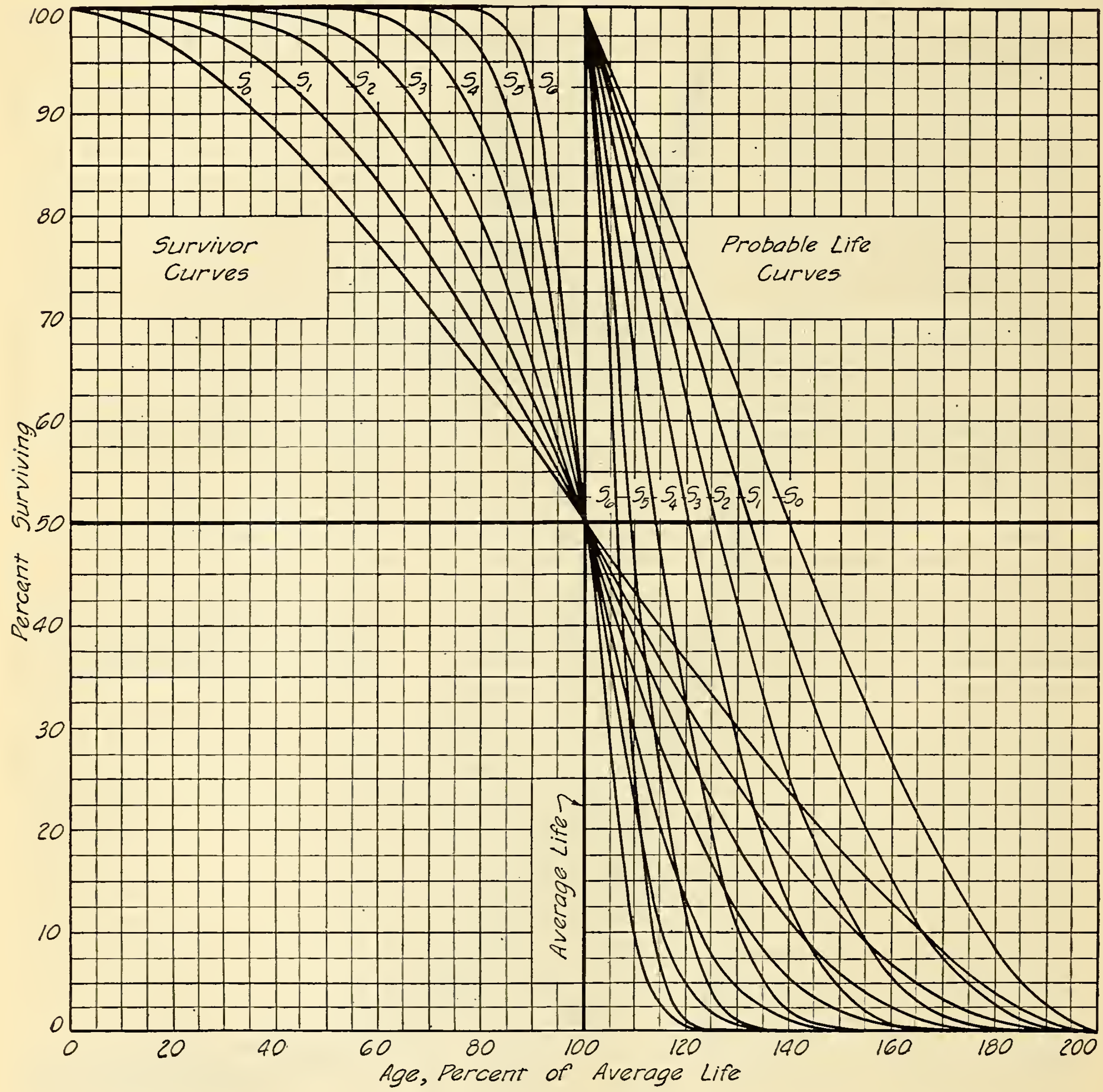


Fig. B.5.—Survivor and probable-life curves of the symmetrical type group.

This can be read direct from the proper curve in Figs. B.4 to B.6 (reducing percents of average life back to years). Figure B.7 illustrates the procedure.

4. *Determine the probable life of the particular unit,* or age-group of like units, under consideration. This must be by judgment, after making actual examinations of the property units; taking into consideration (*a*) the present physical condition (bad, poor, average, good, excellent, FOR THE AGE); (*b*) the probable future service conditions (severe, unfavorable, average, favorable, mild); (*c*) for conditions not better than average, the time until the probable retirement of the average survivor; (*d*) for conditions better than the average, the additional time until the latest probable retirements of a material number of long-lived survivors.

5. *Proceed similarly in estimating the probable future annual renewals,* using Table B.3. The results will be in percentages of renewals during time periods equal to one-tenth of the average life in years; they must be reduced back to numbers of units renewed per year.

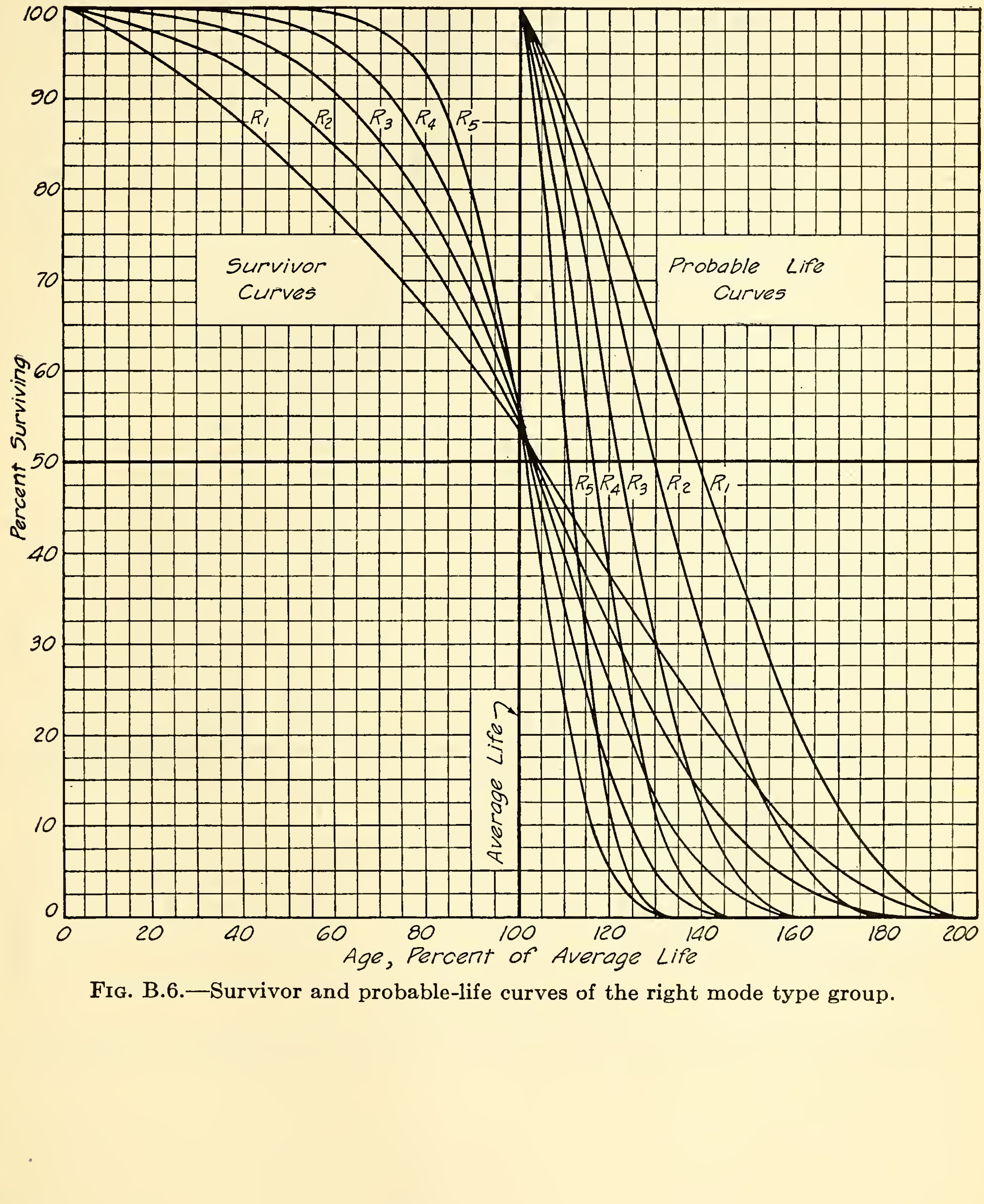


FIG. B.6.—Survivor and probable-life curves of the right mode type group.

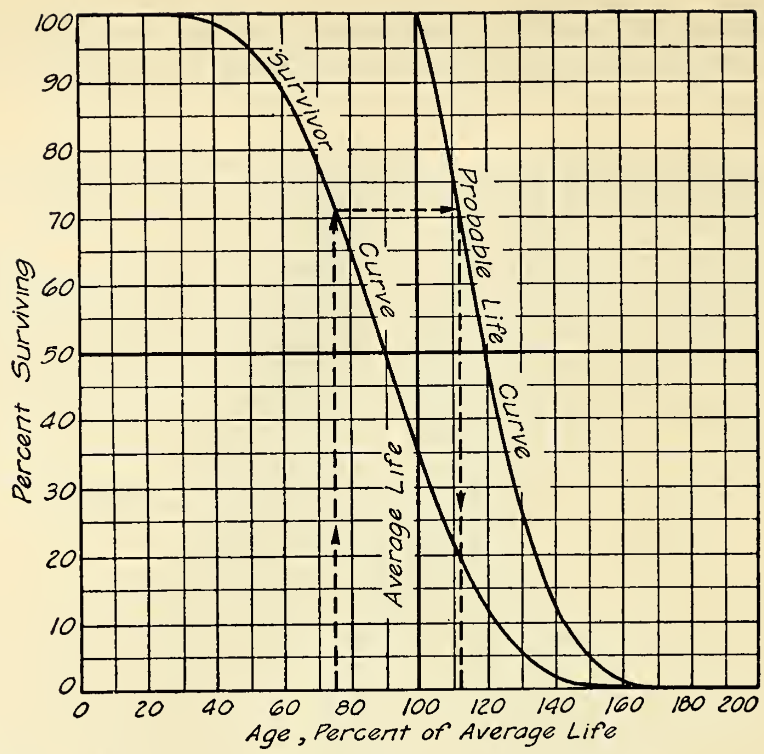


FIG. B.7.—Method of using survivor and probable-life type curves.

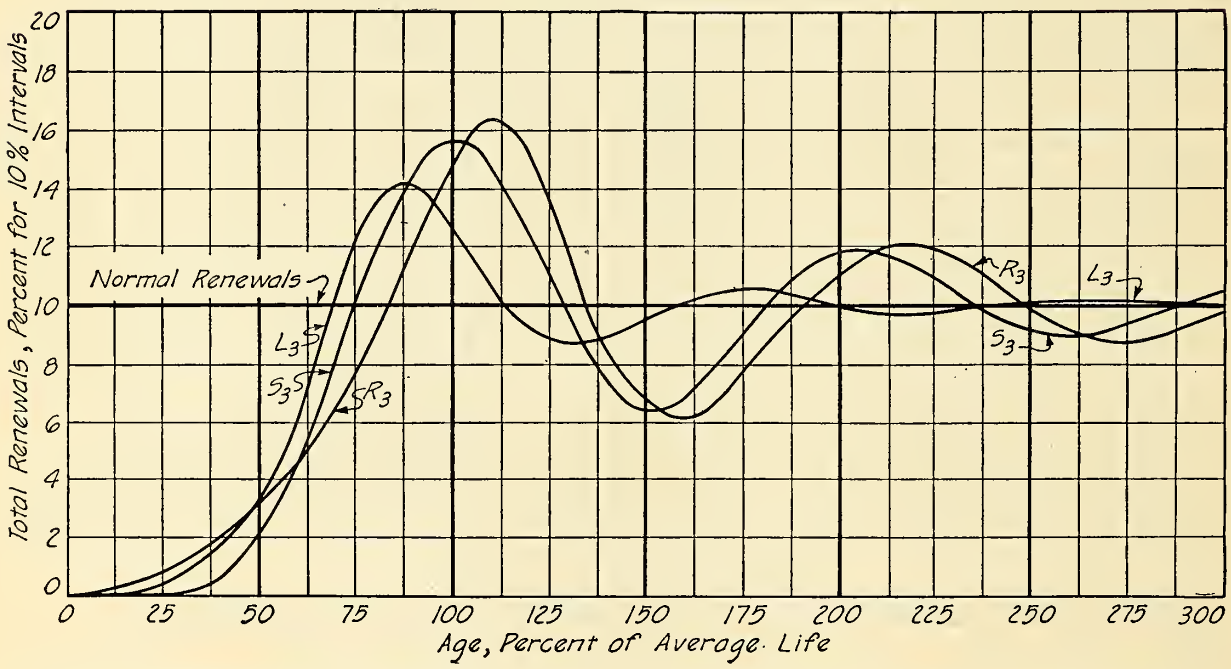


FIG. B.8.—Three typical renewals type curves (renewal periods 10 percent of average life).

Table B.1.—Summary of the Mortality Characteristics of 176 Industrial-property Groups of Different Kinds

Note 1.—The first 65 actual mortality curves listed in Table B.1 were published in Life Characteristics of Physical Property, *Iowa Eng. Exp. Sta., Bull.* 103, by Robley Winfrey and Edwin B. Kurtz in 1931. The remaining 111 curves have been constructed by Winfrey since and are published in *Bull.* 125. The data for them have been collected from many sources, and the work of collecting mortality-curve data by the station is still proceeding actively. The letters A.R., O.G., I.U., respectively, indicate the "annual-rate," "original-group," "individual-unit" methods of compiling mortality curves. (Secs. 3.7, 3.8, 3.9.) The letters M.O.G., C.O.G., indicate the "multiple-original-group," "composite original group" method of compiling mortality curves.

Note 2.—The letters, G, F, and P indicate whether the smoothed frequency curve is a "good-," "fair-," or "poor-shaped" curve, on the basis of general comparison with the 18 type curves. The goodness of fit is based upon the degree (good, fair, poor, and 2 for "second" choice) of coincidence when the individual curve is superimposed on the type curves.

TABLE B.1.—SUMMARY OF THE MORTALITY CHARACTERISTICS OF 176

No.	Class	Kind of property	Years during which renewals observed	No. of units retired	Method of calcu-lation	End point of original data	
						% sur-viving	Age, years
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1-1	W.W.	Waterworks sources	....	18	I.U.	0.0	34.5
2-1	W.W.	Pumping stations	....	23	I.U.	0.0	34.5
3-1	W.W.	Pumps	....	50	I.U.	0.0	50.5
4-1	W.W.	Steam engines	....	17	I.U.	0.0	51.5
5-1	W.W.	Boilers	....	32	I.U.	0.0	23.5
6-1	Tp.	Central-office equipment	....	$1,238,925	I.U.	0.0	13.5
7-1	Tp.	Loading coils	....	$ 265,835	I.U.	0.0	15.5
8-1	Tp.	Poles	....	2,423	I.U.	0.0	26.5
9-1	Tp.	Central-office equipment	....	$ 848,109	I.U.	0.0	24.5
10-1	Tp.	Aerial cable	....	$ 323,890	I.U.	0.0	24.5
11-2	Tp.	Aerial cable	....	$ 386,910	I.U.	0.0	19.5
12-1	Tp.	Submarine cable	....	$ 330,332	I.U.	0.0	17.5
13-2	Tp.	Submarine cable	....	$ 87,296	I.U.	0.0	21.5
14-1	Tp.	Underground cable	....	$1,433,383	I.U.	0.0	27.5
15-2	Tp.	Underground cable	....	$ 759,329	I.U.	0.0	38.5
16-3	Tp.	Underground cable	....	$ 457,251	I.U.	0.0	28.5
17-4	Tp.	Underground cable	....	$ 9,526	I.U.	0.0	23.5
18-5	Tp.	Underground cable	....	$ 31,850	I.U.	0.0	26.5
19-6	Tp.	Underground cable	....	$ 53,101	I.U.	0.0	28.5
20-1	Tg.	Poles	....	5,708	I.U.	0.0	19.5
21-2	Tg.	Poles	....	124,300	I.U.	0.0	23.5
22-3	Tg.	Poles	....	77,606	I.U.	0.0	24.5
23-4	Tg.	Poles	....	11,084	I.U.	0.0	21.5
24-5	Tg.	Poles	....	30,009	I.U.	0.0	23.5
25-1	El.	Poles	1910–1915	1,372	I.U.	0.0	24.5
26-2	El.	Poles	....	309	I.U.	0.0	22.5
27-1	El.	Lamps	....	....	I.U.	0.0	14.0†
28-2	El.	Lamps	....	....	I.U.	0.0	20.0†
29-3	El.	Lamps	....	75	I.U.	0.0	17.5†
30-4	El.	Lamps	....	100	I.U.	0.0	18.0†
31-1	El.Ry.	Wheels	1910	939	I.U.	0.0	16.5‡
32-1	R.R.	Stations	....	17	I.U.	0.0	47.5
33-1	R.R.	Locomotives	....	781	I.U.	0.0	50.5
34-1	R.R.	Passenger cars	....	....	I.U.	0.0	54.5
35-1	R.R.	Freight cars	....	15,372	I.U.	0.0	37.5
36-1	R.R.	Box cars	....	8,788	I.U.	0.0	30.5
37-1	R.R.	Stock cars	....	3,351	I.U.	0.0	32.5
38-1	R.R.	Flat cars	....	2,712	I.U.	0.0	32.5
39-1	R.R.	$ZnCl_2$ crossties	....	9,937	I.U.	0.0	15.5
40-2	R.R.	$ZnCl_2$ crossties	....	13,835	I.U.	0.0	15.5

† 100 lamp-hours. ‡ 5,000 car-miles.

INDUSTRIAL-PROPERTY GROUPS OF DIFFERENT KINDS.—(*Continued*)

Smoothed curve										
Year basis				% of average-life basis			Shape of adjusted curve	Type curve	Goodness of fit to type curve	No.
Average life, years	Mode		Maximum life, years	Mode		Maximum life, %				
	Location age, years	Frequency, %		Location age, %	Frequency,* %					
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(1)
17.0	12.5	4.6	34.5	74.0	7.8	202.9	F	L_1	F	1-1
13.5	9.5	7.3	34.5	70.0	9.8	255.6	G	L_2	G	2-1
21.3	16.0	5.0	50.5	76.0	10.6	237.1	G	L_2	G	3-1
31.5	27.5	5.8	51.5	87.0	18.3	163.5	P	L_4	F	4-1
14.8	15.5	8.5	23.5	105.0	12.6	158.8	F	S_2	P	5-1
5.3	6.0	14.5	13.5	110.0	7.7	254.7	P	R_1	F	6-1
11.6	12.5	19.7	15.5	108.0	22.8	133.6	G	R_4	G	7-1
12.5	9.6	5.8	26.5	77.0	7.3	212.0	F	L_1	F	8-1
8.7	5.0	8.5	24.5	57.0	7.4	281.6	G	L_1	F	9-1
10.6	7.0	7.1	24.5	70.0	8.6	231.1	F	L_1	F	10-1
9.7	9.0	10.0	19.5	94.0	9.7	201.0	G	S_1	G	11-2
9.3	11.5	7.1	17.5	124.0	7.5	188.2	F	R_1	G	12-1
11.4	13.0	6.5	21.5	112.0	7.4	188.6	F	R_1	G	13-2
14.2	15.0	6.8	27.5	105.0	9.7	193.7	G	S_1	F	14-1
14.9	13.5	7.6	38.5	91.0	11.3	258.4	F	L_2	P	15-2
16.3	15.5	6.7	28.5	95.0	10.9	174.8	F	S_1	F	16-3
12.3	14.0	6.7	23.5	116.0	8.3	191.1	G	R_1	G	17-4
12.3	8.0	7.2	26.5	65.0	8.8	215.4	P	L_1	P	18-5
11.6	7.0	8.7	28.5	60.0	10.1	245.7	P	L_2	P	19-6
8.3	7.0	15.2	19.5	84.0	12.6	234.9	G	L_3	F	20-1
9.8	8.5	8.9	23.5	89.0	8.7	239.8	G	S_1	F	21-2
9.7	5.5	7.9	24.5	56.0	7.7	252.6	F	L_1	G	22-3
9.2	7.5	11.6	21.5	82.0	10.7	233.7	F	L_2	F	23-4
10.7	10.5	10.2	23.5	99.0	10.9	219.6	G	S_2	G	24-5
9.9	8.5	9.0	24.5	86.0	8.9	247.5	F	L_2	F	25-1
11.5	9.0	10.0	22.5	78.0	11.5	195.7	F	L_2	F	26-2
8.0†	7.5	13.6	14.0†	94.0	10.9	175.0	G	S_2	P	27-1
10.0†	10.0	7.9	20.0†	100.0	7.9	200.0	G	S_0	F	28-2
12.3†	13.5	18.1	17.5†	108.0	22.3	142.3	G	R_4	G	29-3
10.5†	13.0	11.0	18.0†	122.0	11.5	171.4	F	R_2	F	30-4
6.1‡	5.5	14.8	16.5‡	92.0	9.0	270.5	F	S_1	F	31-1
23.6	17.0	4.5	47.5	73.0	10.6	201.3	P	L_2	F	32-1
25.4	21.5	5.1	50.5	85.0	13.0	198.8	G	L_3	G	33-1
33.2	30.5	4.6	54.5	92.0	15.3	164.2	G	S_3	F	34-1
20.0	21.5	9.4	37.5	108.0	18.7	187.5	G	R_4	F	35-1
20.8	22.0	9.3	30.5	105.0	19.3	146.6	G	R_4	F	36-1
18.7	21.0	9.0	32.5	111.0	16.9	173.8	G	R_3	G	37-1
19.3	21.5	9.3	32.5	111.0	17.9	168.4	G	R_3	F	38-1
11.2	12.5	23.9	15.5	112.0	26.8	138.4	G	R_4	G	39-1
9.0	9.5	14.2	15.5	104.0	12.8	172.2	G	S_2	F	40-2

* Frequency intervals 10% of average life.

Table B.1.—Summary of the Mortality Characteristics of 176

No.	Class	Kind of property	Years during which renewals observed	No. of units retired	Method of calculation	End point of original data	
						% surviving	Age, years
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
41-3	R.R.	Untreated crossties	....	1,000	I.U.	0.0	15.5
42-4	R.R.	Untreated crossties	....	2,001	I.U.	0.0	19.5
43-5	R.R.	$ZnCl_2$ crossties	....	26,146	I.U.	0.0	13.5
44-6	R.R.	$ZnCl_2$ crossties	....	43,681	I.U.	0.0	15.5
45-7	R.R.	$ZnCl_2$ crossties	....	12,951	I.U.	0.0	14.5
46-8	R.R.	$ZnCl_2$ crossties	....	20,536	I.U.	0.0	14.5
47-9	R.R.	$ZnCl_2$ crossties	....	4,786	I.U.	0.0	14.5
48-10	R.R.	$ZnCl_2$ crossties	....	1,048	I.U.	0.0	17.5
49-11	R.R.	$ZnCl_2$ crossties	....	1,000	I.U.	0.0	14.5
50-12	R.R.	$ZnCl_2$ crossties	....	5,909	I.U.	0.0	19.5
51-13	R.R.	Untreated crossties	....	137,000	I.U.	0.0	13.5
52-14	R.R.	Untreated crossties	....	2,916	I.U.	0.0	10.5
53-1	R.R.	Ballast cars	....	760	I.U.	0.0	31.0
54-2	R.R.	Box cars	1869–1923	1,107	I.U.	0.0	50.0
55-2-*a*	R.R.	Flat cars	1869–1923	3,114	I.U.	0.0	45.0
56-1	Ag.I.	Cultivators	1875–1924	56	I.U.	0.0	30.5
57-1	Ag.I.	Corn planters	1874–1924	55	I.U.	0.0	29.5
58-1	Ag.I.	Harrows	1875–1924	43	I.U.	0.0	30.5
59-1	Ag.I.	Grain binders	1882–1924	45	I.U.	0.0	30.5
60-1	Ag.I.	Manure spreaders	1890–1924	37	I.U.	0.0	22.5
61-1	Ag.I.	Mowers	1878–1924	37	I.U.	0.0	25.5
62-1	Ag.I.	Plows	1880–1924	30	I.U.	0.0	28.5
63-1	M.V.	Motor cars	1911, 1913	....	I.U.	0.0	12.0
64-2	M.V.	Motor cars	1922	3,124	I.U.	0.0	13.5
65-3	M.V.	Motor cars	1926	9,878	I.U.	0.0	18.5
66-4-*a*	M.V.	Motor cars	1930	84,233	I.U.	0.0	20.5
67-5-*b*	M.V.	Motor cars	1930	84,233	A.R.	0.0	20.5
68-6-*c*	M.V.	Motor cars	1931	86,377	I.U.	0.0	19.5
69-7-*d*	M.V.	Motor cars	1931	86,377	A.R.	0.2	20.5
70-8-*e*	M.V.	Motor cars	1910–1930	679,048	M.O.G.	0.0	20.5
71-9-*f*	M.V.	Motor cars	1910–1931	765,025	M.O.G.	0.0	21.5
72-1-*a*	M.V.	Motor trucks	1929	6,133	I.U.	0.0	12.5
73-2-*b*	M.V.	Motor trucks	1929	6,133	A.R.	21.8	12.5
74-3-*c*	M.V.	Motor trucks	1930	6,272	I.U.	0.0	13.5
75-4-*d*	M.V.	Motor trucks	1930	6,272	A.R.	18.5	13.5
76-5-*e*	M.V.	Motor trucks	1931	10,616	I.U.	0.0	14.5
77-6-*f*	M.V.	Motor trucks	1931	10,616	A.R.	4.0	14.5
78-7-*g*	M.V.	Motor trucks	1916–1929	31,827	M.O.G.	32.8	13.5
79-8-*h*	M.V.	Motor trucks	1916–1930	35,636	M.O.G.	16.9	14.5
80-9-*i*	M.V.	Motor trucks	1916–1931	44,712	M.O.G.	15.7	15.5

INDUSTRIAL-PROPERTY GROUPS OF DIFFERENT KINDS.—(*Continued*)

Smoothed curve										
Year basis				% of average-life basis			Shape of adjusted curve	Type curve	Goodness of fit to type curve	No.
Average life, years	Mode		Maximum life, years	Mode		Maximum life, %				
	Location age, years	Frequency, %		Location age, %	Frequency,* %					
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(1)
13.4	13.5	22.8	15.5	100.0	30.5	115.7	G	S_5	F	41-3
17.2	18.5	22.8	19.5	107.0	39.3	113.4	G	R_5	G	42-4
9.3	9.0	23.7	13.5	98.0	22.0	145.2	G	S_4	F	43-5
11.0	12.0	22.4	15.5	108.0	24.6	140.9	G	R_4	F	44-6
10.5	11.0	17.6	14.5	106.0	18.5	138.1	G	S_4	F	45-7
11.2	12.5	17.3	14.5	111.0	19.4	129.5	G	R_4	F	46-8
10.2	10.5	20.8	14.5	102.0	21.2	142.2	F	S_4	G	47-9
9.8	9.5	22.2	17.5	97.0	21.8	178.6	F	S_4	G	48-10
11.4	12.5	17.5	14.5	111.0	19.9	127.2	G	R_4	F	49-11
10.8	9.5	21.6	19.5	88.0	23.3	180.6	P	L_4	F	50-12
8.3	8.5	17.8	13.5	102.0	14.8	162.6	G	S_3	G	51-13
8.2	8.5	26.1	10.5	103.0	21.4	128.0	G	S_4	G	52-14
19.8	22.0	12.4	31.0	110.0	24.6	156.6	G	R_4	P	53-1
28.2	33.5	5.5	50.0	119.0	15.4	177.3	G	R_3	F	54-2
25.8	29.5	4.8	45.0	115.0	12.5	174.4	G	R_2	F	55-2-*a*
13.1	11.5	11.8	30.5	88.0	15.4	232.8	G	L_3	G	56-1
10.8	6.0	6.8	29.5	56.0	7.3	273.1	F	L_1	G	57-1
13.3	6.5	8.1	30.5	48.0	10.8	229.3	F	L_2	F	58-1
14.5	10.5	6.8	30.5	72.0	9.8	210.3	F	L_2	G	59-1
10.4	10.5	8.4	22.5	100.0	8.7	216.3	P	S_1	F	60-1
11.5	9.0	7.2	25.5	79.0	8.3	221.7	G	L_1	F	61-1
11.9	9.0	8.7	28.5	74.0	10.4	239.5	F	L_2	G	62-1
5.1	2.0	12.5	12.0	35.0	6.4	235.3	F	L_1	P	63-1
6.3	6.5	17.9	13.5	104.0	11.3	214.3	G	S_2	G	64-2
7.8	8.5	15.0	18.5	109.0	11.7	237.2	G	R_2	F	65-3
7.2	6.5	20.8	20.0	90.3	15.0	277.8	F	L_3	P	66-4-*a*
6.9	6.5	21.5	20.0	94.2	14.8	289.9	G	L_3	P	67-5-*b*
6.4	5.5	17.3	20.0	85.9	11.1	312.5	G	L_2	F	68-6-*c*
6.7	6.5	13.6	21.0	97.0	9.1	313.4	G	L_2	P	69-7-*d*
7.1	6.5	17.9	21.0	91.5	12.7	295.8	G	L_3	P	70-8-*e*
7.0	6.5	20.0	20.0	92.9	14.0	285.7	G	L_3	P	71-9-*f*
5.1	2.5	14.1	17.0	49.0	7.2	333.3	G	L_0	P	72-1-*a*
8.0	3.5	7.5	25.0	43.8	6.0	312.5	G	L_0	G	73-2-*b*
5.8	4.5	10.5	16.0	77.6	6.1	275.9	G	L_0	G	74-3-*c*
8.6	6.5	8.2	23.0	75.6	7.1	267.4	G	L_1	F	75-4-*d*
6.4	4.5	10.7	17.0	70.3	6.8	265.6	G	L_1	F	76-5-*e*
6.4	4.0	11.5	21.0	62.5	7.4	328.1	G	L_1	G	77-6-*f*
8.8	6.0	6.4	24.0	68.2	5.6	272.7	G	L_0	G	78-7-*g*
8.7	6.5	6.3	22.0	74.7	5.5	252.9	G	L_0	F	79-8-*h*
8.2	3.5	7.0	24.0	42.7	5.7	292.7	G	L_0	G	80-9-*i*

* Frequency intervals 10% of average life.

Table B.1.—Summary of the Mortality Characteristics of 176

No.	Class	Kind of property	Years during which renewals observed	No. of units retired	Method of calcu- lation	End point of original data	
						% sur- viving	Age, years
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
81-1-*a*	El.	Power transformers	1922–1932	312	I.U.	0.0	19.5
82-2-*b*	El.	Power transformers	1922–1932	312	A.R.	36.3	22.5
83-1-*a*	El.	Distribution transformers	1931–1932	394	I.U.	0.0	38.5
84-2-*b*	El.	Distribution transformers	1931–1932	394	A.R.	0.5	38.5
85-15-*a*	R.R.	$ZnCl_2$ crossties	....	14,402	I.U.	0.0	10.0
86-16-*b*	R.R.	$ZnCl_2$ crossties	....	14,402	O.G.	58.0	10.0
87-17-*a*	R.R.	Creosoted crossties	....	25,430	I.U.	0.0	10.0
88-18-*b*	R.R.	Creosoted crossties	....	25,430	O.G.	89.1	10.0
89-19-*a*	R.R.	Untreated crossties	....	10,329	I.U.	0.0	10.0
90-20-*b*	R.R.	Untreated crossties	....	10,329	O.G.	82.4	10.0
91-21-*a*	R.R.	Untreated crossties	....	46,603	I.U.	0.0	10.0
92-22-*b*	R.R.	Untreated crossties	....	46,603	O.G.	19.0	10.0
93-2	R.R.	Locomotives	1920–1927	....	A.R.	10.8	44.5
94-4	R.R.	Flat cars	1897–1927	331	O.G.	39.7	30.5
95-3	R.R.	Box cars	1898–1925	1,600	O.G.	0.0	27.5
96-4	R.R.	Box cars	1900–1927	700	O.G.	30.0	27.5
97-5	R.R.	Box cars	1901–1926	2,750	O.G.	0.0	25.5
98-2	R.R.	Stock cars	1901–1927	333	O.G.	16.8	27.5
99-6	R.R.	Box cars	1904–1927	1,159	O.G.	14.2	23.5
100-1	R.R.	Gondolas	1905–1927	492	O.G.	1.6	22.5
101-1	R.R.	Refrigerator	1912–1927	1,500	O.G.	0.0	16.5
102-7	Tp.	Underground cable	1923–1925	$649,210	A.R.	14.5	29.5
103-2	Tp.	Poles	1924–1926	$384,563	A.R.	6.2	29.5
104-2	Tp.	Aerial cable	1924–1926	$244,425	A.R.	22.8	23.5
105-1	Tp.	P.B.X.'s	1924–1926	$1,578,932	A.R.	4.5	23.5
106-1-*a*	El.	A-c. watthour meters	1905–1933	7,328	I.U.	0.0	28.5
107-2-*b*	El.	A-c. watthour meters	1906–1933	435	I.U.	0.0	27.5
108-3-*c*	El.	A-c. watthour meters	1902–1933	8,072	I.U.	0.0	29.5
109-4-*a*	El.	A-c. watthour meters	1906–1933	156	I.U.	0.0	27.5
110-5-*b*	El.	A-c. watthour meters	1906–1933	147	I.U.	0.0	27.5
111-6-*c*	El.	A-c. watthour meters	1906–1933	625	I.U.	0.0	27.5
112-3	El.	Poles	1903–1933	525	I.U.	0.0	30.5
113-23	R.R.	Creosoted crossties	1914–1930	44,125	O.G.	8.2	15.5
114-24	R.R.	$ZnCl_2$ crossties	1914–1930	213,250	O.G.	0.0	16.5
115-25	R.R.	$ZnCl_2$ crossties	1915–1930	325,288	O.G.	13.7	15.5
116-26	R.R.	Creosoted crossties	1915–1930	46,916	O.G.	64.9	15.5
117-27	R.R.	$ZnCl_2$ crossties	1916–1930	234,852	O.G.	5.5	13.5
118-28	R.R.	Creosoted crossties	1916–1930	43,129	O.G.	81.9	14.5
119-29	R.R.	$ZnCl_2$ crossties	1917–1930	45,610	O.G.	73.3	13.5
120-30	R.R.	Creosoted crossties	1918–1930	19,067	O.G.	90.1	12.5

INDUSTRIAL-PROPERTY GROUPS OF DIFFERENT KINDS.—(*Continued*)

Smoothed curve										
Year basis				% of average-life basis						
Average life, years	Mode: Location age, years	Mode: Frequency, %	Maximum life, years	Mode: Location age, %	Mode: Frequency,* %	Maximum life, %	Shape of adjusted curve	Type curve	Goodness of fit to type curve	No.
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(1)
10.2	9.5	11.0	21.0	93.1	11.2	210.8	G	S_2	G	81-1-*a*
17.9	14.5	5.5	41.0	81.0	9.8	229.1	G	L_2	G	82-2-*b*
22.7	25.5	6.1	38.0	112.3	13.8	167.4	G	R_3	P	83-1-*a*
27.1	29.5	10.3	42.0	108.9	27.9	155.0	F	R_4	F	84-2-*b*
8.4	8.5	25.5	11.0	101.2	21.4	130.9	G	R_4	P	85-15-*a*
10.0	11.5	26.9	14.0	115.0	26.9	140.0	G	R_4	P	86-16-*b*
7.2	7.5	14.4	13.0	104.1	10.4	180.4	G	S_1	P	87-17-*a*
20.0	19.5	4.6	39.0	97.5	9.2	195.0	G	S_1	G	88-18-*b*
7.5	7.5	18.7	14.0	100.0	14.0	186.7	G	S_3	F	89-19-*a*
13.6	15.5	10.1	22.0	114.0	13.7	161.8	G	R_3	F	90-20-*b*
6.5	6.5	18.9	11.0	100.0	12.3	169.2	G	S_2	G	91-21-*a*
7.4	6.5	16.7	16.0	87.8	12.4	216.2	G	L_3	P	92-22-*b*
33.3	28.5	4.9	58.0	85.6	16.3	174.2	F	L_4	P	93-2
27.8	31.5	6.6	48.0	113.3	18.3	172.7	F	R_3	F	94-4
21.9	24.5	31.0	28.0	111.9	67.9	127.8	F	R_5	P	95-3
24.1	23.5	19.1	36.0	97.5	46.0	149.4	F	L_5	F	96-4
20.2	21.5	37.5	25.0	106.4	75.8	123.8	F	R_5	P	97-5
19.5	20.5	11.8	38.0	105.1	23.0	194.9	F	R_4	P	98-2
17.9	18.5	18.8	28.0	103.4	33.7	156.4	F	S_5	P	99-6
17.0	17.5	33.2	24.0	102.9	56.4	141.2	G	R_5	F	100-1
10.7	10.5	43.1	17.0	98.1	46.1	158.9	G	S_6	G	101-1
20.0	19.5	4.3	47.0	66.1	8.6	159.3	G	S_1	G	102-7
15.4	11.5	5.6	40.0	74.7	8.6	259.7	G	L_1	F	103-2
18.4	17.5	6.3	35.0	95.1	11.6	190.2	F	S_2	P	104-2
7.2	0.0	9.1	36.0	0.0	6.6	500.0	G	L_0	P	105-1
18.3	21.0	8.9	29.0	114.7	16.3	158.5	G	R_3	F	106-1-*a*
19.6	20.5	12.9	28.0	104.6	25.3	142.9	F	R_4	F	107-2-*b*
18.3	20.5	9.2	29.0	112.0	16.8	158.5	G	R_3	F	108-3-*c*
20.3	21.5	16.1	28.0	105.9	32.7	137.9	F	R_4	F	109-4-*a*
19.6	20.5	15.3	28.0	104.6	30.0	142.9	F	R_5	F	110-5-*b*
19.6	20.0	10.1	28.0	102.0	19.8	142.9	F	S_4	F	111-6-*c*
18.3	22.5	11.9	31.0	122.9	28.8	169.4	F	R_4	P	112-3
14.0	10.5	9.1	27.0	65.0	14.7	192.8	G	L_3	P	113-23
10.6	9.5	19.9	17.0	89.6	21.1	160.4	G	L_4	F	114-24
11.7	10.5	16.1	23.0	89.7	18.8	196.6	F	L_4	G	115-25
19.0	18.0	4.7	37.0	94.7	8.9	194.7	G	S_1	G	116-26
13.0	9.5	12.7	30.0	73.1	16.5	230.8	F	L_3	P	117-27
19.4	20.0	8.1	32.0	103.1	15.7	164.9	G	R_3	G	118-28
15.4	18.5	11.3	24.0	120.1	17.4	155.8	G	R_3	F	119-29
20.0	21.5	7.8	33.0	107.5	15.6	165.0	G	R_3	G	120-30

* Frequency intervals 10% of average life.

Table B.1.—Summary of the Mortality Characteristics of 176

No.	Class	Kind of property	Years during which renewals observed	No. of units retired	Method of calculation	End point of original data	
						% surviving	Age, years
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
121-31	R.R.	$ZnCl_2$ crossties	1918–1930	117,046	O.G.	66.9	12.5
122-32	R.R.	$ZnCl_2$ crossties	1919–1930	39,782	O.G.	82.1	11.5
123-33	R.R.	$ZnCl_2$ crossties	1920–1930	15,424	O.G.	82.2	10.5
124-34	R.R.	$ZnCl_2$ crossties	1921–1930	14,136	O.G.	74.0	9.5
125-7	R.R.	Box cars	1907–1931	178	O.G.	13.7	25.5
126-3	R.R.	Flat cars	1907–1931	100	O.G.	0.0	25.5
127-3	El.	Distribution transformers	1927–1930	....	A.R.	51.7	24.5
128-1	El.	Overhead conductors	1927–1930	....	A.R.	0.0	23.5
129-1	El.	Poles and fixtures	1927–1930	....	A.R.	20.4	22.5
130-4-*b*	R.R.	Flat cars	1869–1923	3,114	A.R.	3.0	45.0
131-5-*c*	R.R.	Flat cars	1869–1923	3,114	M.O.G.	0.0	47.0
132-8-*a*	R.R.	Box cars	1880–1923	4,259	I.U.	0.0	42.0
133-9-*b*	R.R.	Box cars	1880–1923	4,259	A.R.	25.3	42.0
134-3-*a*	R.R.	Stock cars	1880–1923	473	I.U.	0.0	36.0
135-4-*b*	R.R.	Stock cars	1880–1923	473	A.R.	27.0	36.0
136-3-*a*	El.	Power transformers	1925–1932	103	I.U.	0.0	30.5
137-4-*b*	El.	Power transformers	1925–1932	103	A.R.	55.6	32.5
138-4-*a*	El.	Distribution transformers	1928–1932	4,365	I.U.	0.0	34.5
139-5-*b*	El.	Distribution transformers	1928–1932	4,365	A.R.	11.9	35.5
140-6	El.	Distribution transformers	1928–1931	3,878	I.U.	0.0	32.5
141-1-*a*	El.	D-c. watthour meters	1925–1932	2,125	I.U.	0.0	33.5
142-2-*b*	El.	D-c. watthour meters	1925–1932	2,125	A.R.	0.0	34.5
143-7-*a*	El.	A-c. watthour meters	1925–1932	1,043	I.U.	0.0	27.5
144-8-*b*	El.	A-c. watthour meters	1925–1932	1,043	A.R.	70.0	27.5
145-7-*a*	El.	Distribution transformers	1918–1922	216	I.U.	0.0	13.5
146-8-*b*	El.	Distribution transformers	1918–1922	216	A.R.	61.6	15.5
147-9-*c*	El.	Distribution transformers	1923–1927	116	I.U.	0.0	18.5
148-10-*d*	El.	Distribution transformers	1923–1927	116	A.R.	74.6	20.5
149-11-*e*	El.	Distribution transformers	1928–1932	134	I.U.	0.0	23.5
150-12-*f*	El.	Distribution transformers	1928–1932	134	A.R.	74.7	25.5
151-1-*a*	Gas	Meters	1928–1932	2,863	I.U.	0.0	34.5
152-2-*b*	Gas	Meters	1928–1932	2,863	A.R.	50.8	34.5
153-3	Gas	Meters	1898–1932	5,143	M.O.G.	57.4	33.5
154-1	W.W.	Meters	1921–1933	1,237	M.O.G.	48.5	5.0
155-1-*a*	Hy.	Brick pavement	1915–1924	193,989§	I.U.	0.0	26.5
156-2-*b*	Hy.	Brick pavement	1915–1924	193,989§	A.R.	47.1	33.5
157-3-*a*	Hy.	Sheet-asphalt pavement	1925–1934	908,758§	I.U.	0.0	47.5
158-4-*b*	Hy.	Sheet-asphalt pavement	1925–1934	908,758§	A.R.	9.6	51.5
159-4	Gas	Meters	....	21,329	I.U.	0.0	39.0
160-1-*a*	R.R.	Pile and frame trestles	1881–1890	1,377	I.U.	0.0	18.5

§ Square yards.

INDUSTRIAL-PROPERTY GROUPS OF DIFFERENT KINDS.—(*Continued*)

Smoothed curve										
Year basis				% of average-life basis			Shape of adjusted curve	Type curve	Goodness of fit to type curve	No.
Average life, years	Mode		Maximum life, years	Mode		Maximum life, %				
	Location age, years	Frequency, %		Location age, %	Frequency,* %					
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(1)
13.5	13.5	8.6	28.0	100.0	11.6	207.4	G	S_2	G	121-31
15.0	16.5	10.3	25.0	110.0	15.5	166.7	G	R_3	G	122-32
15.2	16.5	10.8	26.0	108.9	16.4	171.1	G	R_3	F	123-33
12.5	10.5	11.7	28.0	84.0	14.6	224.0	G	L_3	G	124-34
18.8	20.5	6.2	33.0	109.0	11.7	175.5	G	R_2	F	125-7
14.2	14.5	6.3	26.0	102.1	8.9	183.1	F	S_1	G	126-3
25.0	25.0	4.1	50.0	100.0	10.3	200.0	G	S_1	F	127-3
17.4	19.5	11.4	24.0	112.1	19.8	137.9	P	R_4	P	128-1
18.0	21.5	8.3	31.0	119.4	14.9	172.2	F	R_3	F	129-1
25.7	28.5	5.0	57.0	110.8	12.9	217.8	F	R_2	P	130-4-*b*
28.6	30.5	8.1	47.0	106.6	23.2	164.3	F	R_4	F	131-5-*c*
25.9	30.5	7.8	42.0	117.8	20.2	162.2	F	R_4	P	132-8-*a*
30.6	31.0	5.3	54.0	101.3	16.2	176.5	F	S_3	F	133-9-*b*
24.5	32.5	7.9	40.0	132.7	19.4	163.3	P	R_4	P	134-3-*a*
29.6	32.5	5.4	51.0	109.8	16.0	172.3	F	R_3	F	135-4-*b*
14.9	10.5	4.8	36.0	70.5	7.2	241.6	G	L_1	G	136-3-*a*
36.2	26.0	2.2	91.0	71.8	8.0	251.4	G	L_1	G	137-4-*b*
21.0	21.0	6.4	36.0	100.0	13.4	171.4	G	S_3	F	138-4-*a*
25.0	19.5	4.5	50.0	78.0	11.3	200.0	G	L_2	P	139-5-*b*
20.8	20.0	7.0	34.0	96.2	14.6	163.5	G	S_3	F	140-6
22.4	26.5	10.8	34.0	118.3	24.2	151.8	G	R_4	P	141-1-*a*
20.2	25.5	8.3	35.0	126.2	16.8	173.3	F	R_3	P	142-2-*b*
14.8	17.5	5.6	28.0	118.2	8.3	189.2	G	R_1	G	143-7-*a*
28.2	30.5	15.0	38.0	108.2	42.3	134.8	F	R_5	G	144-8-*b*
5.8	5.5	10.1	14.0	94.8	5.9	174.1	G	S_0	P	145-7-*a*
19.8	11.5	3.4	60.0	58.1	6.7	303.0	G	L_0	F	146-8-*b*
8.9	9.5	7.2	20.0	106.7	6.4	224.7	G	S_0	F	147-9-*c*
30.4	30.5	2.4	61.0	100.3	7.3	200.7	G	S_0	G	148-10-*d*
11.0	10.5	5.3	26.0	95.5	5.8	236.4	G	S_0	P	149-11-*e*
28.6	33.0	5.2	47.0	115.4	14.9	164.3	G	R_3	G	150-12-*f*
24.7	27.5	7.5	37.0	111.3	18.5	149.8	G	R_4	F	151-1-*a*
34.8	34.5	3.4	65.0	99.1	11.8	186.8	G	S_2	G	152-2-*b*
36.9	37.5	3.1	71.0	101.6	11.4	192.4	G	S_2	G	153-3
35.5	38.0	4.6	56.0	107.0	16.3	157.7	G	R_3	G	154-1
17.2	21.0	8.7	27.0	122.0	15.0	162.7	G	R_3	F	155-1-*a*
24.5	24.5	3.5	48.0	100.0	8.6	195.9	G	S_1	G	156-2-*b*
35.1	36.5	11.6	47.0	104.0	40.5	133.9	G	R_5	G	157-3-*a*
37.2	36.5	5.9	65.0	98.1	22.0	174.6	G	S_4	F	158-4-*b*
19.4	20.5	11.1	34.0	105.6	21.5	175.2	G	R_4	G	159-4
8.0	8.5	32.2	18.0	106.8	25.6	226.8	G	R_4	F	160-1-*a*

* Frequency intervals 10% of average life.

TABLE B.1.—SUMMARY OF THE MORTALITY CHARACTERISTICS OF 176

No.	Class	Kind of property	Years during which renewals observed	No. of units retired	Method of calculation	End point of original data	
						% surviving	Age, years
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
161-2-*b*	R.R.	Pile and frame trestles	1881–1890	1,377	A.R.	5.8	18.5
162-3-*c*	R.R.	Pile and frame trestles	1891–1900	2,555	I.U.	0.0	28.5
163-4-*d*	R.R.	Pile and frame trestles	1891–1900	2,555	A.R.	0.2	28.5
164-5-*e*	R.R.	Pile and frame trestles	1901–1910	1,465	I.U.	0.0	32.5
165-6-*f*	R.R.	Pile and frame trestles	1901–1910	1,465	A.R.	0.0	32.5
166-7-*g*	R.R.	Pile and frame trestles	1911–1920	917	I.U.	0.0	34.5
167-8-*h*	R.R.	Pile and frame trestles	1911–1920	917	A.R.	14.3	36.5
168-9-*i*	R.R.	Pile and frame trestles	1921–1931	956	I.U.	0.0	42.5
169-10-*j*	R.R.	Pile and frame trestles	1921–1931	956	A.R.	7.7	48.5
170-1-*a*	R.R.	Timber box culverts	1881–1895	1,035	I.U.	0.0	17.5
171-2-*b*	R.R.	Timber box culverts	1881–1895	1,035	A.R.	3.1	23.5
172-3-*c*	R.R.	Timber box culverts	1896–1910	1,837	I.U.	0.0	31.5
173-4-*d*	R.R.	Timber box culverts	1896–1910	1,837	A.R.	0.0	31.5
174-5-*e*	R.R.	Timber box culverts	1911–1931	460	I.U.	0.0	41.5
175-6-*f*	R.R.	Timber box culverts	1911–1931	460	A.R.	7.6	50.5
176-1	R.R.	Stone box culverts	1856–1918	148	I.U.	0.0	74.5

INDUSTRIAL-PROPERTY GROUPS OF DIFFERENT KINDS.—(*Concluded*)

Smoothed curve										
Year basis				% of average-life basis			Shape of adjusted curve	Type curve	Goodness of fit to type curve	No.
Average life, years	Mode		Maximum life, years	Mode		Maximum life, %				
	Location age, years	Frequency, %		Location age, %	Frequency,* %					
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(1)
8.8	8.5	34.3	22.0	96.6	30.2	250.0	F	S_5	F	161-2-*b*
9.8	9.5	26.1	25.0	96.9	25.6	255.1	G	L_5	F	162-3-*c*
10.0	9.5	27.6	26.0	95.0	27.6	260.0	G	L_5	P	163-4-*d*
10.8	10.5	18.0	31.0	97.2	19.4	287.0	G	S_4	P	164-5-*e*
13.0	11.5	13.0	34.0	88.5	16.9	261.5	F	L_3	P	165-6-*f*
13.8	13.5	10.9	33.0	97.8	15.0	239.1	F	S_3	P	166-7-*g*
18.1	12.5	5.5	44.0	69.1	10.0	243.1	F	L_2	P	167-8-*h*
20.6	18.5	6.1	40.0	89.8	12.6	194.2	G	L_3	F	168-9-*i*
22.4	16.5	5.2	49.0	73.7	11.6	218.7	F	L_2	F	169-10-*j*
10.3	9.5	21.2	18.0	92.2	21.8	174.8	F	L_4	G	170-1-*a*
11.1	9.5	20.1	23.0	85.6	22.3	207.2	G	L_4	G	171-2-*b*
10.0	10.5	21.8	23.0	105.0	21.8	230.0	G	S_4	G	172-3-*c*
9.8	9.5	21.8	23.0	96.9	21.4	234.7	G	S_4	G	173-4-*d*
10.7	9.5	20.5	29.0	88.9	21.8	271.4	F	L_4	G	174-5-*e*
10.5	8.5	14.9	31.0	81.0	15.6	295.6	F	L_3	P	175-6-*f*
37.0	30.0	3.5	77.0	81.1	13.0	208.1	G	L_3	G	176-1

* Frequency intervals 10% of average life.

TABLE B.2.—SUMMARY OF CHARACTERISTICS OF THE TYPE MORTALITY CURVES

Type curve	Average life, %	Mode		Maximum life,* %
		Location age, %	Frequency, %	
L_0	100	49.40	6.243	408.50
L_1	100	60.00	7.451	316.50
L_2	100	78.10	10.204	282.50
L_3	100	86.90	14.089	238.50
L_4	100	94.30	20.914	217.50
L_5	100	97.13	30.308	191.50
S_0	100	100.00	6.952	200.00
S_1	100	100.00	9.080	199.99
S_2	100	100.00	11.911	198.75
S_3	100	100.00	15.611	192.75
S_4	100	100.00	22.329	175.50
S_5	100	100.00	33.220	156.50
S_6	100	100.00	52.473	139.50
R_1	100	118.80	7.838	200.83
R_2	100	114.03	11.010	185.81
R_3	100	109.60	15.528	164.50
R_4	100	106.00	21.823	152.85
R_5	100	103.75	30.992	137.30

* Frequency ordinate equals 0.00001 percent.

TABLE B.3.—TOTAL RENEWALS IN PERCENT FOR TYPE CURVES
Intervals corresponding to 10 percent of average life

Age interval, percent of average life	Type curve																	
	L_0*	L_1†	L_2†	L_3†	L_4*	L_5*	S_0*	S_1†	S_2†	S_3†	S_4*	S_5*	S_6*	R_1†	R_2*	R_3*	R_4*	R_5*
0– 10	2.93	0.95	0.11	0.00	0.00	0.00	1.17	0.16	0.00	0.00	0.00	0.00	0.00	2.78	1.14	0.15	0.02	0.00
10– 20	4.82	2.09	0.68	0.08	0.00	0.00	2.68	0.89	0.12	0.00	0.00	0.00	0.00	3.23	1.57	0.40	0.06	0.00
20– 30	5.92	3.64	1.60	0.47	0.00	0.00	3.84	2.02	0.58	0.06	0.00	0.00	0.00	3.69	2.12	0.88	0.19	0.00
30– 40	6.72	5.35	2.78	1.22	0.16	0.00	4.83	3.36	1.59	0.38	0.00	0.00	0.00	4.18	2.81	1.60	0.51	0.00
40– 50	7.32	6.90	4.83	2.40	0.95	0.01	5.71	4.78	3.16	1.34	0.10	0.00	0.00	4.76	3.67	2.59	1.18	0.05
50– 60	7.77	7.95	7.42	4.63	2.64	0.46	6.52	6.17	5.18	3.32	0.79	0.02	0.00	5.47	4.73	3.83	2.45	0.46
60– 70	8.18	8.45	9.50	8.28	5.00	2.64	7.25	7.48	7.39	6.36	3.28	0.46	0.00	6.31	6.01	5.37	4.53	1.96
70– 80	8.54	8.82	10.62	12.11	8.66	6.70	7.94	8.63	9.49	10.00	8.66	4.05	0.36	7.25	7.50	7.50	7.49	5.59
80– 90	8.87	9.16	10.85	14.12	16.35	14.73	8.56	9.61	11.20	13.32	15.88	15.63	8.93	8.25	9.17	10.38	11.23	13.40
90–100	9.16	9.47	10.58	13.60	20.53	28.50	9.14	10.37	12.30	15.36	21.28	29.85	40.71	9.24	10.85	13.57	17.14	24.92
100–110	9.41	9.73	10.20	11.66	16.77	23.71	9.67	10.92	12.71	15.52	21.28	29.85	40.71	10.16	12.32	15.94	21.62	29.98
110–120	9.62	9.93	9.93	9.80	11.27	12.43	10.14	11.24	12.45	13.88	15.91	15.63	8.93	10.94	13.23	16.20	18.76	18.70
120–130	9.78	10.08	9.86	8.80	7.93	6.23	10.54	11.34	11.68	11.17	8.80	4.05	0.36	11.52	13.26	13.79	11.69	4.71
130–140	9.92	10.18	9.94	8.70	6.40	3.26	10.86	11.24	10.64	8.49	3.79	0.47	0.00	11.84	12.34	9.97	5.69	0.49
140–150	10.01	10.24	10.06	9.14	6.00	2.09	11.08	10.96	9.61	6.79	2.20	0.14	0.00	11.86	10.85	7.63	3.08	0.65
150–160	10.08	10.25	10.16	9.76	6.57	2.78	11.20	10.54	8.84	6.50	3.31	0.74	0.01	11.56	9.54	6.34	3.94	1.78
160–170	10.12	10.24	10.19	10.26	8.12	5.42	11.17	10.05	8.52	7.44	6.10	2.95	0.30	10.97	8.66	6.31	5.87	4.13
170–180	10.15	10.21	10.16	10.48	10.34	9.68	10.95	9.55	8.69	8.99	9.76	8.16	3.18	10.18	8.04	7.68	8.19	8.17
180–190	10.15	10.16	10.09	10.42	12.28	14.97	10.48	9.17	9.23	10.50	13.32	15.90	14.83	9.39	8.17	9.13	10.67	13.68
190–200	10.14	10.11	10.02	10.18	12.93	18.28	9.55	9.08	9.89	11.51	15.55	22.11	31.68	8.87	8.94	10.46	12.94	18.94
200–210	10.12	10.06	9.96	9.94	12.22	16.98	8.86	9.43	10.38	11.87	15.61	22.11	31.68	8.74	9.66	11.48	14.43	20.78
210–220	10.10	10.01	9.93	9.80	10.88	12.84	9.22	9.81	10.62	11.60	13.57	15.90	14.83	9.15	10.24	12.00	14.40	17.13
220–230	10.08	9.97	9.92	9.80	9.62	8.75	9.51	10.06	10.62	10.91	10.39	8.19	3.18	9.52	10.62	11.92	12.67	10.08
230–240	10.05	9.95	9.94	9.89	8.78	5.94	9.73	10.20	10.48	10.07	7.45	3.11	0.30	9.84	10.79	11.29	10.00	4.35
240–250	10.03	9.94	9.96	10.00	8.47	4.73	9.90	10.26	10.25	9.37	5.82	1.35	0.02	10.08	10.77	10.35	7.64	2.27
250–260	10.01	9.94	9.99	10.07	8.69	5.25	10.02	10.25	10.02	8.99	5.88	2.06	0.15	10.25	10.58	9.44	6.54	3.02
260–270	10.00	9.94	10.00	10.10	9.33	7.28	10.10	10.20	9.84	8.99	7.34	4.82	1.23	10.34	10.30	8.84	6.85	5.40
270–280	9.98	9.96	10.02	10.07	10.12	10.24	10.15	10.13	9.75	9.31	9.53	9.42	5.79	10.36	10.00	8.70	8.11	8.85
280–290	9.98	9.97	10.02	10.03	10.73	13.08	10.17	10.05	9.74	9.78	11.62	14.70	16.08	10.31	9.76	8.97	9.70	12.74
290–300	9.98	9.99	10.02	9.98	10.96	14.53	10.16	9.99	9.79	10.23	12.94	18.34	26.73	10.23	9.61	9.49	11.13	15.91
300–310	9.98	10.00	10.01	9.96	10.79	13.98	10.14	9.94	9.89	10.52	13.10	18.35	26.73	10.13	9.59	10.06	12.06	16.93
310–320	9.98	10.01	10.00	9.96	10.38	11.95	10.10	9.91	9.99	10.59	12.15	14.73	16.08	10.02	9.67	10.50	12.29	15.07
320–330	9.98	10.01	10.00	9.98	9.93	9.53	10.06	9.91	10.07	10.47	10.54	9.55	5.79	9.93	9.82	10.73	11.77	11.12
330–340	9.99	10.01	10.00	10.00	9.60	7.61	10.02	9.92	10.12	10.22	8.92	5.26	1.24	9.86	9.98	10.71	10.70	7.04
340–350	9.99	10.01	10.00	10.01	9.46	6.72	9.98	9.95	10.12	9.96	7.86	3.21	0.22	9.84	10.11	10.50	9.49	4.65

350–360	10.00	10.01	10.00	10.02	9.52	7.04	9.94	9.98	10.09	9.76	7.67	3.60	0.51	9.84	10.19	10.18	8.57	4.54
360–370	10.00	10.00	10.00	10.01	9.75	8.36	9.92	10.01	10.04	9.67	8.33	6.01	2.36	9.88	10.20	9.86	8.26	6.24
370–380	10.00	10.00	10.00	10.01	10.02	10.19	9.92	10.03	10.00	9.70	9.49	9.73	7.47	9.93	10.17	9.63	8.58	8.94
380–390	10.00	10.00	10.00	10.00	10.23	11.82	9.93	10.04	9.96	9.82	10.70	13.56	16.07	9.99	10.10	9.54	9.33	11.83
390–400	10.00	10.00	10.00	9.99	10.32	12.63	9.96	10.04	9.95	9.97	11.53	16.02	23.53	10.03	10.03	9.61	10.18	14.02
400–410	10.00	10.00	10.00	9.99	10.28	12.36	9.99	10.02	9.95	10.10	11.73	16.04	23.53	10.06	9.96	9.77	10.86	14.74
410–420	10.00	10.00	10.00	10.00	10.15	11.24	10.02	10.01	9.96	10.17	11.30	13.66	16.07	10.07	9.92	9.97	11.18	13.67
420–430	10.00	10.00	10.00	10.00	9.99	9.80	10.03	10.00	9.98	10.18	10.47	10.02	7.48	10.06	9.90	10.15	11.09	11.21
430–440	10.00	10.00	10.00	10.00	9.87	8.60	10.03	9.99	10.00	10.13	9.56	6.70	2.39	10.04	9.91	10.25	10.66	8.43
440–450	10.00	10.00	10.00	10.00	9.81	8.02	10.03	9.99	10.02	10.05	8.91	4.88	0.70	10.02	9.94	10.26	10.06	6.47
450–460	10.00			10.00	9.83	8.19	10.02		10.02	9.97	8.71	5.02	1.05	10.00	9.98	10.20	9.51	6.01
460–470	10.00			10.00	9.90	9.00	10.01		10.02	9.92	8.99	6.89	3.38	9.99	10.01	10.09	9.18	7.01
470–480	10.00			10.00	10.00	10.10	10.01		10.02	9.90	9.59	9.76	8.49	9.98	10.04	9.97	9.16	8.95
480–490	10.00			10.00	10.07	11.07	10.00		10.01	9.91	10.26	12.63	15.66	9.97	10.05	9.88	9.42	11.09
490–500	10.00			10.00	10.11	11.55	9.99		10.00	9.95	10.77	14.42	21.26	9.97	10.04	9.84	9.84	12.73
500–510					10.10	11.42	9.99		9.99	10.00	10.95	14.47	21.26	9.98	10.03	9.85	10.26	13.32
510–520					10.06	10.77	9.99		9.99	10.04	10.78	12.81	15.66	9.98	10.01	9.90	10.54	12.69
520–530					10.00	9.92	9.99		9.99	10.06	10.35	10.19	8.50	9.99	10.00	9.98	10.61	11.10
530–540					9.96	9.18			9.99	10.05	9.85	7.67	3.46	10.00	9.98	10.04	10.47	9.18
540–550					9.94	8.81			10.00	10.04	9.46	6.19	1.40	10.01	9.98	10.08	10.20	7.70
550–560					9.94	8.90			10.00	10.01	9.30	6.20	1.72	10.01	9.98	10.10	9.90	7.18
560–570					9.96	9.39			10.00	9.98	9.40	7.57	4.25	10.01	9.98	10.08	9.66	7.72
570–580					10.00	10.05			10.00	9.97	9.70	9.75	9.09	10.01		10.04	9.53	9.03
580–590					10.02	10.63			10.00	9.97	10.08	11.91	15.14	10.00		10.00	9.62	10.58
590–600					10.04	10.92			10.00	9.98	10.38	13.27	19.53	10.00		9.96	9.79	11.82
600–610					10.04	10.85			10.00	9.99	10.52	13.34	19.53	10.00		9.94	10.02	12.34
610–620					10.02	10.48			10.00	10.00	10.46	12.16	15.14	9.99		9.94	10.20	11.98
620–630					10.00	9.97				10.01	10.24	10.24	9.12	9.99		9.96	10.30	10.94
630–640					9.99	9.52				10.02	9.97	8.35	4.39	9.99		9.98	10.29	9.61
640–650					9.98	9.29				10.02	9.74	7.18	2.18	9.99		10.01	10.18	8.51
650–660					9.98	9.34				10.01	9.62	7.12	2.44	10.00		10.03	10.03	8.03
660–670					9.99	9.62				10.00	9.65	8.12	4.97	10.00		10.03	9.88	8.30
670–680					10.00	10.02				9.99	9.80	9.74	9.44	10.00		10.03	9.79	9.16
680–690					10.01	10.37				9.99	10.00	11.38	14.60	10.00		10.02	9.78	10.26
690–700					10.01	10.55				9.99	10.18	12.42	18.17	10.00		10.00	9.84	11.19
700–710					10.01	10.51				9.99	10.28	12.51	18.17	10.00		9.99	9.94	11.63
710–720					10.01	10.29				10.00	10.26	11.66	14.62	10.00		9.98	10.05	11.46
720–730					10.00	9.99				10.00	10.16	10.25	9.50	10.00		9.98	10.13	10.77
730–740					10.00	9.72				10.00	10.01	8.82	5.18	10.00		9.98	10.16	9.85
740–750					9.99	9.58				10.00	9.88	7.92	2.98	10.00		9.99	10.13	9.04
750–760					9.99	9.60				10.00	9.80	7.83	3.18			10.00	10.06	8.63
760–770					9.99	9.77				10.00	9.80	8.54	5.58			10.01	9.98	8.74
770–780					10.00	10.00				10.00	9.87	9.75	9.64			10.01	9.91	9.30
780–790					10.00	10.22				10.00	9.98	10.99	14.09			10.01	9.88	10.08
790–800					10.00	10.33				10.00	10.08	11.79	17.06			10.01	9.89	10.76

* Multipliers were the summations of the frequencies within the intervals 0–½; ½–1½; 1½–2½; etc. for each one percent of average life.
† Multipliers were determined by averaging frequencies for adjacent intervals as 0 and 0–1; 0–1 and 1–2; 1–2 and 2–3; etc.

TABLE B.3.—TOTAL RENEWALS IN PERCENT FOR TYPE CURVES.—(*Concluded*)

Age interval, percent of average life	Type curve L_5*	S_4*	S_5*	S_6*	R_5*
800– 810	10.31	10.14	11.88	17.06	11.13
810– 820	10.18	10.15	11.28	14.12	11.07
820– 830	10.00	10.10	10.24	11.07	10.62
830– 840	9.84	10.02	9.16	4.51	9.98
840– 850	9.75	9.94	8.46	3.75	9.39
850– 860	9.76	9.90	8.37	3.88	9.06
860– 870	9.86	9.89	8.88	6.10	9.09
870– 880	10.00	9.92	9.77	9.75	9.44
880– 890	10.12	9.98	10.70	13.62	9.98
890– 900	10.19	10.04	11.32	16.13	10.48
900– 910	10.19	10.08	11.42	16.17	10.78
910– 920	10.11	10.08	10.98	13.94	10.78
920– 930	10.00	10.06	10.21	10.23	10.49
930– 940	9.90	10.02	9.41	6.06	10.05
940– 950	9.85	9.98	8.86	4.18	9.62
950– 960	9.85	9.95	8.77	4.52	9.36
960– 970	9.91	9.94	9.14	6.55	9.34
970– 980	10.00	9.95	9.80	9.81	9.56
980– 990	10.07	9.98	10.50	13.19	9.93
990–1,000	10.12	10.01	10.97	15.36	10.30
1,000–1,010	10.11	10.04	11.06	15.45	10.53
1,010–1,020	10.07	10.05	10.76	13.46	10.56
1,020–1,030	10.00	10.04	10.19	10.12	10.38
1,030–1,040	9.94	10.02	9.57	6.74	10.08
1,040–1,050	9.91	9.99	9.16	4.89	9.77
1,050–1,060	9.91	9.97	9.08	5.07	9.56
1,060–1,070	9.95	9.96	9.34	6.94	9.53
1,070–1,080	10.00	9.97	9.83	9.85	9.66
1,080–1,090	10.04		10.35	12.81	9.91
1,090–1,100	10.07		10.72	14.70	10.18
1,100–1,110	10.07		10.80	14.79	10.36
1,110–1,120	10.04		10.58	13.04	10.40
1,120–1,130	10.00		10.16	10.11	10.29
1,130–1,140	9.97		9.70	7.21	10.09
1,140–1,150	9.95		9.38	5.53	9.87
1,150–1,160	9.95		9.31	5.61	9.71
1,160–1,170	9.97		9.49	7.27	9.66
1,170–1,180	10.00		9.85	9.86	9.74
1,180–1,190	10.02		10.25	12.47	9.91
1,190–1,200	10.04		10.53	14.13	10.10

Age interval, percent of average life	Type curve L_5*	S_5*	S_6*	R_5*
1,200–1,210	10.04	10.60	14.21	10.24
1,210–1,220	10.02	10.44	12.67	10.28
1,220–1,230	10.00	10.13	10.12	10.22
1,230–1,240	9.98	9.79	7.59	10.08
1,240–1,250	9.97	9.54	6.09	9.93
1,250–1,260	9.97	9.48	6.11	9.80
1,260–1,270	9.98	9.61	7.56	9.76
1,270–1,280		9.88	9.86	9.81
1,280–1,290		10.17	12.17	9.92
1,290–1,300		10.39	13.63	10.05
1,300–1,310		10.45	13.70	10.16
1,310–1,320		10.34	12.35	10.20
1,320–1,330		10.11	10.13	10.17
1,330–1,340		9.85	7.91	10.08
1,340–1,350		9.66	6.57	9.96
1,350–1,360		9.61	6.56	9.87
1,360–1,370		9.70	7.83	9.83
1,370–1,380		9.90	9.87	9.86
1,380–1,390		10.12	11.90	9.93
1,390–1,400		10.28	13.19	10.02
1,400–1,410		10.34	13.25	10.10
1,410–1,420		10.26	12.07	10.14
1,420–1,430		10.09	10.13	10.12
1,430–1,440		9.90	8.19	10.06
1,440–1,450		9.75	6.99	9.98
1,450–1,460		9.71	6.97	9.92
1,460–1,470		9.77	8.08	9.88
1,470–1,480		9.92	9.87	9.89
1,480–1,490		10.08	11.67	9.94
1,490–1,500		10.21	12.80	10.01
1,500–1,510		10.25	12.86	10.07
1,510–1,520		10.20	11.83	10.10
1,520–1,530		10.07	10.13	10.09
1,530–1,540		9.93	8.42	10.05
1,540–1,550		9.82	7.36	10.00
1,550–1,560		9.78	7.33	9.95
1,560–1,570		9.82	8.29	9.92
1,570–1,580		9.93	9.87	9.92
1,580–1,590		10.06	11.46	9.95
1,590–1,600		10.15	12.46	10.00

Age interval, percent of average life	Type curve S_5*	S_6*	R_5*
1,600–1,610	10.19	12.52	10.04
1,610–1,620	10.15	11.62	10.07
1,620–1,630	10.06	10.13	10.06
1,630–1,640	9.95	8.63	10.04
1,640–1,650	9.87	7.68	10.00
1,650–1,660	9.84	7.64	9.97
1,660–1,670	9.87	8.49	9.94
1,670–1,680	9.94	9.87	9.94
1,680–1,690	10.04		9.96
1,690–1,700	10.11		9.99
1,700–1,710	10.14		10.02
1,710–1,720	10.11		10.04
1,720–1,730	10.05		10.05
1,730–1,740	9.97		10.03
1,740–1,750	9.90		10.01
1,750–1,760	9.88		9.98
1,760–1,770	9.90		9.96
1,770–1,780	9.96		9.96
1,780–1,790	10.02		9.97
1,790–1,800	10.08		9.99
1,800–1,810	10.10		10.02
1,810–1,820	10.09		10.03
1,820–1,830	10.04		10.03
1,830–1,840	9.98		10.02
1,840–1,850	9.93		
1,850–1,860	9.91		
1,860–1,870	9.92		
1,870–1,880	9.96		
1,880–1,890	10.02		
1,890–1,900	10.06		
1,900–1,910	10.08		
1,910–1,920	10.06		
1,920–1,930	10.03		
1,930–1,940	9.98		
1,940–1,950	9.95		
1,950–1,960	9.93		
1,960–1,970	9.94		
1,970–1,980	9.97		
1,980–1,990	10.01		
1,990–2,000	10.04		

Age interval, percent of average life	Type S_5*
2,000–2,010	10.06
2,010–2,020	10.05
2,020–2,030	10.02
2,030–2,040	9.99
2,040–2,050	9.96
2,050–2,060	9.95
2,060–2,070	9.95
2,070–2,080	9.98
2,080–2,090	10.01
2,090–2,100	10.03
2,100–2,110	10.04
2,110–2,120	10.04
2,120–2,130	10.02
2,130–2,140	9.99
2,140–2,150	9.97
2,150–2,160	9.96
2,160–2,170	9.96
2,170–2,180	
2,180–2,190	
2,190–2,200	
2,200–2,210	
2,210–2,220	
2,220–2,230	
2,230–2,240	
2,240–2,250	
2,250–2,260	
2,260–2,270	
2,270–2,280	
2,280–2,290	
2,290–2,300	

* Multipliers were the summations of the frequencies within the intervals 0–½; ½–1½; 1½–2½; etc. for each one percent of average life.

APPENDIX C

0% INTEREST RATE

Straight-line Method

1–10 Years Probable Life

Age, years	Condition-percent, %										Age, years
	1	2	3	4	5	6	7	8	9	10	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	0.0000	50.0000	66.6667	75.0000	80.0000	83.3333	85.7143	87.5000	88.8889	90.0000	1
2		0.0000	33.3333	50.0000	60.0000	66.6667	71.4286	75.0000	77.7778	80.0000	2
3			0.0000	25.0000	40.0000	50.0000	57.1429	62.5000	66.6667	70.0000	3
4				0.0000	20.0000	33.3333	42.8571	50.0000	55.5556	60.0000	4
5					0.0000	16.6667	28.5714	37.5000	44.4444	50.0000	5
6						0.0000	14.2857	25.0000	33.3333	40.0000	6
7							0.0000	12.5000	22.2222	30.0000	7
8								0.0000	11.1111	20.0000	8
9									0.0000	10.0000	9
10										0.0000	10

11–20 Years Probable Life

Age, years	11	12	13	14	15	16	17	18	19	20	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	90.9091	91.6667	92.3077	92.8571	93.3333	93.7500	94.1176	94.4444	94.7368	95.0000	1
2	81.8182	83.3333	84.6154	85.7143	86.6667	87.5000	88.2353	88.8889	89.4737	90.0000	2
3	72.7273	75.0000	76.9231	78.5714	80.0000	81.2500	82.3529	83.3333	84.2105	85.0000	3
4	63.6364	66.6667	69.2308	71.4286	73.3333	75.0000	76.4706	77.7778	78.9474	80.0000	4
5	54.5455	58.3333	61.5385	64.2857	66.6667	68.7500	70.5882	72.2222	73.6842	75.0000	5
6	45.4545	50.0000	53.8462	57.1429	60.0000	62.5000	64.7059	66.6667	68.4211	70.0000	6
7	36.3636	41.6667	46.1538	50.0000	53.3333	56.2500	58.8235	61.1111	63.1579	65.0000	7
8	27.2727	33.3333	38.4615	42.8571	46.6667	50.0000	52.9412	55.5556	57.8947	60.0000	8
9	18.1818	25.0000	30.7692	35.7143	40.0000	43.7500	47.0588	50.0000	52.6316	55.0000	9
10	9.0909	16.6667	23.0769	28.5714	33.3333	37.5000	41.1765	44.4444	47.3684	50.0000	10
11	0.0000	8.3333	15.3846	21.4286	26.6667	31.2500	35.2941	38.8889	42.1053	45.0000	11
12		0.0000	7.6923	14.2857	20.0000	25.0000	29.4118	33.3333	36.8421	40.0000	12
13			0.0000	7.1429	13.3333	18.7500	23.5294	27.7778	31.5789	35.0000	13
14				0.0000	6.6667	12.5000	17.6471	22.2222	26.3158	30.0000	14
15					0.0000	6.2500	11.7647	16.6667	21.0526	25.0000	15
16						0.0000	5.8824	11.1111	15.7895	20.0000	16
17							0.0000	5.5556	10.5263	15.0000	17
18								0.0000	5.2632	10.0000	18
19									0.0000	5.0000	19
20										0.0000	20

21–30 Years Probable Life

Age, years	21	22	23	24	25	26	27	28	29	30	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	95.2381	95.4545	95.6522	95.8333	96.0000	96.1538	96.2963	96.4286	96.5517	96.6667	1
2	90.4762	90.9091	91.3043	91.6667	92.0000	92.3077	92.5926	92.8571	93.1034	93.3333	2
3	85.7143	86.3636	86.9565	87.5000	88.0000	88.4615	88.8889	89.2857	89.6552	90.0000	3
4	80.9524	81.8182	82.6087	83.3333	84.0000	84.6154	85.1852	85.7143	86.2069	86.6667	4
5	76.1905	77.2727	78.2609	79.1667	80.0000	80.7692	81.4815	82.1429	82.7586	83.3333	5
6	71.4286	72.7273	73.9130	75.0000	76.0000	76.9231	77.7778	78.5714	79.3103	80.0000	6
7	66.6667	68.1818	69.5652	70.8333	72.0000	73.0769	74.0741	75.0000	75.8621	76.6667	7
8	61.9048	63.6364	65.2174	66.6667	68.0000	69.2308	70.3704	71.4286	72.4138	73.3333	8
9	57.1429	59.0909	60.8696	62.5000	64.0000	65.3846	66.6667	67.8571	68.9655	70.0000	9
10	52.3810	54.5455	56.5217	58.3333	60.0000	61.5385	62.9630	64.2857	65.5172	66.6667	10
11	47.6190	50.0000	52.1739	54.1667	56.0000	57.6923	59.2593	60.7143	62.0690	63.3333	11
12	42.8571	45.4545	47.8261	50.0000	52.0000	53.8462	55.5556	57.1429	58.6207	60.0000	12
13	38.0952	40.9091	43.4783	45.8333	48.0000	50.0000	51.8519	53.5714	55.1724	56.6667	13
14	33.3333	36.3636	39.1304	41.6667	44.0000	46.1538	48.1481	50.0000	51.7241	53.3333	14
15	28.5714	31.8182	34.7826	37.5000	40.0000	42.3077	44.4444	46.4286	48.2759	50.0000	15

0% INTEREST RATE

Straight-line Method

21–30 Years Probable Life

Age, years	Condition-percent, %										Age, years
	21	22	23	24	25	26	27	28	29	30	
16	23.8095	27.2727	30.4348	33.3333	36.0000	38.4615	40.7407	42.8571	44.8276	46.6667	16
17	19.0476	22.7273	26.0870	29.1667	32.0000	34.6154	37.0370	39.2857	41.3793	43.3333	17
18	14.2857	18.1818	21.7391	25.0000	28.0000	30.7692	33.3333	35.7143	37.9310	40.0000	18
19	9.5238	13.6364	17.3913	20.8333	24.0000	26.9231	29.6296	32.1429	34.4828	36.6667	19
20	4.7619	9.0909	13.0435	16.6667	20.0000	23.0769	25.9259	28.5714	31.0345	33.3333	20
21	0.0000	4.5455	8.6957	12.5000	16.0000	19.2308	22.2222	25.0000	27.5862	30.0000	21
22		0.0000	4.3478	8.3333	12.0000	15.3846	18.5185	21.4286	24.1379	26.6667	22
23			0.0000	4.1667	8.0000	11.5385	14.8148	17.8571	20.6897	23.3333	23
24				0.0000	4.0000	7.6923	11.1111	14.2857	17.2414	20.0000	24
25					0.0000	3.8462	7.4074	10.7143	13.7931	16.6667	25
26						0.0000	3.7037	7.1429	10.3448	13.3333	26
27							0.0000	3.5714	6.8966	10.0000	27
28								0.0000	3.4483	6.6667	28
29									0.0000	3.3333	29
30										0.0000	30

31–40 Years Probable Life

Age, years	31	32	33	34	35	36	37	38	39	40	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	96.7742	96.8750	96.9697	97.0588	97.1429	97.2222	97.2973	97.3684	97.4359	97.5000	1
2	93.5484	93.7500	93.9394	94.1176	94.2857	94.4444	94.5946	94.7368	94.8718	95.0000	2
3	90.3226	90.6250	90.9091	91.1765	91.4286	91.6667	91.8919	92.1053	92.3077	92.5000	3
4	87.0968	87.5000	87.8788	88.2353	88.5714	88.8889	89.1892	89.4737	89.7436	90.0000	4
5	83.8710	84.3750	84.8485	85.2941	85.7143	86.1111	86.4865	86.8421	87.1795	87.5000	5
6	80.6452	81.2500	81.8182	82.3529	82.8571	83.3333	83.7838	84.2105	84.6154	85.0000	6
7	77.4194	78.1250	78.7879	79.4118	80.0000	80.5556	81.0811	81.5789	82.0513	82.5000	7
8	74.1935	75.0000	75.7576	76.4706	77.1429	77.7778	78.3784	78.9474	79.4872	80.0000	8
9	70.9677	71.8750	72.7273	73.5294	74.2857	75.0000	75.6757	76.3158	76.9231	77.5000	9
10	67.7419	68.7500	69.6970	70.5882	71.4286	72.2222	72.9730	73.6842	74.3590	75.0000	10
11	64.5161	65.6250	66.6667	67.6471	68.5714	69.4444	70.2703	71.0526	71.7949	72.5000	11
12	61.2903	62.5000	63.6364	64.7059	65.7143	66.6667	67.5676	68.4211	69.2308	70.0000	12
13	58.0645	59.3750	60.6061	61.7647	62.8571	63.8889	64.8649	65.7895	66.6667	67.5000	13
14	54.8387	56.2500	57.5758	58.8235	60.0000	61.1111	62.1622	63.1579	64.1026	65.0000	14
15	51.6129	53.1250	54.5455	55.8824	57.1429	58.3333	59.4595	60.5263	61.5385	62.5000	15
16	48.3871	50.0000	51.5152	52.9412	54.2857	55.5556	56.7568	57.8947	58.9744	60.0000	16
17	45.1613	46.8750	48.4848	50.0000	51.4286	52.7778	54.0541	55.2632	56.4103	57.5000	17
18	41.9355	43.7500	45.4545	47.0588	48.5714	50.0000	51.3514	52.6316	53.8462	55.0000	18
19	38.7097	40.6250	42.4242	44.1176	45.7143	47.2222	48.6486	50.0000	51.2821	52.5000	19
20	35.4839	37.5000	39.3939	41.1765	42.8571	44.4444	45.9459	47.3684	48.7179	50.0000	20
21	32.2581	34.3750	36.3636	38.2353	40.0000	41.6667	43.2432	44.7368	46.1538	47.5000	21
22	29.0323	31.2500	33.3333	35.2941	37.1429	38.8889	40.5405	42.1053	43.5897	45.0000	22
23	25.8065	28.1250	30.3030	32.3529	34.2857	36.1111	37.8378	39.4737	41.0256	42.5000	23
24	22.5806	25.0000	27.2727	29.4118	31.4286	33.3333	35.1351	36.8421	38.4615	40.0000	24
25	19.3548	21.8750	24.2424	26.4706	28.5714	30.5556	32.4324	34.2105	35.8974	37.5000	25
26	16.1290	18.7500	21.2121	23.5294	25.7143	27.7778	29.7297	31.5789	33.3333	35.0000	26
27	12.9032	15.6250	18.1818	20.5882	22.8571	25.0000	27.0270	28.9474	30.7692	32.5000	27
28	9.6774	12.5000	15.1515	17.6471	20.0000	22.2222	24.3243	26.3158	28.2051	30.0000	28
29	6.4516	9.3750	12.1212	14.7059	17.1429	19.4444	21.6216	23.6842	25.6410	27.5000	29
30	3.2258	6.2500	9.0909	11.7647	14.2857	16.6667	18.9189	21.0526	23.0769	25.0000	30
31	0.0000	3.1250	6.0606	8.8235	11.4286	13.8889	16.2162	18.4211	20.5128	22.5000	31
32		0.0000	3.0303	5.8824	8.5714	11.1111	13.5135	15.7895	17.9487	20.0000	32
33			0.0000	2.9412	5.7143	8.3333	10.8108	13.1579	15.3846	17.5000	33
34				0.0000	2.8571	5.5556	8.1081	10.5263	12.8205	15.0000	34
35					0.0000	2.7778	5.4054	7.8947	10.2564	12.5000	35
36						0.0000	2.7027	5.2632	7.6923	10.0000	36
37							0.0000	2.6316	5.1282	7.5000	37
38								0.0000	2.5641	5.0000	38
39									0.0000	2.5000	39
40										0.0000	40

0% INTEREST RATE

Straight-line Method

41–50 Years Probable Life

Age, years	Condition-percent, % 41	42	43	44	45	46	47	48	49	50	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	97.5610	97.6190	97.6744	97.7273	97.7778	97.8261	97.8723	97.9167	97.9592	98.0000	1
2	95.1220	95.2381	95.3488	95.4545	95.5556	95.6522	95.7447	95.8333	95.9184	96.0000	2
3	92.6829	92.8571	93.0233	93.1818	93.3333	93.4783	93.6170	93.7500	93.8776	94.0000	3
4	90.2439	90.4762	90.6977	90.9091	91.1111	91.3043	91.4894	91.6667	91.8367	92.0000	4
5	87.8049	88.0952	88.3721	88.6364	88.8889	89.1304	89.3617	89.5833	89.7959	90.0000	5
6	85.3659	85.7143	86.0465	86.3636	86.6667	86.9565	87.2340	87.5000	87.7551	88.0000	6
7	82.9268	83.3333	83.7209	84.0909	84.4444	84.7826	85.1064	85.4167	85.7143	86.0000	7
8	80.4878	80.9524	81.3953	81.8182	82.2222	82.6087	82.9787	83.3333	83.6735	84.0000	8
9	78.0488	78.5714	79.0698	79.5455	80.0000	80.4348	80.8511	81.2500	81.6327	82.0000	9
10	75.6098	76.1905	76.7442	77.2727	77.7778	78.2609	78.7234	79.1667	79.5918	80.0000	10
11	73.1707	73.8095	74.4186	75.0000	75.5556	76.0870	76.5957	77.0833	77.5510	78.0000	11
12	70.7317	71.4286	72.0930	72.7273	73.3333	73.9130	74.4681	75.0000	75.5102	76.0000	12
13	68.2927	69.0476	69.7674	70.4545	71.1111	71.7391	72.3404	72.9167	73.4694	74.0000	13
14	65.8537	66.6667	67.4419	68.1818	68.8889	69.5652	70.2128	70.8333	71.4286	72.0000	14
15	63.4146	64.2857	65.1163	65 9091	66.6667	67.3913	68.0851	68.7500	69.3878	70.0000	15
16	60.9756	61.9048	62.7907	63.6364	64.4444	65.2174	65.9574	66.6667	67.3469	68.0000	16
17	58.5366	59.5238	60.4651	61.3636	62.2222	63.0435	63.8298	64.5833	65.3061	66.0000	17
18	56.0976	57.1429	58.1395	59.0909	60.0000	60.8696	61.7021	62.5000	63.2653	64.0000	18
19	53.6585	54.7619	55.8140	56.8182	57.7778	58.6957	59.5745	60.4167	61.2245	62.0000	19
20	51.2195	52.3810	53.4884	54.5455	55.5556	56.5217	57.4468	58.3333	59.1837	60.0000	20
21	48.7805	50.0000	51.1628	52.2727	53.3333	54.3478	55.3191	56.2500	57.1429	58.0000	21
22	46.3415	47.6190	48.8372	50.0000	51.1111	52.1739	53.1915	54.1667	55.1020	56.0000	22
23	43.9024	45.2381	46.5116	47.7273	48.8889	50.0000	51.0638	52.0833	53.0612	54.0000	23
24	41.4634	42.8571	44.1860	45.4545	46.6667	47.8261	48.9362	50.0000	51.0204	52.0000	24
25	39.0244	40.4762	41.8605	43.1818	44.4444	45.6522	46.8085	47.9167	48.9796	50.0000	25
26	36.5854	38.0952	39.5349	40.9091	42.2222	43.4783	44.6809	45.8333	46.9388	48.0000	26
27	34.1463	35.7143	37.2093	38.6364	40.0000	41.3043	42.5532	43.7500	44.8980	46.0000	27
28	31.7073	33.3333	34.8837	36.3636	37.7778	39.1304	40.4255	41.6667	42.8571	44.0000	28
29	29.2683	30.9524	32.5581	34.0909	35.5556	36.9565	38.2979	39.5833	40.8163	42.0000	29
30	26.8293	28.5714	30.2326	31.8182	33.3333	34.7826	36.1702	37.5000	38.7755	40.0000	30
31	24.3902	26.1905	27.9070	29.5455	31.1111	32.6087	34.0426	35.4167	36.7347	38.0000	31
32	21.9512	23.8095	25.5814	27.2727	28.8889	30.4348	31.9149	33.3333	34.6939	36.0000	32
33	19.5122	21.4286	23.2558	25.0000	26.6667	28.2609	29.7872	31.2500	32.6531	34.0000	33
34	17.0732	19.0476	20.9302	22.7273	24.4444	26.0870	27.6596	29.1667	30.6122	32.0000	34
35	14.6341	16.6667	18.6047	20.4545	22.2222	23.9130	25.5319	27.0833	28.5714	30.0000	35
36	12.1951	14.2857	16.2791	18.1818	20.0000	21.7391	23.4043	25.0000	26.5306	28.0000	36
37	9.7561	11.9048	13.9535	15.9091	17.7778	19.5652	21.2766	22.9167	24.4898	26.0000	37
38	7.3171	9.5238	11.6279	13.6364	15.5556	17.3913	19.1489	20.8333	22.4490	24.0000	38
39	4.8780	7.1429	9.3023	11.3636	13.3333	15.2174	17.0213	18.7500	20.4082	22.0000	39
40	2.4390	4.7619	6.9767	9.0909	11.1111	13.0435	14.8936	16.6667	18.3673	20.0000	40
41	0.0000	2.3810	4.6512	6.8182	8.8889	10.8696	12.7660	14.5833	16.3265	18.0000	41
42		0.0000	2.3256	4.5455	6.6667	8.6957	10.6383	12.5000	14.2857	16.0000	42
43			0.0000	2.2727	4.4444	6.5217	8.5106	10.4167	12.2449	14.0000	43
44				0.0000	2.2222	4.3478	6.3830	8.3333	10.2041	12.0000	44
45					0.0000	2.1739	4.2553	6.2500	8.1633	10.0000	45
46						0.0000	2.1277	4.1667	6.1225	8.0000	46
47							0.0000	2.0833	4.0816	6.0000	47
48								0.0000	2.0408	4.0000	48
49									0.0000	2.0000	49
50										0.0000	50

0% INTEREST RATE

Straight-line Method

51–60 Years Probable Life

Age, years	Condition-percent, %										Age, years
	51	52	53	54	55	56	57	58	59	60	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.0392	98.0769	98.1132	98.1481	98.1818	98.2143	98.2456	98.2759	98.3051	98.3333	1
2	96.0784	96.1538	96.2264	96.2963	96.3636	96.4286	96.4912	96.5517	96.6102	96.6667	2
3	94.1176	94.2308	94.3396	94.4444	94.5455	94.6429	94.7368	94.8276	94.9153	95.0000	3
4	92.1569	92.3077	92.4528	92.5926	92.7273	92.8571	92.9825	93.1034	93.2203	93.3333	4
5	90.1961	90.3846	90.5660	90.7407	90.9091	91.0714	91.2281	91.3793	91.5254	91.6667	5
6	88.2353	88.4615	88.6792	88.8889	89.0909	89.2857	89.4737	89.6552	89.8305	90.0000	6
7	86.2745	86.5385	86.7925	87.0370	87.2727	87.5000	87.7193	87.9310	88.1356	88.3333	7
8	84.3137	84.6154	84.9057	85.1852	85.4545	85.7143	85.9649	86.2069	86.4407	86.6667	8
9	82.3529	82.6923	83.0189	83.3333	83.6364	83.9286	84.2105	84.4828	84.7458	85.0000	9
10	80.3922	80.7692	81.1321	81.4815	81.8182	82.1429	82.4561	82.7586	83.0508	83.3333	10
11	78.4314	78.8462	79.2453	79.6296	80.0000	80.3571	80.7018	81.0345	81.3559	81.6667	11
12	76.4706	76.9231	77.3585	77.7778	78.1818	78.5714	78.9474	79.3103	79.6610	80.0000	12
13	74.5098	75.0000	75.4717	75.9259	76.3636	76.7857	77.1930	77.5862	77.9661	78.3333	13
14	72.5490	73.0769	73.5849	74.0741	74.5455	75.0000	75.4386	75.8621	76.2712	76.6667	14
15	70.5882	71.1538	71.6981	72.2222	72.7273	73.2143	73.6842	74.1379	74.5763	75.0000	15
16	68.6275	69.2308	69.8113	70.3704	70.9091	71.4286	71.9298	72.4138	72.8814	73.3333	16
17	66.6667	67.3077	67.9245	68.5185	69.0909	69.6429	70.1754	70.6897	71.1864	71.6667	17
18	64.7059	65.3846	66.0377	66.6667	67.2727	67.8571	68.4211	68.9655	69.4915	70.0000	18
19	62.7451	63.4615	64.1509	64.8148	65.4545	66.0714	66.6667	67.2414	67.7966	68.3333	19
20	60.7843	61.5385	62.2642	62.9630	63.6364	64.2857	64.9123	65.5172	66.1017	66.6667	20
21	58.8235	59.6154	60.3774	61.1111	61.8182	62.5000	63.1579	63.7931	64.4068	65.0000	21
22	56.8627	57.6923	58.4906	59.2593	60.0000	60.7143	61.4035	62.0690	62.7119	63.3333	22
23	54.9020	55.7692	56.6038	57.4074	58.1818	58.9286	59.6491	60.3448	61.0169	61.6667	23
24	52.9412	53.8462	54.7170	55.5556	56.3636	57.1429	57.8947	58.6207	59.3220	60.0000	24
25	50.9804	51.9231	52.8302	53.7037	54.5455	55.3571	56.1404	56.8966	57.6271	58.3333	25
26	49.0196	50.0000	50.9434	51.8519	52.7273	53.5714	54.3860	55.1724	55.9322	56.6667	26
27	47.0588	48.0769	49.0566	50.0000	50.9091	51.7857	52.6316	53.4483	54.2373	55.0000	27
28	45.0980	46.1538	47.1698	48.1481	49.0909	50.0000	50.8772	51.7241	52.5424	53.3333	28
29	43.1373	44.2308	45.2830	46.2963	47.2727	48.2143	49.1228	50.0000	50.8475	51.6667	29
30	41.1765	42.3077	43.3962	44.4444	45.4545	46.4286	47.3684	48.2759	49.1525	50.0000	30
31	39.2157	40.3846	41.5094	42.5926	43.6364	44.6429	45.6140	46.5517	47.4576	48.3333	31
32	37.2549	38.4615	39.6226	40.7407	41.8182	42.8571	43.8596	44.8276	45.7627	46.6667	32
33	35.2941	36.5385	37.7358	38.8889	40.0000	41.0714	42.1053	43.1034	44.0678	45.0000	33
34	33.3333	34.6154	35.8491	37.0370	38.1818	39.2857	40.3509	41.3793	42.3729	43.3333	34
35	31.3725	32.6923	33.9623	35.1852	36.3636	37.5000	38.5965	39.6552	40.6780	41.6667	35
36	29.4118	30.7692	32.0755	33.3333	34.5455	35.7143	36.8421	37.9310	38.9830	40.0000	36
37	27.4510	28.8462	30.1887	31.4815	32.7273	33.9286	35.0877	36.2069	37.2881	38.3333	37
38	25.4902	26.9231	28.3019	29.6296	30.9091	32.1429	33.3333	34.4828	35.5932	36.6667	38
39	23.5294	25.0000	26.4151	27.7778	29.0909	30.3571	31.5789	32.7586	33.8983	35.0000	39
40	21.5686	23.0769	24.5283	25.9259	27.2727	28.5714	29.8246	31.0345	32.2034	33.3333	40
41	19.6078	21.1538	22.6415	24.0741	25.4545	26.7857	28.0702	29.3103	30.5085	31.6667	41
42	17.6471	19.2308	20.7547	22.2222	23.6364	25.0000	26.3158	27.5862	28.8136	30.0000	42
43	15.6863	17.3077	18.8679	20.3704	21.8182	23.2143	24.5614	25.8621	27.1186	28.3333	43
44	13.7255	15 3846	16.9811	18.5185	20.0000	21.4286	22.8070	24.1379	25.4237	26.6667	44
45	11.7647	13.4615	15.0943	16.6667	18.1818	19.6429	21.0526	22.4138	23.7288	25.0000	45
46	9.8039	11.5385	13.2075	14.8148	16.3636	17.8571	19.2982	20.6897	22.0339	23.3333	46
47	7.8431	9.6154	11.3208	12.9630	14.5455	16.0714	17.5439	18.9655	20.3390	21.6667	47
48	5.8824	7.6923	9.4340	11.1111	12.7273	14.2857	15.7895	17.2414	18.6441	20.0000	48
49	3.9216	5.7692	7.5472	9.2593	10.9091	12.5000	14.0351	15.5172	16.9492	18.3333	49
50	1.9608	3.8462	5.6604	7.4074	9.0909	10.7143	12.2807	13.7931	15.2542	16.6667	50
51	0.0000	1.9231	3.7736	5.5556	7.2727	8.9286	10.5263	12.0690	13.5593	15.0000	51
52		0.0000	1.8868	3.7037	5.4545	7.1429	8.7719	10.3448	11.8644	13.3333	52
53			0.0000	1.8518	3.6364	5.3571	7.0175	8.6207	10.1695	11.6667	53
54				0.0000	1.8182	3.5714	5.2632	6.8966	8.4746	10.0000	54
55					0.0000	1.7857	3.5088	5.1724	6.7797	8.3333	55
56						0.0000	1.7544	3.4483	5.0847	6.6667	56
57							0.0000	1.7241	3.3898	5.0000	57
58								0.0000	1.6949	3.3333	58
59									0.0000	1.6667	59
60										0.0000	60

0% INTEREST RATE

Straight-line Method

61–70 Years Probable Life

Age, years	Condition-percent, %										Age, years
	61	62	63	64	65	66	67	68	69	70	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.3607	98.3871	98.4127	98.4375	98.4615	98.4848	98.5075	98.5294	98.5507	98.5714	1
2	96.7213	96.7742	96.8254	96.8750	96.9231	96.9697	97.0149	97.0588	97.1014	97.1429	2
3	95.0820	95.1613	95.2381	95.3125	95.3846	95.4545	95.5224	95.5882	95.6522	95.7143	3
4	93.4426	93.5484	93.6508	93.7500	93.8462	93.9394	94.0299	94.1176	94.2029	94.2857	4
5	91.8033	91.9355	92.0635	92.1875	92.3077	92.4242	92.5373	92.6471	92.7536	92.8571	5
6	90.1639	90.3226	90.4762	90.6250	90.7692	90.9091	91.0448	91.1765	91.3043	91.4286	6
7	88.5246	88.7097	88.8889	89.0625	89.2308	89.3939	89.5522	89.7059	89.8551	90.0000	7
8	86.8852	87.0968	87.3016	87.5000	87.6923	87.8788	88.0597	88.2353	88.4058	88.5714	8
9	85.2459	85.4839	85.7143	85.9375	86.1538	86.3636	86.5672	86.7647	86.9565	87.1429	9
10	83.6066	83.8710	84.1270	84.3750	84.6154	84.8485	85.0746	85.2941	85.5072	85.7143	10
11	81.9672	82.2581	82.5397	82.8125	83.0769	83.3333	83.5821	83.8235	84.0580	84.2857	11
12	80.3279	80.6452	80.9524	81.2500	81.5385	81.8182	82.0896	82.3529	82.6087	82.8571	12
13	78.6885	79.0323	79.3651	79.6875	80.0000	80.3030	80.5970	80.8824	81.1594	81.4286	13
14	77.0492	77.4194	77.7778	78.1250	78.4615	78.7879	79.1045	79.4118	79.7101	80.0000	14
15	75.4098	75.8065	76.1905	76.5625	76.9231	77.2727	77.6119	77.9412	78.2609	78.5714	15
16	73.7705	74.1935	74.6032	75.0000	75.3846	75.7576	76.1194	76.4706	76.8116	77.1429	16
17	72.1311	72.5806	73.0159	73.4375	73.8462	74.2424	74.6269	75.0000	75.3623	75.7143	17
18	70.4918	70.9677	71.4286	71.8750	72.3077	72.7273	73.1343	73.5294	73.9130	74.2857	18
19	68.8525	69.3548	69.8413	70.3125	70.7692	71.2121	71.6418	72.0588	72.4638	72.8571	19
20	67.2131	67.7419	68.2540	68.7500	69.2308	69.6970	70.1493	70.5882	71.0145	71.4286	20
21	65.5738	66.1290	66.6667	67.1875	67.6923	68.1818	68.6567	69.1176	69.5652	70.0000	21
22	63.9344	64.5161	65.0794	65.6250	66.1538	66.6667	67.1642	67.6471	68.1159	68.5714	22
23	62.2951	62.9032	63.4921	64.0625	64.6154	65.1515	65.6716	66.1765	66.6667	67.1429	23
24	60.6557	61.2903	61.9048	62.5000	63.0769	63.6364	64.1791	64.7059	65.2174	65.7143	24
25	59.0164	59.6774	60.3175	60.9375	61.5385	62.1212	62.6866	63.2353	63.7681	64.2857	25
26	57.3770	58.0645	58.7302	59.3750	60.0000	60.6061	61.1940	61.7647	62.3188	62.8571	26
27	55.7377	56.4516	57.1429	57.8125	58.4615	59.0909	59.7015	60.2941	60.8696	61.4286	27
28	54.0984	54.8387	55.5556	56.2500	56.9231	57.5758	58.2090	58.8235	59.4203	60.0000	28
29	52.4590	53.2258	53.9683	54.6875	55.3846	56.0606	56.7164	57.3529	57.9710	58.5714	29
30	50.8197	51.6129	52.3810	53.1250	53.8462	54.5455	55.2239	55.8824	56.5217	57.1429	30
31	49.1803	50.0000	50.7937	51.5625	52.3077	53.0303	53.7313	54.4118	55.0725	55.7143	31
32	47.5410	48.3871	49.2063	50.0000	50.7692	51.5152	52.2388	52.9412	53.6232	54.2857	32
33	45.9016	46.7742	47.6190	48.4375	49.2308	50.0000	50.7463	51.4706	52.1739	52.8571	33
34	44.2623	45.1613	46.0317	46.8750	47.6923	48.4848	49.2537	50.0000	50.7246	51.4286	34
35	42.6229	43.5484	44.4444	45.3125	46.1538	46.9697	47.7612	48.5294	49.2754	50.0000	35
36	40.9836	41.9355	42.8571	43.7500	44.6154	45.4545	46.2687	47.0588	47.8261	48.5714	36
37	39.3443	40.3226	41.2698	42.1875	43.0769	43.9394	44.7761	45.5882	46.3768	47.1429	37
38	37.7049	38.7097	39.6825	40.6250	41.5385	42.4242	43.2836	44.1176	44.9275	45.7143	38
39	36.0656	37.0968	38.0952	39.0625	40.0000	40.9091	41.7910	42.6471	43.4783	44.2857	39
40	34.4262	35.4839	36.5079	37.5000	38.4615	39.3939	40.2985	41.1765	42.0290	42.8571	40
41	32.7869	33.8710	34.9206	35.9375	36.9231	37.8788	38.8060	39.7059	40.5797	41.4286	41
42	31.1475	32.2581	33.3333	34.3750	35.3846	36.3636	37.3134	38.2353	39.1304	40.0000	42
43	29.5082	30.6452	31.7460	32.8125	33.8462	34.8485	35.8209	36.7647	37.6812	38.5714	43
44	27.8689	29.0323	30.1587	31.2500	32.3077	33.3333	34.3284	35.2941	36.2319	37.1429	44
45	26.2295	27.4194	28.5714	29.6875	30.7692	31.8182	32.8358	33.8235	34.7826	35.7143	45
46	24.5902	25.8065	26.9841	28.1250	29.2308	30.3030	31.3433	32.3529	33.3333	34.2857	46
47	22.9508	24.1935	25.3968	26.5625	27.6923	28.7879	29.8507	30.8824	31.8841	32.8571	47
48	21.3115	22.5806	23.8095	25.0000	26.1538	27.2727	28.3582	29.4118	30.4348	31.4286	48
49	19.6721	20.9677	22.2222	23.4375	24.6154	25.7576	26.8657	27.9412	28.9855	30.0000	49
50	18.0328	19.3548	20.6349	21.8750	23.0769	24.2424	25.3731	26.4706	27.5362	28.5714	50
51	16.3934	17.7419	19.0476	20.3125	21.5385	22.7273	23.8806	25.0000	26.0870	27.1429	51
52	14.7541	16.1290	17.4603	18.7500	20.0000	21.2121	22.3881	23.5294	24.6377	25.7143	52
53	13.1148	14.5161	15.8730	17.1875	18.4615	19.6970	20.8955	22.0588	23.1884	24.2857	53
54	11.4754	12.9032	14.2857	15.6250	16.9231	18.1818	19.4030	20.5882	21.7391	22.8571	54
55	9.8361	11.2903	12.6984	14.0625	15.3846	16.6667	17.9104	19.1176	20.2899	21.4286	55
56	8.1967	9.6774	11.1111	12.5000	13.8462	15.1515	16.4179	17.6471	18.8406	20.0000	56
57	6.5574	8.0645	9.5238	10.9375	12.3077	13.6364	14.9254	16.1765	17.3913	18.5714	57
58	4.9180	6.4516	7.9365	9.3750	10.7692	12.1212	13.4328	14.7059	15.9420	17.1429	58
59	3.2787	4.8387	6.3492	7.8125	9.2308	10.6061	11.9403	13.2353	14.4928	15.7143	59
60	1.6393	3.2258	4.7619	6.2500	7.6923	9.0909	10.4478	11.7647	13.0435	14.2857	60

0% INTEREST RATE

Straight-line Method

61–70 Years Probable Life

Age, years	Condition-percent, % 61	62	63	64	65	66	67	68	69	70	Age, years
61	0.0000	1.6129	3.1746	4.6875	6.1538	7.5758	8.9552	10.2941	11.5942	12.8571	61
62		0.0000	1.5873	3.1250	4.6154	6.0606	7.4627	8.8235	10.1449	11.4286	62
63			0.0000	1.5625	3.0769	4.5455	5.9702	7.3529	8.6956	10.0000	63
64				0.0000	1.5385	3.0303	4.4776	5.8824	7.2464	8.5714	64
65					0.0000	1.5152	2.9851	4.4118	5.7971	7.1429	65
66						0.0000	1.4925	2.9412	4.3478	5.7143	66
67							0.0000	1.4706	2.8985	4.2857	67
68								0.0000	1.4493	2.8571	68
69									0.0000	1.4286	69
70										0.0000	70

71–80 Years Probable Life

Age, years	71	72	73	74	75	76	77	78	79	80	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.5915	98.6111	98.6301	98.6486	98.6667	98.6842	98.7013	98.7179	98.7342	98.7500	1
2	97.1831	97.2222	97.2603	97.2973	97.3333	97.3684	97.4026	97.4359	97.4684	97.5000	2
3	95.7746	95.8333	95.8904	95.9459	96.0000	96.0526	96.1039	96.1538	96.2025	96.2500	3
4	94.3662	94.4444	94.5205	94.5946	94.6667	94.7368	94.8052	94.8718	94.9367	95.0000	4
5	92.9577	93.0556	93.1507	93.2432	93.3333	93.4211	93.5065	93.5897	93.6709	93.7500	5
6	91.5493	91.6667	91.7808	91.8919	92.0000	92.1053	92.2078	92.3077	92.4051	92.5000	6
7	90.1408	90.2778	90.4110	90.5405	90.6667	90.7895	90.9091	91.0256	91.1392	91.2500	7
8	88.7324	88.8889	89.0411	89.1892	89.3333	89.4737	89.6104	89.7436	89.8734	90.0000	8
9	87.3239	87.5000	87.6712	87.8378	88.0000	88.1579	88.3117	88.4615	88.6076	88.7500	9
10	85.9155	86.1111	86.3014	86.4865	86.6667	86.8421	87.0130	87.1795	87.3418	87.5000	10
11	84.5070	84.7222	84.9315	85.1351	85.3333	85.5263	85.7143	85.8974	86.0759	86.2500	11
12	83.0986	83.3333	83.5616	83.7838	84.0000	84.2105	84.4156	84.6154	84.8101	85.0000	12
13	81.6901	81.9444	82.1918	82.4324	82.6667	82.8947	83.1169	83.3333	83.5443	83.7500	13
14	80.2817	80.5556	80.8219	81.0811	81.3333	81.5789	81.8182	82.0513	82.2785	82.5000	14
15	78.8732	79.1667	79.4521	79.7297	80.0000	80.2632	80.5195	80.7692	81.0127	81.2500	15
16	77.4648	77.7778	78.0822	78.3784	78.6667	78.9474	79.2208	79.4872	79.7468	80.0000	16
17	76.0563	76.3889	76.7123	77.0270	77.3333	77.6316	77.9221	78.2051	78.4810	78.7500	17
18	74.6479	75.0000	75.3425	75.6757	76.0000	76.3158	76.6234	76.9231	77.2152	77.5000	18
19	73.2394	73.6111	73.9726	74.3243	74.6667	75.0000	75.3247	75.6410	75.9494	76.2500	19
20	71.8310	72.2222	72.6027	72.9730	73.3333	73.6842	74.0260	74.3590	74.6835	75.0000	20
21	70.4225	70.8333	71.2329	71.6216	72.0000	72.3684	72.7273	73.0769	73.4177	73.7500	21
22	69.0141	69.4444	69.8630	70.2703	70.6667	71.0526	71.4286	71.7949	72.1519	72.5000	22
23	67.6056	68.0555	68.4932	68.9189	69.3333	69.7368	70.1299	70.5128	70.8861	71.2500	23
24	66.1972	66.6667	67.1233	67.5676	68.0000	68.4211	68.8312	69.2308	69.6203	70.0000	24
25	64.7887	65.2778	65.7534	66.2162	66.6667	67.1053	67.5325	67.9487	68.3544	68.7500	25
26	63.3803	63.8889	64.3836	64.8649	65.3333	65.7895	66.2338	66.6667	67.0886	67.5000	26
27	61.9718	62.5000	63.0137	63.5135	64.0000	64.4737	64.9351	65.3846	65.8228	66.2500	27
28	60.5634	61.1111	61.6438	62.1622	62.6667	63.1579	63.6364	64.1026	64.5570	65.0000	28
29	59.1549	59.7222	60.2740	60.8108	61.3333	61.8421	62.3377	62.8205	63.2911	63.7500	29
30	57.7465	58.3333	58.9041	59.4595	60.0000	60.5263	61.0390	61.5385	62.0253	62.5000	30
31	56.3380	56.9444	57.5342	58.1081	58.6667	59.2105	59.7403	60.2564	60.7595	61.2500	31
32	54.9296	55.5556	56.1644	56.7568	57.3333	57.8947	58.4416	58.9744	59.4937	60.0000	32
33	53.5211	54.1667	54.7945	55.4054	56.0000	56.5789	57.1429	57.6923	58.2278	58.7500	33
34	52.1127	52.7778	53.4247	54.0541	54.6667	55.2632	55.8442	56.4103	56.9620	57.5000	34
35	50.7042	51.3889	52.0548	52.7027	53.3333	53.9474	54.5455	55.1282	55.6962	56.2500	35
36	49.2958	50.0000	50.6849	51.3513	52.0000	52.6316	53.2468	53.8462	54.4304	55.0000	36
37	47.8873	48.6111	49.3151	50.0000	50.6667	51.3158	51.9481	52.5641	53.1646	53.7500	37
38	46.4789	47.2222	47.9452	48.6486	49.3333	50.0000	50.6494	51.2821	51.8987	52.5000	38
39	45.0704	45.8333	46.5753	47.2973	48.0000	48.6842	49.3506	50.0000	50.6329	51.2500	39
40	43.6620	44.4444	45.2055	45.9459	46.6667	47.3684	48.0519	48.7179	49.3671	50.0000	40
41	42.2535	43.0556	43.8356	44.5946	45.3333	46.0526	46.7532	47.4359	48.1013	48.7500	41
42	40.8451	41.6667	42.4658	43.2432	44.0000	44.7368	45.4545	46.1538	46.8354	47.5000	42
43	39.4366	40.2778	41.0959	41.8919	42.6667	43.4211	44.1558	44.8718	45.5696	46.2500	43
44	38.0282	38.8889	39.7260	40.5405	41.3333	42.1053	42.8571	43.5897	44.3038	45.0000	44
45	36.6197	37.5000	38.3562	39.1892	40.0000	40.7895	41.5584	42.3077	43.0380	43.7500	45

0% INTEREST RATE

Straight-line Method

71–80 Years Probable Life

Age, years	Condition-percent, % 71	72	73	74	75	76	77	78	79	80	Age, years
46	35.2113	36.1111	36.9863	37.8378	38.6667	39.4737	40.2597	41.0256	41.7722	42.5000	46
47	33.8028	34.7222	35.6164	36.4865	37.3333	38.1579	38.9610	39.7436	40.5063	41.2500	47
48	32.3944	33.3333	34.2466	35.1351	36.0000	36.8421	37.6623	38.4615	39.2405	40.0000	48
49	30.9859	31.9444	32.8767	33.7838	34.6667	35.5263	36.3636	37.1795	37.9747	38.7500	49
50	29.5775	30.5556	31.5069	32.4324	33.3333	34.2105	35.0649	35.8974	36.7089	37.5000	50
51	28.1690	29.1667	30.1370	31.0811	32.0000	32.8947	33.7662	34.6154	35.4430	36.2500	51
52	26.7606	27.7778	28.7671	29.7297	30.6667	31.5789	32.4675	33.3333	34.1772	35.0000	52
53	25.3521	26.3889	27.3973	28.3784	29.3333	30.2632	31.1688	32.0513	32.9114	33.7500	53
54	23.9437	25.0000	26.0274	27.0270	28.0000	28.9474	29.8701	30.7692	31.6456	32.5000	54
55	22.5352	23.6111	24.6575	25.6757	26.6667	27.6316	28.5714	29.4872	30.3797	31.2500	55
56	21.1268	22.2222	23.2877	24.3243	25.3333	26.3158	27.2727	28.2051	29.1139	30.0000	56
57	19.7183	20.8333	21.9178	22.9730	24.0000	25.0000	25.9740	26.9231	27.8481	28.7500	57
58	18.3099	19.4444	20.5479	21.6216	22.6667	23.6842	24.6753	25.6410	26.5823	27.5000	58
59	16.9014	18.0556	19.1781	20.2703	21.3333	22.3684	23.3766	24.3590	25.3165	26.2500	59
60	15.4930	16.6667	17.8082	18.9189	20.0000	21.0526	22.0779	23.0769	24.0506	25.0000	60
61	14.0845	15.2778	16.4384	17.5676	18.6667	19.7368	20.7792	21.7949	22.7848	23.7500	61
62	12.6761	13.8889	15.0685	16.2162	17.3333	18.4211	19.4805	20.5128	21.5190	22.5000	62
63	11.2676	12.5000	13.6986	14.8649	16.0000	17.1053	18.1818	19.2308	20.2532	21.2500	63
64	9.8592	11.1111	12.3288	13.5135	14.6667	15.7895	16.8831	17.9487	18.9873	20.0000	64
65	8.4507	9.7222	10.9589	12.1622	13.3333	14.4737	15.5844	16.6667	17.7215	18.7500	65
66	7.0423	8.3333	9.5890	10.8108	12.0000	13.1579	14.2857	15.3846	16.4557	17.5000	66
67	5.6338	6.9444	8.2192	9.4595	10.6667	11.8421	12.9870	14.1026	15.1899	16.2500	67
68	4.2254	5.5556	6.8493	8.1081	9.3333	10.5263	11.6883	12.8205	13.9240	15.0000	68
69	2.8169	4.1667	5.4795	6.7568	8.0000	9.2105	10.3896	11.5385	12.6582	13.7500	69
70	1.4085	2.7778	4.1096	5.4054	6.6667	7.8947	9.0909	10.2564	11.3924	12.5000	70
71	0.0000	1.3889	2.7397	4.0541	5.3333	6.5789	7.7922	8.9744	10.1266	11.2500	71
72		0.0000	1.3699	2.7027	4.0000	5.2632	6.4935	7.6923	8.8608	10.0000	72
73			0.0000	1.3513	2.6667	3.9474	5.1948	6.4103	7.5949	8.7500	73
74				0.0000	1.3333	2.6316	3.8961	5.1282	6.3291	7.5000	74
75					0.0000	1.3158	2.5974	3.8462	5.0633	6.2500	75
76						0.0000	1.2987	2.5641	3.7975	5.0000	76
77							0.0000	1.2820	2.5316	3.7500	77
78								0.0000	1.2658	2.5000	78
79									0.0000	1.2500	79
80										0.0000	80

81–90 Years Probable Life

Age, years	81	82	83	84	85	86	87	88	89	90	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.7654	98.7805	98.7952	98.8095	98.8235	98.8372	98.8506	98.8636	98.8764	98.8889	1
2	97.5309	97.5610	97.5904	97.6190	97.6471	97.6744	97.7011	97.7273	97.7528	97.7778	2
3	96.2963	96.3415	96.3855	96.4286	96.4706	96.5116	96.5517	96.5909	96.6292	96.6667	3
4	95.0617	95.1220	95.1807	95.2381	95.2941	95.3488	95.4023	95.4545	95.5056	95.5556	4
5	93.8272	93.9024	93.9759	94.0476	94.1176	94.1860	94.2529	94.3182	94.3820	94.4444	5
6	92.5926	92.6829	92.7711	92.8571	92.9412	93.0233	93.1034	93.1818	93.2584	93.3333	6
7	91.3580	91.4634	91.5663	91.6667	91.7647	91.8605	91.9540	92.0455	92.1348	92.2222	7
8	90.1235	90.2439	90.3614	90.4762	90.5882	90.6977	90.8046	90.9091	91.0112	91.1111	8
9	88.8889	89.0244	89.1566	89.2857	89.4118	89.5349	89.6552	89.7727	89.8876	90.0000	9
10	87.6543	87.8049	87.9518	88.0952	88.2353	88.3721	88.5057	88.6364	88.7640	88.8889	10
11	86.4198	86.5854	86.7470	86.9048	87.0588	87.2093	87.3563	87.5000	87.6404	87.7778	11
12	85.1852	85.3659	85.5422	85.7143	85.8824	86.0465	86.2069	86.3636	86.5169	86.6667	12
13	83.9506	84.1463	84.3373	84.5238	84.7059	84.8837	85.0575	85.2273	85.3933	85.5556	13
14	82.7160	82.9268	83.1325	83.3333	83.5294	83.7209	83.9080	84.0909	84.2697	84.4444	14
15	81.4815	81.7073	81.9277	82.1429	82.3529	82.5581	82.7586	82.9545	83.1461	83.3333	15
16	80.2469	80.4878	80.7229	80.9524	81.1765	81.3953	81.6092	81.8182	82.0225	82.2222	16
17	79.0123	79.2683	79.5181	79.7619	80.0000	80.2326	80.4598	80.6818	80.8989	81.1111	17
18	77.7778	78.0488	78.3133	78.5714	78.8235	79.0698	79.3103	79.5455	79.7753	80.0000	18
19	76.5432	76.8293	77.1084	77.3810	77.6471	77.9070	78.1609	78.4091	78.6517	78.8889	19
20	75.3086	75.6098	75.9036	76.1905	76.4706	76.7442	77.0115	77.2727	77.5281	77.7778	20

0% INTEREST RATE

Straight-line Method

81–90 Years Probable Life

Age, years	Condition-percent, %										Age, years
	81	82	83	84	85	86	87	88	89	90	
21	74.0741	74.3902	74.6988	75.0000	75.2941	75.5814	75.8621	76.1364	76.4045	76.6667	21
22	72.8395	73.1707	73.4940	73.8095	74.1176	74.4186	74.7126	75.0000	75.2809	75.5556	22
23	71.6049	71.9512	72.2892	72.6190	72.9412	73.2558	73.5632	73.8636	74.1573	74.4444	23
24	70.3704	70.7317	71.0843	71.4286	71.7647	72.0930	72.4138	72.7273	73.0337	73.3333	24
25	69.1358	69.5122	69.8795	70.2381	70.5882	70.9302	71.2644	71.5909	71.9101	72.2222	25
26	67.9012	68.2927	68.6747	69.0476	69.4118	69.7674	70.1149	70.4545	70.7865	71.1111	26
27	66.6667	67.0732	67.4699	67.8571	68.2353	68.6047	68.9655	69.3182	69.6629	70.0000	27
28	65.4321	65.8537	66.2651	66.6667	67.0588	67.4419	67.8161	68.1818	68.5393	68.8889	28
29	64.1975	64.6341	65.0602	65.4762	65.8824	66.2791	66.6667	67.0455	67.4157	67.7778	29
30	62.9630	63.4146	63.8554	64.2857	64.7059	65.1163	65.5172	65.9091	66.2921	66.6667	30
31	61.7284	62.1951	62.6506	63.0952	63.5294	63.9535	64.3678	64.7727	65.1685	65.5556	31
32	60.4938	60.9756	61.4458	61.9048	62.3529	62.7907	63.2184	63.6364	64.0449	64.4444	32
33	59.2593	59.7561	60.2410	60.7143	61.1765	61.6279	62.0690	62.5000	62.9213	63.3333	33
34	58.0247	58.5366	59.0361	59.5238	60.0000	60.4651	60.9195	61.3636	61.7978	62.2222	34
35	56.7901	57.3171	57.8313	58.3333	58.8235	59.3023	59.7701	60.2273	60.6742	61.1111	35
36	55.5556	56.0976	56.6265	57.1429	57.6471	58.1395	58.6207	59.0909	59.5506	60.0000	36
37	54.3210	54.8780	55.4217	55.9524	56.4706	56.9767	57.4713	57.9545	58.4270	58.8889	37
38	53.0864	53.6585	54.2169	54.7619	55.2941	55.8140	56.3218	56.8182	57.3034	57.7778	38
39	51.8519	52.4390	53.0120	53.5714	54.1176	54.6512	55.1724	55.6818	56.1798	56.6667	39
40	50.6173	51.2195	51.8072	52.3810	52.9412	53.4884	54.0230	54.5455	55.0562	55.5556	40
41	49.3827	50.0000	50.6024	51.1905	51.7647	52.3256	52.8736	53.4091	53.9326	54.4444	41
42	48.1481	48.7805	49.3976	50.0000	50.5882	51.1628	51.7241	52.2727	52.8090	53.3333	42
43	46.9136	47.5610	48.1928	48.8095	49.4118	50.0000	50.5747	51.1364	51.6854	52.2222	43
44	45.6790	46.3415	46.9880	47.6190	48.2353	48.8372	49.4253	50.0000	50.5618	51.1111	44
45	44.4444	45.1220	45.7831	46.4286	47.0588	47.6744	48.2759	48.8636	49.4382	50.0000	45
46	43.2099	43.9024	44.5783	45.2381	45.8824	46.5116	47.1264	47.7273	48.3146	48.8889	46
47	41.9753	42.6829	43.3735	44.0476	44.7059	45.3488	45.9770	46.5909	47.1910	47.7778	47
48	40.7407	41.4634	42.1687	42.8571	43.5294	44.1860	44.8276	45.4545	46.0674	46.6667	48
49	39.5062	40.2439	40.9639	41.6667	42.3529	43.0233	43.6782	44.3182	44.9438	45.5556	49
50	38.2716	39.0244	39.7590	40.4762	41.1765	41.8605	42.5287	43.1818	43.8202	44.4444	50
51	37.0370	37.8049	38.5542	39.2857	40.0000	40.6977	41.3793	42.0455	42.6966	43.3333	51
52	35.8025	36.5854	37.3494	38.0952	38.8235	39.5349	40.2299	40.9091	41.5730	42.2222	52
53	34.5679	35.3659	36.1446	36.9048	37.6471	38.3721	39.0805	39.7727	40.4494	41.1111	53
54	33.3333	34.1463	34.9398	35.7143	36.4706	37.2093	37.9310	38.6364	39.3258	40.0000	54
55	32.0988	32.9268	33.7349	34.5238	35.2941	36.0465	36.7816	37.5000	38.2022	38.8889	55
56	30.8642	31.7073	32.5301	33.3333	34.1176	34.8837	35.6322	36.3636	37.0787	37.7778	56
57	29.6296	30.4878	31.3253	32.1429	32.9412	33.7209	34.4828	35.2273	35.9551	36.6667	57
58	28.3951	29.2683	30.1205	30.9524	31.7647	32.5581	33.3333	34.0909	34.8315	35.5556	58
59	27.1605	28.0488	28.9157	29.7619	30.5882	31.3953	32.1839	32.9545	33.7079	34.4444	59
60	25.9259	26.8293	27.7108	28.5714	29.4118	30.2326	31.0345	31.8182	32.5843	33.3333	60
61	24.6914	25.6098	26.5060	27.3810	28.2353	29.0698	29.8851	30.6818	31.4607	32.2222	61
62	23.4568	24.3902	25.3012	26.1905	27.0588	27.9070	28.7356	29.5455	30.3371	31.1111	62
63	22.2222	23.1707	24.0964	25.0000	25.8824	26.7442	27.5862	28.4091	29.2135	30.0000	63
64	20.9877	21.9512	22.8916	23.8095	24.7059	25.5814	26.4368	27.2727	28.0899	28.8889	64
65	19.7531	20.7317	21.6867	22.6190	23.5294	24.4186	25.2874	26.1364	26.9663	27.7778	65
66	18.5185	19.5122	20.4819	21.4286	22.3529	23.2558	24.1379	25.0000	25.8427	26.6667	66
67	17.2840	18.2927	19.2771	20.2381	21.1765	22.0930	22.9885	23.8636	24.7191	25.5556	67
68	16.0494	17.0732	18.0723	19.0476	20.0000	20.9302	21.8391	22.7273	23.5955	24.4444	68
69	14.8148	15.8537	16.8675	17.8571	18.8235	19.7674	20.6897	21.5909	22.4719	23.3333	69
70	13.5802	14.6341	15.6626	16.6667	17.6471	18.6047	19.5402	20.4545	21.3483	22.2222	70
71	12.3457	13.4146	14.4578	15.4762	16.4706	17.4419	18.3908	19.3182	20.2247	21.1111	71
72	11.1111	12.1951	13.2530	14.2857	15.2941	16.2791	17.2414	18.1818	19.1011	20.0000	72
73	9.8765	10.9756	12.0482	13.0952	14.1176	15.1163	16.0920	17.0455	17.9775	18.8889	73
74	8.6420	9.7561	10.8434	11.9048	12.9412	13.9535	14.9425	15.9091	16.8539	17.7778	74
75	7.4074	8.5366	9.6386	10.7143	11.7647	12.7907	13.7931	14.7727	15.7303	16.6667	75
76	6.1728	7.3171	8.4337	9.5238	10.5882	11.6279	12.6437	13.6364	14.6067	15.5556	76
77	4.9383	6.0976	7.2289	8.3333	9.4118	10.4651	11.4943	12.5000	13.4831	14.4444	77
78	3.7037	4.8780	6.0241	7.1429	8.2353	9.3023	10.3448	11.3636	12.3596	13.3333	78
79	2.4691	3.6585	4.8193	5.9524	7.0588	8.1395	9.1954	10.2273	11.2360	12.2222	79
80	1.2346	2.4390	3.6145	4.7619	5.8824	6.9767	8.0460	9.0909	10.1124	11.1111	80

0% INTEREST RATE

Straight-line Method

81–90 Years Probable Life

Age, years	Condition-percent, %										Age, years
	81	82	83	84	85	86	87	88	89	90	
81	0.0000	1.2195	2.4096	3.5714	4.7059	5.8140	6.8966	7.9545	8.9888	10.0000	81
82		0.0000	1.2048	2.3810	3.5294	4.6512	5.7471	6.8182	7.8652	8.8889	82
83			0.0000	1.1905	2.3529	3.4884	4.5977	5.6818	6.7416	7.7778	83
84				0.0000	1.1765	2.3256	3.4483	4.5455	5.6180	6.6667	84
85					0.0000	1.1628	2.2988	3.4091	4.4944	5.5556	85
86						0.0000	1.1494	2.2727	3.3708	4.4444	86
87							0.0000	1.1364	2.2472	3.3333	87
88								0.0000	1.1236	2.2222	88
89									0.0000	1.1111	89
90										0.0000	90

91–100 Years Probable Life

Age, years	91	92	93	94	95	96	97	98	99	100	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.9011	98.9130	98.9247	98.9362	98.9474	98.9583	98.9691	98.9796	98.9899	99.0000	1
2	97.8022	97.8261	97.8495	97.8723	97.8947	97.9167	97.9381	97.9592	97.9798	98.0000	2
3	96.7033	96.7391	96.7742	96.8085	96.8421	96.8750	96.9072	96.9388	96.9697	97.0000	3
4	95.6044	95.6522	95.6989	95.7447	95.7895	95.8333	95.8763	95.9184	95.9596	96.0000	4
5	94.5055	94.5652	94.6237	94.6809	94.7368	94.7917	94.8454	94.8980	94.9495	95.0000	5
6	93.4066	93.4783	93.5484	93.6170	93.6842	93.7500	93.8144	93.8776	93.9394	94.0000	6
7	92.3077	92.3913	92.4731	92.5532	92.6316	92.7083	92.7835	92.8571	92.9293	93.0000	7
8	91.2088	91.3043	91.3978	91.4894	91.5789	91.6667	91.7526	91.8367	91.9192	92.0000	8
9	90.1099	90.2174	90.3226	90.4255	90.5263	90.6250	90.7216	90.8163	90.9091	91.0000	9
10	89.0110	89.1304	89.2473	89.3617	89.4737	89.5833	89.6907	89.7959	89.8990	90.0000	10
11	87.9121	88.0435	88.1720	88.2979	88.4211	88.5417	88.6598	88.7755	88.8889	89.0000	11
12	86.8132	86.9565	87.0968	87.2340	87.3684	87.5000	87.6289	87.7551	87.8788	88.0000	12
13	85.7143	85.8696	86.0215	86.1702	86.3158	86.4583	86.5979	86.7347	86.8687	87.0000	13
14	84.6154	84.7826	84.9462	85.1064	85.2632	85.4167	85.5670	85.7143	85.8586	86.0000	14
15	83.5165	83.6957	83.8710	84.0426	84.2105	84.3750	84.5361	84.6939	84.8485	85.0000	15
16	82.4176	82.6087	82.7957	82.9787	83.1579	83.3333	83.5052	83.6735	83.8384	84.0000	16
17	81.3187	81.5217	81.7204	81.9149	82.1053	82.2917	82.4742	82.6531	82.8283	83.0000	17
18	80.2198	80.4348	80.6452	80.8511	81.0526	81.2500	81.4433	81.6327	81.8182	82.0000	18
19	79.1209	79.3478	79.5699	79.7872	80.0000	80.2083	80.4124	80.6122	80.8081	81.0000	19
20	78.0220	78.2609	78.4946	78.7234	78.9474	79.1667	79.3814	79.5918	79.7980	80.0000	20
21	76.9231	77.1739	77.4194	77.6596	77.8947	78.1250	78.3505	78.5714	78.7879	79.0000	21
22	75.8242	76.0870	76.3441	76.5957	76.8421	77.0833	77.3196	77.5510	77.7778	78.0000	22
23	74.7253	75.0000	75.2688	75.5319	75.7895	76.0417	76.2887	76.5306	76.7677	77.0000	23
24	73.6264	73.9130	74.1935	74.4681	74.7368	75.0000	75.2577	75.5102	75.7576	76.0000	24
25	72.5275	72.8261	73.1183	73.4043	73.6842	73.9583	74.2268	74.4898	74.7475	75.0000	25
26	71.4286	71.7391	72.0430	72.3404	72.6316	72.9167	73.1959	73.4694	73.7374	74.0000	26
27	70.3297	70.6522	70.9677	71.2766	71.5789	71.8750	72.1649	72.4490	72.7273	73.0000	27
28	69.2308	69.5652	69.8925	70.2128	70.5263	70.8333	71.1340	71.4286	71.7172	72.0000	28
29	68.1319	68.4783	68.8172	69.1489	69.4737	69.7917	70.1031	70.4082	70.7071	71.0000	29
30	67.0330	67.3913	67.7419	68.0851	68.4211	68.7500	69.0722	69.3878	69.6970	70.0000	30
31	65.9341	66.3043	66.6667	67.0213	67.3684	67.7083	68.0412	68.3673	68.6869	69.0000	31
32	64.8352	65.2174	65.5914	65.9574	66.3158	66.6667	67.0103	67.3469	67.6768	68.0000	32
33	63.7363	64.1304	64.5161	64.8936	65.2632	65.6250	65.9794	66.3265	66.6667	67.0000	33
34	62.6374	63.0435	63.4409	63.8298	64.2105	64.5833	64.9485	65.3061	65.6566	66.0000	34
35	61.5385	61.9565	62.3656	62.7660	63.1579	63.5417	63.9175	64.2857	64.6465	65.0000	35
36	60.4396	60.8696	61.2903	61.7021	62.1053	62.5000	62.8866	63.2653	63.6364	64.0000	36
37	59.3407	59.7826	60.2151	60.6383	61.0526	61.4583	61.8557	62.2449	62.6263	63.0000	37
38	58.2418	58.6957	59.1398	59.5745	60.0000	60.4167	60.8247	61.2245	61.6162	62.0000	38
39	57.1429	57.6087	58.0645	58.5106	58.9474	59.3750	59.7938	60.2041	60.6061	61.0000	39
40	56.0440	56.5217	56.9892	57.4468	57.8947	58.3333	58.7629	59.1837	59.5960	60.0000	40
41	54.9451	55.4348	55.9140	56.3830	56.8421	57.2917	57.7320	58.1633	58.5859	59.0000	41
42	53.8462	54.3478	54.8387	55.3191	55.7895	56.2500	56.7010	57.1429	57.5758	58.0000	42
43	52.7473	53.2609	53.7634	54.2553	54.7368	55.2083	55.6701	56.1224	56.5657	57.0000	43
44	51.6484	52.1739	52.6882	53.1915	53.6842	54.1667	54.6392	55.1020	55.5556	56.0000	44
45	50.5495	51.0870	51.6129	52.1277	52.6316	53.1250	53.6082	54.0816	54.5455	55.0000	45

0% INTEREST RATE
Straight-line Method
91–100 Years Probable Life

Age, years	Condition-percent, %										Age, years
	91	92	93	94	95	96	97	98	99	100	
46	49.4505	50.0000	50.5376	51.0638	51.5789	52.0833	52.5773	53.0612	53.5354	54.0000	46
47	48.3516	48.9130	49.4624	50.0000	50.5263	51.0417	51.5464	52.0408	52.5253	53.0000	47
48	47.2527	47.8261	48.3871	48.9362	49.4737	50.0000	50.5155	51.0204	51.5152	52.0000	48
49	46.1538	46.7391	47.3118	47.8723	48.4211	48.9583	49.4845	50.0000	50.5051	51.0000	49
50	45.0549	45.6522	46.2366	46.8085	47.3684	47.9167	48.4536	48.9796	49.4950	50.0000	50
51	43.9560	44.5652	45.1613	45.7447	46.3158	46.8750	47.4227	47.9592	48.4849	49.0000	51
52	42.8571	43.4783	44.0860	44.6809	45.2632	45.8333	46.3918	46.9388	47.4747	48.0000	52
53	41.7582	42.3913	43.0108	43.6170	44.2105	44.7917	45.3608	45.9184	46.4646	47.0000	53
54	40.6593	41.3043	41.9355	42.5532	43.1579	43.7500	44.3299	44.8980	45.4545	46.0000	54
55	39.5604	40.2174	40.8602	41.4894	42.1053	42.7083	43.2990	43.8775	44.4444	45.0000	55
56	38.4615	39.1304	39.7849	40.4255	41.0526	41.6667	42.2680	42.8571	43.4343	44.0000	56
57	37.3626	38.0435	38.7097	39.3617	40.0000	40.6250	41.2371	41.8367	42.4242	43.0000	57
58	36.2637	36.9565	37.6344	38.2979	38.9474	39.5833	40.2062	40.8163	41.4141	42.0000	58
59	35.1648	35.8696	36.5591	37.2340	37.8947	38.5417	39.1753	39.7959	40.4040	41.0000	59
60	34.0659	34.7826	35.4839	36.1702	36.8421	37.5000	38.1443	38.7755	39.3939	40.0000	60
61	32.9670	33.6957	34.4086	35.1064	35.7895	36.4583	37.1134	37.7551	38.3838	39.0000	61
62	31.8681	32.6087	33.3333	34.0426	34.7368	35.4167	36.0825	36.7347	37.3737	38.0000	62
63	30.7692	31.5217	32.2581	32.9787	33.6842	34.3750	35.0515	35.7143	36.3636	37.0000	63
64	29.6703	30.4348	31.1828	31.9149	32.6316	33.3333	34.0206	34.6939	35.3535	36.0000	64
65	28.5714	29.3478	30.1075	30.8511	31.5789	32.2917	32.9897	33.6735	34.3434	35.0000	65
66	27.4725	28.2609	29.0323	29.7872	30.5263	31.2500	31.9588	32.6531	33.3333	34.0000	66
67	26.3736	27.1739	27.9570	28.7234	29.4737	30.2083	30.9278	31.6327	32.3232	33.0000	67
68	25.2747	26.0870	26.8817	27.6596	28.4211	29.1667	29.8969	30.6122	31.3131	32.0000	68
69	24.1758	25.0000	25.8065	26.5957	27.3684	28.1250	28.8660	29.5918	30.3030	31.0000	69
70	23.0769	23.9130	24.7312	25.5319	26.3158	27.0833	27.8351	28.5714	29.2929	30.0000	70
71	21.9780	22.8261	23.6559	24.4681	25.2632	26.0417	26.8041	27.5510	28.2828	29.0000	71
72	20.8791	21.7391	22.5806	23.4043	24.2105	25.0000	25.7732	26.5306	27.2727	28.0000	72
73	19.7802	20.6522	21.5054	22.3404	23.1579	23.9583	24.7423	25.5102	26.2626	27.0000	73
74	18.6813	19.5652	20.4301	21.2766	22.1053	22.9167	23.7113	24.4898	25.2525	26.0000	74
75	17.5824	18.4783	19.3548	20.2128	21.0526	21.8750	22.6804	23.4694	24.2424	25.0000	75
76	16.4835	17.3913	18.2796	19.1489	20.0000	20.8333	21.6495	22.4490	23.2323	24.0000	76
77	15.3846	16.3043	17.2043	18.0851	18.9474	19.7917	20.6186	21.4286	22.2222	23.0000	77
78	14.2857	15.2174	16.1290	17.0213	17.8947	18.7500	19.5876	20.4082	21.2121	22.0000	78
79	13.1868	14.1304	15.0538	15.9574	16.8421	17.7083	18.5567	19.3878	20.2020	21.0000	79
80	12.0879	13.0435	13.9785	14.8936	15.7895	16.6667	17.5258	18.3673	19.1919	20.0000	80
81	10.9890	11.9565	12.9032	13.8298	14.7368	15.6250	16.4948	17.3469	18.1818	19.0000	81
82	9.8901	10.8696	11.8280	12.7660	13.6842	14.5833	15.4639	16.3265	17.1717	18.0000	82
83	8.7912	9.7826	10.7527	11.7021	12.6316	13.5417	14.4330	15.3061	16.1616	17.0000	83
84	7.6923	8.6957	9.6774	10.6383	11.5789	12.5000	13.4021	14.2857	15.1515	16.0000	84
85	6.5934	7.6087	8.6022	9.5745	10.5263	11.4583	12.3711	13.2653	14.1414	15.0000	85
86	5.4945	6.5217	7.5269	8.5106	9.4737	10.4167	11.3402	12.2449	13.1313	14.0000	86
87	4.3956	5.4348	6.4516	7.4468	8.4211	9.3750	10.3093	11.2245	12.1212	13.0000	87
88	3.2967	4.3478	5.3763	6.3830	7.3684	8.3333	9.2784	10.2041	11.1111	12.0000	88
89	2.1978	3.2609	4.3011	5.3191	6.3158	7.2917	8.2474	9.1837	10.1010	11.0000	89
90	1.0989	2.1739	3.2258	4.2553	5.2632	6.2500	7.2165	8.1633	9.0909	10.0000	90
91	0.0000	1.0870	2.1505	3.1915	4.2105	5.2083	6.1856	7.1429	8.0808	9.0000	91
92		0.0000	1.0753	2.1277	3.1579	4.1667	5.1546	6.1224	7.0707	8.0000	92
93			0.0000	1.0638	2.1053	3.1250	4.1237	5.1020	6.0606	7.0000	93
94				0.0000	1.0526	2.0833	3.0928	4.0816	5.0505	6.0000	94
95					0.0000	1.0417	2.0619	3.0612	4.0404	5.0000	95
96						0.0000	1.0309	2.0408	3.0303	4.0000	96
97							0.0000	1.0204	2.0202	3.0000	97
98								0.0000	1.0101	2.0000	98
99									0.0000	1.0000	99
100										0.0000	100

2% INTEREST RATE

1–10 Years Probable Life

Age, years	Condition-percent, %										Age, years
	1	2	3	4	5	6	7	8	9	10	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	0.0000	50.4950	67.3245	75.7376	80.7841	84.1474	86.5488	88.3490	89.7485	90.8673	1
2		0.0000	33.9956	50.9900	61.1840	67.9778	72.8286	76.4650	79.2919	81.5520	2
3			0.0000	25.7474	41.1918	51.4848	58.8340	64.3433	68.6262	72.0504	3
4				0.0000	20.7998	34.6619	44.5594	51.9792	57.7472	62.3588	4
5					0.0000	17.5025	29.9994	39.3678	46.6506	52.4733	5
6						0.0000	15.1482	26.5042	35.3320	42.3901	6
7							0.0000	13.3833	23.7871	32.1053	7
8								0.0000	12.0113	21.6147	8
9									0.0000	10.9144	9
10										0.0000	10

11–20 Years Probable Life

Age, years	11	12	13	14	15	16	17	18	19	20	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	91.7822	92.5440	93.1882	93.7398	94.2175	94.6350	95.0030	95.3298	95.6218	95.8843	1
2	83.4001	84.9390	86.2401	87.3544	88.3193	89.1627	89.9061	90.5662	91.1561	91.6863	2
3	74.8503	77.1818	79.1531	80.8413	82.3031	83.5809	84.7072	85.7073	86.6010	87.4044	3
4	66.1295	69.2695	71.9243	74.1979	76.1666	77.8875	79.4044	80.7512	81.9549	83.0368	4
5	57.2343	61.1989	64.5509	67.4217	69.9074	72.0803	73.9955	75.6960	77.2158	78.5819	5
6	48.1612	52.9669	57.0301	60.5099	63.5230	66.1569	68.4784	70.5398	72.3819	74.0378	6
7	38.9066	44.5703	49.3589	53.4599	57.0109	60.1150	62.8510	65.2803	67.4514	69.4029	7
8	29.4669	36.0057	41.5342	46.2689	50.3686	53.9523	57.1110	59.9157	62.4222	64.6753	8
9	19.8385	27.2699	33.5531	38.9341	43.5934	47.6663	51.2563	54.4438	57.2925	59.8531	9
10	10.0174	18.3593	25.4123	31.4526	36.6827	41.2546	45.2844	48.8625	52.0602	54.9345	10
11	0.0000	9.2705	17.1087	23.8214	29.6338	34.7147	39.1931	43.1695	46.7232	49.9176	11
12		0.0000	8.6391	16.0377	22.4440	28.0440	32.9800	37.3627	41.2795	44.8002	12
13			0.0000	8.0982	15.1103	21.2398	26.6426	31.4398	35.7269	39.5806	13
14				0.0000	7.6300	14.2996	20.1785	25.3984	30.0633	34.2565	14
15					0.0000	7.2206	13.5851	19.2361	24.2864	28.8260	15
16						0.0000	6.8598	12.9506	18.3939	23.2868	16
17							0.0000	6.5394	12.3836	17.6369	17
18								0.0000	6.2531	11.8730	18
19									0.0000	5.9958	19
20										0.0000	20

21–30 Years Probable Life

Age, years	21	22	23	24	25	26	27	28	29	30	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	96.1215	96.3369	96.5332	96.7129	96.8780	97.0301	97.1707	97.3010	97.4222	97.5350	1
2	92.1655	92.6005	92.9970	93.3600	93.6935	94.0008	94.2848	94.5481	94.7928	95.0207	2
3	88.1303	88.7893	89.3902	89.9401	90.4453	90.9108	91.3412	91.7401	92.1108	92.4561	3
4	84.0144	84.9020	85.7112	86.4518	87.1322	87.7591	88.3387	88.8759	89.3752	89.8403	4
5	79.8162	80.9369	81.9586	82.8937	83.7528	84.5444	85.2762	85.9545	86.5848	87.1721	5
6	75.5341	76.8925	78.1309	79.2645	80.3058	81.2654	82.1524	82.9746	83.7387	84.4505	6
7	71.1663	72.7672	74.2268	75.5627	76.7898	77.9208	78.9661	79.9351	80.8356	81.6746	7
8	66.7112	68.5594	70.2445	71.7868	73.2036	74.5092	75.7161	76.8348	77.8745	78.8430	8
9	62.1669	64.2674	66.1826	67.9355	69.5456	71.0295	72.4011	73.6726	74.8542	75.9549	9
10	57.5318	59.8896	62.0394	64.0071	65.8145	67.4802	69.0199	70.4471	71.7734	73.0090	10
11	52.8039	55.4243	57.8134	60.0001	62.0087	63.8599	65.5709	67.1570	68.6310	70.0042	11
12	47.9815	50.8696	53.5028	55.9130	58.1269	60.1671	62.0531	63.8012	65.4258	66.9393	12
13	43.0627	46.2239	49.1061	51.7441	54.1674	56.4005	58.4648	60.3783	62.1565	63.8131	13
14	38.0454	41.4852	44.6214	47.4919	50.1287	52.5586	54.8048	56.8869	58.8218	60.6244	14
15	32.9279	36.6518	40.0470	43.1546	46.0092	48.6399	51.0716	53.3256	55.4204	57.3719	15

2% INTEREST RATE

21–30 Years Probable Life

Age, years	Condition-percent, % 21	22	23	24	25	26	27	28	29	30	Age, years
16	27.7080	31.7217	35.3811	38.7306	41.8073	44.6428	47.2637	49.6932	51.9510	54.0543	16
17	22.3836	26.6930	30.6220	34.2181	37.5214	40.5657	43.3797	45.9881	48.4122	50.6704	17
18	16.9528	21.5637	25.7676	29.6154	33.1498	36.4071	39.4180	42.2089	44.8026	47.2188	18
19	11.4134	16.3318	20.8161	24.9206	28.6908	32.1653	35.3770	38.3541	41.1208	43.6982	19
20	5.7632	10.9953	15.7656	20.1319	24.1425	27.8387	31.2552	34.4222	37.3654	40.1072	20
21	0.0000	5.5521	10.6141	15.2474	19.5033	23.4255	27.0510	30.4117	33.5348	36.4443	21
22		0.0000	5.3596	10.2652	14.7714	18.9241	22.7627	26.3209	29.6277	32.7082	22
23			0.0000	5.1834	9.9448	14.3327	18.3887	22.1484	25.6424	28.8974	23
24				0.0000	5.0216	9.6494	13.9272	17.8924	21.5774	25.0104	24
25					0.0000	4.8725	9.3764	13.5513	17.4311	21.0456	25
26						0.0000	4.7346	9.1233	13.2019	17.0015	26
27							0.0000	4.6068	8.8881	12.8765	27
28								0.0000	4.4881	8.6691	28
29									0.0000	4.3774	29
30										0.0000	30

31–40 Years Probable Life

Age, years	31	32	33	34	35	36	37	38	39	40	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	97.6404	97.7389	97.8313	97.9181	97.9998	98.0767	98.1493	98.2179	98.2829	98.3444	1
2	95.2335	95.4327	95.6193	95.7946	95.9596	96.1150	96.2616	96.4002	96.5314	96.6557	2
3	92.7786	93.0802	93.3631	93.6287	93.8785	94.1140	94.3362	94.5462	94.7449	94.9333	3
4	90.2745	90.6808	91.0617	91.4194	91.7559	92.0730	92.3722	92.6551	92.9227	93.1764	4
5	87.7204	88.2333	88.7142	89.1659	89.5908	89.9911	90.3690	90.7261	91.0641	91.3843	5
6	85.1151	85.7370	86.3199	86.8673	87.3824	87.8677	88.3257	88.7586	89.1682	89.5564	6
7	82.4578	83.1906	83.8776	84.5228	85.1298	85.7018	86.2415	86.7517	87.2345	87.6920	7
8	79.7473	80.5934	81.3865	82.1314	82.8322	83.4925	84.1157	84.7047	85.2621	85.7903	8
9	76.9826	77.9442	78.8456	79.6922	80.4886	81.2391	81.9473	82.6167	83.2502	83.8505	9
10	74.1627	75.2420	76.2539	77.2041	78.0981	78.9406	79.7356	80.4870	81.1981	81.8719	10
11	71.2863	72.4858	73.6103	74.6664	75.6599	76.5961	77.4796	78.3147	79.1049	79.8538	11
12	68.3524	69.6744	70.9138	72.0778	73.1729	74.2047	75.1785	76.0989	76.9699	77.7953	12
13	65.3598	66.8069	68.1635	69.4375	70.6361	71.7655	72.8314	73.8388	74.7922	75.6956	13
14	62.3073	63.8820	65.3581	66.7444	68.0486	69.2776	70.4374	71.5335	72.5709	73.5539	14
15	59.1938	60.8985	62.4966	63.9974	65.4093	66.7398	67.9955	69.1821	70.3052	71.3695	15
16	56.0181	57.8554	59.5779	61.1955	62.7173	64.1513	65.5047	66.7837	67.9942	69.1413	16
17	52.7788	54.7515	56.6008	58.3375	59.9714	61.5111	62.9641	64.3374	65.6370	66.8685	17
18	49.4748	51.5855	53.5641	55.4224	57.1706	58.8180	60.3727	61.8420	63.2326	64.5503	18
19	46.1046	48.3561	50.4668	52.4490	54.3138	56.0711	57.7295	59.2968	60.7802	62.1858	19
20	42.6671	45.0622	47.3074	49.4161	51.3999	53.2692	55.0334	56.7007	58.2786	59.7739	20
21	39.1608	41.7024	44.0849	46.3226	48.4277	50.4113	52.2834	54.0527	55.7271	57.3138	21
22	35.5844	38.2753	40.7980	43.1671	45.3960	47.4963	49.4784	51.3517	53.1245	54.8045	22
23	31.9364	34.7798	37.4453	39.9486	42.3037	44.5229	46.6173	48.5966	50.4699	52.2450	23
24	28.2155	31.2143	34.0255	36.6657	39.1496	41.4901	43.6989	45.7865	47.7622	49.6343	24
25	24.4202	27.5775	30.5374	33.3172	35.9323	38.3966	40.7222	42.9202	45.0003	46.9714	25
26	20.5490	23.8680	26.9795	29.9017	32.6507	35.2413	37.6860	39.9965	42.1832	44.2553	26
27	16.6003	20.0843	23.3504	26.4178	29.3035	32.0228	34.5890	37.0144	39.3098	41.4848	27
28	12.5727	16.2250	19.6488	22.8643	25.8894	28.7400	31.4301	33.9726	36.3788	38.6589	28
29	8.4645	12.2884	15.8731	19.2397	22.4070	25.3915	28.2081	30.8700	33.3893	35.7766	29
30	4.2742	8.2731	12.0219	15.5427	18.8549	21.9760	24.9215	27.7054	30.3400	32.8365	30
31	0.0000	4.1775	8.0937	11.7716	15.2318	18.4923	21.5693	24.4774	27.2297	29.8377	31
32		0.0000	4.0869	7.9252	11.5362	14.9388	18.1500	21.1849	24.0571	26.7788	32
33			0.0000	4.0018	7.7667	11.3143	14.6623	17.8266	20.8212	23.6588	33
34				0.0000	3.9218	7.6173	11.1049	14.4010	17.5205	20.4764	34
35					0.0000	3.8464	7.4763	10.9070	14.1538	17.2304	35
36						0.0000	3.7752	7.3431	10.7197	13.9194	36
37							0.0000	3.7079	7.2170	10.5422	37
38								0.0000	3.6442	7.0975	38
39									0.0000	3.5839	39
40										0.0000	40

2% INTEREST RATE

41–50 Years Probable Life

Age, years	Condition-percent, %										Age, years
	41	42	43	44	45	46	47	48	49	50	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	1000.000	100.0000	0
1	98.4028	98.4583	98.5110	98.5612	98.6090	98.6547	98.6982	98.7398	98.7796	98.8177	1
2	96.7737	96.8857	96.9922	97.0936	97.1903	97.2824	97.3704	97.4544	97.5348	97.6117	2
3	95.1120	95.2817	95.4431	95.5967	95.7431	95.8827	96.0160	96.1433	96.2651	96.3816	3
4	93.4170	93.6456	93.8630	94.0699	94.2670	94.4550	94.6345	94.8060	94.9700	95.1269	4
5	91.6882	91.9768	92.2512	92.5125	92.7614	92.9988	93.2254	93.4420	93.6490	93.8472	5
6	89.9247	90.2746	90.6073	90.9239	91.2256	91.5134	91.7881	92.0506	92.3016	92.5418	6
7	88.1261	88.5383	88.9304	89.3036	89.6592	89.9983	90.3221	90.6314	90.9272	91.2103	7
8	86.2914	86.7674	87.2200	87.6509	88.0614	88.4530	88.8268	89.1839	89.5254	89.8522	8
9	84.4200	84.9610	85.4754	85.9651	86.4317	86.8767	87.3015	87.7074	88.0955	88.4669	9
10	82.5112	83.1185	83.6959	84.2456	84.7694	85.2689	85.7457	86.2013	86.6370	87.0539	10
11	80.5643	81.2391	81.8809	82.4917	83.0738	83.6289	84.1589	84.6652	85.1493	85.6127	11
12	78.5784	79.3222	80.0295	80.7028	81.3443	81.9562	82.5402	83.0983	83.6319	84.1426	12
13	76.5528	77.3669	78.1411	78.8780	79.5802	80.2499	80.8893	81.5001	82.0842	82.6441	13
14	74.4866	75.3725	76.2149	77.0168	77.7809	78.5096	79.2053	79.8699	80.5055	81.1137	14
15	72.3792	73.3382	74.2502	75.1183	75.9472	76.7345	77.4876	78.2071	78.8952	79.5536	15
16	70.2296	71.2633	72.2462	73.1819	74.0735	74.9238	75.7355	76.5111	77.2527	77.9624	16
17	68.0370	69.1468	70.2022	71.2068	72.1640	73.0769	73.9484	74.7811	75.5773	76.3393	17
18	65.8005	66.9880	68.1172	69.1921	70.2163	71.1931	72.1256	73.0165	73.8685	74.6838	18
19	63.5193	64.7860	65.9906	67.1371	68.2297	69.2716	70.2663	71.2167	72.1255	72.9951	19
20	61.1925	62.5400	63.8214	65.0411	66.2033	67.3117	68.3699	69.3808	70.3476	71.2727	20
21	58.8192	60.2491	61.6088	62.9031	64.1364	65.3126	66.4355	67.5083	68.5341	69.5158	21
22	56.3984	57.9124	59.3520	60.7224	62.0282	63.2735	64.4624	65.5983	66.6844	67.7238	22
23	53.9292	55.5289	57.0501	58.4980	59.8778	61.1937	62.4499	63.6500	64.7977	65.8960	23
24	51.4106	53.0977	54.7021	56.2292	57.6843	59.0722	60.3971	61.6629	62.8733	64.0316	24
25	48.8416	50.6180	52.3071	53.9150	55.4471	56.9083	58.3032	59.6359	60.9104	62.1299	25
26	46.2212	48.0886	49.8643	51.5545	53.1650	54.7011	56.1675	57.5685	58.9082	60.1902	26
27	43.5485	45.5086	47.3725	49.1468	50.8374	52.4498	53.9890	55.4597	56.8659	58.2117	27
28	40.8223	42.8771	44.8310	46.6909	48.4632	50.1535	51.7670	53.3087	54.7828	56.1936	28
29	38.0415	40.1929	42.2386	44.1860	46.0415	47.8112	49.5006	51.1146	52.6581	54.1351	29
30	35.2051	37.4550	39.5944	41.6309	43.5713	45.4221	47.1888	48.8768	50.4909	52.0355	30
31	32.3121	34.6624	36.8973	39.0247	41.0518	42.9852	44.8308	46.5941	48.2803	49.8939	31
32	29.3611	31.8139	34.1462	36.3664	38.4819	40.4995	42.4256	44.2658	46.0255	47.7095	32
33	26.3512	28.9085	31.3402	33.6550	35.8606	37.9642	39.9723	41.8909	43.7256	45.4813	33
34	23.2810	25.9449	28.4780	30.8893	33.1868	35.3781	37.4700	39.4686	41.3797	43.2086	34
35	20.1494	22.9221	25.5586	28.0682	30.4596	32.7404	34.9176	36.9978	38.9869	40.8905	35
36	16.9552	19.8388	22.5808	25.1908	27.6778	30.0498	32.3141	34.4775	36.5463	38.5260	36
37	13.6971	16.6938	19.5434	22.2558	24.8404	27.3054	29.6596	31.9069	34.0568	36.1142	37
38	10.3739	13.4860	16.4452	19.2622	21.9463	24.5062	26.9500	29.2849	31.5175	33.6541	38
39	6.9842	10.2140	13.2852	16.2086	18.9942	21.6510	24.1872	26.6104	28.9275	31.1449	39
40	3.5267	6.8765	10.0619	13.0940	15.9831	18.7387	21.3692	23.8824	26.2857	28.5855	40
41	0.0000	3.4723	6.7741	9.9171	12.9118	15.7681	18.4948	21.0999	23.5910	25.9749	41
42		0.0000	3.4206	6.6766	9.7791	12.7381	15.5629	18.2617	20.8424	23.3120	42
43			0.0000	3.3714	6.5837	9.6475	12.5723	15.3667	18.0389	20.5960	43
44				0.0000	3.3245	6.4952	9.5220	12.4139	15.1792	17.8255	44
45					0.0000	3.2797	6.4106	9.4020	12.2624	14.9997	45
46						0.0000	3.2371	6.3298	9.2873	12.1174	46
47							0.0000	3.1963	6.2526	9.1774	47
48								0.0000	3.1573	6.1787	48
49									0.0000	3.1199	49
50										0.0000	50

2% INTEREST RATE
51–60 Years Probable Life

Age, years	Condition-percent, %										Age, years
	51	52	53	54	55	56	57	58	59	60	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.8541	98.8891	98.9226	98.9548	98.9857	99.0153	99.0439	99.0713	99.0978	99.1232	1
2	97.6854	97.7560	97.8237	97.8886	97.9510	98.0110	98.0686	98.1152	98.1775	98.2289	2
3	96.4932	96.6002	96.7028	96.8012	96.8957	96.9866	97.0739	97.1579	97.2388	97.3167	3
4	95.2772	95.4217	95.5594	95.6920	95.8193	95.9417	96.0592	96.1724	96.2813	96.3862	4
5	94.0369	94.2193	94.3932	94.5606	94.7213	94.8758	95.0243	95.1672	95.3047	95.4371	5
6	92.7718	92.9923	93.2037	93.4066	93.6014	93.7887	93.9687	94.1419	94.3085	94.4691	6
7	91.4814	91.7412	91.9904	92.2295	92.4591	92.6798	92.8919	93.0960	93.2925	93.4816	7
8	90.1652	90.4651	90.7528	91.0289	91.2940	91.5487	91.7937	92.0293	92.2561	92.4745	8
9	88.8226	89.1635	89.4904	89.8042	90.1055	90.3950	90.6734	90.9412	91.1989	91.4472	9
10	87.4532	87.8359	88.2029	88.5551	88.8933	89.2183	89.5308	89.8314	90.1207	90.3993	10
11	86.0564	86.4817	86.8895	87.2809	87.6568	88.0180	88.3652	88.6993	89.0209	89.3305	11
12	84.6317	85.1004	85.5499	85.9813	86.3956	86.7937	87.1764	87.5446	87.8990	88.2403	12
13	83.1785	83.6915	84.1835	84.6557	85.1092	85.5449	85.9638	86.3669	86.7548	87.1283	13
14	81.6962	82.2544	82.7898	83.3036	83.7970	84.2712	84.7270	85.1655	85.5876	85.9941	14
15	80.1842	80.7886	81.3682	81.9245	82.4586	82.9719	83.4654	83.9402	84.3971	84.8372	15
16	78.6421	79.2935	79.9182	80.5177	81.0935	81.6467	82.1786	82.6903	83.1828	83.6571	16
17	77.0690	77.7684	78.4392	79.0829	79.7010	80.2950	80.8661	81.4154	81.9442	82.4535	17
18	75.4646	76.2129	76.9305	77.6193	78.2807	78.9162	79.5273	80.1151	80.6809	81.2257	18
19	73.8280	74.6262	75.3918	76.1265	76.8320	77.5099	78.1617	78.7887	79.3923	79.9735	19
20	72.1587	73.0078	73.8222	74.6038	75.3543	76.0754	76.7688	77.4358	78.0779	78.6961	20
21	70.4560	71.3571	72.2213	73.0506	73.8470	74.6123	75.3481	76.0559	76.7372	77.3933	21
22	68.7193	69.6733	70.5883	71.4664	72.3096	73.1199	73.8989	74.6483	75.3697	76.0643	22
23	66.9478	67.9559	68.9227	69.8505	70.7415	71.5976	72.4208	73.2126	73.9748	74.7088	23
24	65.1409	66.2041	67.2237	68.2023	69.1420	70.0449	70.9130	71.7482	72.5521	73.3262	24
25	63.2979	64.4173	65.4908	66.5211	67.5105	68.4612	69.3752	70.2545	71.1009	71.9159	25
26	61.4180	62.5947	63.7232	64.8063	65.8464	66.8457	67.8066	68.7309	69.6207	70.4775	26
27	59.5005	60.7357	61.9203	63.0572	64.1489	65.1980	66.2065	67.1769	68.1108	69.0102	27
28	57 5446	58.8395	60.0813	61.2731	62.4176	63.5173	64.5746	65.5918	66.5708	67.5136	28
29	55.5497	56.9054	58.2056	59.4533	60.6516	61.8030	62.9100	63.9749	65.0000	65.9871	29
30	53.5148	54.9326	56.2923	57.5972	58.8503	60.0544	61.2120	62.3258	63.3977	64.4300	30
31	51.4393	52.9203	54.3407	55.7039	57.0130	58.2708	59.4802	60.6436	61.7634	62.8418	31
32	49.3222	50.8678	52.3502	53.7728	55.1389	56.4516	57.7137	58.9278	60.0965	61.2219	32
33	47.1628	48.7742	50.3198	51.8030	53.2273	54.5960	55.9118	57.1777	58.3962	59.5695	33
34	44.9602	46.6388	48.2488	49.7938	51.2775	52.7032	54.0739	55.3926	56.6618	57.8841	34
35	42.7135	44.4607	46.1363	47.7445	49.2888	50.7726	52.1993	53.5718	54.8928	56.1650	35
36	40.4219	42.2390	43.9817	45.6541	47.2602	48.8034	50.2872	51.7145	53.0884	54.4115	36
37	38.0845	39.9729	41.7839	43.5220	45.1911	46.7948	48.3368	49.8202	51.2480	52.6229	37
38	35.7004	37.6614	39.5422	41.3472	43.0805	44.7461	46.3474	47.8879	49.3707	50.7986	38
39	33.2685	35.3037	37.2557	39.1289	40.9278	42.6564	44.3182	45.9170	47.4559	48.9378	39
40	30.7880	32.8989	34.9234	36.8663	38.7320	40.5248	42.2485	43.9067	45.5027	47.0397	40
41	28.2579	30.4460	32.5445	34.5584	36.4923	38.3507	40.1373	41.8561	43.5105	45.1037	41
42	25.6772	27.9440	30.1180	32.2043	34.2078	36.1330	37.9839	39.7646	41.4785	43.1290	42
43	23.0449	25.3919	27.6429	29.8032	31.8777	33.8710	35.7875	37.6312	39.4058	41.1148	43
44	20.3600	22.7889	25.1184	27.3540	29.5009	31.5638	33.5471	35.4552	37.2917	39.0603	44
45	17.6213	20.1337	22.5434	24.8559	27.0766	29.2104	31.2619	33.2356	35.1353	36.9647	45
46	14.8279	17.4255	19.9168	22.3077	24.6038	26.8100	28.9311	30.9716	32.9358	34.8272	46
47	11.9786	14.6631	17.2378	19.7087	22.0815	24.3615	26.5536	28.6624	30.6922	32.6469	47
48	9.0723	11.8455	14.5052	17.0576	19.5088	21.8641	24.1285	26.3070	28.4038	30.4231	48
49	6.1079	8.9714	11.7179	14.3536	16.8846	19.3167	21.6550	23.9045	26.0697	28.1548	49
50	3.0842	6.0400	8.8748	11.5954	14.2080	16.7184	19.1319	21.4539	23.6888	25.8410	50
51	0.0000	3.0499	5.9749	8.7821	11.4778	14.0681	16.5585	18.9543	21.2604	23.4811	51
52		0.0000	3.0171	5.9125	8.6930	11.3648	13.9334	16.4047	18.7833	21.0739	52
53			0.0000	2.9855	5.8525	8.6074	11.2560	13.8042	16.2567	18.6186	53
54				0.0000	2.9553	5.7949	8.5250	11.1516	13.6796	16.1141	54
55					0.0000	2.9262	5.7394	8.4459	11.0510	13.5596	55
56						0.0000	2.8982	5.6862	8.3698	10.9540	56
57							0.0000	2.8712	5.6349	8.2963	57
58								0.0000	2.8454	5.5854	58
59									0.0000	2.8204	59
60										0.0000	60

2% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, %										Age, years
	61	62	63	64	65	66	67	68	69	70	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.1477	99.1714	99.1942	99.2161	99.2374	99.2579	99.2777	99.2968	99.3153	99.3332	1
2	98.2784	98.3261	98.3722	98.4166	98.4595	98.5009	98.5409	98.5796	98.6170	98.6531	2
3	97.3917	97.4640	97.5338	97.6011	97.6661	97.7288	97.7894	97.8480	97.9047	97.9594	3
4	96.4872	96.5847	96.6786	96.7693	96.8568	96.9413	97.0229	97.1018	97.1781	97.2519	4
5	95.5647	95.6877	95.8063	95.9208	96.0313	96.1380	96.2410	96.3407	96.4370	96.5301	5
6	94.6237	94.7728	94.9166	95.0553	95.1893	95.3186	95.4435	95.5643	95.6811	95.7940	6
7	93.6639	93.8396	94.0091	94.1726	94.3304	94.4829	94.6301	94.7724	94.9100	95.0431	7
8	92.6849	92.8878	93.0834	93.2722	93.4544	93.6304	93.8004	93.9647	94.1235	94.2772	8
9	91.6863	91.9169	92.1393	92.3538	92.5609	92.7609	92.9541	93.1408	93.3213	93.4960	9
10	90.6678	90.9266	91.1762	91.4170	91.6495	91.8740	92.0908	92.3005	92.5031	92.6991	10
11	89.6289	89.9165	90.1939	90.4615	90.7198	90.9693	91.2103	91.4433	91.6685	91.8863	11
12	88.5692	88.8862	89.1919	89.4869	89.7716	90.0466	90.3122	90.5690	90.8172	91.0573	12
13	87.4883	87.8352	88.1699	88.4928	88.8044	89.1054	89.3962	89.6772	89.9489	90.2117	13
14	86.3858	86.7633	87.1274	87.4787	87.8179	88.1454	88.4618	88.7676	89.0632	89.3491	14
15	85.2612	85.6699	86.0641	86.4445	86.8116	87.1662	87.5087	87.8397	88.1598	88.4694	15
16	84.1141	84.5547	84.9796	85.3895	85.7852	86.1674	86.5365	86.8934	87.2383	87.5720	16
17	82.9442	83.4171	83.8733	84.3134	84.7383	85.1486	85.5450	85.9281	86.2984	86.6567	17
18	81.7508	82.2568	82.7449	83.2159	83.6705	84.1095	84.5335	84.9434	85.3397	85.7230	18
19	80.5335	81.0733	81.5940	82.0963	82.5812	83.0495	83.5019	83.9391	84.3619	84.7707	19
20	79.2919	79.8662	80.4200	80.9544	81.4702	81.9684	82.4496	82.9147	83.3644	83.7994	20
21	78.0254	78.6348	79.2226	79.7896	80.3370	80.8656	81.3763	81.8699	82.3471	82.8086	21
22	76.7337	77.3789	78.0012	78.6016	79.1811	79.7408	80.2815	80.8041	81.3093	81.7980	22
23	75.4161	76.0978	76.7553	77.3897	78.0021	78.5935	79.1648	79.7170	80.2509	80.7672	23
24	74.0721	74.7911	75.4846	76.1537	76.7996	77.4233	78.0258	78.6082	79.1712	79.7158	24
25	72.7013	73.4583	74.1884	74.8929	75.5729	76.2296	76.8640	77.4772	78.0700	78.6433	25
26	71.3030	72.0988	72.8664	73.6069	74.3218	75.0121	75.6790	76.3235	76.9467	77.5494	26
27	69.8768	70.7122	71.5178	72.2952	73.0456	73.7702	74.4702	75.1468	75.8010	76.4336	27
28	68.4221	69.2978	70.1424	70.9572	71.7439	72.5035	73.2373	73.9466	74.6323	75.2955	28
29	66.9382	67.8551	68.7394	69.5925	70.4161	71.2114	71.9797	72.7223	73.4403	74.1347	29
30	65.4247	66.3835	67.3083	68.2005	69.0618	69.8935	70.6970	71.4736	72.2245	72.9506	30
31	63.8809	64.8826	65.8486	66.7807	67.6804	68.5493	69.3887	70.1999	70.9843	71.7429	31
32	62.3063	63.3516	64.3597	65.3324	66.2714	67.1781	68.0541	68.9007	69.7193	70.5110	32
33	60.7001	61.7900	62.8411	63.8552	64.8342	65.7796	66.6929	67.5756	68.4290	69.2544	33
34	59.0618	60.1971	61.2921	62.3485	63.3683	64.3531	65.3044	66.2239	67.1129	67.9727	34
35	57.3908	58.5724	59.7121	60.8116	61.8730	62.8980	63.8882	64.8452	65.7705	66.6654	35
36	55.6863	56.9152	58.1004	59.2440	60.3478	61.4138	62.4437	63.4390	64.4013	65.3320	36
37	53.9478	55.2249	56.4566	57.6450	58.7922	59.9000	60.9702	62.0046	63.0046	63.9719	37
38	52.1745	53.5007	54.7799	56.0140	57.2054	58.3559	59.4673	60.5415	61.5801	62.5845	38
39	50.3657	51.7421	53.0696	54.3504	55.5869	56.7809	57.9343	59.0491	60.1270	61.1694	39
40	48.5207	49.9483	51.3252	52.6536	53.9360	55.1744	56.3707	57.5270	58.6449	59.7261	40
41	46.6389	48.1186	49.5458	50.9228	52.2521	53.5357	54.7758	55.9743	57.1331	58.2538	41
42	44.7194	46.2524	47.7309	49.1574	50.5345	51.8643	53.1490	54.3906	55.5911	56.7521	42
43	42.7615	44.3488	45.8797	47.3567	48.7826	50.1595	51.4896	52.7753	54.0183	55.2204	43
44	40.7644	42.4071	43.9914	45.5200	46.9956	48.4205	49.7971	51.1276	52.4140	53.6581	44
45	38.7274	40.4266	42.0654	43.6465	45.1729	46.6468	48.0707	49.4470	50.7776	52.0645	45
46	36.6497	38.4065	40.1009	41.7356	43.3137	44.8377	46.3099	47.7328	49.1085	50.4390	46
47	34.5304	36.3460	38.0970	39.7865	41.4174	42.9923	44.5137	45.9842	47.4060	48.7810	47
48	32.3687	34.2443	36.0531	37.7983	39.4831	41.1100	42.6817	44.2007	45.6694	47.0909	48
49	30.1638	32.1005	33.9683	35.7705	37.5101	39.1901	40.8130	42.3816	43.8981	45.3649	49
50	27.9148	29.9139	31.8419	33.7020	35.4977	37.2318	38.9070	40.5260	42.0914	43.6054	50
51	25.6209	27.6835	29.6728	31.5922	33.4450	35.2343	36.9628	38.6334	40.2486	41.8108	51
52	23.2810	25.4085	27.4604	29.4402	31.3513	33.1968	34.9797	36.7029	38.3689	39.9802	52
53	20.8943	23.0881	25.2038	27.2452	29.2157	31.1187	32.9570	34.7338	36.4516	38.1130	53
54	18.4599	20.7212	22.9020	25.0062	27.0374	28.9989	30.8938	32.7253	34.4960	36.2085	54
55	15.9769	18.3070	20.5542	22.7225	24.8155	26.8368	28.7894	30.6766	32.5013	34.2659	55
56	13.4441	15.8445	18.1595	20.3930	22.5492	24.6314	26.6429	28.5870	30.4666	32.2845	56
57	10.8607	13.3327	15.7168	18.0171	20.2376	22.3819	24.4534	26.4555	28.3913	30.2634	57
58	8.2257	10.7707	13.2253	15.5935	17.8797	20.0874	22.2201	24.2815	26.2744	28.2019	58
59	5.5379	8.1575	10.6840	13.1216	15.4747	17.7470	19.9422	22.0639	24.1153	26.0992	59
60	2.7964	5.4920	8.0918	10.6001	13.0216	15.3598	17.6187	19.8020	21.9129	23.9544	60

2% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, % 61	62	63	64	65	66	67	68	69	70	Age, years
61	0.0000	2.7732	5.4478	8.0283	10.5194	12.9249	15.2488	17.4949	19.6665	21.7667	61
62		0.0000	2.7508	5.4050	7.9671	10.4413	12.8315	15.1416	17.3752	19.5353	62
63			0.0000	2.7293	5.3638	7.9080	10.3658	12.7413	15.0380	17.2593	63
64				0.0000	2.7085	5.3240	7.8508	10.2929	12.6541	14.9377	64
65					0.0000	2.6884	5.2855	7.7956	10.2225	12.5697	65
66						0.0000	2.6689	5.2484	7.7423	10.1543	66
67							0.0000	2.6502	5.2125	7.6906	67
68								0.0000	2.6321	5.1777	68
69									0.0000	2.6145	69
70										0.0000	70

71–80 Years Probable Life

Age, years	71	72	73	74	75	76	77	78	79	80	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.3506	99.3673	99.3835	99.3993	99.4145	99.4292	99.4436	99.4574	99.4709	99.4839	1
2	98.6881	98.7220	98.7548	98.7865	98.8173	98.8471	98.8760	98.9040	98.9312	98.9575	2
3	98.0124	98.0637	98.1134	98.1615	98.2081	98.2533	98.2970	98.3395	98.3807	98.4206	3
4	97.3232	97.3923	97.4592	97.5240	97.5868	97.6476	97.7065	97.7637	97.8192	97.8730	4
5	96.6203	96.7075	96.7919	96.8737	96.9530	97.0298	97.1042	97.1764	97.2464	97.3143	5
6	95.9032	96.0090	96.1113	96.2105	96.3065	96.3996	96.4899	96.5774	96.6622	96.7446	6
7	95.1718	95.2965	95.4171	95.5340	95.6472	95.7569	95.8632	95.9663	96.0663	96.1634	7
8	94.4258	94.5697	94.7090	94.8439	94.9746	95.1013	95.2240	95.3431	95.4586	95.5706	8
9	93.6649	93.8284	93.9867	94.1400	94.2886	94.4325	94.5721	94.7074	94.8386	94.9659	9
10	92.8888	93.0723	93.2500	93.4221	93.5889	93.7504	93.9071	94.0589	94.2063	94.3492	10
11	92.0971	92.3011	92.4985	92.6898	92.8751	93.0547	93.2288	93.3975	93.5613	93.7201	11
12	91.2896	91.5144	91.7321	91.9429	92.1471	92.3450	92.5369	92.7229	92.9034	93.0784	12
13	90.4659	90.7120	90.9503	91.1810	91.4045	91.6212	91.8312	92.0348	92.2323	92.4239	13
14	89.6258	89.8936	90.1528	90.4039	90.6471	90.8829	91.1114	91.3329	91.5478	91.7563	14
15	88.7689	89.0587	89.3394	89.6112	89.8746	90.1298	90.3771	90.6170	90.8497	91.0754	15
16	87.8948	88.2072	88.5097	88.8027	89.0865	89.3616	89.6282	89.8868	90.1375	90.3808	16
17	87.0032	87.3387	87.6635	87.9780	88.2828	88.5781	88.8643	89.1419	89.4112	89.6724	17
18	86.0939	86.4528	86.8003	87.1369	87.4629	87.7789	88.0852	88.3822	88.6703	88.9497	18
19	85.1663	85.5492	85.9198	86.2789	86.6267	86.9637	87.2904	87.6073	87.9145	88.2127	19
20	84.2202	84.6275	85.0218	85.4037	85.7737	86.1323	86.4798	86.8178	87.1437	87.4608	20
21	83.2551	83.6873	84.1058	84.5110	84.9037	85.2841	85.6530	86.0106	86.3575	86.6940	21
22	82.2708	82.7284	83.1714	83.6005	84.0162	84.4191	84.8096	85.1882	85.5555	85.9118	22
23	81.2668	81.7503	82.2184	82.6718	83.1110	83.5367	83.9493	84.3494	84.7375	85.1140	23
24	80.2426	80.7526	81.2463	81.7245	82.1878	82.6367	83.0718	83.4938	83.9031	84.3002	24
25	79.1980	79.7350	80.2548	80.7582	81.2460	81.7187	82.1768	82.6211	83.0520	83.4701	25
26	78.1326	78.6970	79.2434	79.7727	80.2854	80.7823	81.2639	81.7310	82.1839	82.6234	26
27	77.0458	77.6382	78.2119	78.7709	79.3056	79.8272	80.3328	80.8230	81.2985	81.7598	27
28	75.9372	76.5583	77.1597	77.7420	78.3062	78.8530	79.3830	79.8969	80.3954	80.8789	28
29	74.8065	75.4568	76.0864	76.6961	77.2868	77.8593	78.4142	78.9523	79.4741	79.9805	29
30	73.6532	74.3333	74.9916	75.6293	76.2471	76.8457	77.4260	77.9887	78.5345	79.0640	30
31	72.4768	73.1872	73.8750	74.5411	75.1865	75.8119	76.4181	77.0059	77.5761	78.1292	31
32	71.2769	72.0183	72.7361	73.4312	74.1047	74.7574	75.3900	76.0035	76.5985	77.1757	32
33	70.0530	70.8260	71.5743	72.2991	73.0013	73.6818	74.3414	74.9810	75.6013	76.2032	33
34	68.8046	69.6098	70.3894	71.1444	71.8758	72.5847	73.2717	73.9380	74.5842	75.2111	34
35	67.5313	68.3693	69.1807	69.9665	70.7278	71.4656	72.1807	72.8742	73.5468	74.1993	35
36	66.2325	67.1040	67.9479	68.7651	69.5569	70.3242	71.0679	71.7891	72.4886	73.1672	36
37	64.9077	65.8134	66.6904	67.5397	68.3625	69.1599	69.9328	70.6823	71.4092	72.1145	37
38	63.5564	64.4970	65.4077	66.2897	67.1443	67.9723	68.7750	69.5534	70.3083	71.0407	38
39	62.1781	63.1543	64.0994	65.0148	65.9016	66.7610	67.5941	68.4019	69.1853	69.9455	39
40	60.7722	61.7847	62.7650	63.7144	64.6342	65.5255	66.3895	67.2279	68.0399	68.8283	40
41	59.3382	60.3877	61.4038	62.3879	63.3413	64.2653	65.1608	66.0293	66.8716	67.6888	41
42	57.8755	58.9628	60.0154	61.0349	62.0226	62.9798	63.9076	64.8073	65.6799	66.5265	42
43	56.3836	57.5093	58.5993	59.6549	60.6776	61.6687	62.6293	63.5609	64.4644	65.3409	43
44	54.8618	56.0269	57.1548	58.2473	59.3056	60.3313	61.3255	62.2895	63.2245	64.1317	44
45	53.3096	54.5147	55.6815	56.8115	57.9062	58.9671	59.9955	60.9927	61.9599	62.8983	45

2% INTEREST RATE

71–80 Years Probable Life

Age, years	Condition-percent, % 71	72	73	74	75	76	77	78	79	80	Age, years
46	51.7263	52.9723	54.1786	55.3470	56.4789	57.5758	58.6390	59.6700	60.6700	61.6402	46
47	50.1114	51.3991	52.6457	53.8532	55.0229	56.1565	57.2552	58.3208	59.3543	60.3569	47
48	48.4642	49.7944	51.0822	52.3295	53.5379	54.7090	55.8440	56.9447	58.0122	59.0479	48
49	46.7840	48.1576	49.4874	50.7753	52.0231	53.2323	54.4044	55.5410	56.6433	57.7129	49
50	45.0703	46.4881	47.8607	49.1901	50.4781	51.7262	52.9360	54.1092	55.2471	56.3510	50
51	43.3222	44.7851	46.2014	47.5732	48.9021	50.1900	51.4383	52.6488	53.8229	54.9620	51
52	41.5392	43.0481	44.5090	45.9239	47.2947	48.6230	49.9106	51.1593	52.3702	53.5451	52
53	39.7205	41.2764	42.7828	44.2416	45.6551	47.0248	48.3524	49.6399	50.8885	52.1000	53
54	37.8655	39.4693	41.0220	42.5257	43.9826	45.3945	46.7630	48.0901	49.3772	50.6259	54
55	35.9734	37.6260	39.2259	40.7755	42.2768	43.7316	45.1418	46.5093	47.8356	49.1223	55
56	34.0434	35.7458	37.3940	38.9902	40.5368	42.0355	43.4882	44.8969	46.2632	47.5887	56
57	32.0748	33.8280	35.5254	37.1694	38.7621	40.3055	41.8015	43.2523	44.6593	46.0244	57
58	30.0669	31.8719	33.6195	35.3120	36.9518	38.5408	40.0811	41.5748	43.0234	44.4288	58
59	28.0188	29.8767	31.6754	33.4175	35.1053	36.7409	38.3263	39.8637	41.3547	42.8014	59
60	25.9297	27.8415	29.6925	31.4851	33.2219	34.9050	36.5364	38.1184	39.6527	41.1413	60
61	23.7988	25.7657	27.6699	29.5141	31.3008	33.0323	34.7106	36.3382	37.9166	39.4481	61
62	21.6254	23.6483	25.6068	27.5036	29.3414	31.1222	32.8484	34.5224	36.1458	37.7210	62
63	19.4084	21.4886	23.5025	25.4530	27.3427	29.1739	30.9489	32.6702	34.3396	35.9593	63
64	17.1471	19.2857	21.3561	23.3613	25.3040	27.1866	29.0115	30.7811	32.4973	34.1624	64
65	14.8406	17.0387	19.1668	21.2278	23.2246	25.1596	27.0352	28.8541	30.6181	32.3296	65
66	12.4880	14.7468	16.9336	19.0516	21.1036	23.0921	25.0196	26.8886	28.7014	30.4601	66
67	10.0883	12.4091	14.6559	16.8319	18.9401	20.9831	22.9634	24.8838	26.7463	28.5532	67
68	7.6406	10.0246	12.3325	14.5678	16.7334	18.8321	20.8663	22.8389	24.7521	26.6082	68
69	5.1440	7.5924	9.9627	12.2584	14.4826	16.6379	18.7271	20.7531	22.7180	24.6244	69
70	2.5975	5.1115	7.5455	9.9029	12.1867	14.4000	16.5452	18.6256	20.6432	22.6007	70
71	0.0000	2.5811	5.0800	7.5002	9.8450	12.1172	14.3197	16.4555	18.5270	20.5367	71
72		0.0000	2.5652	5.0495	7.4564	9.7888	12.0496	14.2421	16.3684	18.4314	72
73			0.0000	2.5497	5.0200	7.4138	9.7342	11.9843	14.1666	16.2839	73
74				0.0000	2.5348	4.9913	7.3724	9.6814	11.9209	14.0935	74
75					0.0000	2.5203	4.9634	7.3325	9.6301	11.8593	75
76						0.0000	2.5063	4.9366	7.2936	9.5804	76
77							0.0000	2.4927	4.9104	7.2560	77
78								0.0000	2.4795	4.8850	78
79									0.0000	2.4667	79
80										0.0000	80

81–90 Years Probable Life

Age, years	81	82	83	84	85	86	87	88	89	90	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.4966	99.5089	99.5208	99.5324	99.5437	99.5546	99.5653	99.5756	99.5856	99.5954	1
2	98.9831	99.0080	99.0321	99.0555	99.0782	99.1003	99.1218	99.1427	99.1630	99.1827	2
3	98.4594	98.4970	98.5335	98.5690	98.6035	98.6370	98.6695	98.7011	98.7319	98.7618	3
4	97.9252	97.9758	98.0250	98.0728	98.1192	98.1643	98.2081	98.2507	98.2921	98.3324	4
5	97.3803	97.4442	97.5064	97.5667	97.6253	97.6822	97.7375	97.7913	97.8436	97.8944	5
6	96.8245	96.9020	96.9773	97.0504	97.1215	97.1905	97.2575	97.3227	97.3861	97.4477	6
7	96.2575	96.3489	96.4377	96.5239	96.6076	96.6889	96.7680	96.8448	96.9194	96.9921	7
8	95.6793	95.7848	95.8873	95.9868	96.0834	96.1773	96.2686	96.3573	96.4435	96.5273	8
9	95.0895	95.2094	95.3258	95.4389	95.5488	95.6555	95.7592	95.8600	95.9580	96.0533	9
10	94.4879	94.6225	94.7532	94.8801	95.0034	95.1232	95.2396	95.3528	95.4628	95.5697	10
11	93.8742	94.0238	94.1691	94.3101	94.4472	94.5803	94.7097	94.8354	94.9576	95.0765	11
12	93.2483	93.4132	93.5733	93.7288	93.8798	94.0265	94.1691	94.3077	94.4424	94.5734	12
13	92.6098	92.7903	92.9656	93.1358	93.3011	93.4617	93.6177	93.7694	93.9169	94.0603	13
14	91.9586	92.1550	92.3457	92.5309	92.7108	92.8855	93.0553	93.2204	93.3809	93.5369	14
15	91.2944	91.5070	91.7135	91.9139	92.1087	92.2978	92.4817	92.6604	92.8341	93.0030	15
16	90.6169	90.8461	91.0686	91.2846	91.4945	91.6984	91.8966	92.0892	92.2764	92.4585	16
17	89.9258	90.1719	90.4108	90.6427	90.8681	91.0870	91.2998	91.5066	91.7076	91.9031	17
18	89.2209	89.4842	89.7398	89.9880	90.2291	90.4634	90.6910	90.9123	91.1274	91.3365	18
19	88.5019	88.7828	89.0554	89.3202	89.5774	89.8272	90.0701	90.3061	90.5356	90.7587	19
20	87.7686	88.0673	88.3574	88.6390	88.9126	89.1784	89.4367	89.6878	89.9319	90.1692	20

2% INTEREST RATE
81–90 Years Probable Life

Age, years	Condition-percent, %										Age, years
	81	82	83	84	85	86	87	88	89	90	
21	87.0205	87.3376	87.6453	87.9442	88.2345	88.5166	88.7907	89.0571	89.3161	89.5680	21
22	86.2576	86.5932	86.9191	87.2355	87.5429	87.8416	88.1317	88.4139	88.6881	88.9548	22
23	85.4793	85.8339	86.1783	86.5126	86.8374	87.1530	87.4596	87.7577	88.0475	88.3293	23
24	84.6855	85.0595	85.4227	85.7753	86.1179	86.4507	86.7741	87.0885	87.3941	87.6913	24
25	83.8758	84.2696	84.6519	85.0232	85.3839	85.7343	86.0748	86.4058	86.7276	87.0405	25
26	83.0499	83.4639	83.8658	84.2561	84.6353	85.0036	85.3616	85.7095	86.0478	86.3767	26
27	82.2075	82.6420	83.0640	83.4737	83.8716	84.2583	84.6341	84.9993	85.3544	85.6996	27
28	81.3482	81.8038	82.2461	82.6756	83.0928	83.4981	83.8920	84.2749	84.6471	85.0090	28
29	80.4718	80.9487	81.4118	81.8615	82.2983	82.7227	83.1351	83.5360	83.9257	84.3046	29
30	79.5779	80.0766	80.5609	81.0311	81.4879	81.9317	82.3630	82.7823	83.1898	83.5861	30
31	78.6660	79.1870	79.6929	80.1842	80.6614	81.1250	81.5755	82.0135	82.4392	82.8532	31
32	77.7359	78.2797	78.8076	79.3203	79.8183	80.3021	80.7723	81.2293	81.6736	82.1057	32
33	76.7872	77.3541	77.9046	78.4391	78.9583	79.4628	79.9530	80.4295	80.8927	81.3432	33
34	75.8195	76.4101	76.9835	77.5403	78.0812	78.6067	79.1173	79.6137	80.0962	80.5654	34
35	74.8325	75.4472	76.0440	76.6235	77.1865	77.7334	78.2649	78.7815	79.2838	79.7721	35
36	73.8258	74.4650	75.0857	75.6884	76.2739	76.8427	77.3955	77.9328	78.4551	78.9630	36
37	72.7989	73.4632	74.1082	74.7346	75.3430	75.9342	76.5086	77.0670	77.6098	78.1376	37
38	71.7515	72.4414	73.1112	73.7617	74.3936	75.0075	75.6040	76.1839	76.7477	77.2958	38
39	70.6831	71.3991	72.0943	72.7694	73.4251	74.0622	74.6814	75.2832	75.8682	76.4371	39
40	69.5934	70.3360	71.0570	71.7572	72.4373	73.0981	73.7402	74.3644	74.9712	75.5612	40
41	68.4818	69.2516	69.9990	70.7247	71.4297	72.1147	72.7803	73.4273	74.0563	74.6679	41
42	67.3480	68.1455	68.9198	69.6717	70.4020	71.1116	71.8011	72.4714	73.1230	73.7566	42
43	66.1916	67.0173	67.8190	68.5975	69.3537	70.0884	70.8024	71.4964	72.1711	72.8271	43
44	65.0120	65.8666	66.6962	67.5019	68.2845	69.0448	69.7837	70.5020	71.2002	71.8791	44
45	63.8089	64.6928	65.5510	66.3843	67.1938	67.9804	68.7446	69.4876	70.2098	70.9121	45
46	62.5816	63.4955	64.3828	65.2444	66.0814	66.8946	67.6848	68.4529	69.1996	69.9257	46
47	61.3298	62.2742	63.1913	64.0817	64.9467	65.7871	66.6037	67.3976	68.1693	68.9196	47
48	60.0530	61.0287	61.9759	62.8958	63.7893	64.6575	65.5011	66.3211	67.1183	67.8934	48
49	58.7507	59.7582	60.7363	61.6861	62.6088	63.5052	64.3763	65.2231	66.0463	66.8467	49
50	57.4224	58.4622	59.4718	60.4523	61.4046	62.3299	63.2291	64.1031	64.9528	65.7790	50
51	56.0673	57.1403	58.1821	59.1937	60.1764	61.1312	62.0589	62.9608	63.8375	64.6900	51
52	54.6853	55.7920	56.8666	57.9100	58.9236	59.9084	60.8654	61.7956	62.6999	63.5792	52
53	53.2756	54.4168	55.5247	56.6007	57.6458	58.6612	59.6479	60.6071	61.5395	62.4462	53
54	51.8377	53.0140	54.1560	55.2651	56.3424	57.3890	58.4061	59.3948	60.3559	61.2905	54
55	50.3710	51.5832	52.7600	53.9028	55.0129	56.0914	57.1395	58.1583	59.1487	60.1117	55
56	48.8751	50.1237	51.3360	52.5133	53.6568	54.7679	55.8476	56.8970	57.9173	58.9093	56
57	47.3492	48.6351	49.8836	51.0960	52.2736	53.4179	54.5297	55.6106	56.6613	57.6829	57
58	45.7927	47.1167	48.4021	49.6503	50.8628	52.0408	53.1856	54.2984	55.3801	56.4320	58
59	44.2052	45.5679	46.8909	48.1757	49.4237	50.6363	51.8146	52.9599	54.0734	55.1560	59
60	42.5859	43.9881	45.3496	46.6717	47.9559	49.2036	50.4161	51.5947	52.7405	53.8545	60
61	40.9342	42.3768	43.7774	45.1375	46.4587	47.7423	48.9897	50.2022	51.3809	52.5270	61
62	39.2495	40.7332	42.1737	43.5727	44.9315	46.2518	47.5347	48.7818	49.9940	51.1729	62
63	37.5311	39.0568	40.5381	41.9766	43.3738	44.7314	46.0507	47.3330	48.5797	49.7918	63
64	35.7783	37.3468	38.8697	40.3485	41.7850	43.1807	44.5369	45.8553	47.1369	48.3830	64
65	33.9904	35.6026	37.1679	38.6879	40.1644	41.5989	42.9929	44.3480	45.6653	46.9461	65
66	32.1668	33.8236	35.4321	36.9941	38.5113	39.9855	41.4180	42.8105	44.1642	45.4804	66
67	30.3068	32.0089	33.6615	35.2664	36.8253	38.3398	39.8116	41.2423	42.6331	43.9854	67
68	28.4095	30.1580	31.8556	33.5041	35.1054	36.6612	38.1732	39.6427	41.0714	42.4605	68
69	26.4743	28.2701	30.0135	31.7066	33.3512	34.9491	36.5018	38.0112	39.4785	40.9052	69
70	24.5004	26.3443	28.1346	29.8732	31.5619	33.2027	34.7970	36.3470	37.8537	39.3186	70
71	22.4870	24.3801	26.2182	28.0031	29.7368	31.4214	33.0583	34.6495	36.1964	37.7004	71
72	20.4333	22.3766	24.2633	26.0955	27.8753	29.6044	31.2847	32.9181	34.5059	36.0498	72
73	18.3386	20.3330	22.2694	24.1499	25.9764	27.7511	29.4757	31.1520	32.7817	34.3662	73
74	16.2019	18.2486	20.2357	22.1653	24.0397	25.8608	27.6304	29.3506	31.0229	32.6489	74
75	14.0226	16.1225	18.1612	20.1410	22.0641	23.9326	25.7483	27.5132	29.2290	30.8973	75
76	11.7996	13.9538	16.0452	18.0763	20.0491	21.9659	23.8285	25.6391	27.3992	29.1107	76
77	9.5322	11.7418	13.8870	15.9702	17.9937	19.9598	21.8703	23.7274	25.5327	27.2883	77
78	7.2194	9.4855	11.6855	13.8220	15.8973	17.9136	19.8730	21.7776	23.6291	25.4294	78
79	4.8604	7.1841	9.4401	11.6309	13.7589	15.8265	17.8357	19.7887	21.6873	23.5335	79
80	2.4543	4.8367	7.1497	9.3959	11.5778	13.6977	15.7576	17.7601	19.7067	21.5995	80

2% INTEREST RATE

81–90 Years Probable Life

Age, years	Condition-percent, % 81	82	83	84	85	86	87	88	89	90	Age, years
81	0.0000	2.4423	4.8135	7.1163	9.3530	11.5262	13.6378	15.6909	17.6866	19.6269	81
82		0.0000	2.4306	4.7910	7.0838	9.3114	11.4761	13.5803	15.6258	17.6148	82
83			0.0000	2.4192	4.7691	7.0522	9.2706	11.4275	13.5240	15.5625	83
84				0.0000	2.4082	4.7479	7.0215	9.2316	11.3801	13.4691	84
85					0.0000	2.3975	4.7272	6.9918	9.1933	11.3339	85
86						0.0000	2.3870	4.7072	6.9626	9.1560	86
87							0.0000	2.3769	4.6877	6.9345	87
88								0.0000	2.3670	4.6686	88
89									0.0000	2.3575	89
90										0.0000	90

91–100 Years Probable Life

Age, years	91	92	93	94	95	96	97	98	99	100	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.6049	99.6141	99.6231	99.6319	99.6404	99.6487	99.6568	99.6646	99.6723	99.6797	1
2	99.2019	99.2206	99.2387	99.2564	99.2736	99.2903	99.3067	99.3225	99.3380	99.3530	2
3	98.7908	98.8191	98.8466	98.8734	98.9056	98.9248	98.9495	98.9736	98.9970	99.0198	3
4	98.3715	98.4096	98.4467	98.4828	98.5179	98.5520	98.5853	98.6177	98.6492	98.6800	4
5	97.9439	97.9920	98.0388	98.0843	98.1286	98.1718	98.2138	98.2547	98.2945	98.3333	5
6	97.5077	97.5660	97.6227	97.6779	97.7316	97.7839	97.8348	97.8844	97.9326	97.9797	6
7	97.0627	97.1314	97.1983	97.2633	97.3266	97.3882	97.4482	97.5067	97.5636	97.6190	7
8	96.6089	96.6882	96.7654	96.8405	96.9135	96.9847	97.0540	97.1214	97.1871	97.2511	8
9	96.1459	96.2361	96.3238	96.4091	96.4922	96.5731	96.6518	96.7285	96.8031	96.8758	9
10	95.6738	95.7750	95.8734	95.9692	96.0625	96.1532	96.2416	96.3276	96.4115	96.4931	10
11	95.1921	95.3046	95.4140	95.5205	95.6241	95.7250	95.8232	95.9188	96.0120	96.1027	11
12	94.7009	94.8248	94.9454	95.0628	95.1770	95.2882	95.3964	95.5018	95.6045	95.7045	12
13	94.1998	94.3355	94.4675	94.5959	94.7209	94.8426	94.9611	95.0765	95.1888	95.2983	13
14	93.6887	93.8363	93.9799	94.1197	94.2557	94.3881	94.5171	94.6426	94.7649	94.8840	14
15	93.1674	93.3272	93.4827	93.6340	93.7813	93.9246	94.0642	94.2001	94.3324	94.4614	15
16	92.6356	92.8079	92.9754	93.1385	93.2973	93.4518	93.6022	93.7487	93.8914	94.0303	16
17	92.0932	92.2782	92.4581	92.6332	92.8036	92.9695	93.1310	93.2883	93.4415	93.5907	17
18	91.5400	91.7379	91.9304	92.1177	92.3001	92.4776	92.6504	92.8187	92.9826	93.1422	18
19	90.9757	91.1868	91.3921	91.5920	91.7865	91.9758	92.1602	92.3397	92.5145	92.6848	19
20	90.4001	90.6246	90.8431	91.0557	91.2626	91.4640	91.6601	91.8511	92.0370	92.2182	20
21	89.8130	90.0513	90.2831	90.5087	90.7283	90.9420	91.1501	91.3527	91.5501	91.7423	21
22	89.2141	89.4664	89.7119	89.9508	90.1832	90.4095	90.6298	90.8444	91.0533	91.2568	22
23	88.6033	88.8699	89.1293	89.3816	89.6273	89.8664	90.0992	90.3259	90.5467	90.7617	23
24	87.9803	88.2615	88.5350	88.8012	89.0602	89.3124	89.5579	89.7970	90.0299	90.2567	24
25	87.3448	87.6408	87.9288	88.2091	88.4818	88.7474	89.0059	89.2576	89.5027	89.7412	25
26	86.6966	87.0078	87.3105	87.6051	87.8919	88.1710	88.4427	88.7074	88.9651	89.2161	26
27	86.0354	86.3621	86.6799	86.9891	87.2901	87.5831	87.8683	88.1461	88.4166	88.6801	27
28	85.3610	85.7035	86.0366	86.3608	86.6763	86.9834	87.2825	87.5737	87.8572	88.1335	28
29	84.6731	85.0317	85.3804	85.7199	86.0502	86.3718	86.6849	86.9897	87.2867	87.5758	29
30	83.9715	84.3464	84.7112	85.0662	85.4116	85.7479	86.0753	86.3942	86.7047	87.0071	30
31	83.2558	83.6475	84.0285	84.3994	84.7603	85.1116	85.4536	85.7867	86.1110	86.4270	31
32	82.5259	82.9346	83.3323	83.7192	84.0959	84.4625	84.8194	85.1670	85.5055	85.8352	32
33	81.7813	82.2074	82.6220	83.0255	83.4182	83.8004	84.1726	84.5350	84.8879	85.2317	33
34	81.0218	81.4657	81.8976	82.3179	82.7269	83.1251	83.5128	83.8903	84.2579	84.6160	34
35	80.2471	80.7092	81.1587	81.5961	82.0219	82.4363	82.8398	83.2327	83.6155	83.9881	35
36	79.4570	79.9375	80.4050	80.8599	81.3027	81.7337	82.1533	82.5620	82.9599	83.3476	36
37	78.6510	79.1504	79.6362	80.1090	80.5692	81.0171	81.4532	81.8778	82.2915	82.6942	37
38	77.8289	78.3475	78.8520	79.3431	79.8210	80.2861	80.7390	81.1800	81.6095	82.0278	38
39	76.9904	77.5286	78.0522	78.5618	79.0578	79.5405	80.0105	80.4682	80.9140	81.3481	39
40	76.1351	76.6934	77.2364	77.7649	78.2793	78.7800	79.2675	79.7422	80.2045	80.6548	40
41	75.2627	75.8413	76.4043	76.9521	77.4853	78.0043	78.5096	79.0017	79.4809	79.9476	41
42	74.3729	74.9723	75.5555	76.1230	76.6754	77.2131	77.7366	78.2463	78.7428	79.2263	42
43	73.4652	74.0859	74.6897	75.2774	75.8493	76.4060	76.9480	77.4759	77.9899	78.4906	43
44	72.5394	73.1818	73.8067	74.4148	75.0067	75.5828	76.1438	76.6900	77.2220	77.7401	44
45	71.5951	72.2596	72.9059	73.5350	74.1472	74.7432	75.3234	75.8884	76.4386	76.9746	45

2% INTEREST RATE
91–100 Years Probable Life

Age, years	Condition-percent, %										Age, years
	91	92	93	94	95	96	97	98	99	100	
46	70.6319	71.3189	71.9872	72.6376	73.2706	73.8867	74.4866	75.0708	75.6397	76.1939	46
47	69.6495	70.3594	71.0500	71.7222	72.3764	73.0131	73.6331	74.2368	74.8248	75.3975	47
48	68.6473	69.3807	70.0942	70.7885	71.4643	72.1221	72.7625	73.3862	73.9935	74.5851	48
49	67.6252	68.3825	69.1192	69.8362	70.5340	71.2132	71.8745	72.5185	73.1457	73.7565	49
50	66.5826	67.3643	68.1247	68.8648	69.5851	70.2862	70.9688	71.6335	72.2809	72.9114	50
51	65.5191	66.3257	67.1104	67.8740	68.6172	69.3406	70.0449	70.7308	71.3988	72.0494	51
52	64.4344	65.2664	66.0757	66.8633	67.6299	68.3761	69.1026	69.8100	70.4990	71.1701	52
53	63.3280	64.1858	65.0203	65.8325	66.6229	67.3923	68.1414	68.8708	69.5813	70.2732	53
54	62.1995	63.0837	63.9439	64.7810	65.5958	66.3888	67.1610	67.9129	68.6452	69.3584	54
55	61.0484	61.9595	62.8459	63.7085	64.5481	65.3653	66.1610	66.9358	67.6903	68.4253	55
56	59.8742	60.8128	61.7259	62.6146	63.4794	64.3213	65.1409	65.9391	66.7164	67.4735	56
57	58.6766	59.6432	60.5836	61.4987	62.3894	63.2564	64.1005	64.9225	65.7230	66.5027	57
58	57.4550	58.4502	59.4184	60.3606	61.2776	62.1702	63.0393	63.8855	64.7097	65.5125	58
59	56.2090	57.2334	58.2299	59.1997	60.1436	61.0623	61.9568	62.8279	63.6762	64.5025	59
60	54.9381	55.9922	57.0176	58.0156	58.9868	59.9322	60.8527	61.7491	62.6220	63.4723	60
61	53.6418	54.7261	55.7811	56.8078	57.8070	58.7795	59.7265	60.6487	61.5467	62.4214	61
62	52.3195	53.4348	54.5199	55.5758	56.6035	57.6038	58.5778	59.5262	60.4499	61.3496	62
63	50.9708	52.1177	53.2334	54.3192	55.3760	56.4046	57.4061	58.3814	59.3312	60.2563	63
64	49.5951	50.7742	51.9212	53.0375	54.1239	55.1814	56.2110	57.2136	58.1901	59.1412	64
65	48.1919	49.4038	50.5827	51.7301	52.8468	53.9337	54.9920	56.0225	57.0262	58.0037	65
66	46.7607	48.0060	49.2175	50.3966	51.5441	52.6610	53.7486	54.8076	55.8390	56.8435	66
67	45.3008	46.5803	47.8250	49.0364	50.2154	51.3629	52.4803	53.5683	54.6280	55.6601	67
68	43.8117	45.1260	46.4047	47.6490	48.8601	50.0389	51.1867	52.3043	53.3928	54.4530	68
69	42.2928	43.6427	44.9559	46.2339	47.4777	48.6884	49.8672	51.0150	52.1330	53.2218	69
70	40.7436	42.1297	43.4781	44.7904	46.0676	47.3108	48.5213	49.7000	50.8479	51.9660	70
71	39.1633	40.5864	41.9708	43.3181	44.6294	45.9057	47.1485	48.3586	49.5371	50.6850	71
72	37.5515	39.0123	40.4334	41.8164	43.1624	44.4725	45.7482	46.9904	48.2001	49.3784	72
73	35.9074	37.4067	38.8652	40.2846	41.6660	43.0107	44.3199	45.5948	46.8364	48.0457	73
74	34.2305	35.7689	37.2656	38.7221	40.1397	41.5196	42.8631	44.1713	45.4454	46.6864	74
75	32.5200	34.0985	35.6340	37.1285	38.5829	39.9987	41.3771	42.7193	44.0266	45.2998	75
76	30.7753	32.3946	33.9699	35.5029	36.9950	38.4473	39.8614	41.2383	42.5794	43.8856	76
77	28.9957	30.6566	32.2724	33.8449	35.3753	36.8649	38.3154	39.7277	41.1032	42.4430	77
78	27.1805	28.8839	30.5410	32.1536	33.7232	35.2509	36.7384	38.1869	39.5975	40.9716	78
79	25.3290	27.0757	28.7749	30.4286	32.0381	33.6046	35.1300	36.6153	38.0618	39.4707	79
80	23.4405	25.2313	26.9735	28.6691	30.3192	31.9254	33.4893	35.0122	36.4953	37.9399	80
81	21.5142	23.3500	25.1362	26.8743	28.5660	30.2126	31.8159	33.3770	34.8975	36.3784	81
82	19.5494	21.4313	23.2620	25.0437	26.7777	28.4655	30.1089	31.7092	33.2677	34.7857	82
83	17.5453	19.4740	21.3504	23.1764	24.9537	26.6835	28.3679	30.0080	31.6053	33.1611	83
84	15.5011	17.4776	19.4005	21.2719	23.0931	24.8659	26.5920	28.2728	29.9097	31.5041	84
85	13.4160	15.4413	17.4117	19.3292	21.1954	23.0119	24.7806	26.5028	28.1802	29.8139	85
86	11.2892	13.3643	15.3830	17.3476	19.2597	21.1208	22.9330	24.6975	26.4161	28.0899	86
87	9.1199	11.2457	13.3138	15.3265	17.2853	19.1919	21.0484	22.8561	24.6166	26.3314	87
88	6.9072	9.0848	11.2032	13.2649	15.2714	17.2244	19.1261	20.9778	22.7811	24.5378	88
89	4.6503	6.8806	9.0504	11.1621	13.2172	15.2176	17.1654	19.0620	20.9091	22.7082	89
90	2.3481	4.6324	6.8546	9.0172	11.1220	13.1707	15.1655	17.1079	18.9996	20.8421	90
91	0.0000	2.3391	4.6148	6.8294	8.9848	11.0828	13.1255	15.1146	17.0519	18.9387	91
92		0.0000	2.3303	4.5979	6.8049	8.9531	11.0448	13.0815	15.0652	16.9972	92
93			0.0000	2.3217	4.5814	6.7808	8.9224	11.0078	13.0387	15.0169	93
94				0.0000	2.3133	4.5651	6.7577	8.8926	10.9718	12.9970	94
95					0.0000	2.3052	4.5496	6.7350	8.8635	10.9366	95
96						0.0000	2.2973	4.5343	6.7130	8.8351	96
97							0.0000	2.2896	4.5196	6.6915	97
98								0.0000	2.2821	4.5051	98
99									0.0000	2.2748	99
100										0.0000	100

3% INTEREST RATE

1–10 Years Probable Life

Age, years	Condition-percent, %										Age, years
	1	2	3	4	5	6	7	8	9	10	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	0.0000	50.7389	67.6470	76.0973	81.1645	84.5403	86.9494	88.7544	90.1566	91.2769	1
2		0.0000	34.3233	51.4775	61.7640	68.6167	73.5072	77.1714	80.0179	82.2922	2
3			0.0000	26.1191	41.7815	52.2155	59.6618	65.2409	69.5751	73.0379	3
4				0.0000	21.1995	35.3222	45.4010	52.9524	58.8189	63.5060	4
5					0.0000	17.9221	30.7124	40.2954	47.7401	53.6881	5
6						0.0000	15.5831	27.2586	36.3289	43.5757	6
7							0.0000	13.8307	24.5754	33.1600	7
8								0.0000	12.4693	22.4317	8
9									0.0000	11.3816	9
10										0.0000	10

11–20 Years Probable Life

Age, years	11	12	13	14	15	16	17	18	19	20	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	92.1923	92.9538	93.5970	94.1474	94.6233	95.0389	95.4047	95.7291	96.0186	96.2784	1
2	84.1503	85.6962	87.0000	88.1192	89.0854	89.9290	90.6716	91.3301	91.9178	92.4452	2
3	75.8670	78.2209	80.2091	81.9101	83.3813	84.6658	85.7965	86.7992	87.6939	88.4970	3
4	67.3353	70.5213	73.2124	75.5148	77.5061	79.2447	80.7752	82.1323	83.3434	84.4303	4
5	58.5476	62.5907	66.0058	68.9276	71.4546	73.6609	75.6032	77.3254	78.8623	80.2417	5
6	49.4963	54.4222	58.5831	62.1428	65.2216	67.9097	70.2760	72.3743	74.2468	75.9274	6
7	40.1734	46.0087	50.9376	55.1544	58.8016	61.9859	64.7890	67.2746	69.4928	71.4836	7
8	30.5709	37.3427	43.0628	47.9564	52.1890	55.8844	59.1375	62.0220	64.5962	66.9065	8
9	20.6803	28.4168	34.9517	40.5425	45.3780	49.5998	53.3163	56.6118	59.5527	62.1922	9
10	10.4930	19.2231	26.5973	32.9061	38.3626	43.1267	47.3206	51.0393	54.3578	57.3364	10
11	0.0000	9.7536	17.9923	25.0407	31.1369	36.4594	41.1449	45.2996	49.0072	52.3349	11
12		0.0000	9.1291	16.9392	23.6943	29.5921	34.7840	39.3877	43.4960	47.1834	12
13			0.0000	8.5948	16.0285	22.5188	28.2323	33.2984	37.8195	41.8773	13
14				0.0000	8.1327	15.2333	21.4840	27.0265	31.9727	36.4120	14
15					0.0000	7.7292	14.5333	20.5665	25.9505	30.7828	15
16						0.0000	7.3740	13.9126	19.7476	24.9847	16
17							0.0000	7.0591	13.3587	19.0127	17
18								0.0000	6.7780	12.8615	18
19									0.0000	6.5258	19
20										0.0000	20

21–30 Years Probable Life

Age, years	21	22	23	24	25	26	27	28	29	30	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	96.5128	96.7253	96.9186	97.0953	97.2572	97.4062	97.5436	97.6707	97.7885	97.8981	1
2	92.9210	93.3523	93.7448	94.1034	94.4321	94.7345	95.0135	95.2715	95.5107	95.7331	2
3	89.2215	89.8781	90.4757	91.0217	91.5223	91.9827	92.4074	92.8003	93.1646	93.5032	3
4	85.4109	86.2997	87.1086	87.8476	88.5252	89.1484	89.7233	90.2550	90.7480	91.2063	4
5	81.4861	82.6140	83.6405	84.5783	85.4382	86.2290	86.9585	87.6333	88.2590	88.8406	5
6	77.4435	78.8176	80.0683	81.2109	82.2585	83.2221	84.1109	84.9330	85.6953	86.4039	6
7	73.2796	74.9074	76.3890	77.7425	78.9835	80.1249	81.1778	82.1517	83.0547	83.8941	7
8	68.9908	70.8799	72.5992	74.1701	75.6102	76.9348	78.1567	79.2869	80.3349	81.3090	8
9	64.5734	66.7316	68.6958	70.4904	72.1357	73.6490	75.0450	76.3362	77.5335	78.6463	9
10	60.0234	62.4588	64.6753	66.7004	68.5570	70.2647	71.8399	73.2969	74.6480	75.9038	10
11	55.3369	58.0578	60.5342	62.7967	64.8709	66.7788	68.5387	70.1665	71.6760	73.0790	11
12	50.5099	53.5248	56.2688	58.7758	61.0743	63.1883	65.1384	66.9422	68.6148	70.1694	12
13	45.5380	48.8558	51.8755	54.6343	57.1637	59.4901	61.6361	63.6211	65.4618	67.1726	13
14	40.4169	44.0467	47.3504	50.3686	53.1359	55.6810	58.0288	60.2004	62.2142	64.0858	14
15	35.1423	39.0934	42.6895	45.9750	48.9871	51.7576	54.3132	56.6771	58.8691	60.9065	15

3% INTEREST RATE

21–30 Years Probable Life

Age, years	Condition-percent, % 21	22	23	24	25	26	27	28	29	30	Age, years
16	29.7094	33.9915	37.8888	41.4495	44.7140	47.7165	50.4862	53.0481	55.4237	57.6317	16
17	24.1135	28.7365	32.9441	36.7882	40.3126	43.5542	46.5444	49.3102	51.8750	54.2588	17
18	18.3497	23.3238	27.8510	31.9871	35.7792	39.2670	42.4843	45.4602	48.2197	50.7846	18
19	12.4130	17.7488	22.6051	27.0420	31.1098	34.8511	38.3024	41.4947	44.4549	47.2062	19
20	6.2982	12.0065	17.2019	21.9485	26.3003	30.3028	33.9950	37.4102	40.5771	43.5205	20
21	0.0000	6.0920	11.6366	16.7022	21.3465	25.6181	29.5585	33.2032	36.5829	39.7241	21
22		0.0000	5.9043	11.2985	16.2444	20.7928	24.9888	28.8700	32.4689	35.8139	22
23			0.0000	5.7327	10.9887	15.8228	20.2821	24.4068	28.2315	31.7864	23
24				0.0000	5.5755	10.7036	15.4341	19.8096	23.8670	27.6381	24
25					0.0000	5.4309	10.4407	15.0746	19.3715	23.3653	25
26						0.0000	5.2975	10.1975	14.7412	18.9644	26
27							0.0000	5.1741	9.9720	14.4314	27
28								0.0000	5.0597	9.7624	28
29									0.0000	4.9533	29
30										0.0000	30

31–40 Years Probable Life

Age, years	31	32	33	34	35	36	37	38	39	40	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.0001	98.0953	98.1844	98.2678	98.3461	98.4196	98.4888	98.5541	98.6156	98.6738	1
2	95.9402	96.1335	96.3143	96.4836	96.6425	96.7918	96.9323	97.0648	97.1897	97.3077	2
3	93.8185	94.1129	94.3881	94.6460	94.8879	95.1152	95.3291	95.5308	95.7210	95.9007	3
4	91.6332	92.0316	92.4042	92.7531	93.0806	93.3883	93.6779	93.9508	94.2082	94.4515	4
5	89.3823	89.8879	90.3607	90.8035	91.2191	91.6096	91.9770	92.3233	92.6501	92.9588	5
6	87.0639	87.6799	88.2559	88.7954	89.3017	89.7775	90.2252	90.6471	91.0452	91.4213	6
7	84.6759	85.4056	86.0879	86.7271	87.3268	87.8904	88.4208	88.9206	89.3922	89.8378	7
8	82.2163	83.0631	83.8550	84.5967	85.2927	85.9467	86.5622	87.1423	87.6896	88.2066	8
9	79.6829	80.6503	81.5550	82.4024	83.1976	83.9448	84.6479	85.3106	85.9359	86.5266	9
10	77.0735	78.1652	79.1860	80.1423	81.0396	81.8827	82.6762	83.4240	84.1296	84.7962	10
11	74.3858	75.6055	76.7460	77.8144	78.8168	79.7588	80.6453	81.4808	82.2691	83.0138	11
12	71.6175	72.9690	74.2328	75.4166	76.5274	77.5712	78.5535	79.4793	80.3528	81.1780	12
13	68.7661	70.2534	71.6442	72.9469	74.1693	75.3180	76.3990	77.4177	78.3790	79.2871	13
14	65.8292	67.4563	68.9779	70.4031	71.7404	72.9971	74.1798	75.2943	76.3459	77.3395	14
15	62.8042	64.5754	66.2316	67.7830	69.2387	70.6067	71.8940	73.1072	74.2519	75.3334	15
16	59.6884	61.6080	63.4029	65.0843	66.6619	68.1445	69.5397	70.8545	72.0951	73.2672	16
17	56.4792	58.5515	60.4894	62.3047	64.0079	65.6084	67.1147	68.5342	69.8736	71.1390	17
18	53.1736	55.4034	57.4885	59.4416	61.2742	62.9963	64.6170	66.1443	67.5854	68.9469	18
19	49.7690	52.1609	54.3975	56.4927	58.4585	60.3058	62.0443	63.6827	65.2286	66.6891	19
20	46.2621	48.8210	51.2138	53.4552	55.5583	57.5346	59.3945	61.1472	62.8011	64.3635	20
21	42.6501	45.3810	47.9346	50.3267	52.5711	54.6803	56.6652	58.5357	60.3007	61.9682	21
22	38.9297	41.8378	44.5571	47.1043	49.4943	51.7403	53.8540	55.8458	57.7253	59.5010	22
23	35.0977	38.1882	41.0782	43.7852	46.3252	48.7121	50.9584	53.0753	55.0727	56.9598	23
24	31.1507	34.4292	37.4949	40.3666	43.0611	45.5931	47.9760	50.2216	52.3405	54.3423	24
25	27.0854	30.5574	33.8041	36.8454	39.6990	42.3805	44.9041	47.2823	49.5263	51.6463	25
26	22.8980	26.5695	30.0026	33.2186	36.2360	39.0716	41.7401	44.2548	46.6277	48.8695	26
27	18.5851	22.4619	26.0871	29.4829	32.6692	35.6634	38.4811	41.1366	43.6422	46.0093	27
28	14.1427	18.2311	22.0541	25.6352	28.9953	32.1529	35.1244	37.9247	40.5671	43.0634	28
29	9.5671	13.8734	17.9001	21.6721	25.2112	28.5371	31.6670	34.6165	37.3997	40.0291	29
30	4.8543	9.3849	13.6215	17.5901	21.3136	24.8128	28.1058	31.2091	34.1373	36.9037	30
31	0.0000	4.7618	9.2145	13.3856	17.2991	20.9768	24.4379	27.6994	30.7770	33.6845	31
32		0.0000	4.6754	9.0549	13.1642	17.0258	20.6598	24.0845	27.3160	30.3688	32
33			0.0000	4.5944	8.9052	12.9561	16.7685	20.3611	23.7511	26.9537	33
34				0.0000	4.5184	8.7644	12.7604	16.5260	20.0792	23.4360	34
35					0.0000	4.4470	8.6320	12.5758	16.2972	19.8129	35
36						0.0000	4.3798	8.5072	12.4017	16.0810	36
37							0.0000	4.3164	8.3894	12.2372	37
38								0.0000	4.2567	8.2781	38
39									0.0000	4.2002	39
40										0.0000	40

3% INTEREST RATE
41–50 Years Probable Life

Age, years	Condition-percent, %										Age, years
	41	42	43	44	45	46	47	48	49	50	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.7288	98.7808	98.8342	98.8770	98.9215	98.9637	99.0035	99.0422	99.0787	99.1135	1
2	97.4194	97.5251	97.6253	97.7203	97.8106	97.8964	97.9780	98.0557	98.1297	98.2003	2
3	96.0707	96.2317	96.3842	96.5290	96.6664	96.7970	96.9213	97.0396	97.1523	97.2598	3
4	94.6816	94.8995	95.1059	95.3019	95.4879	95.6647	95.8329	95.9930	96.1456	96.2910	4
5	93.2508	93.5273	93.7893	94.0379	94.2740	94.4984	94.7118	94.9150	95.1086	95.2932	5
6	91.7771	92.1139	92.4332	92.7361	93.0244	93.2971	93.5571	93.8047	94.0406	94.2654	6
7	90.2592	90.6582	91.0364	91.3952	91.7359	92.0597	92.3678	92.6611	92.9405	93.2068	7
8	88.6957	89.1588	89.5976	90.0140	90.4095	90.7853	91.1428	91.4831	91.8074	92.1165	8
9	87.0853	87.6144	88.1158	88.5915	89.0432	89.4726	89.8810	90.2698	90.6403	90.9934	9
10	85.4266	86.0236	86.5894	87.1262	87.6360	88.1205	88.5814	89.0202	89.4382	89.8367	10
11	83.7182	84.3852	85.0173	85.6170	86.1866	86.7279	87.2428	87.7330	88.2000	88.6452	11
12	81.9585	82.6975	83.3980	84.0626	84.6936	85.2935	85.8640	86.4072	86.9247	87.4181	12
13	80.1460	80.9593	81.7301	82.4615	83.1559	83.8160	84.4439	85.0416	85.6111	86.1540	13
14	78.2792	79.1689	80.0122	80.8123	81.5721	82.2942	82.9811	83.6351	84.2581	84.8521	14
15	76.3563	77.3248	78.2428	79.1137	79.9407	80.7268	81.4745	82.1864	82.8646	83.5111	15
16	74.3757	75.4254	76.4203	77.3641	78.2604	79.1124	79.9227	80.6942	81.4292	82.1299	16
17	72.3358	73.4690	74.5431	75.5621	76.5297	77.4495	78.3243	79.1573	79.9507	80.7073	17
18	70.2346	71.4539	72.6095	73.7059	74.7471	75.7367	76.6780	77.5742	78.4280	79.2419	18
19	68.0704	69.3783	70.6180	71.7941	72.9110	73.9725	74.9823	75.9436	76.8595	77.7326	19
20	65.8413	67.2405	68.5667	69.8250	71.0198	72.1555	73.2357	74.2642	75.2439	76.1781	20
21	63.5453	65.0386	66.4539	67.7967	69.0719	70.2839	71.4368	72.5343	73.5800	74.5769	21
22	61.1804	62.7706	64.2777	65.7077	67.0655	68.3561	69.5838	70.7526	71.8660	72.9276	22
23	58.7446	60.4345	62.0363	63.5559	64.9990	66.3706	67.6753	68.9174	70.1007	71.2289	23
24	56.2357	58.0284	59.7275	61.3396	62.8704	64.3254	65.7095	67.0271	68.2824	69.4792	24
25	53.6515	55.5501	57.3496	59.0568	60.6780	62.2189	63.6847	65.0802	66.4096	67.6770	25
26	50.9898	52.9974	54.9002	56.7055	58.4198	60.0493	61.5992	63.0748	64.4805	65.8208	26
27	48.2482	50.3682	52.3774	54.2837	56.0939	57.8145	59.4511	61.0092	62.4936	63.9088	27
28	45.4245	47.6600	49.7789	51.7892	53.6982	55.5127	57.2386	58.8817	60.4471	61.9396	28
29	42.5159	44.8707	47.1025	49.2199	51.2307	53.1418	54.9597	56.6904	58.3392	59.9112	29
30	39.5202	41.9976	44.3458	46.5735	48.6891	50.6998	52.6125	54.4334	56.1681	57.8220	30
31	36.4345	39.0384	41.5063	43.8477	46.0712	48.1845	50.1948	52.1086	53.9318	55.6701	31
32	33.2563	35.9904	38.5817	41.0402	43.3748	45.5938	47.7046	49.7141	51.6285	53.4537	32
33	29.9828	32.8509	35.5693	38.1484	40.5976	42.9254	45.1397	47.2477	49.2560	51.1707	33
34	26.6110	29.6173	32.4666	35.1699	37.7370	40.1769	42.4978	44.7074	46.8124	48.8193	34
35	23.1381	26.2866	29.2708	32.1020	34.7906	37.3459	39.7767	42.0908	44.2954	46.3973	35
36	19.5610	22.8561	25.9791	28.9421	31.7558	34.4300	36.9739	39.3958	41.7030	43.9027	36
37	15.8766	19.3226	22.5887	25.6873	28.6299	31.4267	34.0871	36.6199	39.0328	41.3332	37
38	12.0817	15.6831	19.0965	22.3350	25.4103	28.3332	31.1137	33.7607	36.2824	38.6867	38
39	8.1729	11.9344	15.4996	18.8820	22.0941	25.1470	28.0510	30.8157	33.4496	35.9608	39
40	4.1468	8.0733	11.7948	15.3255	18.6784	21.8651	24.8965	27.7824	30.5318	33.1530	40
41	0.0000	4.0963	7.9788	11.6623	15.1602	18.4848	21.6473	24.6581	27.5264	30.2611	41
42		0.0000	4.0484	7.8892	11.5365	15.0031	18.3007	21.4401	24.4309	27.2823	42
43			0.0000	4.0029	7.8041	11.4170	14.8537	18.1255	21.2425	24.2143	43
44				0.0000	3.9597	7.7232	11.3032	14.7115	17.9585	21.0541	44
45					0.0000	3.9187	7.6463	11.1950	14.5759	17.7992	45
46						0.0000	3.8796	7.5731	11.0919	14.4466	46
47							0.0000	3.8425	7.5033	10.9935	47
48								0.0000	3.8071	7.4367	48
49									0.0000	3.7733	49
50										0.0000	50

3% INTEREST RATE
51–60 Years Probable Life

Age, years	Condition-percent, %										Age, years
	51	52	53	54	55	56	57	58	59	60	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.1466	99.1783	99.2085	99.2374	99.2651	99.2916	99.3169	99.3412	99.3644	99.3867	1
2	98.2676	98.3319	98.3933	98.4520	98.5081	98.5619	98.6133	98.6625	98.7097	98.7550	2
3	97.3623	97.4602	97.5536	97.6430	97.7285	97.8103	97.8886	97.9636	98.0354	98.1044	3
4	96.4298	96.5622	96.6888	96.8097	96.9254	97.0361	97.1421	97.2436	97.3409	97.4342	4
5	95.4693	95.6374	95.7980	95.9515	96.0983	96.2388	96.3733	96.5021	96.6256	96.7439	5
6	94.4800	94.6848	94.8804	95.0675	95.2463	95.4175	95.5813	95.7383	95.8887	96.0330	6
7	93.4610	93.7036	93.9354	94.1569	94.3688	94.5716	94.7657	94.9516	95.1298	95.3006	7
8	92.4114	92.6930	92.9620	93.2191	93.4650	93.7003	93.9255	94.1413	94.3481	94.5464	8
9	91.3304	91.6521	91.9594	92.2531	92.5340	92.8028	93.0602	93.3067	93.5429	93.7695	9
10	90.2169	90.5799	90.9267	91.2581	91.5751	91.8784	92.1689	92.4470	92.7136	92.9692	10
11	89.0701	89.4756	89.8630	90.2333	90.5875	90.9264	91.2508	91.5616	91.8595	92.1450	11
12	87.8888	88.3382	88.7674	89.1778	89.5702	89.9457	90.3052	90.6496	90.9796	91.2961	12
13	86.6721	87.1666	87.6390	88.0905	88.5224	88.9356	89.3313	89.7102	90.0734	90.4217	13
14	85.4188	85.9599	86.4767	86.9707	87.4431	87.8952	88.3281	88.7427	89.1401	89.5210	14
15	84.1280	84.7169	85.2795	85.8172	86.3315	86.8237	87.2948	87.7461	88.1787	88.5934	15
16	82.7985	83.4367	84.0464	84.6292	85.1866	85.7199	86.2306	86.7197	87.1884	87.6379	16
17	81.4291	82.1181	82.7763	83.4055	84.0073	84.5831	85.1344	85.6624	86.1685	86.6537	17
18	80.0185	80.7599	81.4682	82.1451	82.7926	83.4121	84.0053	84.5734	85.1179	85.6400	18
19	78.5657	79.3610	80.1207	80.8469	81.5414	82.2060	82.8423	83.4518	84.0359	84.5959	19
20	77.0693	77.9201	78.7329	79.5098	80.2528	80.9638	81.6445	82.2965	82.9214	83.5205	20
21	75.5280	76.4360	77.3034	78.1325	78.9254	79.6842	80.4107	81.1066	81.7734	82.4128	21
22	73.9405	74.9074	75.8310	76.7139	77.5583	78.3663	79.1399	79.8809	80.5910	81.2719	22
23	72.3053	73.3329	74.3145	75.2528	76.1501	77.0089	77.8310	78.6185	79.3732	80.0968	23
24	70.6211	71.7111	72.7524	73.7478	74.6997	75.6107	76.4828	77.3182	78.1188	78.8864	24
25	68.8863	70.0408	71.1435	72.1977	73.2058	74.1705	75.0942	75.9789	76.8267	77.6397	25
26	67.0995	68.3203	69.4864	70.6011	71.6671	72.6872	73.6639	74.5994	75.4960	76.3556	26
27	65.2591	66.5482	67.7795	68.9565	70.0822	71.1594	72.1907	73.1785	74.1252	75.0329	27
28	63.3635	64.7229	66.0214	67.2627	68.4498	69.5857	70.6733	71.7150	72.7134	73.6706	28
29	61.4110	62.8429	64.2106	65.5180	66.7684	67.9648	69.1104	70.2077	71.2592	72.2675	29
30	59.4000	60.9064	62.3454	63.7210	65.0365	66.2953	67.5006	68.6550	69.7614	70.8222	30
31	57.3286	58.9119	60.4243	61.8701	63.2527	64.5758	65.8425	67.0558	68.2186	69.3336	31
32	55.1951	56.8575	58.4456	59.9636	61.4154	62.8046	64.1346	65.4087	66.6296	67.8003	32
33	52.9976	54.7415	56.4075	58.0000	59.5229	60.9803	62.3756	63.7121	64.9929	66.2210	33
34	50.7341	52.5621	54.3082	55.9774	57.5737	59.1012	60.5637	61.9646	63.3071	64.5943	34
35	48.4027	50.3172	52.1460	53.8942	55.5660	57.1658	58.6975	60.1647	61.5707	62.9188	35
36	46.0014	48.0050	49.9189	51.7484	53.4981	55.1723	56.7753	58.3108	59.7823	61.1931	36
37	43.5281	45.6235	47.6250	49.5383	51.3681	53.1191	54.7955	56.4012	57.9401	59.4156	37
38	40.9806	43.1704	45.2623	47.2619	49.1742	51.0042	52.7562	54.4344	56.0427	57.5848	38
39	38.3566	40.6438	42.8287	44.9172	46.9146	48.8259	50.6558	52.4086	54.0884	55.6990	39
40	35.6539	38.0414	40.3221	42.5022	44.5871	46.5822	48.4923	50.3220	52.0755	53.7567	40
41	32.8702	35.3610	37.7403	40.0147	42.1898	44.2712	46.2640	48.1728	50.0022	51.7561	41
42	30.0029	32.6001	35.0810	37.4526	39.7206	41.8909	43.9688	45.9592	47.8666	49.6955	42
43	27.0496	29.7563	32.3420	34.8136	37.1773	39.4392	41.6048	43.6791	45.6670	47.5731	43
44	24.0077	26.8273	29.5208	32.0954	34.5577	36.9139	39.1698	41.3306	43.4015	45.3870	44
45	20.8745	23.8104	26.6149	29.2957	31.8595	34.3129	36.6618	38.9117	41.0679	43.1353	45
46	17.6474	20.7030	23.6219	26.4120	29.0804	31.6338	34.0785	36.4202	38.6643	40.8160	46
47	14.3234	17.5024	20.5391	23.4418	26.2179	28.8744	31.4177	33.8540	36.1887	38.4272	47
48	10.8998	14.2057	17.3638	20.3825	23.2695	26.0322	28.6772	31.2107	33.6388	35.9667	48
49	7.3734	10.8102	14.0932	17.2315	20.2327	23.1047	25.8544	28.4882	31.0123	33.4325	49
50	3.7412	7.3128	10.7246	13.9858	17.1048	20.0894	22.9469	25.6840	28.3071	30.8221	50
51	0.0000	3.7104	7.2548	10.6429	13.8830	16.9836	19.9522	22.7957	25.5207	28.1335	51
52		0.0000	3.6810	7.1996	10.5646	13.7847	16.8676	19.8207	22.6507	25.3642	52
53			0.0000	3.6530	7.1466	10.4898	13.6905	16.7565	19.6947	22.5118	53
54				0.0000	3.6261	7.0960	10.4181	13.6003	16.6499	19.5739	54
55					0.0000	3.6005	7.0476	10.3495	13.5138	16.5478	55
56						0.0000	3.5759	7.0011	10.2837	13.4310	56
57							0.0000	3.5523	6.9566	10.2206	57
58								0.0000	3.5297	6.9139	58
59									0.0000	3.5080	59
60										0.0000	60

3% INTEREST RATE
61–70 Years Probable Life

Age, years	Condition-percent, % 61	62	63	64	65	66	67	68	69	70	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.4081	99.4286	99.4483	99.4672	99.4854	99.5029	99.5197	99.5358	99.5514	99.5663	1
2	98.7984	98.8401	98.8801	98.9185	98.9554	98.9909	99.0250	99.0578	99.0893	99.1197	2
3	98.1705	98.2339	98.2948	98.3533	98.4095	98.4635	98.5154	98.5653	98.6134	98.6596	3
4	97.5237	97.6095	97.6920	97.7711	97.8472	97.9203	97.9905	98.0581	98.1231	98.1857	4
5	96.8575	96.9664	97.0710	97.1715	97.2680	97.3608	97.4500	97.5357	97.6182	97.6976	5
6	96.1713	96.3040	96.4315	96.5539	96.6715	96.7845	96.8931	96.9976	97.0982	97.1949	6
7	95.4645	95.6218	95.7728	95.9177	96.0570	96.1909	96.3196	96.4434	96.5625	96.6771	7
8	94.7366	94.9191	95.0943	95.2625	95.4242	95.5795	95.7289	95.8725	96.0107	96.1437	8
9	93.9867	94.1952	94.3954	94.5876	94.7723	94.9498	95.1204	95.2846	95.4424	95.5944	9
10	93.2144	93.4497	93.6756	93.8925	94.1009	94.3012	94.4937	94.6789	94.8571	95.0285	10
11	92.4190	92.6818	92.9342	93.1765	93.4094	93.6331	93.8482	94.0551	94.2542	94.4457	11
12	91.5996	91.8909	92.1705	92.4391	92.6971	92.9450	93.1834	93.4126	93.6332	93.8454	12
13	90.7557	91.0762	91.3839	91.6795	91.9634	92.2362	92.4986	92.7509	92.9936	93.2271	13
14	89.8865	90.2371	90.5738	90.8971	91.2077	91.5062	91.7932	92.0692	92.3348	92.5903	14
15	88.9912	89.3729	89.7393	90.0912	90.4294	90.7543	91.0667	91.3671	91.6562	91.9343	15
16	88.0690	88.4827	88.8798	89.2612	89.6277	89.9798	90.3184	90.6440	90.9573	91.2587	16
17	87.1191	87.5658	87.9945	88.4063	88.8019	89.1821	89.5476	89.8992	90.2374	90.5628	17
18	86.1408	86.6213	87.0827	87.5257	87.9514	88.3605	88.7537	89.1320	89.4959	89.8460	18
19	85.1331	85.6486	86.1435	86.6187	87.0753	87.5142	87.9360	88.3418	88.7321	89.1077	19
20	84.0952	84.6467	85.1761	85.6845	86.1730	86.6425	87.0938	87.5279	87.9455	88.3473	20
21	83.0262	83.6147	84.1797	84.7223	85.2436	85.7446	86.2263	86.6896	87.1352	87.5641	21
22	81.9250	82.5517	83.1534	83.7312	84.2864	84.8199	85.3328	85.8261	86.3007	86.7573	22
23	80.7909	81.4569	82.0963	82.7104	83.3004	83.8673	84.4125	84.9367	85.4411	85.9264	23
24	79.6227	80.3292	81.0075	81.6590	82.2848	82.8863	83.4645	84.0206	84.5557	85.0705	24
25	78.4195	79.1677	79.8861	80.5760	81.2388	81.8757	82.4881	83.0771	83.6437	84.1890	25
26	77.1801	77.9714	78.7310	79.4604	80.1613	80.8349	81.4825	82.1053	82.7044	83.2810	26
27	75.9036	76.7391	77.5412	78.3115	79.0516	79.7628	80.4466	81.1043	81.7369	82.3457	27
28	74.5888	75.4699	76.3158	77.1281	77.9086	78.6586	79.3797	80.0732	80.7404	81.3824	28
29	73.2346	74.1626	75.0536	75.9092	76.7312	77.5213	78.2808	79.0113	79.7140	80.3903	29
30	71.8397	72.8161	73.7535	74.6537	75.5186	76.3498	77.1489	77.9174	78.6568	79.3683	30
31	70.4030	71.4292	72.4144	73.3606	74.2696	75.1432	75.9831	76.7908	77.5679	78.3157	31
32	68.9232	70.0007	71.0352	72.0286	72.9831	73.9004	74.7822	75.6304	76.4463	77.2315	32
33	67.3990	68.5293	69.6145	70.6567	71.6580	72.6203	73.5454	74.4351	75.2911	76.1148	33
34	65.8290	67.0138	68.1513	69.2437	70.2931	71.3018	72.2714	73.2040	74.1012	74.9646	34
35	64.2120	65.4529	66.6442	67.7882	68.8874	69.9437	70.9593	71.9360	72.8756	73.7798	35
36	62.5465	63.8451	65.0918	66.2891	67.4394	68.5449	69.6077	70.6299	71.6133	72.5596	36
37	60.8309	62.1890	63.4929	64.7450	65.9480	67.1041	68.2157	69.2846	70.3131	71.3027	37
38	59.0640	60.4833	61.8460	63.1546	64.4119	65.6201	66.7818	67.8990	68.9738	70.0081	38
39	57.2440	58.7264	60.1497	61.5165	62.8296	64.0916	65.3050	66.4718	67.5944	68.6747	39
40	55.3694	56.9169	58.4025	59.8292	61.1999	62.5173	63.7838	65.0018	66.1736	67.3013	40
41	53.4386	55.0530	56.6029	58.0913	59.5214	60.8957	62.2170	63.4877	64.7102	65.8866	41
42	51.4498	53.1332	54.7493	56.3013	57.7924	59.2254	60.6032	61.9282	63.2029	64.4296	42
43	49.4014	51.1558	52.8401	54.4576	56.0116	57.5051	58.9410	60.3219	61.6504	62.9288	43
44	47.2915	49.1191	50.8736	52.5586	54.1774	55.7331	57.2289	58.6674	60.0513	61.3830	44
45	45.1184	47.0213	48.8481	50.6026	52.2881	53.9080	55.4654	56.9632	58.4042	59.7908	45
46	42.8800	44.8605	46.7619	48.5879	50.3422	52.0282	53.6491	55.2080	56.7077	58.1509	46
47	40.5745	42.6349	44.6131	46.5128	48.3379	50.0919	51.7783	53.4001	54.9603	56.4618	47
48	38.1998	40.3426	42.3998	44.3754	46.2734	48.0975	49.8513	51.5379	53.1605	54.7219	48
49	35.7539	37.9815	40.1201	42.1739	44.1470	46.0434	47.8665	49.6199	51.3067	52.9300	49
50	33.2346	35.5496	37.7720	39.9063	41.9569	43.9275	45.8222	47.6443	49.3973	51.0842	50
51	30.6398	33.0447	35.3535	37.5708	39.7010	41.7483	43.7166	45.6095	47.4306	49.1830	51
52	27.9670	30.4646	32.8624	35.1651	37.3774	39.5036	41.5477	43.5136	45.4049	47.2249	52
53	25.2142	27.8072	30.2966	32.6873	34.9842	37.1916	39.3139	41.3549	43.3184	45.2079	53
54	22.3787	25.0700	27.6538	30.1352	32.5191	34.8102	37.0130	39.1314	41.1694	43.1305	54
55	19.4581	22.2507	24.9317	27.5065	29.9801	32.3574	34.6430	36.8411	38.9558	40.9908	55
56	16.4500	19.3469	22.1280	24.7989	27.3650	29.8311	32.2020	34.4822	36.6759	38.7868	56
57	13.3515	16.3559	19.2402	22.0101	24.6713	27.2289	29.6878	32.0525	34.3275	36.5168	57
58	10.1602	13.2752	16.2657	19.1377	21.8969	24.5486	27.0981	29.5499	31.9087	34.1786	58
59	6.8731	10.1020	13.2020	16.1790	19.0392	21.7880	24.4307	26.9723	29.4174	31.7704	59
60	3.4873	6.8337	10.0464	13.1317	16.0958	18.9445	21.6833	24.3173	26.8513	29.2898	60

3% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, % 61	62	63	64	65	66	67	68	69	70	Age, years
61	0.0000	3.4673	6.7961	9.9928	13.0641	16.0157	18.8535	21.5826	24.2082	26.7348	61
62		0.0000	3.4483	6.7597	9.9414	12.9991	15.9388	18.7660	21.4858	24.1032	62
63			0.0000	3.4299	6.7251	9.8920	12.9366	15.8648	18.6818	21.3926	63
64				0.0000	3.4123	6.6916	9.8444	12.8766	15.7936	18.6007	64
65					0.0000	3.3952	6.6595	9.7987	12.8188	15.7251	65
66						0.0000	3.3789	6.6285	9.7548	12.7632	66
67							0.0000	3.3632	6.5988	9.7124	67
68								0.0000	3.3482	6.5701	68
69									0.0000	3.3336	69
70										0.0000	70

71–80 Years Probable Life

Age, years	71	72	73	74	75	76	77	78	79	80	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.5807	99.5946	99.6079	99.6208	99.6332	99.6452	99.6567	99.6678	99.6785	99.6888	1
2	99.1489	99.1770	99.2041	99.2302	99.2554	99.2797	99.3030	99.3256	99.3473	99.3683	2
3	98.7041	98.7469	98.7882	98.8280	98.8663	98.9032	98.9388	98.9731	99.0062	99.0382	3
4	98.2459	98.3039	98.3598	98.4136	98.4655	98.5154	98.5636	98.6101	98.6549	98.6982	4
5	97.7741	97.8476	97.9185	97.9868	98.0526	98.1161	98.1772	98.2362	98.2931	98.3479	5
6	97.2880	97.3777	97.4640	97.5472	97.6274	97.7047	97.7792	97.8510	97.9203	97.9872	6
7	96.7874	96.8936	96.9959	97.0945	97.1894	97.2810	97.3692	97.4543	97.5364	97.6156	7
8	96.2717	96.3950	96.5137	96.6281	96.7383	96.8446	96.9470	97.0458	97.1410	97.2329	8
9	95.7406	95.8814	96.0171	96.1478	96.2737	96.3951	96.5121	96.6249	96.7337	96.8387	9
10	95.1936	95.3525	95.5056	95.6530	95.7951	95.9321	96.0641	96.1914	96.3142	96.4327	10
11	94.6301	94.8077	94.9787	95.1434	95.3022	95.4552	95.6027	95.7449	95.8822	96.0145	11
12	94.0498	94.2465	94.4360	94.6185	94.7944	94.9640	95.1274	95.2851	95.4371	95.5838	12
13	93.4520	93.6685	93.8770	94.0779	94.2715	94.4580	94.6379	94.8114	94.9787	95.1401	13
14	92.8363	93.0731	93.3013	93.5210	93.7328	93.9369	94.1337	94.3235	94.5066	94.6832	14
15	92.2021	92.4599	92.7082	92.9475	93.1780	93.4002	93.6144	93.8210	94.0203	94.2125	15
16	91.5489	91.8283	92.0974	92.3567	92.6065	92.8474	93.0795	93.3034	93.5194	93.7277	16
17	90.8761	91.1777	91.4683	91.7482	92.0179	92.2779	92.5286	92.7703	93.0034	93.2283	17
18	90.1831	90.5077	90.8203	91.1215	91.4117	91.6914	91.9611	92.2212	92.4720	92.7140	18
19	89.4694	89.8175	90.1529	90.4759	90.7872	91.0873	91.3766	91.6556	91.9247	92.1843	19
20	88.7342	89.1066	89.4654	89.8110	90.1441	90.4651	90.7746	91.0730	91.3609	91.6386	20
21	87.9769	88.3744	88.7573	89.1261	89.4816	89.8242	90.1545	90.4730	90.7802	91.0766	21
22	87.1970	87.6202	88.0280	88.4207	88.7992	89.1641	89.5158	89.8549	90.1821	90.4977	22
23	86.3936	86.8434	87.2767	87.6942	88.0964	88.4841	88.8579	89.2184	89.5661	89.9015	23
24	85.5662	86.0433	86.5030	86.9458	87.3725	87.7838	88.1803	88.5627	88.9315	89.2873	24
25	84.7139	85.2192	85.7060	86.1750	86.6269	87.0625	87.4824	87.8874	88.2780	88.6548	25
26	83.8360	84.3704	84.8852	85.3810	85.8589	86.3195	86.7635	87.1918	87.6048	88.0033	26
27	82.9318	83.4961	84.0397	84.5633	85.0679	85.5542	86.0231	86.4753	86.9114	87.3322	27
28	82.0005	82.5956	83.1688	83.7210	84.2531	84.7660	85.2605	85.7373	86.1973	86.6410	28
29	81.0413	81.6681	82.2718	82.8534	83.4139	83.9541	84.4750	84.9772	85.4617	85.9290	29
30	80.0532	80.7127	81.3479	81.9598	82.5495	83.1179	83.6659	84.1943	84.7040	85.1957	30
31	79.0356	79.7287	80.3963	81.0394	81.6592	82.2566	82.8325	83.3879	83.9236	84.4404	31
32	77.9874	78.7151	79.4161	80.0914	80.7422	81.3694	81.9742	82.5573	83.1198	83.6625	32
33	76.9077	77.6712	78.4066	79.1150	79.7977	80.4557	81.0901	81.7018	82.2919	82.8612	33
34	75.7957	76.5959	77.3667	78.1092	78.8248	79.5145	80.1794	80.8206	81.4392	82.0358	34
35	74.6503	75.4884	76.2957	77.0733	77.8228	78.5451	79.2415	79.9130	80.5608	81.1857	35
36	73.4705	74.3476	75.1925	76.0063	76.7906	77.5466	78.2754	78.9782	79.6562	80.3101	36
37	72.2554	73.1726	74.0562	74.9073	75.7276	76.5181	77.2803	78.0154	78.7243	79.4082	37
38	71.0038	71.9624	72.8858	73.7754	74.6326	75.4588	76.2554	77.0236	77.7645	78.4793	38
39	69.7146	70.7159	71.6804	72.6094	73.5048	74.3678	75.1997	76.0021	76.7760	77.5225	39
40	68.3868	69.4320	70.4387	71.4085	72.3431	73.2439	74.1124	74.9499	75.7577	76.5370	40
41	67.0192	68.1095	69.1598	70.1716	71.1466	72.0864	72.9925	73.8662	74.7090	75.5219	41
42	65.6105	66.7474	67.8425	68.8975	69.9142	70.8942	71.8389	72.7499	73.6287	74.4764	42
43	64.1595	65.3444	66.4858	67.5853	68.6449	69.6661	70.6507	71.6002	72.5161	73.3995	43
44	62.6650	63.8993	65.0883	66.2336	67.3374	68.4013	69.4269	70.4160	71.3700	72.2904	44
45	61.1257	62.4109	63.6489	64.8415	65.9907	67.0984	68.1664	69.1962	70.1896	71.1479	45

3% INTEREST RATE

71–80 Years Probable Life

Age, years	Condition-percent, % 71	72	73	74	75	76	77	78	79	80	Age, years
46	59.5402	60.8778	62.1663	63.4075	64.6037	65.7566	66.8681	67.9399	68.9738	69.9712	46
47	57.9072	59.2988	60.6393	61.9306	63.1750	64.3744	65.5308	66.6459	67.7215	68.7591	47
48	56.2251	57.6723	59.0664	60.4093	61.7034	62.9508	64.1534	65.3130	66.4316	67.5107	48
49	54.4926	55.9971	57.4463	58.8424	60.1877	61.4845	62.7346	63.9402	65.1031	66.2249	49
50	52.7081	54.2716	55.7776	57.2284	58.6266	59.9741	61.2733	62.5262	63.7347	64.9004	50
51	50.8701	52.4944	54.0589	55.5661	57.0186	58.4185	59.7682	61.0697	62.3252	63.5363	51
52	48.9769	50.6638	52.2886	53.8539	55.3623	56.8162	58.2179	59.5696	60.8735	62.1312	52
53	47.0270	48.7783	50.4652	52.0903	53.6564	55.1659	56.6211	58.0245	59.3781	60.6839	53
54	45.0185	46.8362	48.5872	50.2738	51.8993	53.4660	54.9764	56.4330	57.8380	59.1933	54
55	42.9498	44.8359	46.6527	48.4029	50.0895	51.7151	53.2824	54.7937	56.2516	57.6579	55
56	40.8190	42.7756	44.6602	46.4758	48.2254	49.9117	51.5375	53.1053	54.6177	56.0765	56
57	38.6243	40.6534	42.6080	44.4908	46.3053	48.0542	49.7403	51.3663	52.9347	54.4476	57
58	36.3638	38.4676	40.4942	42.4464	44.3277	46.1410	47.8892	49.5750	51.2012	52.7698	58
59	34.0354	36.2162	38.3170	40.3406	42.2907	44.1704	45.9826	47.7301	49.4157	51.0418	59
60	31.6372	33.8973	36.0744	38.1716	40.1927	42.1407	44.0187	45.8297	47.5767	49.2618	60
61	29.1671	31.5089	33.7646	35.9376	38.0316	40.0500	41.9959	43.8724	45.6825	47.4285	61
62	26.6228	29.0487	31.3854	33.6365	35.8058	37.8967	39.9125	41.8564	43.7314	45.5402	62
63	24.0023	26.5148	28.9350	31.2664	33.5132	35.6787	37.7665	39.7798	41.7219	43.5952	63
64	21.3031	23.9048	26.4110	28.8252	31.1517	33.3942	35.5562	37.6410	39.6520	41.5919	64
65	18.5229	21.2165	23.8113	26.3108	28.7195	31.0412	33.2795	35.4380	37.5201	39.5285	65
66	15.6593	18.4476	21.1335	23.7209	26.2143	28.6176	30.9346	33.1689	35.3242	37.4032	66
67	12.7098	15.5956	18.3755	21.0533	23.6339	26.1213	28.5193	30.8318	33.0624	35.2141	67
68	9.6718	12.6581	15.5347	18.3057	20.9762	23.5501	26.0316	28.4245	30.7327	32.9594	68
69	6.5427	9.6324	12.6087	15.4757	18.2387	20.9017	23.4692	25.9450	28.3332	30.6370	69
70	3.3197	6.5160	9.5949	12.5608	15.4190	18.1739	20.8299	23.3911	25.8617	28.2449	70
71	0.0000	3.3062	6.4907	9.5584	12.5148	15.3643	18.1115	20.7606	23.3160	25.7811	71
72		0.0000	3.2933	6.4660	9.5234	12.4704	15.3115	18.0512	20.6940	23.2433	72
73			0.0000	3.2808	6.4423	9.4896	12.4275	15.2605	17.9933	20.6294	73
74				0.0000	3.2687	6.4195	9.4570	12.3861	15.2116	17.9372	74
75					0.0000	3.2571	6.3974	9.4255	12.3465	15.1641	75
76						0.0000	3.2460	6.3760	9.3953	12.3078	76
77							0.0000	3.2351	6.3557	9.3659	77
78								0.0000	3.2248	6.3357	78
79									0.0000	3.2147	79
80										0.0000	80

81–90 Years Probable Life

Age, years	81	82	83	84	85	86	87	88	89	90	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.6988	99.7084	99.7177	99.7267	99.7354	99.7437	99.7518	99.7596	99.7672	99.7744	1
2	99.3886	99.4081	99.4270	99.4452	99.4628	99.4797	99.4961	99.5120	99.5273	99.5421	2
3	99.0690	99.0988	99.1275	99.1552	99.1820	99.2079	99.2328	99.2570	99.2803	99.3028	3
4	98.7399	98.7802	98.8190	98.8566	98.8928	98.9278	98.9616	98.9943	99.0259	99.0564	4
5	98.4009	98.4520	98.5013	98.5489	98.5949	98.6394	98.6823	98.7237	98.7638	98.8025	5
6	98.0517	98.1140	98.1741	98.2321	98.2881	98.3423	98.3945	98.4450	98.4938	98.5410	6
7	97.6921	97.7658	97.8370	97.9057	97.9721	98.0362	98.0982	98.1580	98.2158	98.2717	7
8	97.3216	97.4072	97.4898	97.5696	97.6466	97.7210	97.7929	97.8623	97.9294	97.9943	8
9	96.9401	97.0378	97.1322	97.2234	97.3114	97.3964	97.4785	97.5578	97.6345	97.7086	9
10	96.5471	96.6574	96.7639	96.8668	96.9661	97.0620	97.1546	97.2442	97.3307	97.4143	10
11	96.1423	96.2656	96.3846	96.4995	96.6104	96.7176	96.8211	96.9211	97.0177	97.1111	11
12	95.7253	95.8619	95.9938	96.1211	96.2441	96.3628	96.4775	96.5883	96.6954	96.7989	12
13	95.2959	95.4462	95.5913	95.7315	95.8668	95.9974	96.1236	96.2456	96.3634	96.4773	13
14	94.8536	95.0180	95.1768	95.3301	95.4781	95.6210	95.7591	95.8926	96.0215	96.1461	14
15	94.3980	94.5770	94.7498	94.9167	95.0778	95.2334	95.3837	95.5290	95.6693	95.8049	15
16	93.9287	94.1227	94.3100	94.4909	94.6655	94.8341	94.9970	95.1544	95.3065	95.4535	16
17	93.4454	93.6548	93.8571	94.0523	94.2408	94.4229	94.5987	94.7687	94.9329	95.0915	17
18	92.9475	93.1729	93.3905	93.6005	93.8034	93.9993	94.1885	94.3713	94.5480	94.7187	18
19	92.4348	92.6765	92.9099	93.1352	93.3528	93.5629	93.7659	93.9621	94.1516	94.3347	19
20	91.9066	92.1652	92.4149	92.6560	92.8888	93.1136	93.3307	93.5406	93.7433	93.9392	20

3% INTEREST RATE

81–90 Years Probable Life

Age, years	Condition-percent, %										Age, years
	81	82	83	84	85	86	87	88	89	90	
21	91.3626	91.6386	91.9051	92.1623	92.4108	92.6507	92.8824	93.1064	93.3227	93.5318	21
22	90.8023	91.0962	91.3800	91.6539	91.9184	92.1739	92.4207	92.6592	92.8896	93.1122	22
23	90.2252	90.5375	90.8391	91.1302	91.4113	91.6828	91.9451	92.1986	92.4434	92.6801	23
24	89.6307	89.9621	90.2820	90.5908	90.8890	91.1770	91.4553	91.7241	91.9839	92.2349	24
25	89.0184	89.3694	89.7081	90.0352	90.3511	90.6561	90.9507	91.2354	91.5105	91.7764	25
26	88.3878	88.7589	89.1171	89.4630	89.7969	90.1195	90.4311	90.7321	91.0230	91.3041	26
27	87.7382	88.1300	88.5083	88.8735	89.2262	89.5668	89.8958	90.2137	90.5208	90.8177	27
28	87.0692	87.4824	87.8813	88.2664	88.6383	88.9975	89.3445	89.6797	90.0036	90.3167	28
29	86.3800	86.8153	87.2354	87.6411	88.0328	88.4111	88.7766	89.1297	89.4709	89.8006	29
30	85.6702	86.1282	86.5702	86.9970	87.4092	87.8072	88.1917	88.5632	88.9222	89.2691	30
31	84.9391	85.4204	85.8850	86.3336	86.7668	87.1851	87.5892	87.9797	88.3570	88.7216	31
32	84.1861	84.6915	85.1793	85.6503	86.1051	86.5444	86.9687	87.3787	87.7748	88.1577	32
33	83.4105	83.9406	84.4524	84.9465	85.4236	85.8844	86.3296	86.7597	87.1752	87.5769	33
34	82.6116	83.1673	83.7037	84.2216	84.7217	85.2047	85.6713	86.1221	86.5576	86.9786	34
35	81.7888	82.3707	82.9325	83.4749	83.9987	84.5045	84.9932	85.4653	85.9215	86.3624	35
36	80.9412	81.5503	82.1382	82.7059	83.2540	83.7834	84.2948	84.7889	85.2663	85.7277	36
37	80.0683	80.7052	81.3201	81.9137	82.4870	83.0406	83.5754	84.0922	84.5915	85.0740	37
38	79.1691	79.8348	80.4774	81.0978	81.6969	82.2755	82.8345	83.3746	83.8964	84.4007	38
39	78.2430	78.9382	79.6094	80.2574	80.8832	81.4875	82.0713	82.6354	83.1804	83.7071	39
40	77.2891	78.0148	78.7154	79.3919	80.0451	80.6759	81.2853	81.8741	82.4430	82.9928	40
41	76.3065	77.0637	77.7946	78.5003	79.1818	79.8399	80.4756	81.0899	81.6834	82.2570	41
42	75.2945	76.0840	76.8462	77.5820	78.2926	78.9788	79.6417	80.2822	80.9011	81.4992	42
43	74.2522	75.0749	75.8693	76.6361	77.3767	78.0918	78.7827	79.4503	80.0952	80.7186	43
44	73.1785	74.0356	74.8631	75.6619	76.4333	77.1783	77.8980	78.5934	79.2653	79.9146	44
45	72.0727	72.9651	73.8267	74.6585	75.4617	76.2374	76.9867	77.7108	78.4104	79.0865	45
46	70.9337	71.8625	72.7592	73.6249	74.4609	75.2682	76.0482	76.8017	77.5298	78.2335	46
47	69.7605	70.7268	71.6597	72.5603	73.4300	74.2700	75.0814	75.8654	76.6229	77.3550	47
48	68.5521	69.5570	70.5272	71.4638	72.3683	73.2418	74.0856	74.9010	75.6887	76.4501	48
49	67.3074	68.3522	69.3607	70.3344	71.2747	72.1828	73.0600	73.9076	74.7265	75.5180	49
50	66.0255	67.1111	68.1592	69.1711	70.1482	71.0920	72.0036	72.8844	73.7355	74.5580	50
51	64.7050	65.8329	66.9219	67.9730	68.9881	69.9684	70.9155	71.8306	72.7147	73.5692	51
52	63.3450	64.5163	65.6471	66.7388	67.7931	68.8112	69.7948	70.7451	71.6633	72.5507	52
53	61.9441	63.1602	64.3342	65.4677	66.5622	67.6193	68.6404	69.6270	70.5803	71.5017	53
54	60.5013	61.7635	62.9820	64.1584	65.2944	66.3915	67.4514	68.4755	69.4649	70.4212	54
55	59.0151	60.3248	61.5892	62.8099	63.9885	65.1271	66.2268	67.2893	68.3160	69.3082	55
56	57.4843	58.8429	60.1545	61.4208	62.6437	63.8245	64.9654	66.0676	67.1326	68.1619	56
57	55.9077	57.3167	58.6769	59.9902	61.2583	62.4830	63.6661	64.8093	65.9138	66.9812	57
58	54.2837	55.7446	57.1549	58.5165	59.8314	61.1012	62.3279	63.5131	64.6583	65.7651	58
59	52.6110	54.1253	55.5873	56.9987	58.3617	59.6780	60.9495	62.1781	63.3652	64.5125	59
60	50.8881	52.4575	53.9726	55.4354	56.8479	58.2120	59.5298	60.8031	62.0333	63.2223	60
61	49.1136	50.7397	52.3095	53.8251	55.2887	56.7021	58.0675	59.3868	60.6615	61.8934	61
62	47.2858	48.9703	50.5965	52.1666	53.6827	55.1469	56.5613	57.9280	59.2485	60.5247	62
63	45.4032	47.1478	48.8321	50.4582	52.0285	53.5450	55.0100	56.4255	57.7931	59.1149	63
64	43.4641	45.2707	47.0148	48.6987	50.3247	51.8951	53.4121	54.8778	56.2940	57.6628	64
65	41.4668	43.3372	45.1430	46.8863	48.5698	50.1957	51.7662	53.2838	54.7500	56.1671	65
66	39.4096	41.3458	43.2150	45.0196	46.7623	48.4452	50.0711	51.6419	53.1597	54.6265	66
67	37.2907	39.2946	41.2291	43.0969	44.9005	46.6423	48.3249	49.9507	51.5216	53.0398	67
68	35.1082	37.1818	39.1837	41.1165	42.9829	44.7853	46.5265	48.2089	49.8344	51.4054	68
69	32.8602	35.0057	37.0770	39.0767	41.0077	42.8725	44.6741	46.4148	48.0966	49.7220	69
70	30.5448	32.7643	34.9070	36.9756	38.9733	40.9024	42.7661	44.5668	46.3066	47.9881	70
71	28.1600	30.4557	32.6719	34.8116	36.8778	38.8732	40.8009	42.6634	44.4630	46.2022	71
72	25.7036	28.0778	30.3698	32.5826	34.7195	36.7831	38.7767	40.7029	42.5640	44.3627	72
73	23.1735	25.6285	27.9986	30.2868	32.4964	34.6303	36.6918	38.6836	40.6081	42.4680	73
74	20.5675	23.1058	25.5563	27.9221	30.2066	32.4130	34.5443	36.6037	38.5935	40.5165	74
75	17.8833	20.5074	23.0407	25.4864	27.8482	30.1291	32.3325	34.4615	36.5185	38.5065	75
76	15.1186	17.8310	20.4496	22.9777	25.4190	27.7767	30.0544	32.2549	34.3812	36.4361	76
77	12.2710	15.0744	17.7808	20.3937	22.9170	25.3537	27.7077	29.9821	32.1797	34.3037	77
78	9.3379	12.2350	15.0320	17.7322	20.3398	22.8580	25.2907	27.6412	29.9123	32.1072	78
79	6.3168	9.3105	12.2006	14.9909	17.6854	20.2875	22.8012	25.2301	27.5768	29.8449	79
80	3.2051	6.2983	9.2844	12.1673	14.9513	17.6398	20.2371	22.7466	25.1713	27.5146	80

3% INTEREST RATE

81–90 Years Probable Life

Age, years	Condition-percent, %										Age, years
	81	82	83	84	85	86	87	88	89	90	
81	0.0000	3.1957	6.2806	9.2590	12.1352	14.9127	17.5960	20.1886	22.6936	25.1145	81
82		0.0000	3.1867	6.2635	9.2346	12.1038	14.8756	17.5538	20.1415	22.6424	82
83			0.0000	3.1780	6.2470	9.2106	12.0737	14.8401	17.5129	20.0961	83
84				0.0000	3.1696	6.2307	9.1877	12.0449	14.8054	17.4734	84
85					0.0000	3.1614	6.2151	9.1658	12.0168	14.7721	85
86						0.0000	3.1536	6.2004	9.1444	11.9897	86
87							0.0000	3.1460	6.1859	9.1238	87
88								0.0000	3.1387	6.1720	88
89									0.0000	3.1316	89
90										0.0000	90

91–100 Years Probable Life

Age, years	91	92	93	94	95	96	97	98	99	100	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.7815	99.7883	99.7949	99.8013	99.8074	99.8134	99.8191	99.8247	99.8301	99.8353	1
2	99.5564	99.5703	99.5836	99.5966	99.6091	99.6212	99.6329	99.6442	99.6551	99.6657	2
3	99.3246	99.3457	99.3660	99.3857	99.4048	99.4232	99.4410	99.4582	99.4749	99.4910	3
4	99.0858	99.1144	99.1419	99.1686	99.1944	99.2193	99.2434	99.2667	99.2893	99.3111	4
5	98.8399	98.8761	98.9111	98.9449	98.9776	99.0092	99.0398	99.0694	99.0981	99.1258	5
6	98.5866	98.6307	98.6733	98.7145	98.7543	98.7929	98.8301	98.8662	98.9011	98.9349	6
7	98.3257	98.3779	98.4284	98.4772	98.5244	98.5700	98.6142	98.6569	98.6983	98.7382	7
8	98.0570	98.1175	98.1761	98.2328	98.2875	98.3405	98.3918	98.4413	98.4893	98.5357	8
9	97.7802	97.8494	97.9163	97.9810	98.0436	98.1041	98.1627	98.2193	98.2741	98.3271	9
10	97.4951	97.5732	97.6487	97.7217	97.7923	97.8606	97.9267	97.9906	98.0524	98.1123	10
11	97.2014	97.2887	97.3730	97.4546	97.5335	97.6098	97.6836	97.7550	97.8241	97.8910	11
12	96.8989	96.9956	97.0891	97.1795	97.2669	97.3515	97.4333	97.5124	97.5890	97.6630	12
13	96.5874	96.6938	96.7967	96.8962	96.9924	97.0854	97.1754	97.2625	97.3468	97.4282	13
14	96.2665	96.3829	96.4955	96.6043	96.7096	96.8114	96.9098	97.0051	97.0973	97.1864	14
15	95.9360	96.0627	96.1852	96.3037	96.4183	96.5291	96.6363	96.7400	96.8403	96.9374	15
16	95.5956	95.7329	95.8657	95.9941	96.1182	96.2383	96.3545	96.4669	96.5756	96.6808	16
17	95.2449	95.3932	95.5365	95.6752	95.8092	95.9389	96.0643	96.1856	96.3030	96.4166	17
18	94.8838	95.0433	95.1975	95.3467	95.4909	95.6304	95.7654	95.8959	96.0222	96.1444	18
19	94.5118	94.6829	94.8484	95.0083	95.1631	95.3127	95.4575	95.5975	95.7330	95.8641	19
20	94.1286	94.3117	94.4887	94.6599	94.8254	94.9855	95.1403	95.2901	95.4351	95.5753	20
21	93.7340	93.9293	94.1183	94.3009	94.4775	94.6484	94.8137	94.9736	95.1283	95.2779	21
22	93.3275	93.5355	93.7367	93.9312	94.1193	94.3013	94.4772	94.6475	94.8122	94.9716	22
23	92.9088	93.1299	93.3437	93.5504	93.7503	93.9437	94.1307	94.3116	94.4867	94.6561	23
24	92.4775	92.7121	92.9389	93.1582	93.3702	93.5754	93.7738	93.9657	94.1514	94.3311	24
25	92.0334	92.2818	92.5219	92.7542	92.9788	93.1960	93.4061	93.6094	93.8061	93.9964	25
26	91.5759	91.8386	92.0925	92.3381	92.5756	92.8053	93.0274	93.2424	93.4504	93.6516	26
27	91.1046	91.3820	91.6502	91.9095	92.1603	92.4028	92.6374	92.8644	93.0840	93.2965	27
28	90.6192	90.9118	91.1946	91.4680	91.7325	91.9883	92.2357	92.4750	92.7066	92.9307	28
29	90.1193	90.4274	90.7253	91.0133	91.2919	91.5613	91.8219	92.0740	92.3180	92.5540	29
30	89.6044	89.9286	90.2419	90.5450	90.8381	91.1215	91.3957	91.6609	91.9176	92.1659	30
31	89.0740	89.4147	89.7441	90.0626	90.3706	90.6686	90.9567	91.2355	91.5052	91.7662	31
32	88.5277	88.8855	89.2313	89.5658	89.8892	90.2020	90.5046	90.7973	91.0805	91.3545	32
33	87.9651	88.3404	88.7031	89.0540	89.3933	89.7214	90.0388	90.3459	90.6430	90.9305	33
34	87.3855	87.7789	88.1591	88.5269	88.8825	89.2265	89.5592	89.8810	90.1925	90.4937	34
35	86.7886	87.2005	87.5988	87.9839	88.3564	88.7166	89.0651	89.4021	89.7283	90.0439	35
36	86.1737	86.6049	87.0217	87.4247	87.8145	88.1915	88.5562	88.9089	89.2503	89.5805	36
37	85.5404	85.9913	86.4272	86.8487	87.2564	87.6506	88.0320	88.4009	88.7579	89.1033	37
38	84.8881	85.3594	85.8149	86.2555	86.6815	87.0935	87.4921	87.8777	88.2508	88.6117	38
39	84.2162	84.7084	85.1843	85.6444	86.0893	86.5197	86.9360	87.3387	87.7284	88.1054	39
40	83.5242	84.0380	84.5347	85.0150	85.4795	85.9287	86.3632	86.7836	87.1904	87.5839	40
41	82.8114	83.3474	83.8656	84.3667	84.8513	85.3199	85.7733	86.2118	86.6362	87.0468	41
42	82.0773	82.6362	83.1765	83.6990	84.2042	84.6929	85.1656	85.6229	86.0654	86.4935	42
43	81.3211	81.9036	82.4667	83.0112	83.5378	84.0471	84.5397	85.0163	85.4775	85.9236	43
44	80.5422	81.1490	81.7355	82.3028	82.8513	83.3819	83.8951	84.3915	84.8719	85.3367	44
45	79.7400	80.3718	80.9825	81.5731	82.1443	82.6967	83.2311	83.7480	84.2482	84.7321	45

3% INTEREST RATE

91–100 Years Probable Life

Age, years	Condition-percent, %										Age, years
	91	92	93	94	95	96	97	98	99	100	
46	78.9137	79.5712	80.2069	80.8216	81.4160	81.9910	82.5471	83.0851	83.6058	84.1094	46
47	78.0626	78.7467	79.4080	80.0475	80.6659	81.2641	81.8427	82.4024	82.9440	83.4680	47
48	77.1859	77.8974	78.5851	79.2502	79.8933	80.5154	81.1171	81.6992	82.2624	82.8074	48
49	76.2830	77.0226	77.7376	78.4290	79.0976	79.7443	80.3698	80.9749	81.5605	82.1270	49
50	75.3530	76.1216	76.8646	77.5831	78.2779	78.9500	79.6000	80.2289	80.8374	81.4261	50
51	74.3951	75.1935	75.9654	76.7118	77.4337	78.1318	78.8072	79.4604	80.0926	80.7042	51
52	73.4084	74.2376	75.0393	75.8145	76.5641	77.2892	77.9905	78.6690	79.3255	79.9607	52
53	72.3922	73.2531	74.0853	74.8901	75.6685	76.4212	77.1494	77.8538	78.5354	79.1948	53
54	71.3454	72.2390	73.1028	73.9381	74.7459	75.5273	76.2830	77.0141	77.7216	78.4060	54
55	70.2673	71.1945	72.0908	72.9575	73.7957	74.6064	75.3906	76.1492	76.8833	77.5935	55
56	69.1568	70.1186	71.0484	71.9475	72.8170	73.6580	74.4715	72.2584	76.0200	76.7566	56
57	68.0130	69.0105	69.9747	70.9072	71.8090	72.6811	73.5248	74.3409	75.1307	75.8947	57
58	66.8348	67.8691	68.8689	69.8357	70.7707	71.6750	72.5497	73.3958	74.2146	75.0068	58
59	65.6214	66.6935	67.7298	68.7320	69.7012	70.6386	71.5453	72.4224	73.2713	74.0924	59
60	64.3715	65.4826	66.5566	67.5952	68.5997	69.5711	70.5108	71.4198	72.2995	73.1505	60
61	63.0842	64.2353	65.3482	66.4244	67.4651	68.4716	69.4453	70.3871	71.2986	72.1803	61
62	61.7582	62.9507	64.1035	65.2184	66.2964	67.3392	68.3478	69.3235	70.2677	71.1811	62
63	60.3924	61.6275	62.8215	63.9761	65.0928	66.1727	67.2174	68.2279	69.2058	70.1518	63
64	58.9857	60.2647	61.5011	62.6967	63.8529	64.9713	66.0530	67.0995	68.1121	69.0917	64
65	57.5367	58.8609	60.1410	61.3789	62.5760	63.7337	64.8538	65.9372	66.9856	67.9998	65
66	56.0443	57.4151	58.7401	60.0215	61.2607	62.4592	63.6185	64.7400	65.8253	66.8751	66
67	54.5072	55.9258	57.2972	58.6234	59.9059	61.1464	62.3462	63.5070	64.6301	65.7167	67
68	52.9239	54.3919	55.8110	57.1834	58.5105	59.7941	61.0357	62.2368	63.3991	64.5236	68
69	51.2931	52.8120	54.2802	55.7002	57.0733	58.4013	59.6860	60.9287	62.1312	63.2947	69
70	49.6134	51.1846	52.7035	54.1724	55.5929	56.9668	58.2957	59.5812	60.8253	62.0288	70
71	47.8832	49.5083	51.0795	52.5989	54.0681	55.4892	56.8637	58.1934	59.4802	60.7250	71
72	46.1012	47.7820	49.4068	50.9781	52.4976	53.9672	55.3887	56.7639	58.0947	59.3820	72
73	44.2658	46.0038	47.6839	49.3086	50.8799	52.3996	53.8696	55.2916	56.6676	57.9988	73
74	42.3752	44.1722	45.9093	47.5892	49.2137	50.7850	52.3048	53.7750	55.1978	56.5741	74
75	40.4280	42.2857	44.0815	45.8182	47.4976	49.1218	50.6931	52.2130	53.6838	55.1067	75
76	38.4223	40.3426	42.1988	43.9940	45.7299	47.4089	49.0330	50.6041	52.1245	53.5952	76
77	36.3565	38.3411	40.2597	42.1151	43.9092	45.6446	47.3231	48.9470	50.5183	52.0384	77
78	34.2287	36.2797	38.2624	40.1798	42.0339	43.8273	45.5620	47.2401	48.8639	50.4349	78
79	32.0370	34.1564	36.2051	38.1864	40.1024	41.9555	43.7480	45.4820	47.1600	48.7834	79
80	29.7796	31.9694	34.0862	36.1333	38.1129	40.0276	41.8796	43.6712	45.4049	47.0821	80
81	27.4545	29.7166	31.9037	34.0185	36.0637	38.0418	39.9551	41.8060	43.5972	45.3299	81
82	25.0596	27.3966	29.6557	31.8404	33.9530	35.9964	37.9729	39.8849	41.7352	43.5251	82
83	22.5929	25.0068	27.3402	29.5966	31.7790	33.8897	35.9312	37.9062	39.8174	41.6662	83
84	20.0522	22.5453	24.9553	27.2860	29.5397	31.7197	33.8283	35.8681	37.8420	39.7515	84
85	17.4352	20.0100	22.4989	24.9058	27.2334	29.4846	31.6623	33.7689	35.8074	37.7794	85
86	14.7398	17.3986	19.9688	22.4543	24.8579	27.1826	29.4313	31.6066	33.7117	35.7481	86
87	11.9635	14.7088	17.3627	19.9292	22.4110	24.8115	27.1334	29.3797	31.5532	33.6559	87
88	9.1039	11.9384	14.6785	17.3283	19.8908	22.3692	24.7665	27.0857	29.3298	31.5009	88
89	6.1585	9.0848	11.9137	14.6494	17.2949	19.8537	22.3287	24.7230	27.0399	29.2815	89
90	3.1247	6.1457	9.0660	11.8902	14.6212	17.2627	19.8177	22.2894	24.6812	26.9951	90
91	0.0000	3.1181	6.1329	9.0481	11.8673	14.5939	17.2314	19.7828	22.2518	24.6403	91
92		0.0000	3.1118	6.1208	9.0307	11.8451	14.5675	17.2010	19.7494	22.2148	92
93			0.0000	3.1056	6.1090	9.0139	11.8236	14.5417	17.1720	19.7166	93
94				0.0000	3.0996	6.0977	8.9975	11.8027	14.5173	17.1434	94
95					0.0000	3.0938	6.0865	8.9815	11.7830	14.4930	95
96						0.0000	3.0882	6.0756	8.9666	11.7632	96
97							0.0000	3.0828	6.0657	8.9514	97
98								0.0000	3.0776	6.0553	98
99									0.0000	3.0725	99
100										0.0000	100

4% INTEREST RATE

1–10 Years Probable Life

Age, years	Condition-percent, % 1	2	3	4	5	6	7	8	9	10	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	0.0000	50.9804	67.9651	76.4510	81.5373	84.9238	87.3390	89.1472	90.5507	91.6709	1
2		0.0000	34.6489	51.9600	62.3361	69.2446	74.1716	77.8603	80.7234	83.0086	2
3			0.0000	26.4894	42.3668	52.9382	60.4775	66.1220	70.5031	73.9999	3
4				0.0000	21.5988	35.9795	46.2357	53.9140	59.8739	64.6308	4
5					0.0000	18.3425	31.4242	41.2178	48.8195	54.8869	5
6						0.0000	16.0202	28.0138	37.3230	44.7533	6
7							0.0000	14.2815	25.3667	34.2144	7
8								0.0000	12.9320	23.2538	8
9									0.0000	11.8549	9
10										0.0000	10

11–20 Years Probable Life

Age, years	11	12	13	14	15	16	17	18	19	20	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	92.5851	93.3448	93.9856	94.5331	95.0059	95.4180	95.7801	96.1007	96.3861	96.6418	1
2	84.8736	86.4234	87.7307	88.8475	89.8120	90.6527	91.3915	92.0454	92.6277	93.1493	2
3	76.8536	79.2251	81.2255	82.9345	84.4104	85.6968	86.8273	87.8278	88.7190	89.5171	3
4	68.5129	71.7389	74.4602	76.7850	78.7927	80.5427	82.0806	83.4416	84.6539	85.7396	4
5	59.8385	63.9532	67.4242	70.3895	72.9503	75.1824	77.1439	78.8800	80.4262	81.8110	5
6	50.8171	55.8561	60.1068	63.7382	66.8742	69.6077	72.0098	74.1358	76.0293	77.7253	6
7	41.4349	47.4351	52.4967	56.8208	60.5550	63.8100	66.6704	69.2019	71.4567	73.4761	7
8	31.6774	38.6773	44.5822	49.6268	53.9831	57.7804	61.1173	64.0707	66.7011	69.0570	8
9	21.5296	29.5692	36.3511	42.1449	47.1484	51.5096	55.3422	58.7342	61.7552	64.4611	9
10	10.9759	20.0968	27.7908	34.3638	40.0402	44.9880	49.3360	53.1842	56.6116	59.6814	10
11	0.0000	10.2454	18.8881	26.2715	32.6477	38.2055	43.0896	47.4122	51.2622	54.7105	11
12		0.0000	9.6292	17.8555	24.9595	31.1518	36.5933	41.4094	45.6988	49.5407	12
13			0.0000	9.1028	16.9637	23.8158	29.8372	35.1664	39.9129	44.1642	13
14				0.0000	8.6482	16.1865	22.8108	28.6737	33.8956	38.5726	14
15					0.0000	8.2519	15.5034	21.9214	27.6375	32.7573	15
16						0.0000	7.9037	14.8989	21.1292	26.7094	16
17							0.0000	7.5955	14.3605	20.4196	17
18								0.0000	7.3210	13.8782	18
19									0.0000	7.0752	19
20										0.0000	20

21–30 Years Probable Life

Age, years	21	22	23	24	25	26	27	28	29	30	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	96.8720	97.0801	97.2691	97.4413	97.5988	97.7433	97.8761	97.9987	98.1100	98.2170	1
2	93.6189	94.0434	94.4290	94.7803	95.1016	95.3963	95.6673	95.9174	96.1485	96.3627	2
3	90.2356	90.8853	91.4752	92.0128	92.5044	92.9554	93.3702	93.7524	94.1064	94.4342	3
4	86.7170	87.6008	88.4033	89.1346	89.8034	90.4168	90.9811	91.5016	91.9827	92.4285	4
5	83.0577	84.1850	85.2085	86.1413	86.9943	87.7768	88.4965	89.1603	89.7740	90.3426	5
6	79.2520	80.6325	81.8860	83.0283	84.0729	85.0311	85.9125	86.7254	87.4770	88.1733	6
7	75.2940	76.9379	78.4305	79.7908	81.0346	82.1756	83.2252	84.1932	85.0881	85.9173	7
8	71.1778	73.0956	74.8368	76.4237	77.8748	79.2059	80.4303	81.5596	82.6036	83.5709	8
9	66.8969	69.0995	71.0994	72.9220	74.5886	76.1174	77.5237	78.8207	80.0198	81.1308	9
10	62.4448	64.9436	67.2125	69.2802	71.1710	72.9054	74.5008	75.9722	77.3326	78.5930	10
11	57.8145	60.6215	63.1700	65.4927	67.6166	69.5648	71.3570	73.0098	74.5379	75.9537	11
12	52.9991	56.1264	58.9659	61.5537	63.9201	66.0907	68.0874	69.9289	71.6314	73.2088	12
13	47.9911	51.4516	54.5937	57.4572	60.0757	62.4776	64.6870	66.7247	68.6086	70.3542	13
14	42.7827	46.5898	50.0465	53.1968	56.0775	58.7199	61.1507	63.3924	65.4650	67.3853	14
15	37.3660	41.5335	45.3175	48.7660	51.9194	54.8120	57.4728	59.9268	62.1956	64.2977	15

4% INTEREST RATE

21–30 Years Probable Life

Age, years	Condition-percent, %										Age, years
	21	22	23	24	25	26	27	28	29	30	
16	31.7326	36.2750	40.3993	44.1579	47.5950	50.7478	53.6479	56.3228	58.7954	61.0866	16
17	25.8739	30.8061	35.2843	39.3656	43.0976	46.5209	49.6699	52.5742	55.2593	57.7471	17
18	19.7809	25.1184	29.9648	34.3815	38.4203	42.1250	45.5329	48.6759	51.5816	54.2740	18
19	13.4441	19.2033	24.4325	29.1981	33.5560	37.5533	41.2303	44.6216	47.7569	50.6619	19
20	6.8539	13.0516	18.6789	23.8073	28.4970	32.7987	36.7557	40.4052	43.7792	46.9054	20
21	0.0000	6.6537	12.6951	18.2009	23.2357	27.8539	32.1021	36.0201	39.6424	42.9986	21
22		0.0000	6.4720	12.3713	17.7639	22.7113	27.2623	31.4596	35.3401	38.9355	22
23			0.0000	6.3064	12.0733	17.3630	22.2290	26.7167	30.8657	34.7099	23
24				0.0000	6.1550	11.8008	16.9943	21.7841	26.2123	30.3153	24
25					0.0000	6.0161	11.5502	16.6541	21.3728	25.7449	25
26						0.0000	5.8883	11.3190	16.3397	20.9917	26
27							0.0000	5.7705	11.1053	16.0484	27
28								0.0000	5.6615	10.9073	28
29									0.0000	5.5606	29
30										0.0000	30

31–40 Years Probable Life

Age, years	31	32	33	34	35	36	37	38	39	40	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.3145	98.4051	98.4896	98.5685	98.6423	98.7113	98.7760	98.8368	98.8939	98.9477	1
2	96.5615	96.7465	96.9189	97.0798	97.2302	97.3711	97.5031	97.6271	97.7436	97.8532	2
3	94.7384	95.0215	95.2853	95.5315	95.7617	95.9772	96.1793	96.3690	96.5473	96.7150	3
4	92.8424	93.2275	93.5863	93.9213	94.2344	94.5276	94.8025	95.0605	95.3031	95.5312	4
5	90.8706	91.3617	91.8194	92.2467	92.6461	93.0201	93.3707	93.6998	94.0091	94.3001	5
6	88.8199	89.4213	89.9818	90.5050	90.9942	91.4522	91.8815	92.2846	92.6634	93.0198	6
7	86.6871	87.4033	88.0708	88.6938	89.2762	89.8216	90.3328	90.8128	91.2638	91.6882	7
8	84.4691	85.3046	86.0832	86.8100	87.4896	88.1257	88.7222	89.2821	89.8083	90.3034	8
9	82.1623	83.1219	84.0162	84.8510	85.6314	86.3621	87.0471	87.6902	88.2946	88.8632	9
10	79.7633	80.8520	81.8665	82.8135	83.6989	84.5279	85.3050	86.0346	86.7202	87.3654	10
11	77.2683	78.4912	79.6308	80.6946	81.6891	82.6203	83.4933	84.3128	85.0830	85.8077	11
12	74.6735	76.0360	77.3057	78.4909	79.5990	80.6364	81.6091	82.5221	83.3802	84.1876	12
13	71.9749	73.4825	74.8875	76.1991	77.4252	78.5732	79.6495	80.6598	81.6093	82.5028	13
14	69.1683	70.8270	72.3727	73.8155	75.1645	76.4274	77.6115	78.7230	79.7676	80.7505	14
15	66.2495	68.0652	69.7572	71.3367	72.8133	74.1958	75.4920	76.7087	77.8523	78.9282	15
16	63.2140	65.1930	67.0372	68.7587	70.3681	71.8750	73.2877	74.6139	75.8603	77.0330	16
17	60.0570	62.2058	64.2083	66.0776	67.8251	69.4613	70.9953	72.4352	73.7886	75.0619	17
18	56.7738	59.0992	61.2663	63.2892	65.1804	66.9511	68.6111	70.1695	71.6340	73.0121	18
19	53.3592	55.8683	58.2066	60.3893	62.4299	64.3404	66.1316	67.8130	69.3933	70.8802	19
20	49.8080	52.5082	55.0245	57.3734	59.5693	61.6253	63.5529	65.3624	67.0630	68.6631	20
21	46.1148	49.0136	51.7151	54.2368	56.5944	58.8017	60.8711	62.8137	64.6394	66.3572	21
22	42.2738	45.3793	48.2733	50.9748	53.5004	55.8651	58.0820	60.1630	62.1189	63.9592	22
23	38.2793	41.5996	44.6939	47.5823	50.2827	52.8110	55.1813	57.4064	59.4976	61.4652	23
24	34.1249	37.6688	40.9713	44.0542	46.9363	49.6347	52.1646	54.5394	56.7714	58.8715	24
25	29.8047	33.5810	37.1001	40.3851	43.4563	46.3314	49.0274	51.5580	53.9364	56.1742	25
26	25.3110	29.3290	33.0734	36.5688	39.8365	42.8960	45.7643	48.4569	50.9875	53.3686	26
27	20.6379	24.9073	28.8860	32.6000	36.0723	39.3231	42.3709	45.2320	47.9210	50.4510	27
28	15.7779	20.3088	24.5311	28.4726	32.1574	35.6074	38.8418	41.8781	44.7317	47.4167	28
29	10.7235	15.5262	20.0020	24.1800	28.0860	31.7430	35.1715	38.3900	41.4149	44.2610	29
30	5.4669	10.5524	15.2917	19.7157	23.8517	27.7240	31.3544	34.7624	37.9654	40.9791	30
31	0.0000	5.3797	10.3930	15.0729	19.4480	23.5443	27.3847	30.9898	34.3779	37.5659	31
32		0.0000	5.2984	10.2443	14.8682	19.1974	23.2561	27.0062	30.6470	34.0162	32
33			0.0000	5.2226	10.1052	14.6766	18.9624	22.9856	26.7668	30.3245	33
34				0.0000	5.1517	9.9750	14.4969	18.7418	22.7314	26.4851	34
35					0.0000	5.0853	9.8528	14.3283	18.5345	22.4922	35
36						0.0000	5.0230	9.7383	14.1698	18.3395	36
37							0.0000	4.9646	9.6305	14.0207	37
38								0.0000	4.9097	9.5292	38
39									0.0000	4.8580	39
40										0.0000	40

4% INTEREST RATE

41–50 Years Probable Life

Age, years	Condition-percent, %										Age, years
	41	42	43	44	45	46	47	48	49	50	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.9983	99.0460	99.0910	99.1335	99.1738	99.2118	99.2478	99.2819	99.3143	99.3450	1
2	97.9565	98.0538	98.1457	98.2324	98.3145	98.3921	98.4655	98.5351	98.6011	98.6638	2
3	96.8730	97.0219	97.1625	97.2953	97.4208	97.5395	97.6520	97.7585	97.8595	97.9553	3
4	95.7462	95.9488	96.1400	96.3206	96.4914	96.6529	96.8059	96.9508	97.0881	97.2185	4
5	94.5743	94.8327	95.0766	95.3070	95.5248	95.7308	95.9259	96.1107	96.2860	96.4522	5
6	93.3555	93.6720	93.9707	94.2528	94.5195	94.7719	95.0107	95.2371	95.4517	95.6553	6
7	92.0880	92.4649	92.8205	93.1565	93.4741	93.7745	94.0590	94.3285	94.5840	94.8265	7
8	90.7698	91.2094	91.6244	92.0163	92.3868	92.7373	93.0692	93.3836	93.6817	93.9645	8
9	89.3988	89.9038	90.3804	90.8305	91.2560	91.6586	92.0397	92.4009	92.7433	93.0681	9
10	87.9730	88.5459	89.0866	89.5973	90.0800	90.5367	90.9691	91.3711	91.7673	92.1358	10
11	86.4902	87.1338	87.7411	88.3147	88.8570	89.3700	89.8557	90.3159	90.7522	91.1662	11
12	84.9481	85.6651	86.3417	86.9808	87.5850	88.1566	88.6978	89.2105	89.6966	90.1578	12
13	83.3443	84.1377	84.8864	85.5936	86.2621	86.8946	87.4935	88.0608	88.5988	89.1091	13
14	81.6763	82.5492	83.3729	84.1509	84.8864	85.5822	86.2410	86.8652	87.4570	88.0184	14
15	79.9416	80.8971	81.7988	82.6505	83.4556	84.2173	84.9385	85.6218	86.2696	86.8841	15
16	78.1375	79.1790	80.1617	81.0900	81.9676	82.7978	83.5838	84.3286	85.0347	85.7045	16
17	76.2613	77.3921	78.4592	79.4672	80.4200	81.3215	82.1750	82.9836	83.7503	84.4777	17
18	74.3100	75.5338	76.6886	77.7794	78.8106	79.7862	80.7098	81.5849	82.4146	83.2017	18
19	72.2807	73.6011	74.8472	76.0242	77.1368	78.1894	79.1860	80.1303	81.0255	81.8748	19
20	70.1702	71.5911	72.9321	74.1987	75.3960	76.5288	77.6013	78.6174	79.5808	80.4948	20
21	67.9752	69.5007	70.9403	72.3002	73.5856	74.8017	75.9531	77.0440	78.0783	79.0595	21
22	65.6925	67.3268	68.8690	70.3257	71.7028	73.0056	74.2391	75.4077	76.5158	77.5669	22
23	63.3185	65.0658	66.7147	68.2723	69.7446	71.1376	72.4564	73.7060	74.8907	76.0146	23
24	60.8495	62.7144	64.4743	66.1367	67.7082	69.1949	70.6025	71.9361	73.2006	74.4001	24
25	58.2817	60.2689	62.1443	63.9157	65.5902	67.1745	68.6744	70.0955	71.4429	72.7211	25
26	55.6112	57.7257	59.7211	61.6059	63.3876	65.0733	66.6692	68.1813	69.6149	70.9749	26
27	52.8340	55.0807	57.2010	59.2037	61.0969	62.8880	64.5838	66.1905	67.7138	69.1589	27
28	49.9456	52.3299	54.5800	56.7054	58.7145	60.6153	62.4149	64.1200	65.7366	67.2702	28
29	46.9417	49.4691	51.8542	54.1071	56.2368	58.2517	60.1593	61.9668	63.6804	65.3060	29
30	43.8176	46.4938	49.0194	51.4050	53.6600	55.7936	57.8135	59.7274	61.5419	63.2632	30
31	40.5686	43.3996	46.0712	48.5947	50.9802	53.2371	55.3739	57.3984	59.3178	61.1388	31
32	37.1896	40.1815	43.0051	45.6721	48.1932	50.5784	52.8366	54.9763	57.0049	58.9293	32
33	33.6754	36.8348	39.8163	42.6325	45.2946	47.8133	50.1979	52.4573	54.5993	56.6314	33
34	30.0207	33.3542	36.4999	39.4713	42.2802	44.9376	47.4536	49.8375	52.0976	54.2417	34
35	26.2198	29.7343	33.0509	36.1837	39.1451	41.9469	44.5996	47.1129	49.4958	51.7563	35
36	22.2668	25.9697	29.4640	32.7646	35.8847	38.8366	41.6314	44.2794	46.7899	49.1716	36
37	18.1558	22.0544	25.7336	29.2088	32.4938	35.6019	38.5445	41.3325	43.9758	46.4834	37
38	13.8802	17.9826	21.8539	25.5107	28.9674	32.2378	35.3341	38.2677	41.0491	43.6877	38
39	9.4337	13.7479	17.8191	21.6646	25.2998	28.7391	31.9952	35.0804	38.0054	40.7802	39
40	4.8094	9.3438	13.6229	17.6648	21.4855	25.1004	28.5229	31.7655	34.8399	37.7564	40
41	0.0000	4.7635	9.2588	13.5049	17.5187	21.3162	24.9116	28.3181	31.5477	34.6116	41
42		0.0000	4.7202	9.1786	13.3932	17.3807	21.1559	24.7327	28.1239	31.3411	42
43			0.0000	4.6793	9.1027	13.2877	17.2499	21.0040	24.5632	27.9397	43
44				0.0000	4.6406	9.0310	13.1877	17.1261	20.8600	24.4023	44
45					0.0000	4.6040	8.9630	13.0930	17.0087	20.7233	45
46						0.0000	4.5694	8.8987	13.0033	16.8972	46
47							0.0000	4.5366	8.8378	12.9181	47
48								0.0000	4.5055	8.7798	48
49									0.0000	4.4760	49
50										0.0000	50

4% INTEREST RATE

51–60 Years Probable Life

Age, years	Condition-percent, %										Age, years
	51	52	53	54	55	56	57	58	59	60	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.3741	99.4018	99.4281	99.4531	99.4769	99.4995	99.5211	99.5416	99.5612	99.5798	1
2	98.7232	98.7796	98.8333	98.8843	98.9328	98.9790	99.0230	99.0649	99.1048	99.1428	2
3	98.0462	98.1326	98.2147	98.2928	98.3670	98.4377	98.5050	98.5691	98.6301	98.6884	3
4	97.3422	97.4597	97.5714	97.6776	97.7786	97.8747	97.9662	98.0534	98.1365	98.2157	4
5	96.6100	96.7599	96.9023	97.0378	97.1666	97.2892	97.4059	97.5172	97.6231	97.7241	5
6	95.8485	96.0321	96.2065	96.3724	96.5301	96.6803	96.8233	96.9594	97.0892	97.2129	6
7	95.0566	95.2751	95.4828	95.6803	95.8682	96.0470	96.2173	96.3794	96.5339	96.6813	7
8	94.2330	94.4879	94.7302	94.9606	95.1798	95.3884	95.5870	95.7762	95.9565	96.1283	8
9	93.3764	93.6692	93.9475	94.2122	94.4639	94.7034	94.9316	95.1488	95.3559	95.5533	9
10	92.4856	92.8178	93.1335	93.4337	93.7193	93.9911	94.2499	94.4964	94.7313	94.9552	10
11	91.5591	91.9323	92.2870	92.6242	92.9450	93.2503	93.5410	93.8178	94.0817	94.3332	11
12	90.5956	91.0114	91.4065	91.7822	92.1396	92.4798	92.8037	93.1122	93.4061	93.6864	12
13	89.5935	90.0536	90.4909	90.9066	91.3021	91.6785	92.0369	92.3782	92.7035	93.0137	13
14	88.5514	89.0576	89.5386	89.9960	90.4311	90.8451	91.2394	91.6150	91.9729	92.3140	14
15	87.4675	88.0216	88.5482	89.0489	89.5252	89.9785	90.4101	90.8212	91.2129	91.5864	15
16	86.3404	86.9443	87.5182	88.0639	88.5831	89.0771	89.5475	89.9956	90.4226	90.8297	16
17	85.1681	85.8239	86.4470	87.0396	87.6033	88.1397	88.6505	89.1370	89.6007	90.0427	17
18	83.9489	84.6586	85.3330	85.9743	86.5843	87.1648	87.7176	88.2441	88.7459	89.2242	18
19	82.6810	83.4467	84.1744	84.8663	85.5245	86.1509	86.7474	87.3155	87.8569	88.3730	19
20	81.3623	82.1864	82.9695	83.7141	84.4224	85.0965	85.7383	86.3497	86.9323	87.4877	20
21	79.9910	80.8756	81.7163	82.5157	83.2761	83.9998	84.6889	85.3453	85.9708	86.5670	21
22	78.5647	79.5125	80.4131	81.2694	82.0841	82.8593	83.5976	84.3007	84.9708	85.6095	22
23	77.0814	78.0947	79.0577	79.9733	80.8443	81.6732	82.4625	83.2143	83.9308	84.6137	23
24	75.5388	76.6203	77.6481	78.6253	79.5550	80.4397	81.2821	82.0845	82.8491	83.5781	24
25	73.9345	75.0869	76.1821	77.2234	78.2140	79.1568	80.0544	80.9095	81.7243	82.5010	25
26	72.2659	73.4922	74.6574	75.7655	76.8195	77.8225	78.7777	79.6875	80.5544	81.3809	26
27	70.5307	71.8337	73.0718	74.2492	75.3691	76.4350	77.4499	78.4165	79.3378	80.2159	27
28	68.7260	70.1088	71.4228	72.6722	73.8608	74.9919	76.0689	77.0948	78.0724	79.0044	28
29	66.8492	68.3149	69.7078	71.0322	72.2921	73.4911	74.6328	75.7202	76.7565	77.7444	29
30	64.8973	66.4493	67.9242	69.3266	70.6606	71.9302	73.1391	74.2906	75.3879	76.4340	30
31	62.8673	64.5091	66.0692	67.5527	68.9639	70.3069	71.5858	72.8038	73.9646	75.0711	31
32	60.7561	62.4912	64.1401	65.7079	67.1994	68.6187	69.9703	71.2576	72.4843	73.6538	32
33	58.5604	60.3927	62.1338	63.7893	65.3642	66.8630	68.2901	69.6495	70.9449	72.1797	33
34	56.2770	58.2102	60.0472	61.7940	63.4557	65.0370	66.5428	67.9771	69.3438	70.6468	34
35	53.9022	55.9404	57.8772	59.7188	61.4708	63.1380	64.7256	66.2377	67.6788	69.0524	35
36	51.4324	53.5798	55.6204	56.5607	59.4065	61.1631	62.8357	64.4289	65.9471	67.3944	36
37	48.8638	51.1247	53.2733	55.3162	57.2596	59.1091	60.8702	62.5476	64.1461	65.6699	37
38	46.1925	48.5715	50.8323	52.9820	55.0269	56.9730	58.8261	60.5911	62.2731	63.8766	38
39	43.4143	45.9162	48.2937	50.5543	52.7048	54.7514	56.7002	58.5564	60.3252	62.0114	39
40	40.5250	43.1546	45.6535	48.0296	50.2899	52.4410	54.4892	56.4402	58.2994	60.0717	40
41	37.5201	40.2826	42.9077	45.4039	47.7783	50.0381	52.1899	54.2394	56.1925	58.0544	41
42	34.3950	37.2957	40.0521	42.6731	45.1664	47.5392	49.7985	51.9506	54.0014	55.9564	42
43	31.1449	34.1893	37.0823	39.8331	42.4499	44.9402	47.3116	49.5702	51.7226	53.7744	43
44	27.7648	30.9586	33.9936	36.8795	39.6248	42.2374	44.7251	47.0946	49.3527	51.5052	44
45	24.2495	27.5988	30.7815	33.8078	36.6866	39.4264	42.0352	44.5200	46.8879	49.1453	45
46	20.5936	24.1045	27.4408	30.6132	33.6310	36.5029	39.2376	41.8424	44.3246	46.6909	46
47	16.7915	20.4705	23.9665	27.2908	30.4531	33.4626	36.3282	39.0577	41.6588	44.1383	47
48	12.8372	16.6911	20.3533	23.8356	27.1481	30.3006	33.3024	36.1616	38.8863	41.4837	48
49	8.7249	12.7605	16.5955	20.2421	23.7110	27.0121	30.1556	33.1497	36.0029	38.7229	49
50	4.4480	8.6727	12.6874	16.5048	20.1362	23.5921	26.8828	30.0173	33.0042	35.8516	50
51	0.0000	4.4214	8.6230	12.6181	16.4185	20.0353	23.4792	26.7595	29.8855	32.8655	51
52		0.0000	4.3960	8.5759	12.5521	16.3362	19.9395	23.3715	26.6421	29.7599	52
53			0.0000	4.3721	8.5311	12.4892	16.2581	19.8480	23.2690	26.5301	53
54				0.0000	4.3492	8.4883	12.4295	16.1835	19.7609	23.1711	54
55					0.0000	4.3274	8.4478	12.3724	16.1125	19.6778	55
56						0.0000	4.3067	8.4089	12.3181	16.0447	56
57							0.0000	4.2869	8.3720	12.2663	57
58								0.0000	4.2681	8.3368	58
59									0.0000	4.2501	59
60										0.0000	60

4% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, %										Age, years
	61	62	63	64	65	66	67	68	69	70	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.5976	99.6146	99.6308	99.6462	99.6610	99.6751	99.6885	99.7014	99.7137	99.7255	1
2	99.1791	99.2137	99.2468	99.2783	99.3084	99.3372	99.3646	99.3909	99.4160	99.4400	2
3	98.7439	98.7968	98.8474	98.8956	98.9417	98.9857	99.0278	99.0680	99.1064	99.1431	3
4	98.2912	98.3633	98.4320	98.4977	98.5604	98.6202	98.6774	98.7321	98.7844	98.8343	4
5	97.8205	97.9124	98.0001	98.0838	98.1638	98.2401	98.3131	98.3828	98.4495	98.5132	5
6	97.3309	97.4435	97.5509	97.6534	97.7513	97.8448	97.9342	98.0195	98.1012	98.1792	6
7	96.8217	96.9558	97.0837	97.2057	97.3223	97.4337	97.5401	97.6417	97.7389	97.8319	7
8	96.2922	96.4486	96.5978	96.7402	96.8762	97.0061	97.2488	97.2488	97.3622	97.4706	8
9	95.7415	95.9211	96.0924	96.2560	96.4122	96.5614	96.7040	96.8402	96.9704	97.0950	9
10	95.1688	95.3725	95.5669	95.7525	95.9297	96.0990	96.2607	96.4152	96.5630	96.7043	10
11	94.5731	94.8020	95.0203	95.2288	95.4279	95.6180	95.7997	95.9733	96.1392	96.2979	11
12	93.9536	94.2086	94.4519	94.6842	94.9060	95.1178	95.3202	95.5136	95.6985	95.8753	12
13	93.3094	93.5915	93.8608	94.1178	94.3632	94.5976	94.8216	95.0356	95.2402	95.4358	13
14	92.6394	92.9497	93.2459	93.5287	93.7987	94.0566	94.3030	94.5384	94.7635	94.9788	14
15	91.9427	92.2823	92.6066	92.9161	93.2116	93.4939	93.7636	94.0214	94.2678	94.5034	15
16	91.2179	91.5882	91.9416	92.2789	92.6011	92.9088	93.2027	93.4837	93.7522	94.0090	16
17	90.4642	90.8663	91.2500	91.6163	91.9661	92.3002	92.6194	92.9244	93.2161	93.4949	17
18	89.6803	90.1155	90.5308	90.9272	91.3057	91.6673	92.0127	92.3428	92.6584	92.9602	18
19	88.8652	89.3347	89.7828	90.2105	90.6189	91.0090	91.3818	91.7380	92.0785	92.4041	19
20	88.0174	88.5226	89.0048	89.4651	89.9047	90.3245	90.7256	91.1089	91.4754	91.8257	20
21	87.1357	87.6781	88.1958	88.6899	89.1618	89.6125	90.0432	90.4547	90.8481	91.2243	21
22	86.2187	86.7998	87.3544	87.8838	88.3893	88.8721	89.3334	89.7743	90.1958	90.5987	22
23	85.2650	85.8864	86.4793	87.0453	87.5858	88.1021	88.5953	89.0667	89.5173	89.9482	23
24	84.2736	84.9364	85.5693	86.1734	86.7503	87.3012	87.8277	88.3308	88.8117	89.2716	24
25	83.2418	83.9484	84.6228	85.2665	85.8812	86.4684	87.0293	87.5650	88.0779	88.5679	25
26	82.1690	82.9209	83.6385	84.3234	84.9775	85.6022	86.1991	86.7695	87.3148	87.8362	26
27	81.0534	81.8523	82.6148	83.3425	84.0376	84.7013	85.3356	85.9417	86.5211	87.0751	27
28	79.8931	80.7410	81.5501	82.3225	83.0600	83.7645	84.4375	85.0808	85.6957	86.2836	28
29	78.6865	79.5852	80.4429	81.2616	82.0434	82.7901	83.5036	84.1854	84.8372	85.4604	29
30	77.4315	78.3832	79.2914	80.1583	80.9861	81.7768	82.5323	83.2543	83.9445	84.6043	30
31	76.1264	77.1331	78.0938	79.0108	79.8866	80.7230	81.5221	82.2859	83.0160	83.7140	31
32	74.7691	75.8330	76.8483	77.8175	78.7430	79.6270	80.4716	81.2787	82.0503	82.7881	32
33	73.3574	74.4809	75.5530	76.5764	77.5537	78.4871	79.3790	80.2313	81.0461	81.8251	33
34	71.8893	73.0747	74.2059	75.2857	76.3168	77.3017	78.2427	79.1420	80.0016	80.8236	34
35	70.3625	71.6122	72.8049	73.9433	75.0305	76.0688	77.0609	78.0091	78.9154	79.7820	35
36	68.7746	70.0913	71.3478	72.5473	73.6927	74.7867	75.8319	76.8301	77.7858	78.6988	36
37	67.1232	68.5095	69.8325	71.0954	72.3014	73.4532	74.5538	75.6055	76.6109	77.5722	37
38	65.4057	66.8645	68.2566	69.5854	70.8544	72.0664	73.2245	74.3311	75.3891	76.4006	38
39	63.6204	65.1536	66.6176	68.0151	69.3496	70.6242	71.8420	73.0058	74.1184	75.1821	39
40	61.7619	63.3743	64.9131	66.3819	67.7845	69.1242	70.4042	71.6275	72.7969	73.9149	40
41	59.8300	61.5239	63.1403	64.6834	66.1569	67.5643	68.9089	70.1940	71.4225	72.5970	41
42	57.8208	59.5994	61.2967	62.9169	64.4642	65.9419	67.3538	68.7032	69.9931	71.2264	42
43	55.7312	57.5979	59.3794	61.0799	62.7037	64.2547	65.7365	67.1527	68.5065	69.8009	43
44	53.5581	55.5163	57.3853	59.1692	60.8728	62.4999	64.0545	65.5403	66.9605	68.3185	44
45	51.2980	53.3516	55.3115	57.1822	58.9687	60.6750	62.3053	63.8633	65.3527	66.7767	45
46	48.9475	51.1003	53.1547	55.1157	56.9885	58.7771	60.4860	62.1192	63.6805	65.1733	46
47	46.5030	48.7589	50.9116	52.9666	54.9290	56.8033	58.5940	60.3054	61.9415	63.5057	47
48	43.9608	46.3238	48.5789	50.7315	52.7871	54.7505	56.6263	58.4191	60.1328	61.7714	48
49	41.3168	43.7913	46.1528	48.4069	50.5596	52.6156	54.5799	56.4573	58.2519	59.9678	49
50	38.5671	41.1575	43.6297	45.9894	48.2430	50.3953	52.4517	54.4170	56.2957	58.0920	50
51	35.7074	38.4184	41.0056	43.4752	45.8337	48.0861	50.2383	52.2951	54.2612	56.1411	51
52	32.7333	35.5697	38.2766	40.8605	43.3280	45.6847	47.9364	50.0883	52.1454	54.1123	52
53	29.6402	32.6071	35.4384	38.1412	40.7221	43.1871	45.5424	47.7915	49.9450	52.0023	53
54	26.4234	29.5259	32.4867	35.3130	38.0119	40.5897	43.0526	45.4064	47.6565	49.8079	54
55	23.0779	26.3215	29.4170	32.3717	35.1934	37.8884	40.4633	42.9241	45.2765	47.5257	55
56	19.5987	22.9890	26.2244	29.3128	32.2621	35.0790	37.7704	40.3425	42.8013	45.1522	56
57	15.9802	19.5231	22.9041	26.1315	29.2136	32.1572	34.9697	37.6576	40.2270	42.6838	57
58	12.2170	15.9186	19.4511	22.8230	26.0431	29.1186	32.0571	34.8653	37.5498	40.1166	58
59	8.3033	12.1699	15.8599	19.3821	22.7458	25.9584	29.0279	31.9613	34.7656	37.4467	59
60	4.2330	8.2713	12.1250	15.8034	19.3166	22.6718	25.8776	28.9412	31.8699	34.6701	60

4% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, % 61	62	63	64	65	66	67	68	69	70	Age, years
61	0.0000	4.2167	8.2408	12.0820	15.7503	19.2538	22.6012	25.8003	28.8585	31.7824	61
62		0.0000	4.2012	8.2115	12.0413	15.6990	19.1938	22.5337	25.7265	28.7792	62
63			0.0000	4.1863	8.1839	12.0021	15.6501	19.1365	22.4693	25.6559	63
64				0.0000	4.1722	8.1572	11.9647	15.6034	19.0818	22.4076	64
65					0.0000	4.1586	8.1318	11.9289	15.5588	19.0294	65
66						0.0000	4.1456	8.1075	11.8949	15.5161	66
67							0.0000	4.1332	8.0844	11.8622	67
68								0.0000	4.1214	8.0622	68
69									0.0000	4.1101	69
70										0.0000	70

71–80 Years Probable Life

Age, years	71	72	73	74	75	76	77	78	79	80	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.7367	99.7475	99.7578	99.7677	99.7771	99.7861	99.7931	99.8031	99.8110	99.8186	1
2	99.4630	99.4849	99.5059	99.5260	99.5453	99.5637	99.5813	99.5982	99.6144	99.6299	2
3	99.1782	99.2118	99.2440	99.2747	99.3042	99.3324	99.3594	99.3852	99.4100	99.4337	3
4	98.8821	98.9278	98.9715	99.0134	99.0535	99.0918	99.1285	99.1637	99.1974	99.2297	4
5	98.5741	98.6324	98.6882	98.7416	98.7927	98.8416	98.8885	98.9333	98.9763	99.0174	5
6	98.2538	98.3252	98.3936	98.4589	98.5215	98.5814	98.6388	98.6937	98.7463	98.7967	6
7	97.9208	98.0058	98.0871	98.1650	98.2395	98.3108	98.3791	98.4445	98.5072	98.5672	7
8	97.5743	97.6735	97.7684	97.8592	97.9461	98.0294	98.1090	98.1854	98.2584	98.3285	8
9	97.2140	97.3280	97.4370	97.5413	97.6411	97.7367	97.8282	97.9158	97.9998	98.0802	9
10	96.8394	96.9686	97.0922	97.2106	97.3238	97.4323	97.5361	97.6355	97.7308	97.8220	10
11	96.4497	96.5948	96.7337	96.8667	96.9939	97.1157	97.2323	97.3440	97.4510	97.5535	11
12	96.0444	96.2061	96.3609	96.5090	96.6507	96.7864	96.9164	97.0408	97.1600	97.2742	12
13	95.6229	95.8019	95.9731	96.1370	96.2939	96.4440	96.5878	96.7255	96.8574	96.9838	13
14	95.1846	95.3815	95.5699	95.7502	95.9227	96.0879	96.2461	96.3976	96.5427	96.6817	14
15	94.7287	94.9443	95.1505	95.3478	95.5367	95.7176	95.8907	96.0566	96.2154	96.3676	15
16	94.2546	94.4895	94.7143	94.9294	95.1353	95.3324	95.5211	95.7019	95.8750	96.0409	16
17	93.7616	94.0166	94.2607	94.4943	94.7178	94.9318	95.1368	95.3330	95.5210	95.7011	17
18	93.2488	93.5248	93.7889	94.0417	94.2836	94.5152	94.7370	94.9494	95.1528	95.3477	18
19	92.7155	93.0133	93.2983	93.5710	93.8321	94.0820	94.3213	94.5505	94.7699	94.9802	19
20	92.1608	92.4814	92.7880	93.0815	93.3625	93.6314	93.8889	94.1355	94.3717	94.5980	20
21	91.5840	91.9281	92.2574	92.5725	92.8741	93.1628	93.4392	93.7040	93.9576	94.2005	21
22	90.9841	91.3528	91.7055	92.0430	92.3661	92.6764	92.9716	93.2552	93.5269	93.7871	22
23	90.3602	90.7544	91.1315	91.4924	91.8379	92.1686	92.4852	92.7885	93.0790	93.3572	23
24	89.7114	90.1321	90.5346	90.9198	91.2885	91.6414	91.9794	92.3031	92.6131	92.9101	24
25	89.0366	89.4849	89.9138	90.3242	90.7171	91.0932	91.4534	91.7983	92.1286	92.4451	25
26	88.3348	88.8118	89.2681	89.7049	90.1229	90.5231	90.9063	91.2733	91.6248	91.9615	26
27	87.6049	88.1118	88.5967	89.0607	89.5049	89.9301	90.3373	90.7273	91.1008	91.4585	27
28	86.8459	87.3837	87.8984	88.3908	88.8622	89.3135	89.7456	90.1594	90.5558	90.9355	28
29	86.0565	86.6266	87.1721	87.6941	88.1938	88.6722	89.1302	89.5689	89.9890	90.3915	29
30	85.2355	85.8392	86.4168	86.9696	87.4987	88.0052	88.4902	88.9547	89.3996	89.8257	30
31	84.3816	85.0202	85.6313	86.2160	86.7757	87.3115	87.8246	88.3159	88.7865	89.2374	31
32	83.4937	84.1686	84.8143	85.4323	86.0238	86.5901	87.1323	87.6516	88.1490	88.6255	32
33	82.5701	83.2828	83.9647	84.6173	85.2419	85.8398	86.4124	86.9608	87.4859	87.9891	33
34	81.6097	82.3616	83.0811	83.7696	84.4287	85.0596	85.6637	86.2423	86.7964	87.3272	34
35	80.6108	81.4036	82.1622	82.8881	83.5829	84.2481	84.8850	85.4950	86.0792	86.6389	35
36	79.5720	80.4073	81.2065	81.9713	82.7033	83.4041	84.0752	84.7179	85.3334	85.9230	36
37	78.4916	79.3711	80.2125	81.0178	81.7886	82.5264	83.2330	83.9096	84.5577	85.1786	37
38	77.3681	78.2934	79.1788	80.0262	80.8372	81.6136	82.3571	83.0691	83.7510	84.4043	38
39	76.1995	77.1727	78.1038	78.9949	79.8478	80.6643	81.4461	82.1949	82.9120	83.5991	39
40	74.9842	76.0071	76.9858	77.9223	78.8188	79.6770	80.4988	81.2858	82.0395	82.7616	40
41	73.7204	74.7949	75.8230	76.8069	77.7487	78.6502	79.5135	80.3403	81.1321	81.8907	41
42	72.4059	73.5342	74.6137	75.6468	76.6357	77.5823	78.4888	79.3569	80.1884	80.9849	42
43	71.0389	72.2231	73.3561	74.4404	75.4782	76.4718	77.4231	78.3343	79.2069	80.0429	43
44	69.6172	70.8595	72.0482	73.1857	74.2745	75.3168	76.3149	77.2707	78.1862	79.0632	44
45	68.1386	69.4414	70.6879	71.8808	73.0225	74.1156	75.1622	76.1646	77.1246	78.0443	45

4% INTEREST RATE

71–80 Years Probable Life

Age, years	Condition-percent, % 71	72	73	74	75	76	77	78	79	80	Age, years
46	66.6009	67.9666	69.2732	70.5237	71.7206	72.8663	73.9635	75.0142	76.0206	76.9847	46
47	65.0017	66.4328	67.8020	69.1123	70.3665	71.5671	72.7168	73.8179	74.8724	75.8826	47
48	63.3385	64.8376	66.2718	67.6444	68.9582	70.2159	71.4203	72.5736	73.6783	74.7365	48
49	61.6088	63.1786	64.6805	66.1179	67.4937	68.8107	70.0719	71.2797	72.4364	73.5446	49
50	59.8099	61.4532	63.0256	64.5302	65.9705	67.3492	68.6695	69.9339	71.1449	72.3050	50
51	57.9391	59.6589	61.3044	62.8791	64.3864	65.8293	67.2111	68.5343	69.8016	71.0158	51
52	55.9934	57.7927	59.5144	61.1620	62.7390	64.2486	65.6943	67.0787	68.4047	69.6750	52
53	53.9699	55.8520	57.6528	59.3761	61.0256	62.6047	64.1168	65.5650	66.9519	68.2806	53
54	51.8654	53.8336	55.7167	57.5188	59.2438	60.8950	62.4763	63.9906	65.4409	66.8304	54
55	49.6768	51.7344	53.7032	55.5872	57.3906	59.1170	60.7701	62.3533	63.8696	65.3222	55
56	47.4006	49.5513	51.6091	53.5784	55.4633	57.2678	58.9957	60.6505	62.2354	63.7537	56
57	45.0334	47.2809	49.4313	51.4892	53.4590	55.3446	57.1503	58.8795	60.5358	62.1224	57
58	42.5714	44.9196	47.1663	49.3164	51.3744	53.3445	55.2311	57.0378	58.7682	60.4259	58
59	40.0110	42.4639	44.8108	47.0567	49.2065	51.2645	53.2351	55.1224	56.9299	58.6616	59
60	37.3481	39.9100	42.3610	44.7067	46.9519	49.1012	51.1593	53.1304	55.0181	56.8266	60
61	34.5789	37.2538	39.8133	42.2626	44.6071	46.8513	49.0005	51.0586	53.0298	54.9182	61
62	31.6988	34.4915	37.1636	39.7208	42.1684	44.5115	46.7553	48.9040	50.9620	52.9336	62
63	28.7035	31.6187	34.4080	37.0773	39.6323	42.0781	44.4203	46.6633	48.8115	50.8695	63
64	25.5884	28.6310	31.5421	34.3280	36.9946	39.5474	41.9918	44.3328	46.5749	48.7229	64
65	22.3487	25.5237	28.5616	31.4688	34.2515	36.9155	39.4663	41.9092	44.2489	46.4904	65
66	18.9794	22.2922	25.4619	28.4952	31.3987	34.1782	36.8397	39.3887	41.8299	44.1686	66
67	15.4753	18.9314	22.2382	25.4027	28.4318	31.3314	34.1081	36.7672	39.3140	41.7539	67
68	11.8311	15.4362	18.8855	22.1865	25.3461	28.3708	31.2672	34.0410	36.6977	39.2427	68
69	8.0411	11.8011	15.3987	18.8416	22.1371	25.2918	28.3127	31.2057	33.9765	36.6311	69
70	4.0993	8.0207	11.7725	15.3629	18.7997	22.0896	25.2399	28.2570	31.1466	33.9148	70
71	0.0000	4.0890	8.0012	11.7451	15.3287	18.7593	22.0443	25.1903	28.2034	31.0900	71
72		0.0000	4.0790	7.9826	11.7190	15.2958	18.7209	22.0010	25.1425	28.1522	72
73			0.0000	4.0696	7.9649	11.6937	15.2645	18.6841	21.9592	25.0969	73
74				0.0000	4.0605	7.9476	11.6698	15.2345	18.6486	21.9193	74
75					0.0000	4.0518	7.9314	11.6470	15.2055	18.6147	75
76						0.0000	4.0435	7.9159	11.6247	15.1779	76
77							0.0000	4.0355	7.9007	11.6036	77
78								0.0000	4.0279	7.8863	78
79									0.0000	4.0206	79
80										0.0000	80

81–90 Years Probable Life

Age, years	81	82	83	84	85	86	87	88	89	90	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.8259	99.8329	99.8395	99.8459	99.8521	99.8580	99.8636	99.8690	99.8742	99.8792	1
2	99.6448	99.6590	99.6727	99.6857	99.6983	99.7103	99.7218	99.7329	99.7435	99.7536	2
3	99.4564	99.4782	99.4991	99.5191	99.5383	99.5567	99.5743	99.5912	99.6074	99.6230	3
4	99.2606	99.2902	99.3186	99.3458	99.3719	99.3969	99.4209	99.4439	99.4660	99.4871	4
5	99.0569	99.0947	99.1309	99.1656	99.1989	99.2308	99.2614	99.2907	99.3189	99.3458	5
6	98.8450	98.8913	98.9357	98.9782	99.0189	99.0580	99.0955	99.1314	99.1659	99.1989	6
7	98.6247	98.6798	98.7326	98.7832	98.8318	98.8783	98.9229	98.9657	99.0067	99.0461	7
8	98.3956	98.4598	98.5215	98.5805	98.6371	98.6914	98.7435	98.7934	98.8412	98.8872	8
9	98.1572	98.2311	98.3019	98.3697	98.4347	98.4971	98.5568	98.6142	98.6691	98.7219	9
10	97.9094	97.9932	98.0735	98.1504	98.2242	98.2949	98.3627	98.4278	98.4901	98.5500	10
11	97.6517	97.7458	97.8359	97.9224	98.0052	98.0847	98.1609	98.2339	98.3040	98.3712	11
12	97.3836	97.4884	97.5889	97.6852	97.7775	97.8661	97.9509	98.0323	98.1104	98.1853	12
13	97.1048	97.2208	97.3320	97.4386	97.5407	97.6387	97.7326	97.8227	97.9090	97.9919	13
14	96.8149	96.9425	97.0648	97.1821	97.2945	97.4022	97.5055	97.6046	97.6996	97.7908	14
15	96.5134	96.6531	96.7870	96.9153	97.0383	97.1563	97.2694	97.3779	97.4819	97.5817	15
16	96.1998	96.3520	96.4980	96.6378	96.7720	96.9005	97.0238	97.1420	97.2554	97.3641	16
17	95.8736	96.0390	96.1974	96.3493	96.4949	96.6345	96.7684	96.8967	97.0198	97.1379	17
18	95.5344	95.7134	95.8849	96.0492	96.2068	96.3579	96.5027	96.6417	96.7749	96.9027	18
19	95.1817	95.3748	95.5598	95.7371	95.9072	96.0702	96.2265	96.3764	96.5201	96.6580	19
20	94.8148	95.0226	95.2217	95.4126	95.5956	95.7710	95.9392	96.1005	96.2552	96.4036	20

4% INTEREST RATE

81–90 Years Probable Life

Age, years	Condition-percent, %										Age, years
	81	82	83	84	85	86	87	88	89	90	
21	94.4333	94.6563	94.8701	95.0750	95.2715	95.4598	95.6404	95.8135	95.9796	96.1389	21
22	94.0365	94.2755	94.5045	94.7240	94.9344	95.1362	95.3296	95.5151	95.6930	95.8637	22
23	93.6238	93.8793	94.1242	94.3589	94.5839	94.7996	95.0064	95.2048	95.3950	95.5775	23
24	93.1947	93.4673	93.7287	93.9792	94.2193	94.4496	94.6703	94.8820	95.0850	95.2798	24
25	92.7483	93.0389	93.3174	93.5843	93.8402	94.0855	94.3207	94.5463	94.7627	94.9702	25
26	92.2841	92.5933	92.8896	93.1736	93.4459	93.7069	93.9572	94.1972	94.4274	94.6482	26
27	91.8014	92.1299	92.4447	92.7465	93.0358	93.3132	93.5791	93.8342	94.0788	94.3134	27
28	91.2993	91.6479	91.9821	92.3023	92.6093	92.9037	93.1859	93.4566	93.7162	93.9652	28
29	90.7771	91.1466	91.5009	91.8403	92.1658	92.4778	92.7770	93.0639	93.3391	93.6030	29
30	90.2341	90.6254	91.0004	91.3599	91.7045	92.0349	92.3517	92.6555	92.9469	93.2263	30
31	89.6693	90.0833	90.4800	90.8602	91.2248	91.5743	91.9094	92.2308	92.5390	92.8346	31
32	89.0820	89.5194	89.9387	90.3406	90.7259	91.0953	91.4494	91.7891	92.1148	92.4272	32
33	88.4711	88.9331	89.3758	89.8002	90.2070	90.5971	90.9710	91.3297	91.6736	92.0035	33
34	87.8359	88.3233	88.7904	89.2381	89.6674	90.0789	90.4735	90.8519	91.2148	91.5629	34
35	87.1752	87.6890	88.1815	88.6536	89.1062	89.5400	89.9560	90.3550	90.7376	91.1047	35
36	86.4880	87.0294	87.5483	88.0457	88.5225	88.9796	89.4179	89.8383	90.2414	90.6281	36
37	85.7734	86.3435	86.8898	87.4134	87.9155	88.3968	88.8583	89.3008	89.7253	90.1324	37
38	85.0302	85.6301	86.2049	86.7559	87.2842	87.7907	88.2762	88.7419	89.1885	89.6169	38
39	84.2573	84.8881	85.4927	86.0721	86.6277	87.1603	87.6709	88.1607	88.6303	89.0808	39
40	83.4535	84.1165	84.7519	85.3609	85.9449	86.5047	87.0414	87.5561	88.0498	88.5233	40
41	82.6175	83.3140	83.9815	84.6213	85.2348	85.8228	86.3867	86.9274	87.4460	87.9435	41
42	81.7481	82.4794	83.1803	83.8521	84.4962	85.1137	85.7058	86.2736	86.8181	87.3404	42
43	80.8439	81.6114	82.3471	83.0522	83.7282	84.3763	84.9976	85.5935	86.1651	86.7133	43
44	79.9035	80.7087	81.4805	82.2202	82.9294	83.6093	84.2611	84.8863	85.4859	86.0610	44
45	78.9255	79.7699	80.5793	81.3549	82.0987	82.8116	83.4952	84.1508	84.7796	85.3827	45
46	77.9084	78.7936	79.6420	80.4551	81.2347	81.9821	82.6987	83.3859	84.0450	84.6772	46
47	76.8506	77.7782	78.6672	79.5193	80.3362	81.1194	81.8702	82.5904	83.2811	83.9435	47
48	75.7505	76.7222	77.6534	78.5460	79.4017	80.2221	81.0087	81.7631	82.4865	83.1805	48
49	74.6064	75.6240	76.5991	77.5338	78.4299	79.2890	80.1127	80.9026	81.6602	82.3869	49
50	73.4166	74.4818	75.5026	76.4811	77.4192	78.3185	79.1808	80.0078	80.8009	81.5616	50
51	72.1791	73.2939	74.3622	75.3863	76.3680	77.3092	78.2117	79.0771	79.9072	80.7033	51
52	70.8921	72.0585	73.1762	74.2477	75.2748	76.2595	77.2037	78.1093	78.9777	79.8107	52
53	69.5537	70.7737	71.9428	73.0635	74.1379	75.1679	76.1555	77.1027	78.0110	78.8823	53
54	68.1617	69.4375	70.6601	71.8320	72.9555	74.0326	75.0654	76.0559	77.0057	77.9168	54
55	66.7140	68.0478	69.3260	70.5513	71.7258	72.8519	73.9316	74.9671	75.9601	76.9127	55
56	65.2085	66.6026	67.9386	69.2193	70.4470	71.6239	72.7525	73.8349	74.8728	75.8684	56
57	63.6428	65.0996	66.4957	67.8340	69.1169	70.3469	71.5262	72.6573	73.7419	74.7824	57
58	62.0143	63.5364	64.9950	66.3933	67.7337	69.0187	70.2509	71.4326	72.5659	73.6529	58
59	60.3208	61.9107	63.4344	64.8950	66.2951	67.6374	68.9246	70.1590	71.3427	72.4782	59
60	58.5595	60.2200	61.8113	63.3367	64.7990	66.2009	67.5452	68.8344	70.0707	71.2566	60
61	56.7277	58.4616	60.1233	61.7161	63.2430	64.7069	66.1106	67.4568	68.7478	69.9861	61
62	54.8227	56.6329	58.3678	60.0307	61.6248	63.1532	64.6187	66.0241	67.3719	68.6647	62
63	52.8415	54.7311	56.5420	58.2779	59.9419	61.5373	63.0671	64.5341	65.9410	67.2905	63
64	50.7810	52.7532	54.6432	56.4549	58.1917	59.8568	61.4534	62.9845	64.4529	65.8614	64
65	48.6382	50.6962	52.6685	54.5591	56.3714	58.1090	59.7752	61.3729	62.9053	64.3751	65
66	46.4096	48.5569	50.6148	52.5874	54.4784	56.2914	58.0298	59.6969	61.2957	62.8293	66
67	44.0918	46.3320	48.4789	50.5368	52.5096	54.4010	56.2146	57.9538	59.6218	61.2217	67
68	41.6814	44.0181	46.2576	48.4043	50.4621	52.4350	54.3268	56.1410	57.8809	59.5498	68
69	39.1745	41.6117	43.9474	46.1864	48.3327	50.3904	52.3635	54.2557	56.0704	57.8110	69
70	36.5673	39.1090	41.5449	43.8798	46.1181	48.2640	50.3217	52.2950	54.1875	56.0027	70
71	33.8559	36.5062	39.0462	41.4809	43.8149	46.0526	48.1982	50.2558	52.2292	54.1220	71
72	31.0360	33.7993	36.4476	38.9861	41.4196	43.7526	45.9898	48.1351	50.1926	52.1661	72
73	28.1033	30.9841	33.7450	36.3915	38.9284	41.3607	43.6930	45.9295	48.0746	50.1320	73
74	25.0533	28.0564	30.9344	33.6931	36.3377	38.8731	41.3043	43.6358	45.8718	48.0165	74
75	21.8813	25.0115	28.0113	30.8868	33.6433	36.2861	38.8201	41.2502	43.5809	45.8164	75
76	18.5825	21.8448	24.9713	27.9682	30.8411	33.5955	36.2366	38.7693	41.1984	43.5283	76
77	15.1516	18.5514	21.8096	24.9329	27.9268	30.7973	33.5497	36.1891	38.7205	41.1486	77
78	11.5836	15.1263	18.5216	21.7761	24.8960	27.8872	30.7553	33.5057	36.1436	38.6738	78
79	7.8728	11.5642	15.1020	18.4931	21.7439	24.8606	27.8491	30.7150	33.4636	36.1000	79
80	4.0136	7.8596	11.5457	15.0788	18.4658	21.7130	24.8267	27.8127	30.6764	33.4232	80

4% INTEREST RATE

81–90 Years Probable Life

Age, years	Condition-percent, % 81	82	83	84	85	86	87	88	89	90	Age, years
81	0.0000	4.0069	7.8470	11.5279	15.0565	18.4395	21.6834	24.7942	27.7777	30.6393	81
82		0.0000	4.0004	7.8349	11.5108	15.0351	18.4144	21.6550	24.7630	27.7441	82
83			0.0000	3.9943	7.8233	11.4945	15.0146	18.3903	21.6278	24.7331	83
84				0.0000	3.9884	7.8122	11.4788	14.9949	18.3671	21.6017	84
85					0.0000	3.9827	7.8016	11.4638	14.9761	18.3450	85
86						0.0000	3.9773	7.7918	11.4499	14.9580	86
87							0.0000	3.9721	7.7816	11.4355	87
88								0.0000	3.9671	7.7722	88
89									0.0000	3.9623	89
90										0.0000	90

91–100 Years Probable Life

Age, years	91	92	93	94	95	96	97	98	99	100	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.8840	99.8886	99.8930	99.8972	99.9013	99.9052	99.9089	99.9125	99.9159	99.9192	1
2	99.7634	99.7727	99.7817	99.7903	99.7986	99.8065	99.8141	99.8214	99.8284	99.8352	2
3	99.6379	99.6522	99.6660	99.6791	99.6918	99.7039	99.7156	99.7267	99.7375	99.7478	3
4	99.5074	99.5269	99.5456	99.5635	99.5807	99.5972	99.6131	99.6283	99.6429	99.6569	4
5	99.3717	99.3966	99.4204	99.4433	99.4652	99.4863	99.5065	99.5259	99.5445	99.5624	5
6	99.2306	99.2610	99.2902	99.3182	99.3451	99.3709	99.3956	99.4194	99.4422	99.4641	6
7	99.0838	99.1201	99.1548	99.1881	99.2201	99.2508	99.2803	99.3086	99.3358	99.3618	7
8	98.9312	98.9734	99.0140	99.0529	99.0902	99.1260	99.1604	99.1934	99.2251	99.2555	8
9	98.7724	98.8210	98.8675	98.9122	98.9551	98.9962	99.0357	99.0736	99.1100	99.1449	9
10	98.6073	98.6624	98.7152	98.7659	98.8145	98.8612	98.9060	98.9490	98.9903	99.0299	10
11	98.4357	98.4975	98.5568	98.6138	98.6684	98.7208	98.7711	98.8194	98.8658	98.9103	11
12	98.2571	98.3260	98.3921	98.4555	98.5164	98.5748	98.6309	98.6847	98.7363	98.7859	12
13	98.0714	98.1476	98.2208	98.2909	98.3583	98.4229	98.4850	98.5445	98.6017	98.6566	13
14	97.8782	97.9621	98.0426	98.1198	98.1939	98.2650	98.3333	98.3988	98.4617	98.5220	14
15	97.6774	97.7692	97.8573	97.9418	98.0229	98.1008	98.1755	98.2472	98.3160	98.3821	15
16	97.4685	97.5685	97.6646	97.7567	97.8451	97.9299	98.0114	98.0895	98.1646	98.2366	16
17	97.2512	97.3599	97.4641	97.5642	97.6602	97.7523	97.8407	97.9256	98.0070	98.0852	17
18	97.0253	97.1428	97.2557	97.3639	97.4678	97.5675	97.6632	97.7551	97.8432	97.9279	18
19	96.7903	96.9171	97.0389	97.1557	97.2678	97.3754	97.4786	97.5777	97.6728	97.7642	19
20	96.5459	96.6824	96.8134	96.9391	97.0598	97.1755	97.2867	97.3933	97.4957	97.5939	20
21	96.2917	96.4383	96.5790	96.7139	96.8434	96.9677	97.0870	97.2015	97.3114	97.4169	21
22	96.0274	96.1844	96.3351	96.4797	96.6184	96.7516	96.8794	97.0020	97.1197	97.2328	22
23	95.7525	95.9204	96.0815	96.2361	96.3844	96.5268	96.6634	96.7945	96.9204	97.0413	23
24	95.4666	95.6458	95.8178	95.9827	96.1411	96.2930	96.4388	96.5788	96.7132	96.8421	24
25	95.1693	95.3602	95.5435	95.7193	95.8880	96.0499	96.2053	96.3544	96.4976	96.6350	25
26	94.8601	95.0632	95.2582	95.4452	95.6247	95.7970	95.9624	96.1210	96.2734	96.4196	26
27	94.5385	94.7543	94.9615	95.1603	95.3510	95.5341	95.7097	95.8783	96.0402	96.1956	27
28	94.2040	94.4331	94.6530	94.8639	95.0663	95.2606	95.4470	95.6259	95.7977	95.9626	28
29	93.8562	94.0990	94.3321	94.5556	94.7702	94.9761	95.1738	95.3634	95.5455	95.7203	29
30	93.4944	93.7516	93.9984	94.2351	94.4623	94.6803	94.8896	95.0904	95.2833	95.4683	30
31	93.1182	93.3902	93.6513	93.9017	94.1420	94.3727	94.5941	94.8065	95.0105	95.2063	31
32	92.7269	93.0144	93.2903	93.5550	93.8090	94.0528	94.2867	94.5112	94.7268	94.9337	32
33	92.3200	92.6236	92.9149	93.1944	93.4626	93.7200	93.9671	94.2042	94.4318	94.6503	33
34	91.8968	92.2171	92.5245	92.8194	93.1024	93.3740	93.6346	93.8848	94.1249	94.3555	34
35	91.4567	91.7944	92.1185	92.4294	92.7277	93.0141	93.2889	93.5526	93.8058	94.0489	35
36	90.9990	91.3548	91.6962	92.0237	92.3381	92.6398	92.9293	93.2072	93.4740	93.7301	36
37	90.5229	90.8976	91.2570	91.6019	91.9329	92.2505	92.5554	92.8480	93.1288	93.3985	37
38	90.0279	90.4220	90.8003	91.1632	91.5115	91.8457	92.1665	92.4743	92.7699	93.0536	38
39	89.5130	89.9275	90.3253	90.7069	91.0732	91.4247	91.7620	92.0858	92.3966	92.6949	39
40	88.9775	89.4132	89.8313	90.2324	90.6174	90.9868	91.3414	91.6817	92.0084	92.3219	40
41	88.4206	88.8783	89.3176	89.7389	90.1433	90.5315	90.9039	91.2614	91.6046	91.9340	41
42	87.8415	88.3220	88.7832	89.2257	89.6503	90.0579	90.4489	90.8243	91.1847	91.5306	42
43	87.2391	87.7435	88.2276	88.6920	89.1376	89.5653	89.9758	90.3697	90.7480	91.1110	43
44	86.6127	87.1419	87.6497	88.1368	88.6044	89.0531	89.4837	89.8970	90.2938	90.6746	44
45	85.9612	86.5161	87.0486	87.5595	88.0498	88.5204	88.9719	89.4053	89.8214	90.2208	45

4% INTEREST RATE
91–100 Years Probable Life

Age, years	Condition-percent, %										Age, years
	91	92	93	94	95	96	97	98	99	100	
46	85.2836	85.8654	86.4236	86.9591	87.4731	87.9663	88.4397	88.8940	89.3302	89.7489	46
47	84.5790	85.1886	85.7735	86.3347	86.8732	87.3901	87.8861	88.3623	88.8193	89.2580	47
48	83.8462	84.4847	85.0974	85.6853	86.2494	86.7909	87.3105	87.8092	88.2880	88.7475	48
49	83.0840	83.7527	84.3943	85.0099	85.6007	86.1677	86.7118	87.2340	87.7354	88.2166	49
50	82.2913	82.9914	83.6631	84.3075	84.9260	85.5195	86.0891	86.6358	87.1607	87.6645	50
51	81.4670	82.1996	82.9026	83.5770	84.2243	84.8455	85.4416	86.0137	86.5630	87.0903	51
52	80.6096	81.3762	82.1117	82.8173	83.4945	84.1444	84.7681	85.3667	85.9414	86.4931	52
53	79.7180	80.5198	81.2892	82.0273	82.7355	83.4154	84.0677	84.6939	85.2950	85.8720	53
54	78.7908	79.6292	80.4337	81.2056	81.9463	82.6571	83.3393	83.9941	84.6227	85.2261	54
55	77.8264	78.7030	79.5441	80.3510	81.1254	81.8685	82.5818	83.2663	83.9235	84.5543	55
56	76.8235	77.7397	78.6188	79.4623	80.2717	81.0484	81.7939	82.5094	83.1964	83.8557	56
57	75.7804	76.7378	77.6566	78.5380	79.3838	80.1955	80.9746	81.7223	82.4401	83.1291	57
58	74.6956	75.6959	76.6558	77.5767	78.4604	79.3085	80.1224	80.9037	81.6536	82.3735	58
59	73.5674	74.6124	75.6150	76.5770	77.5001	78.3860	79.2362	80.0523	80.8357	81.5876	59
60	72.3942	73.4855	74.5326	75.5373	76.5014	77.4266	78.3145	79.1669	79.9850	80.7704	60
61	71.1739	72.3135	73.4069	74.4560	75.4627	76.4288	77.3560	78.2460	79.1003	79.9204	61
62	69.9049	71.0946	72.2361	73.3314	74.3824	75.3911	76.3591	77.2883	78.1802	79.0364	62
63	68.5851	69.8270	71.0186	72.1619	73.2590	74.3119	75.3224	76.2923	77.2233	78.1170	63
64	67.2125	68.5086	69.7523	70.9456	72.0906	73.1895	74.2441	75.2565	76.2281	77.1609	64
65	65.7850	67.1376	68.4354	69.6806	70.8755	72.0222	73.1228	74.1792	75.1932	76.1666	65
66	64.3004	65.7117	67.0658	68.3650	69.6118	70.8083	71.9566	73.0588	74.1168	75.1324	66
67	62.7564	64.2287	65.6414	66.9968	68.2975	69.5458	70.7437	71.8936	72.9974	74.0569	67
68	61.1507	62.6865	64.1600	65.5739	66.9307	68.2328	69.4823	70.6818	71.8332	72.9384	68
69	59.4807	61.0825	62.6194	64.0941	65.5092	66.8672	68.1705	69.4216	70.6224	71.7751	69
70	57.7440	59.4144	61.0172	62.5551	64.0308	65.4470	66.8062	68.1109	69.3632	70.5653	70
71	55.9377	57.6796	59.3509	60.9545	62.4933	63.9701	65.3873	66.7478	68.0536	69.3071	71
72	54.0592	55.8754	57.6179	59.2899	60.8943	62.4340	63.9117	65.3302	66.6916	67.9986	72
73	52.1056	53.9990	55.8156	57.5586	59.2313	60.8365	62.3771	63.8558	65.2752	66.6378	73
74	50.0738	52.0476	53.9412	55.7582	57.5019	59.1752	60.7810	62.3225	63.8021	65.2225	74
75	47.9608	50.0181	51.9919	53.8858	55.7032	57.4473	59.1212	60.7279	62.2701	63.7506	75
76	45.7632	47.9074	49.9645	51.9384	53.8326	55.6504	57.3949	59.0695	60.6768	62.2198	76
77	43.4778	45.7122	47.8561	49.9132	51.8871	53.7815	55.5996	57.3447	59.0198	60.6278	77
78	41.1009	43.4293	45.6633	47.8069	49.8639	51.8379	53.7325	55.5510	57.2965	58.9721	78
79	38.6292	41.0551	43.3829	45.6164	47.7597	49.8166	51.7907	53.6855	55.5042	57.2502	79
80	36.0581	38.5858	41.0112	43.3383	45.5714	47.7144	49.7712	51.7454	53.6403	55.4594	80
81	33.3844	36.0179	38.5447	40.9690	43.2955	45.5281	47.6709	49.7276	51.7018	53.5970	81
82	30.6038	33.3472	35.9794	38.5048	40.9286	43.2544	45.4866	47.6292	49.6858	51.6601	82
83	27.7119	30.5697	33.3115	35.9424	38.4668	40.8897	43.2150	45.4468	47.5891	49.6457	83
84	24.7044	27.6811	30.5370	33.2773	35.9069	38.4305	40.8525	43.1772	45.4086	47.5507	84
85	21.5766	24.6769	27.6515	30.5056	33.2444	35.8729	38.3953	40.8167	43.1408	45.3719	85
86	18.3237	21.5526	24.6505	27.6230	30.4755	33.2129	35.8402	38.3615	40.7824	43.1060	86
87	14.9406	18.3033	21.5295	24.6252	27.5958	30.4466	33.1826	35.8088	38.3295	40.7494	87
88	11.4223	14.9240	18.2837	21.5074	24.6008	27.5696	30.4188	33.1536	35.7786	38.2986	88
89	7.7632	11.4095	14.9080	18.2649	21.4861	24.5775	27.5445	30.3922	33.1257	35.7498	89
90	3.9577	7.7545	11.3973	14.8927	18.2469	21.4658	24.5551	27.5204	30.3666	33.0989	90
91	0.0000	3.9533	7.7462	11.3856	14.8780	18.2296	21.4462	24.5336	27.4972	30.3421	91
92		0.0000	3.9490	7.7382	11.3744	14.8639	18.2129	21.4274	24.5130	27.4750	92
93			0.0000	3.9450	7.7306	11.3636	14.8498	18.1970	21.4094	24.4932	93
94				0.0000	3.9411	7.7233	11.3532	14.8373	18.1816	21.3921	94
95					0.0000	3.9374	7.7162	11.3433	14.8249	18.1670	95
96						0.0000	3.9338	7.7095	11.3337	14.8129	96
97							0.0000	3.9303	7.7030	11.3246	97
98								0.0000	3.9270	7.6968	98
99									0.0000	3.9238	99
100										0.0000	100

5% INTEREST RATE

1–10 Years Probable Life

Age, years	Condition-percent, %										Age, years
	1	2	3	4	5	6	7	8	9	10	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	0.0000	51.2195	68.2791	76.7988	81.9025	85.2983	87.7180	89.5278	90.9310	92.0495	1
2		0.0000	34.9722	52.4376	62.9002	69.8614	74.8219	78.5320	81.4085	83.7016	2
3			0.0000	26.8583	42.9477	53.6527	61.2811	66.9864	71.4100	74.9362	3
4				0.0000	21.9976	36.6336	47.0631	54.8636	60.9114	65.7325	4
5					0.0000	18.7636	32.1343	42.1346	49.8880	56.0687	5
6						0.0000	16.4590	28.7691	38.3134	45.9217	6
7							0.0000	14.7354	26.1601	35.2673	7
8								0.0000	13.3991	24.0802	8
9									0.0000	12.3338	9
10										0.0000	10

11–20 Years Probable Life

Age, years	11	12	13	14	15	16	17	18	19	20	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	92.9611	93.7175	94.3544	94.8976	95.3658	95.7730	96.1301	96.4454	96.7255	96.9757	1
2	85.5703	87.1208	88.4266	89.5401	90.4998	91.3347	92.0667	92.7130	93.2873	93.8003	2
3	77.8099	80.1943	82.2023	83.9147	85.3906	86.6744	87.8001	88.7941	89.6771	90.4660	3
4	69.6615	72.9215	75.6669	78.0080	80.0259	81.7811	83.3202	84.6791	85.8865	86.9651	4
5	61.1057	65.2850	68.8046	71.8060	74.3930	76.6432	78.6163	80.3585	81.9063	83.2891	5
6	52.1221	57.2667	61.5993	65.2939	68.4784	71.2484	73.6772	75.8218	77.7271	79.4293	6
7	42.6893	48.8475	54.0337	58.4562	62.2681	65.5838	68.4911	71.0582	73.3390	75.3765	7
8	32.7849	40.0073	46.0898	51.2767	55.7472	59.6360	63.0458	66.0565	68.7314	71.1210	8
9	22.3852	30.7252	37.7487	43.7381	48.9004	53.3908	57.3281	60.8047	63.8935	66.6528	9
10	11.4656	20.9789	28.9905	35.8226	41.7112	46.8334	51.3246	55.2903	58.8137	61.9612	10
11	0.0000	10.7453	19.7945	27.5113	34.1625	39.9480	45.0210	49.5002	53.4799	57.0350	11
12		0.0000	10.1386	18.7845	26.2364	32.7184	38.4021	43.4206	47.8794	51.8625	12
13			0.0000	9.6213	17.9140	25.1274	31.4523	37.0370	41.9988	46.4314	13
14				0.0000	9.1755	17.1568	24.1550	30.3343	35.8243	40.7287	14
15					0.0000	8.7876	16.4928	23.2964	29.3410	34.7408	15
16						0.0000	8.4475	15.9066	22.5335	28.4536	16
17							0.0000	8.1473	15.3857	21.8520	17
18								0.0000	7.8805	14.9204	18
19									0.0000	7.6421	19
20										0.0000	20

21–30 Years Probable Life

Age, years	21	22	23	24	25	26	27	28	29	30	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	97.2004	97.4029	97.5863	97.7529	97.9048	98.0436	98.1708	98.2877	98.3954	98.4949	1
2	94.2608	94.6760	95.0520	95.3935	95.7047	95.9893	96.2502	96.4899	96.7107	96.9145	2
3	91.1742	91.8128	92.3909	92.9160	93.3947	93.8323	94.2335	94.6021	94.9417	95.2550	3
4	87.9333	88.8064	89.5967	90.3148	90.9692	91.5675	92.1160	92.6200	93.0842	93.5126	4
5	84.5304	85.6497	86.6629	87.5834	88.4224	89.1895	89.8926	90.5387	91.1338	91.6831	5
6	80.9573	82.3351	83.5823	84.7155	85.7483	86.6925	87.5580	88.3534	89.0860	89.7621	6
7	77.2055	78.8548	80.3478	81.7042	82.9405	84.0707	85.1068	86.0588	86.9357	87.7451	7
8	73.2662	75.2005	76.9515	78.5423	79.9923	81.3178	82.5329	83.6495	84.6780	85.6272	8
9	69.1299	71.3635	73.3854	75.2223	76.8966	78.4273	79.8304	81.1197	82.3073	83.4034	9
10	64.7868	67.3346	69.6410	71.7363	73.6462	75.3922	76.9927	78.4635	79.8181	81.0685	10
11	60.2265	63.1042	65.7093	68.0761	70.2333	72.2054	74.0131	75.6744	77.2045	78.6167	11
12	55.4382	58.6624	61.5811	64.2328	66.6497	68.8592	70.8846	72.7459	74.4602	76.0424	12
13	50.4105	53.9985	57.2465	60.1973	62.8869	65.3458	67.5997	69.6709	71.5786	73.3394	13
14	45.1315	49.1014	52.6951	55.9601	58.9360	61.6566	64.1505	66.4422	68.5530	70.5012	14
15	39.5884	43.9594	47.9162	51.5110	54.7876	57.7830	60.5288	63.0520	65.3761	67.5212	15

5% INTEREST RATE

21–30 Years Probable Life

Age, years	Condition-percent, % 21	22	23	24	25	26	27	28	29	30	Age, years
16	33.7682	38.5603	42.8983	46.8395	50.4317	53.7157	56.7260	59.4924	62.0403	64.3921	16
17	27.6570	32.8913	37.6296	41.9344	45.8581	49.4451	52.7332	55.7548	58.5378	61.1065	17
18	21.2403	26.9388	32.0974	36.7840	41.0557	44.9609	48.5406	51.8302	54.8602	57.6567	18
19	14.5027	20.6886	26.2886	31.3761	36.0133	40.2525	44.1385	47.7095	50.9986	54.0344	19
20	7.4282	14.1260	20.1893	25.6978	30.7187	35.3087	39.5162	43.3827	46.9440	50.2310	20
21	0.0000	7.2353	13.7851	19.7356	25.1594	30.1177	34.6628	38.8396	42.6866	46.2374	21
22		0.0000	7.0607	13.4753	19.3221	24.6672	29.5668	34.0693	38.2164	42.0441	22
23			0.0000	6.9020	13.1930	18.9441	24.2160	29.0605	33.5227	37.6412	23
24				0.0000	6.7574	12.9349	18.5976	23.8013	28.5943	33.0181	24
25					0.0000	6.6252	12.6983	18.2791	23.4194	28.1638	25
26						0.0000	6.5040	12.4808	17.9859	23.0669	26
27							0.0000	6.3926	12.2806	17.7151	27
28								0.0000	6.2901	12.0957	28
29									0.0000	6.1954	29
30										0.0000	30

31–40 Years Probable Life

Age, years	31	32	33	34	35	36	37	38	39	40	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.5868	98.6720	98.7510	98.8245	98.8928	98.9566	99.0160	99.0716	99.1235	99.1722	1
2	97.1029	97.2775	97.4395	97.5901	97.7303	97.8609	97.9828	98.0967	98.2033	98.3030	2
3	95.5448	95.8133	96.0625	96.2941	96.5096	96.7105	96.8980	97.0731	97.2370	97.3903	3
4	93.9089	94.2760	94.6166	94.9333	95.2280	95.5026	95.7589	95.9984	96.2223	96.4320	4
5	92.1911	92.6617	93.0985	93.5044	93.8822	94.2343	94.5629	94.8699	95.1570	95.4258	5
6	90.3875	90.9668	91.5044	92.0040	92.4691	92.9026	93.3071	93.6849	94.0384	94.3693	6
7	88.4936	89.1871	89.8306	90.4287	90.9854	91.5043	91.9884	92.4408	92.8638	93.2599	7
8	86.5051	87.3184	88.0731	88.7746	89.4275	90.0360	90.6039	91.1344	91.6306	92.0951	8
9	84.4171	85.3563	86.2278	87.0378	87.7917	88.4944	89.1501	89.7627	90.3356	90.8720	9
10	82.2248	83.2960	84.2902	85.2141	86.0741	86.8756	87.6236	88.3224	88.9760	89.5878	10
11	79.9228	81.1328	82.2557	83.2993	84.2707	85.1760	86.0208	86.8101	87.5483	88.2394	11
12	77.5057	78.8614	80.1194	81.2887	82.3770	83.3913	84.3379	85.2222	86.0492	86.8235	12
13	74.9678	76.4764	77.8764	79.1776	80.3887	81.5175	82.5708	83.5549	84.4752	85.3369	13
14	72.3030	73.9722	75.5212	76.9609	78.3010	79.5499	80.7154	81.8042	82.8225	83.7759	14
15	69.5049	71.3428	73.0483	74.6334	76.1088	77.4839	78.7671	79.9660	81.0872	82.1369	15
16	66.5669	68.5819	70.4517	72.1896	73.8071	75.3147	76.7215	78.0358	79.2651	80.4159	16
17	63.4821	65.6829	67.7253	69.6235	71.3903	73.0370	74.5736	76.0092	77.3519	78.6089	17
18	60.2430	62.6390	64.8625	66.9291	68.8526	70.6454	72.3183	73.8812	75.3430	76.7116	18
19	56.8419	59.4429	61.8567	64.1000	66.1881	68.1342	69.9503	71.6469	73.2337	74.7193	19
20	53.2708	56.0870	58.7005	61.1295	63.3903	65.4975	67.4638	69.3008	71.0189	72.6275	20
21	49.5211	52.5633	55.3865	58.0104	60.4527	62.7289	64.8530	66.8374	68.6934	70.4310	21
22	45.5840	48.8635	51.9068	54.7354	57.3681	59.8219	62.1117	64.2509	66.2516	68.1248	22
23	41.4500	44.9786	48.2532	51.2966	54.1294	56.7695	59.2333	61.5350	63.6878	65.7032	23
24	37.1092	40.8995	44.4168	47.6859	50.7287	53.5646	56.2110	58.6833	60.9957	63.1605	24
25	32.5515	36.6164	40.3887	43.8946	47.1579	50.1994	53.0375	55.6891	58.1690	60.4907	25
26	27.7659	32.1192	36.1591	39.9138	43.4087	46.6659	49.7054	52.5451	55.2010	57.6875	26
27	22.7409	27.3971	31.7180	35.7340	39.4719	42.9557	46.2067	49.2439	52.0846	54.7440	27
28	17.4648	22.4389	27.0549	31.3451	35.3384	39.0601	42.5331	45.7777	48.8123	51.6534	28
29	11.9248	17.2328	22.1587	26.7368	30.9981	34.9696	38.6758	42.1382	45.3765	48.4083	29
30	6.1078	11.7664	17.0176	21.8981	26.4408	30.6747	34.6256	38.3166	41.7689	45.0009	30
31	0.0000	6.0267	11.6195	16.8175	21.6557	26.1649	30.3729	34.3041	37.9809	41.4231	31
32		0.0000	5.9514	11.4828	16.6313	21.4297	25.9075	30.0908	34.0034	37.6664	32
33			0.0000	5.8815	11.3557	16.4578	21.2189	25.6670	29.8271	33.7219	33
34				0.0000	5.8163	11.2372	16.2959	21.0219	25.4420	29.5802	34
35					0.0000	5.7556	11.1267	16.1445	20.8377	25.2314	35
36						0.0000	5.6990	11.0234	16.0031	20.6652	36
37							0.0000	5.6461	10.9268	15.8706	37
38								0.0000	5.5966	10.8363	38
39									0.0000	5.5503	39
40										0.0000	40

5% INTEREST RATE

41–50 Years Probable Life

Age, years	Condition-percent, %										Age, years
	41	42	43	44	45	46	47	48	49	50	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.2178	99.2605	99.3007	99.3384	99.3738	99.4072	99.4386	99.4682	99.4960	99.5223	1
2	98.3964	98.4841	98.5664	98.6437	98.7163	98.7847	98.8491	98.9097	98.9669	99.0208	2
3	97.5340	97.6688	97.7954	97.9142	98.0260	98.1311	98.2301	98.3234	98.4113	98.4941	3
4	96.6285	96.8128	96.9858	97.1483	97.3011	97.4449	97.5802	97.7077	97.8278	97.9412	4
5	95.6777	95.9140	96.1357	96.3441	96.5400	96.7243	96.8978	97.0612	97.2153	97.3605	5
6	94.6793	94.9702	95.2432	95.4997	95.7408	95.9677	96.1813	96.3825	96.5721	96.7509	6
7	93.6311	93.9792	94.3060	94.6130	94.9017	95.1733	95.4289	95.6697	95.8967	96.1108	7
8	92.5304	92.9387	93.3220	93.6821	94.0206	94.3391	94.6389	94.9214	95.1876	95.4386	8
9	91.3747	91.8462	92.2888	92.7045	93.0955	93.4632	93.8095	94.1356	94.4430	94.7329	9
10	90.1612	90.6990	91.2039	91.6782	92.1241	92.5436	92.9385	93.3105	93.6612	93.9919	10
11	88.8870	89.4945	90.0647	90.6004	91.1041	91.5779	92.0240	92.4442	92.8403	93.2138	11
12	87.5492	88.2297	88.8686	89.4688	90.0331	90.5640	91.0638	91.5346	91.9783	92.3968	12
13	86.1444	86.9018	87.6127	88.2806	88.9086	89.4994	90.0556	90.5795	91.0733	91.5390	13
14	84.6694	85.5074	86.2940	87.0331	87.7279	88.3815	88.9969	89.5766	90.1230	90.6382	14
15	83.1206	84.0433	84.9094	85.7231	86.4881	87.2078	87.8853	88.5236	89.1252	89.6925	15
16	81.4944	82.5060	83.4555	84.3476	85.1863	85.9754	86.7182	87.4179	88.0775	88.6994	16
17	79.7869	80.8918	81.9290	82.9034	83.8195	84.6813	85.4927	86.2570	86.9774	87.6567	17
18	77.9940	79.1969	80.3261	81.3869	82.3843	83.3226	84.2059	85.0380	85.8223	86.5619	18
19	76.1115	77.4173	78.6431	79.7946	80.8773	81.8959	82.8548	83.7580	84.6094	85.4123	19
20	74.1349	75.5487	76.8759	78.1227	79.2950	80.3978	81.4361	82.4141	83.3359	84.2053	20
21	72.0594	73.5866	75.0203	76.3673	77.6336	78.8249	79.9465	81.0030	81.9987	82.9378	21
22	69.8801	71.5265	73.0720	74.5240	75.8891	77.1733	78.3824	79.5213	80.5947	81.6071	22
23	67.5919	69.3634	71.0263	72.5886	74.0574	75.4392	76.7401	77.9655	79.1205	80.2097	23
24	65.1892	67.0921	68.8783	70.5564	72.1340	73.6183	75.0156	76.3319	77.5726	78.7426	24
25	62.6665	64.7072	66.6229	68.4226	70.1146	71.7064	73.2050	74.6167	75.9472	77.2020	25
26	60.0176	62.2031	64.2547	66.1821	67.9941	69.6989	71.3038	72.8157	74.2406	75.5844	26
27	57.2362	59.5738	61.7681	63.8295	65.7677	67.5910	69.3076	70.9246	72.4487	73.8860	27
28	54.3158	56.8130	59.1571	61.3594	63.4299	65.3778	67.2116	68.9390	70.5671	72.1026	28
29	51.2494	53.9142	56.4157	58.7657	60.9752	63.0538	65.0107	66.8541	68.5915	70.2301	29
30	48.0296	50.8704	53.5371	56.0424	58.3978	60.6137	62.6998	64.6650	66.5171	68.2639	30
31	44.6489	47.6744	50.5146	53.1829	55.6915	58.0516	60.2734	62.3664	64.3390	66.1994	31
32	41.0991	44.3187	47.3410	50.1804	52.8499	55.3613	57.7257	59.9528	62.0520	64.0317	32
33	37.3718	40.7952	44.0088	47.0278	49.8662	52.5366	55.0505	57.4186	59.6507	61.7556	33
34	33.4582	37.0955	40.5099	43.7176	46.7333	49.5706	52.2416	54.7577	57.1292	59.3657	34
35	29.3488	33.2108	36.8360	40.2418	43.4439	46.4563	49.2923	51.9638	54.4817	56.8563	35
36	25.0340	29.1318	32.9785	36.5923	39.9899	43.1863	46.1955	49.0301	51.7018	54.2215	36
37	20.5035	24.8489	28.9281	32.7603	36.3632	39.7528	42.9438	45.9498	48.7830	51.4549	37
38	15.7465	20.3519	24.6752	28.7367	32.5552	36.1476	39.5296	42.7154	45.7182	48.5500	38
39	10.7516	15.6300	20.2096	24.5119	28.5568	32.3622	35.9447	39.3194	42.5001	45.4998	39
40	5.5069	10.6721	15.5207	20.0758	24.3584	28.3875	32.1805	35.7535	39.1211	42.2971	40
41	0.0000	5.4662	10.5974	15.4180	19.9502	24.2140	28.2281	32.0093	35.5732	38.9343	41
42		0.0000	5.4280	10.5273	15.3215	19.8319	24.0781	28.0779	31.8479	35.4033	42
43			0.0000	5.3920	10.4614	15.2307	19.7205	23.9500	27.9364	31.6958	43
44				0.0000	5.3583	10.3994	15.1451	19.6157	23.8292	27.8029	44
45					0.0000	5.3265	10.3410	15.0646	19.5167	23.7154	45
46						0.0000	5.2966	10.2860	14.9886	19.4235	46
47							0.0000	5.2684	10.2340	14.9170	47
48								0.0000	5.2419	10.1852	48
49									0.0000	5.2168	49
50										0.0000	50

5% INTEREST RATE

51–60 Years Probable Life

Age, years	Condition-percent, %										Age, years
	51	52	53	54	55	56	57	58	59	60	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.5471	99.5706	99.5927	99.6136	99.6333	99.6520	99.6697	99.6864	99.7022	99.7172	1
2	99.0716	99.1196	99.1650	99.2078	99.2483	99.2866	99.3228	99.3571	99.3895	99.4202	2
3	98.5723	98.6462	98.7159	98.7818	98.8440	98.9029	98.9586	99.0113	99.0612	99.1084	3
4	98.0481	98.1490	98.2443	98.3344	98.4195	98.5000	98.5762	98.6482	98.7164	98.7810	4
5	97.4976	97.6270	97.7492	97.8647	97.9738	98.0770	98.1746	98.2670	98.3545	98.4373	5
6	96.9196	97.0789	97.2293	97.3715	97.5058	97.6329	97.7530	97.8667	97.9744	98.0763	6
7	96.3128	96.5034	96.6835	96.8536	97.0144	97.1665	97.3103	97.4465	97.5753	97.6973	7
8	95.6755	95.8991	96.1103	96.3099	96.4985	96.6768	96.8455	97.0052	97.1563	97.2993	8
9	95.0064	95.2646	95.5085	95.7389	95.9567	96.1626	96.3575	96.5418	96.7163	96.8815	9
10	94.3039	94.5984	94.8766	95.1394	95.3879	95.6228	95.8450	96.0552	96.2543	96.4427	10
11	93.5662	93.8989	94.2131	94.5100	94.7906	95.0559	95.3069	95.5444	95.7692	95.9821	11
12	92.7917	93.1644	93.5164	93.8490	94.1634	94.4607	94.7419	95.0080	95.2598	95.4984	12
13	91.9784	92.3932	92.7849	93.1550	93.5049	93.8357	94.1486	94.4447	94.7250	94.9904	13
14	91.1244	91.5834	92.0168	92.4263	92.8134	93.1795	93.5257	93.8534	94.1635	94.4572	14
15	90.2278	90.7331	91.2103	91.6612	92.0874	92.4904	92.8717	93.2324	93.5739	93.8972	15
16	89.2863	89.8403	90.3635	90.8578	91.3251	91.7670	92.1849	92.5804	92.9548	93.3092	16
17	88.2978	88.9029	89.4743	90.0143	90.5247	91.0073	91.4638	91.8958	92.3047	92.6919	17
18	87.2598	87.9185	88.5407	89.1286	89.6842	90.2096	90.7067	91.1770	91.6221	92.0437	18
19	86.1699	86.8850	87.5604	88.1986	88.8018	89.3721	89.9117	90.4222	90.9054	91.3630	19
20	85.0255	85.7998	86.5311	87.2221	87.8752	88.4927	89.0769	89.6297	90.1529	90.6484	20
21	83.8239	84.6604	85.4503	86.1967	86.9023	87.5693	88.2004	88.7975	89.3627	89.8980	21
22	82.5623	83.4639	84.3155	85.1201	85.8807	86.5998	87.2801	87.9238	88.5331	89.1100	22
23	81.2375	82.2077	83.1240	83.9897	84.8080	85.5818	86.3137	87.0063	87.6619	88.2827	23
24	79.8465	80.8886	81.8728	82.8027	83.6817	84.5129	85.2991	86.0430	86.7472	87.4140	24
25	78.3860	79.5036	80.5591	81.5564	82.4991	83.3905	84.2337	85.0315	85.7868	86.5019	25
26	76.8524	78.0493	79.1798	80.2478	81.2574	82.2120	83.1150	83.9695	84.7783	85.5442	26
27	75.2422	76.5223	77.7314	78.8738	79.9536	80.9746	81.9404	82.8544	83.7194	84.5386	27
28	73.5514	74.9190	76.2106	77.4310	78.5846	79.6753	80.7071	81.6834	82.6076	83.4827	28
29	71.7761	73.2355	74.6138	75.9161	77.1471	78.3111	79.4121	80.4540	81.4402	82.3740	29
30	69.9120	71.4678	72.9372	74.3255	75.6378	76.8786	78.0524	79.1631	80.2144	81.2099	30
31	67.9548	69.6118	71.1767	72.6553	74.0530	75.3745	76.6247	77.8076	78.9273	79.9876	31
32	65.8996	67.6629	69.3283	70.9017	72.3890	73.7953	75.1256	76.3843	77.5759	78.7041	32
33	63.7418	65.6166	67.3873	69.0603	70.6417	72.1370	73.5515	74.8899	76.1568	77.3565	33
34	61.4760	63.4680	65.3493	67.1269	68.8071	70.3958	71.8987	73.3208	74.6669	75.9415	34
35	59.0969	61.2119	63.2095	65.0968	66.8808	68.5677	70.1633	71.6732	73.1024	74.4558	35
36	56.5989	58.8431	60.9626	62.9652	64.8582	66.6480	68.3411	69.9432	71.4598	72.8958	36
37	53.9760	56.3558	58.6034	60.7270	62.7344	64.6324	66.4279	68.1268	69.7349	71.2577	37
38	51.2219	53.7441	56.1262	58.3769	60.5044	62.5160	64.4189	66.2195	67.9239	69.5378	38
39	48.3301	51.0019	53.5252	55.9093	58.1630	60.2938	62.3095	64.2168	66.0223	67.7318	39
40	45.2938	48.1225	50.7941	53.3184	55.7044	57.9605	60.0946	62.1141	64.0256	65.8356	40
41	42.1056	45.0993	47.9265	50.5979	53.1229	55.5105	57.7690	59.9061	61.9291	63.8446	41
42	38.7580	41.9247	44.9155	47.7413	50.4124	52.9380	55.3271	57.5878	59.7277	61.7540	42
43	35.2430	38.5915	41.7540	44.7420	47.5663	50.2369	52.7632	55.1536	57.4163	59.5589	43
44	31.5523	35.0916	38.4343	41.5926	44.5779	47.4007	50.0709	52.5976	54.9893	57.2540	44
45	27.6771	31.4168	34.9487	38.2858	41.4402	44.4228	47.2442	49.9139	52.4410	54.8339	45
46	23.6081	27.5582	31.2888	34.8136	38.1455	41.2959	44.2760	47.0959	49.7652	52.2928	46
47	19.3356	23.5066	27.4459	31.1679	34.6861	38.0127	41.1595	44.1372	46.9557	49.6246	47
48	14.8495	19.2525	23.4109	27.3398	31.0537	34.5653	37.8871	41.0303	44.0057	46.8230	48
49	10.1391	14.7857	19.1741	23.3204	27.2397	30.9455	34.4511	37.7682	40.9081	43.8813	49
50	5.1932	10.0955	14.7254	19.1000	23.2350	27.1448	30.8434	34.3430	37.6557	40.7926	50
51	0.0000	5.1709	10.0544	14.6685	19.0301	23.1540	27.0552	30.7466	34.2407	37.5494	51
52		0.0000	5.1498	10.0155	14.6149	18.9637	23.0776	26.9703	30.6550	34.1441	52
53			0.0000	5.1299	9.9790	14.5639	18.9011	23.0051	26.8899	30.5685	53
54				0.0000	5.1111	9.9441	14.5158	18.8418	22.9366	26.8141	54
55					0.0000	5.0933	9.9113	14.4702	18.7856	22.8720	55
56						0.0000	5.0765	9.8801	14.4271	18.7327	56
57							0.0000	5.0606	9.8507	14.3866	57
58								0.0000	5.0455	9.8231	58
59									0.0000	5.0313	59
60										0.0000	60

5% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, %										Age, years
	61	62	63	64	65	66	67	68	69	70	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.7314	99.7448	99.7576	99.7696	99.7811	99.7919	99.8022	99.8120	99.8213	99.8301	1
2	99.4493	99.4769	99.5030	99.5278	99.5512	99.5735	99.5946	99.6146	99.6336	99.6517	2
3	99.1532	99.1955	99.2357	99.2738	99.3099	99.3441	99.3766	99.4074	99.4366	99.4643	3
4	98.8422	98.9001	98.9550	99.0071	99.0564	99.1032	99.1476	99.1898	99.2297	99.2676	4
5	98.5157	98.5900	98.6604	98.7271	98.7904	98.8504	98.9073	98.9613	99.0125	99.0611	5
6	98.1728	98.2643	98.3509	98.4331	98.5110	98.5848	98.6549	98.7213	98.7844	98.8443	6
7	97.8128	97.9223	98.0260	98.1244	98.2176	98.3060	98.3898	98.4694	98.5449	98.6166	7
8	97.4349	97.5632	97.6849	97.8002	97.9096	98.0132	98.1116	98.2049	98.2934	98.3775	8
9	97.0380	97.1862	97.3267	97.4599	97.5861	97.7058	97.8194	97.9272	98.0294	98.1264	9
10	96.6212	96.7903	96.9506	97.1025	97.2465	97.3831	97.5126	97.6355	97.7521	97.8628	10
11	96.1837	96.3747	96.5557	96.7273	96.8899	97.0442	97.1905	97.3293	97.4610	97.5861	11
12	95.7242	95.9382	96.1410	96.3333	96.5155	96.6883	96.8523	97.0078	97.1554	97.2954	12
13	95.2418	95.4799	95.7056	95.9195	96.1224	96.3147	96.4971	96.6702	96.8344	96.9903	13
14	94.7353	94.9988	95.2485	95.4852	95.7096	95.9224	96.1242	96.3157	96.4974	96.6699	14
15	94.2034	94.4935	94.7684	95.0291	95.2761	95.5104	95.7327	95.9435	96.1436	96.3335	15
16	93.6450	93.9630	94.2644	94.5501	94.8210	95.0779	95.3216	95.5527	95.7721	95.9802	16
17	93.0586	93.4060	93.7352	94.0473	94.3432	94.6237	94.8899	95.1424	95.3819	95.6093	17
18	92.4429	92.8211	93.1795	93.5193	93.8414	94.1469	94.4366	94.7115	94.9723	95.2199	18
19	91.7964	92.2070	92.5961	92.9649	93.3146	93.6461	93.9607	94.2591	94.5422	94.8110	19
20	91.1176	91.5621	91.9834	92.3828	92.7614	93.1204	93.4610	93.7841	94.0906	94.3816	20
21	90.4048	90.8851	91.3402	91.7715	92.1805	92.5684	92.9363	93.2853	93.6164	93.9308	21
22	89.6565	90.1741	90.6647	91.1297	91.5706	91.9887	92.3853	92.7616	93.1185	93.4574	22
23	88.8707	89.4276	89.9555	90.4559	90.9302	91.3801	91.8068	92.2117	92.5958	92.9603	23
24	88.0456	88.6438	89.2108	89.7483	90.2578	90.7410	91.1994	91.6342	92.0468	92.4384	24
25	87.1792	87.8209	88.4289	89.0053	89.5518	90.0700	90.5616	91.0280	91.4705	91.8905	25
26	86.2695	86.9567	87.6080	88.2252	88.8105	89.3655	89.8919	90.3914	90.8653	91.3151	26
27	85.3144	86.0494	86.7459	87.4061	88.0321	88.6257	89.1888	89.7230	90.2298	90.7109	27
28	84.3115	85.0967	85.8408	86.5461	87.2148	87.8489	88.4505	89.0211	89.5626	90.0765	28
29	83.2584	84.0963	84.8904	85.6430	86.3566	87.0333	87.6752	88.2842	88.8620	89.4104	29
30	82.1527	83.0459	83.8924	84.6948	85.4555	86.1769	86.8613	87.5104	88.1264	88.7110	30
31	80.9917	81.9431	82.8446	83.6992	84.5094	85.2777	86.0066	86.6980	87.3540	87.9767	31
32	79.7727	80.7850	81.7444	82.6538	83.5160	84.3336	85.1091	85.8449	86.5430	87.2056	32
33	78.4927	79.5691	80.5892	81.5561	82.4728	83.3422	84.1668	84.9491	85.6914	86.3960	33
34	77.1487	78.2924	79.3762	80.4035	81.3776	82.3012	83.1774	84.0086	84.7973	85.5458	34
35	75.7375	76.9518	78.1026	79.1934	80.2275	81.2082	82.1385	83.0210	83.8584	84.6532	35
36	74.2558	75.5442	76.7653	77.9227	79.0200	80.0606	81.0476	81.9841	82.8726	83.7160	36
37	72.6999	74.0662	75.3611	76.5884	77.7521	78.8556	79.9023	80.8953	81.8375	82.7319	37
38	71.0663	72.5144	73.8867	75.1875	76.4208	77.5903	78.6996	79.7521	80.7507	81.6985	38
39	69.3510	70.8849	72.3386	73.7165	75.0229	76.2617	77.4368	78.5517	79.6095	80.6135	39
40	67.5499	69.1740	70.7131	72.1720	73.5551	74.8668	76.1109	77.2913	78.4113	79.4743	40
41	65.6588	67.3775	69.0063	70.5502	72.0140	73.4020	74.7187	75.9679	77.1531	78.2781	41
42	63.6731	65.4912	67.2141	68.8473	70.3957	71.8641	73.2569	74.5783	75.8321	77.0221	42
43	61.5881	63.5105	65.3324	67.0593	68.6966	70.2492	71.7220	73.1193	74.4449	75.7033	43
44	59.3989	61.4309	63.3566	65.1819	66.9125	68.5536	70.1103	71.5872	72.9885	74.3185	44
45	57.1002	59.2472	61.2820	63.2107	65.0392	66.7733	68.4181	69.9785	71.4592	72.8645	45
46	54.6866	56.9544	59.1036	61.1408	63.0723	64.9039	66.6412	68.2895	69.8534	71.3379	46
47	52.1523	54.5470	56.8164	58.9675	61.0070	62.9410	64.7755	66.5160	68.1674	69.7348	47
48	49.4913	52.0191	54.4147	56.6855	58.8384	60.8800	62.8166	64.6538	66.3971	68.0517	48
49	46.6972	49.3649	51.8930	54.2894	56.5614	58.7159	60.7596	62.6985	64.5382	66.2843	49
50	43.7635	46.5780	49.2453	51.7735	54.1706	56.4437	58.5999	60.6454	62.5864	64.4286	50
51	40.6830	43.6517	46.4651	49.1319	51.6602	54.0578	56.3321	58.4897	60.5370	62.4802	51
52	37.4485	40.5791	43.5459	46.3581	49.0243	51.5526	53.9509	56.2262	58.3851	60.4342	52
53	34.0523	37.3529	40.4807	43.4456	46.2566	48.9222	51.4507	53.8495	56.1257	58.2860	53
54	30.4862	33.9654	37.2623	40.3875	43.3505	46.1603	48.8255	51.3540	53.7533	56.0303	54
55	26.7419	30.4085	33.8830	37.1765	40.2991	43.2602	46.0690	48.7337	51.2622	53.6619	55
56	22.8104	26.6737	30.3347	33.8050	37.0952	40.2152	43.1747	45.9824	48.6466	51.1751	56
57	18.6823	22.7522	26.6090	30.2648	33.7310	37.0179	40.1357	43.0936	45.9003	48.5640	57
58	14.3478	18.6346	22.6970	26 5477	30.1986	33.6608	36.9447	40.0603	43.0166	45.8223	58
59	9.7965	14.3112	18.5894	22.6447	26.4896	30.1358	33.5942	36.8753	39.9887	42.9435	59
60	5.0177	9.7715	14.2765	18.5466	22.5952	26.4345	30.0762	33.5311	36.8094	39.9207	60

5% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, % 61	62	63	64	65	66	67	68	69	70	Age, years
61	0.0000	5.0049	9.7479	14.2436	18.5060	22.5482	26.3822	30.0196	33.4711	36.7468	61
62		0.0000	4.9928	9.7254	14.2124	18.4675	22.5036	26.3326	29.9660	33.4143	62
63			0.0000	4.9813	9.7041	14.1828	18.4310	22.4613	26.2856	29.9151	63
64				0.0000	4.9704	9.6839	14.1548	18.3963	22.4211	26.2409	64
65					0.0000	4.9601	9.6648	14.1282	18.3635	22.3830	65
66						0.0000	4.9502	9.6466	14.1029	18.3323	66
67							0.0000	4.9409	9.6294	14.0790	67
68								0.0000	4.9321	9.6130	68
69									0.0000	4.9237	69
70										0.0000	70

71–80 Years Probable Life

Age, years	71	72	73	74	75	76	77	78	79	80	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.8384	99.8464	99.8539	99.8610	99.8678	99.8743	99.8804	99.8862	99.8918	99.8970	1
2	99.6688	99.6851	99.7005	99.7151	99.7291	99.7423	99.7549	99.7668	99.7781	99.7889	2
3	99.4907	99.5157	99.5394	99.5620	99.5834	99.6037	99.6230	99.6414	99.6588	99.6754	3
4	99.3036	99.3378	99.3703	99.4011	99.4304	99.4582	99.4846	99.5097	99.5335	99.5562	4
5	99.1073	99.1511	99.1927	99.2322	99.2697	99.3054	99.3392	99.3714	99.4020	99.4311	5
6	98.9011	98.9550	99.0062	99.0549	99.1011	99.1449	99.1866	99.2262	99.2639	99.2997	6
7	98.6846	98.7491	98.8104	98.8686	98.9239	98.9765	99.0264	99.0738	99.1189	99.1617	7
8	98.4572	98.5329	98.6048	98.6731	98.7380	98.8006	98.8581	98.9137	98.9666	99.0168	8
9	98.2185	98.3060	98.3890	98.4678	98.5427	98.6139	98.6814	98.7457	98.8067	98.8647	9
10	97.9679	98.0676	98.1623	98.2523	98.3377	98.4188	98.4959	98.5692	98.6388	98.7050	10
11	97.7047	97.8174	97.9243	98.0259	98.1224	98.2141	98.3012	98.3839	98.4625	98.5372	11
12	97.4284	97.5546	97.6745	97.7883	97.8964	97.9991	98.0966	98.1893	98.2774	98.3611	12
13	97.1382	97.2787	97.4121	97.5387	97.6590	97.7733	97.8819	97.9850	98.0831	98.1762	13
14	96.8336	96.9890	97.1366	97.2767	97.4098	97.5363	97.6564	97.7705	97.8790	97.9821	14
15	96.5137	96.6848	96.8473	97.0016	97.1482	97.2874	97.4196	97.5453	97.6647	97.7782	15
16	96.1778	96.3654	96.5436	96.7127	96.8734	97.0260	97.1710	97.3088	97.4397	97.5642	16
17	95.8252	96.0301	96.2246	96.4094	96.5849	96.7516	96.9100	97.0605	97.2035	97.3394	17
18	95.4548	95.6779	95.8898	96.0909	96.2820	96.4635	96.6359	96.7998	96.9555	97.1034	18
19	95.0660	95.3082	95.5382	95.7565	95.9639	96.1610	96.3481	96.5260	96.6950	96.8556	19
20	94.6578	94.9200	95.1690	95.4054	95.6300	95.8433	96.0460	96.2385	96.4215	96.5955	20
21	94.2291	94.5124	94.7813	95.0367	95.2793	95.5098	95.7287	95.9367	96.1344	96.3223	21
22	93.7790	94.0843	94.3743	94.6496	94.9111	95.1595	95.3955	95.6198	95.8329	96.0354	22
23	93.3064	93.6349	93.9469	94.2431	94.5245	94.7918	95.0457	95.2870	95.5163	95.7342	23
24	92.8101	93.1630	93.4981	93.8163	94.1186	94.4057	94.6785	94.9376	95.1839	95.4180	24
25	92.2891	92.6675	93.0269	93.3682	93.6923	94.0003	94.2928	94.5708	94.8349	95.0859	25
26	91.7420	92.1473	92.5322	92.8976	93.2448	93.5746	93.8879	94.1855	94.4684	94.7373	26
27	91.1675	91.6010	92.0127	92.4036	92.7749	93.1276	93.4627	93.7811	94.0836	94.3712	27
28	90.5643	91.0274	91.4672	91.8848	92.2815	92.6583	93.0162	93.3564	93.6796	93.9868	28
29	89.9310	90.4252	90.8945	91.3401	91.7634	92.1655	92.5475	92.9104	93.2553	93.5831	29
30	89.2660	89.7928	90.2931	90.7681	91.2194	91.6480	92.0552	92.4422	92.8099	93.1593	30
31	88.5677	89.1288	89.6616	90.1676	90.6482	91.1047	91.5384	91.9505	92.3421	92.7143	31
32	87.8345	88.4316	88.9986	89.5370	90.0484	90.5343	90.9958	91.4343	91.8510	92.2471	32
33	87.0647	87.6996	88.3024	88.8749	89.4187	89.9353	90.4260	90.8923	91.3353	91.7565	33
34	86.2563	86.9309	87.5715	88.1797	88.7575	89.3063	89.8277	90.3231	90.7939	91.2413	34
35	85.4076	86.1238	86.8039	87.4497	88.0632	88.6459	89.1995	89.7255	90.2254	90.7004	35
36	84.5164	85.2764	85.9980	86.6833	87.3342	87.9525	88.5399	89.0980	89.6284	90.1325	36
37	83.5807	84.3866	85.1518	85.8785	86.5687	87.2244	87.8473	88.4392	89.0016	89.5362	37
38	82.5981	83.4523	84.2633	85.0334	85.7650	86.4599	87.1201	87.7474	88.3435	88.9100	38
39	81.5665	82.4712	83.3304	84.1462	84.9211	85.6572	86.3565	87.0210	87.6524	88.2525	39
40	80.4833	81.4412	82.3508	83.2145	84.0350	84.8144	85.5548	86.2583	86.9268	87.5622	40
41	79.3458	80.3596	81.3222	82.2363	83.1046	83.9294	84.7129	85.4575	86.1649	86.8374	41
42	78.1516	79.2239	80.2422	81.2092	82.1276	83.0002	83.8290	84.6166	85.3649	86.0763	42
43	76.8976	78.0315	79.1082	80.1307	81.1019	82.0245	82.9009	83.7336	84.5250	85.2771	43
44	75.5809	76.7794	77.9176	78.9983	80.0248	81.0000	81.9263	82.8066	83.6430	84.4380	44
45	74.1984	75.4648	76.6673	77.8093	78.8939	79.9242	80.9031	81.8331	82.7169	83.5570	45

5% INTEREST RATE

71–80 Years Probable Life

Age, years	Condition-percent, % 71	72	73	74	75	76	77	78	79	80	Age, years
46	72.7466	74.0844	75.3546	76.5608	77.7064	78.7948	79.8286	80.8111	81.7446	82.6318	46
47	71.2226	72.6350	73.9762	75.2500	76.4596	77.6088	78.7005	79.7378	80.7236	81.6605	47
48	69.6221	71.1131	72.5289	73.8734	75.1504	76.3635	77.5159	78.6110	79.6515	80.6405	48
49	67.9417	69.5151	71.0093	72.4281	73.7757	75.0560	76.2721	77.4278	78.5259	79.5695	49
50	66.1772	67.8372	69.4136	70.9106	72.3324	73.6831	74.9662	76.1854	77.3439	78.4450	50
51	64.3245	66.0755	67.7382	69.3171	70.8168	72.2415	73.5949	74.8809	76.1029	77.2643	51
52	62.3791	64.2256	65.9790	67.6441	69.2255	70.7278	72.1551	73.5112	74.7998	76.0246	52
53	60.3365	62.2833	64.1319	65.8873	67.5546	69.1385	70.6432	72.0730	73.4316	74.7228	53
54	58.1918	60.2438	62.1924	64.0427	65.8002	67.4697	69.0558	70.5629	71.9950	73.3560	54
55	55.9398	58.1023	60.1559	62.1059	63.9580	65.7175	67.3890	68.9773	70.4865	71.9209	55
56	53.5753	55.8538	58.0176	60.0722	62.0238	63.8777	65.6389	67.3124	68.9026	70.4139	56
57	51.0925	53.4929	55.7723	57.9369	59.9928	61.9458	63.8013	65.5643	67.2395	68.8317	57
58	48.4855	51.0139	53.4148	55.6948	57.8603	59.9174	61.8717	63.7287	65.4933	67.1703	58
59	45.7482	48.4110	50.9395	53.3406	55.6211	57.7876	59.8458	61.8014	63.6597	65.4259	59
60	42.8741	45.6779	48.3403	50.8687	53.2700	55.5513	57.7185	59.7777	61.7345	63.5942	60
61	39.8561	42.8082	45.6112	48.2731	50.8014	53.2031	55.4848	57.6529	59.7130	61.6709	61
62	36.6875	39.7949	42.7457	45.5478	48.2093	50.7376	53.1395	55.4217	57.5904	59.6515	62
63	33.3603	36.6311	39.7369	42.6863	45.4876	48.1487	50.6769	53.0791	55.3617	57.5311	63
64	29.8667	33.3091	36.5776	39.6814	42.6299	45.4304	48.0911	50.6193	53.0216	55.3047	64
65	26.1985	29.8208	33.2604	36.5268	39.6291	42.5763	45.3761	48.0365	50.5644	52.9670	65
66	22.3469	26.1583	29.7773	33.2142	36.4785	39.5795	42.5253	45.3246	47.9844	50.5124	66
67	18.3026	22.3125	26.1200	29.7359	33.1703	36.4326	39.5320	42.4770	45.2754	47.9350	67
68	14.0562	18.2745	22.2799	26.0837	29.6966	33.1285	36.3891	39.4871	42.4310	45.2288	68
69	9.5975	14.0346	18.2478	22.2490	26.0493	29.6593	33.0889	36.3477	39.4443	42.3873	69
70	4.9158	9.5827	14.0141	18.2225	22.2196	26.0165	29.6238	33.0513	36.3083	39.4039	70
71	0.0000	4.9082	9.5687	13.9946	18.1984	22.1916	25.9854	29.5901	33.0155	36.2709	71
72		0.0000	4.9011	9.5554	13.9761	18.1755	22.1651	25.9559	29.5581	32.9815	72
73			0.0000	4.8942	9.5428	13.9586	18.1538	22.1399	25.9278	29.5276	73
74				0.0000	4.8878	9.5308	13.9419	18.1331	22.1159	25.9011	74
75					0.0000	4.8816	9.5194	13.9260	18.1135	22.0932	75
76						0.0000	4.8758	9.5086	13.9110	18.0949	76
77							0.0000	4.8702	9.4983	13.8966	77
78								0.0000	4.8650	9.4885	78
79									0.0000	4.8600	79
80										0.0000	80

81–90 Years Probable Life

Age, years	81	82	83	84	85	86	87	88	89	90	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9020	99.9068	99.9113	99.9156	99.9197	99.9236	99.9273	99.9308	99.9341	99.9373	1
2	99.7992	99.8089	99.8182	99.8270	99.8354	99.8433	99.8509	99.8581	99.8649	99.8714	2
3	99.6912	99.7062	99.7204	99.7339	99.7468	99.7590	99.7707	99.7818	99.7923	99.8023	3
4	99.5778	99.5982	99.6177	99.6362	99.6538	99.6706	99.6865	99.7016	99.7160	99.7297	4
5	99.4587	99.4850	99.5099	99.5336	99.5562	99.5777	99.5981	99.6175	99.6359	99.6535	5
6	99.3337	99.3660	99.3967	99.4259	99.4537	99.4801	99.5052	99.5291	99.5518	99.5734	6
7	99.2024	99.2411	99.2779	99.3128	99.3461	99.3777	99.4078	99.4363	99.4635	99.4894	7
8	99.0645	99.1099	99.1531	99.1941	99.2331	99.2701	99.3054	99.3389	99.3708	99.4012	8
9	98.9198	98.9722	99.0220	99.0694	99.1144	99.1572	99.1979	99.2367	99.2735	99.3085	9
10	98.7678	98.8276	98.8844	98.9384	98.9898	99.0386	99.0851	99.1293	99.1713	99.2112	10
11	98.6083	98.6758	98.7399	98.8010	98.8590	98.9141	98.9666	99.0165	99.0639	99.1091	11
12	98.4407	98.5163	98.5882	98.6566	98.7216	98.7834	98.8422	98.8981	98.9513	99.0018	12
13	98.2648	98.3490	98.4290	98.5050	98.5774	98.6461	98.7116	98.7738	98.8329	98.8892	13
14	98.0801	98.1732	98.2617	98.3459	98.4259	98.5020	98.5744	98.6432	98.7087	98.7710	14
15	97.8861	97.9886	98.0861	98.1788	98.2669	98.3507	98.4304	98.5062	98.5782	98.6468	15
16	97.6824	97.7949	97.9017	98.0033	98.0999	98.1918	98.2792	98.3622	98.4413	98.5164	16
17	97.4686	97.5914	97.7081	97.8191	97.9246	98.0249	98.1204	98.2111	98.2974	98.3795	17
18	97.2441	97.3778	97.5048	97.6257	97.7405	97.8498	97.9537	98.0525	98.1464	98.2358	18
19	97.0083	97.1534	97.2914	97.4225	97.5472	97.6658	97.7786	97.8858	97.9878	98.0849	19
20	96.7608	96.9179	97.0672	97.2093	97.3443	97.4727	97.5948	97.7109	97.8214	97.9264	20

5% INTEREST RATE
81–90 Years Probable Life

Age, years	Condition-percent, %										Age, years
	81	82	83	84	85	86	87	88	89	90	
21	96.5008	96.6706	96.8319	96.9853	97.1312	97.2699	97.4018	97.5272	97.6465	97.7600	21
22	96.2279	96.4109	96.5848	96.7502	96.9074	97.0569	97.1991	97.3344	97.4630	97.5853	22
23	95.9413	96.1382	96.3254	96.5033	96.6725	96.8333	96.9863	97.1319	97.2702	97.4019	23
24	95.6405	95.8519	96.0529	96.2441	96.4258	96.5986	96.7629	96.9192	97.0679	97.2092	24
25	95.3245	95.5513	95.7669	95.9719	96.1668	96.3521	96.5283	96.6959	96.8554	97.0070	25
26	94.9928	95.2357	95.4665	95.6861	95.8948	96.0932	96.2820	96.4615	96.6322	96.7946	26
27	94.6444	94.9042	95.1512	95.3860	95.6092	95.8215	96.0234	96.2154	96.3980	96.5716	27
28	94.2787	94.5562	94.8200	95.0709	95.3093	95.5361	95.7518	95.9569	96.1520	96.3375	28
29	93.8947	94.1908	94.4723	94.7400	94.9945	95.2365	95.4666	95.6855	95.8937	96.0917	29
30	93.4914	93.8072	94.1073	94.3926	94.6639	94.9219	95.1672	95.4006	95.6225	95.8336	30
31	93.0681	93.4043	93.7239	94.0279	94.3168	94.5915	94.8529	95.1014	95.3377	95.5625	31
32	92.6235	92.9813	93.3214	93.6449	93.9523	94.2447	94.5228	94.7872	95.0387	95.2779	32
33	92.1567	92.5372	92.8988	93.2427	93.5696	93.8805	94.1762	94.4573	94.7248	94.9791	33
34	91.6666	92.0708	92.4551	92.8204	93.1678	93.4981	93.8122	94.1110	94.3951	94.6654	34
35	91.1520	91.5812	91.9891	92.3771	92.7458	93.0966	93.4301	93.7473	94.0490	94.3359	35
36	90.6116	91.0670	91.4999	91.9115	92.3028	92.6749	93.0289	93.3654	93.6855	93.9900	36
37	90.0442	90.5271	90.9862	91.4227	91.8376	92.2323	92.6076	92.9645	93.3039	93.6268	37
38	89.4484	89.9603	90.4468	90.9094	91.3492	91.7674	92.1652	92.5435	92.9032	93.2454	38
39	88.8229	89.3651	89.8805	90.3705	90.8363	91.2794	91.7007	92.1014	92.4825	92.8450	39
40	88.1661	88.7401	89.2858	89.8046	90.2978	90.7669	91.2130	91.6373	92.0407	92.4245	40
41	87.4764	88.0839	88.6614	89.2105	89.7324	90.2288	90.7009	91.1499	91.5769	91.9830	41
42	86.7523	87.3948	88.0058	88.5866	89.1387	89.6638	90.1632	90.6382	91.0898	91.5195	42
43	85.9919	86.6714	87.3173	87.9315	88.5153	89.0706	89.5987	90.1009	90.5784	91.0327	43
44	85.1936	85.9117	86.5945	87.2437	87.8608	88.4477	89.0059	89.5367	90.0414	90.5217	44
45	84.3553	85.1141	85.8356	86.5215	87.1735	87.7937	88.3834	88.9443	89.4776	89.9850	45
46	83.4751	84.2766	85.0386	85.7631	86.4519	87.1069	87.7298	88.3223	88.8856	89.4216	46
47	82.5509	83.3972	84.2019	84.9669	85.6942	86.3858	87.0436	87.6691	88.2640	88.8300	47
48	81.5805	82.4739	83.3233	84.1308	84.8986	85.6287	86.3230	86.9833	87.6113	88.2087	48
49	80.5615	81.5044	82.4007	83.2530	84.0632	84.8337	85.5664	86.2633	86.9260	87.5564	49
50	79.4916	80.4864	81.4321	82.3312	83.1861	83.9990	84.7720	85.5072	86.2064	86.8715	50
51	78.3682	79.4175	80.4150	81.3634	82.2651	83.1225	83.9379	84.7133	85.4509	86.1524	51
52	77.1886	78.2951	79.3471	80.3472	81.2980	82.2022	83.0620	83.8798	84.6575	85.3973	52
53	75.9501	77.1167	78.2257	79.2801	80.2826	81.2359	82.1424	83.0045	83.8245	84.6044	53
54	74.6496	75.8793	77.0483	78.1597	79.2164	80.2212	81.1768	82.0855	82.9498	83.7719	54
55	73.2842	74.5801	75.8120	76.9833	78.0969	79.1559	80.1629	81.1206	82.0314	82.8978	55
56	71.8504	73.2159	74.5139	75.7480	76.9214	78.0372	79.0983	80.1074	81.0671	81.9800	56
57	70.3450	71.7834	73.1509	74.4510	75.6872	76.8626	77.9805	79.0435	80.0546	81.0163	57
58	68.7643	70.2794	71.7198	73.0892	74.3912	75.6293	76.8067	77.9265	78.9914	80.0044	58
59	67.1045	68.7002	70.2171	71.6592	73.0305	74.3344	75.5743	76.7536	77.8751	78.9419	59
60	65.3618	67.0419	68.6392	70.1578	71.6017	72.9747	74.2803	75.5220	76.7030	77.8263	60
61	63.5319	65.3008	66.9825	68.5813	70.1015	71.5470	72.9216	74.2289	75.4722	76.6549	61
62	61.6105	63.4727	65.2429	66.9260	68.5262	70.0479	71.4949	72.8711	74.1800	75.4249	62
63	59.5931	61.5531	63.4164	65.1879	66.8722	68.4738	69.9969	71.4454	72.8231	74.1334	63
64	57.4748	59.5375	61.4985	63.3628	65.1355	66.8211	68.4241	69.9485	71.3983	72.7774	64
65	55.2505	57.4212	59.4848	61.4466	63.3120	65.0857	66.7725	68.3767	69.9024	71.3536	65
66	52.9151	55.1990	57.3703	59.4345	61.3972	63.2636	65.0384	66.7263	68.3316	69.8585	66
67	50.4629	52.8658	55.1501	57.3219	59.3868	61.3503	63.2176	64.9933	66.6823	68.2888	67
68	47.8881	50.4159	52.8189	55.1035	57.2758	59.3414	61.3057	63.1738	64.9505	66.6405	68
69	45.1845	47.8434	50.3712	52.7743	55.0593	57.2320	59.2983	61.2632	63.1322	64 9098	69
70	42.3458	45.1424	47.8011	50.3286	52.7319	55.0172	57.1904	59.2572	61.2229	63.0926	70
71	39.3651	42.3063	45.1024	47.7607	50.2882	52.6916	54.9772	57.1508	59.2181	61.1845	71
72	36.2354	39.3284	42.2688	45.0643	47.7223	50.2498	52.6533	54.9391	57.1132	59.1810	72
73	32.9492	36.2016	39.2936	42.2331	45.0281	47.6858	50.2133	52.6169	54.9029	57.0773	73
74	29.4987	32.9185	36.1696	39.2604	42.1992	44.9937	47.6512	50.1785	52.5822	54.8685	74
75	25.8757	29.4712	32.8894	36.1390	39.2289	42.1670	44.9610	47.6182	50.1454	52.5492	75
76	22.0715	25.8516	29.4451	32.8616	36.1100	39.1989	42.1363	44.9298	47.5868	50.1140	76
77	18.0771	22.0509	25.8287	29.4202	32.8352	36.0824	39.1704	42.1071	44.9002	47.5569	77
78	13.8830	18.0603	22.0314	25.8068	29.3966	32.8101	36.0562	39.1432	42.0794	44.8721	78
79	9.4792	13.8701	18.0443	22.0128	25.7861	29.3741	32.7862	36.0312	39.1174	42.0530	79
80	4.8552	9.4704	13.8578	18.0290	21.9951	25.7664	29.3528	32.7635	36.0074	39.0929	80

5% INTEREST RATE

81–90 Years Probable Life

Age, years	81	82	83	84	85	86	87	88	89	90	Age, years
	Condition-percent, %										
81	0.0000	4.8507	9.4620	13.8461	18.0145	21.9783	25.7477	29.3324	32.7419	35.9848	81
82		0.0000	4.8464	9.4540	13.8350	18.0008	21.9623	25.7298	29.3131	32.7214	82
83			0.0000	4.8423	9.4464	13.8244	17.9877	21.9471	25.7129	29.2947	83
84				0.0000	4.8384	9.4392	13.8143	17.9752	21.9326	25.6968	84
85					0.0000	4.8347	9.4323	13.8048	17.9634	21.9189	85
86						0.0000	4.8312	9.4258	13.7957	17.9521	86
87							0.0000	4.8278	9.4196	13.7870	87
88								0.0000	4.8247	9.4137	88
89									0.0000	4.8216	89
90										0.0000	90

91–100 Years Probable Life

Age, years	91	92	93	94	95	96	97	98	99	100	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9403	99.9432	99.9459	99.9485	99.9510	99.9534	99.9556	99.9577	99.9598	99.9617	1
2	99.8776	99.8835	99.8891	99.8945	99.8995	99.9044	99.9090	99.9133	99.9175	99.9215	2
3	99.8118	99.8209	99.8295	99.8377	99.8455	99.8529	99.8600	99.8667	99.8731	99.8792	3
4	99.7427	99.7551	99.7669	99.7781	99.7888	99.7989	99.8086	99.8178	99.8265	99.8349	4
5	99.6702	99.6861	99.7012	99.7156	99.7292	99.7422	99.7546	99.7664	99.7776	99.7883	5
6	99.5940	99.6135	99.6322	99.6499	99.6667	99.6827	99.6979	99.7125	99.7263	99.7394	6
7	99.5140	99.5374	99.5597	99.5809	99.6010	99.6202	99.6384	99.6558	99.6723	99.6880	7
8	99.4300	99.4575	99.4836	99.5084	99.5321	99.5546	99.5760	99.5963	99.6157	99.6341	8
9	99.3418	99.3735	99.4037	99.4324	99.4597	99.4856	99.5103	99.5339	99.5562	99.5775	9
10	99.2492	99.2854	99.3198	99.3525	99.3836	99.4133	99.4415	99.4683	99.4938	99.5181	10
11	99.1520	99.1928	99.2317	99.2687	99.3038	99.3373	99.3691	99.3994	99.4282	99.4557	11
12	99.0499	99.0957	99.1392	99.1806	99.2200	99.2575	99.2932	99.3271	99.3594	99.3902	12
13	98.9427	98.9936	99.0421	99.0882	99.1320	99.1737	99.2134	99.2512	99.2871	99.3213	13
14	98.8302	98.8865	98.9401	98.9911	99.0396	99.0858	99.1297	99.1715	99.2113	99.2491	14
15	98.7120	98.7740	98.8330	98.8892	98.9426	98.9934	99.0418	99.0878	99.1316	99.1732	15
16	98.5879	98.6559	98.7206	98.7822	98.8407	98.8964	98.9494	98.9999	99.0479	99.0936	16
17	98.4576	98.5319	98.6026	98.6698	98.7337	98.7946	98.8525	98.9076	98.9601	99.0100	17
18	98.3208	98.4017	98.4786	98.5518	98.6214	98.6877	98.7507	98.8107	98.8678	98.9221	18
19	98.1772	98.2649	98.3485	98.4279	98.5035	98.5754	98.6439	98.7090	98.7710	98.8299	19
20	98.0263	98.1214	98.2118	98.2978	98.3797	98.4575	98.5316	98.6022	98.6693	98.7331	20
21	97.8680	97.9706	98.0683	98.1612	98.2496	98.3338	98.4138	98.4900	98.5625	98.6315	21
22	97.7017	97.8123	97.9176	98.0178	98.1131	98.2038	98.2901	98.3722	98.4504	98.5247	22
23	97.5271	97.6461	97.7594	97.8672	97.9698	98.0674	98.1602	98.2486	98.3326	98.4126	23
24	97.3437	97.4716	97.5933	97.7091	97.8193	97.9241	98.0238	98.1187	98.2091	98.2950	24
25	97.1512	97.2884	97.4189	97.5431	97.6612	97.7736	97.8806	97.9824	98.0792	98.1714	25
26	96.9491	97.0960	97.2358	97.3688	97.4953	97.6157	97.7302	97.8392	97.9429	98.0416	26
27	96.7369	96.8940	97.0435	97.1857	97.3210	97.4498	97.5723	97.6889	97.7998	97.9054	27
28	96.5140	96.6819	96.8416	96.9936	97.1381	97.2756	97.4065	97.5311	97.6496	97.7624	28
29	96.2800	96.4591	96.6296	96.7918	96.9460	97.0928	97.2324	97.3654	97.4918	97.6122	29
30	96.0343	96.2253	96.4070	96.5799	96.7443	96.9008	97.0497	97.1914	97.3262	97.4545	30
31	95.7763	95.9797	96.1733	96.3574	96.5325	96.6992	96.8577	97.0087	97.1522	97.2889	31
32	95.5054	95.7219	95.9278	96.1238	96.3101	96.4875	96.6562	96.8168	96.9696	97.1150	32
33	95.2210	95.4512	95.6702	95.8785	96.0766	96.2652	96.4446	96.6154	96.7778	96.9324	33
34	94.9223	95.1669	95.3996	95.6209	95.8315	96.0318	96.2224	96.4039	96.5765	96.7407	34
35	94.6088	94.8685	95.1155	95.3505	95.5740	95.7868	95.9891	96.1818	96.3651	96.5395	35
36	94.2795	94.5551	94.8172	95.0665	95.3037	95.5294	95.7442	95.9486	96.1431	96.3281	36
37	93.9338	94.2260	94.5040	94.7684	95.0199	95.2593	95.4870	95.7038	95.9100	96.1062	37
38	93.5708	93.8805	94.1751	94.4553	94.7219	94.9756	95.2169	95.4467	95.6652	95.8732	38
39	93.1897	93.5177	93.8297	94.1266	94.4090	94.6777	94.9334	95.1767	95.4082	95.6286	39
40	92.7895	93.1368	93.4671	93.7815	94.0804	94.3649	94.6356	94.8933	95.1384	95.3717	40
41	92.3693	92.7368	93.0864	93.4191	93.7355	94.0365	94.3230	94.5957	94.8551	95.1019	41
42	91.9281	92.3168	92.6867	93.0385	93.3732	93.6917	93.9948	94.2832	94.5576	94.8187	42
43	91.4648	91.8758	92.2669	92.6390	92.9929	93.3296	93.6501	93.9551	94.2452	94.5214	43
44	90.9783	91.4128	91.8262	92.2195	92.5935	92.9495	93.2882	93.6106	93.9172	94.2091	44
45	90.4676	90.9266	91.3634	91.7790	92.1742	92.5504	92.9082	93.2488	93.5729	93.8812	45

5% INTEREST RATE
91–100 Years Probable Life

Age, years	Condition-percent, %										Age, years
	91	92	93	94	95	96	97	98	99	100	
46	89.9313	90.4162	90.8775	91.3164	91.7339	92.1312	92.5092	92.8690	93.2113	93.5370	46
47	89.3681	89.8801	90.3673	90.8308	91.2716	91.6911	92.0903	92.4702	92.8316	93.1755	47
48	88.7769	89.3173	89.8316	90.3208	90.7862	91.2290	91.6504	92.0514	92.4329	92.7960	48
49	88.1560	88.7264	89.2691	89.7853	90.2765	90.7438	91.1885	91.6117	92.0143	92.3975	49
50	87.5041	88.1059	88.6785	89.2231	89.7413	90.2343	90.7035	91.1500	91.5748	91.9790	50
51	86.8196	87.4544	88.0583	88.6328	89.1794	89.6994	90.1943	90.6652	91.1133	91.5397	51
52	86.1009	86.7703	87.4071	88.0129	88.5893	89.1377	89.6596	90.1562	90.6287	91.0783	52
53	85.3463	86.0520	86.7234	87.3621	87.9698	88.5480	89.0982	89.6218	90.1199	90.5940	53
54	84.5539	85.2978	86.0055	86.6787	87.3193	87.9288	88.5087	89.0606	89.5856	90.0853	54
55	83.7219	84.5058	85.2517	85.9612	86.6363	87.2786	87.8898	88.4713	89.0247	89.5513	55
56	82.8483	83.6743	84.4602	85.2078	85.9191	86.5959	87.2398	87.8526	88.4357	88.9905	56
57	81.9311	82.8012	83.6291	84.4167	85.1660	85.8790	86.5574	87.2030	87.8172	88.4018	57
58	80.9679	81.8845	82.7565	83.5861	84.3753	85.1263	85.8409	86.5209	87.1678	87.7835	58
59	79.9566	80.9219	81.8403	82.7139	83.5451	84.3360	85.0885	85.8046	86.4860	87.1344	59
60	78.8948	79.9112	80.8782	81.7981	82.6733	83.5061	84.2985	85.0526	85.7700	86.4529	60
61	77.7798	78.8500	79.8680	80.8365	81.7580	82.6348	83.4691	84.2630	85.0183	85.7372	61
62	76.6091	77.7356	78.8073	79.8269	80.7969	81.7199	82.5981	83.4338	84.2289	84.9857	62
63	75.3799	76.5656	77.6936	78.7667	79.7877	80.7592	81.6836	82.5632	83.4002	84.1967	63
64	74.0892	75.3371	76.5242	77.6536	78.7281	79.7505	80.7234	81.6491	82.5299	83.3682	64
65	72.7340	74.0471	75.2964	76.4848	77.6155	78.6914	79.7151	80.6892	81.6162	82.4983	65
66	71.3110	72.6927	74.0071	75.2576	76.4473	77.5793	78.6565	79.6814	80.6567	81.5849	66
67	69.8168	71.2705	72.6534	73.9690	75.2207	76.4117	77.5449	78.6232	79.6493	80.6259	67
68	68.2480	69.7772	71.2320	72.6159	73.9327	75.1856	76.3777	77.5121	78.5916	79.6189	68
69	66.6007	68.2092	69.7395	71.1953	72.5803	73.8982	75.1522	76.3454	77.4809	78.5615	69
70	64.8710	66.5629	68.1724	69.7035	71.1603	72.5465	73.8654	75.1204	76.3147	77.4512	70
71	63.0549	64.8342	66.5269	68.1372	69.6694	71.1272	72.5143	73.8342	75.0902	76.2855	71
72	61.1480	63.0191	64.7991	66.4926	68.1038	69.6369	71.0956	72.4836	73.8044	75.0614	72
73	59.1457	61.1132	62.9850	64.7658	66.4600	68.0721	69.6060	71.0655	72.4544	73.7762	73
74	57.0433	59.1121	61.0802	62.9526	64.7340	66.4290	68.0419	69.5765	71.0369	72.4267	74
75	54.8357	57.0109	59.0801	61.0487	62.9217	64.7038	66.3995	68.0131	69.5485	71.0097	75
76	52.5178	54.8046	56.9800	59.0497	61.0188	62.8924	64.6751	66.3715	67.9857	69.5219	76
77	50.0840	52.4880	54.7750	56.9507	59.0207	60.9904	62.8645	64.6478	66.3448	67.9597	77
78	47.5286	50.0556	52.4596	54.7468	56.9227	58.9932	60.9633	62.8379	64.6218	66.3193	78
79	44.8453	47.5016	50.0285	52.4326	54.7199	56.8962	58.9670	60.9375	62.8126	64.5970	79
80	42.0279	44.8198	47.4759	50.0028	52.4069	54.6944	56.8710	58.9421	60.9130	62.7885	80
81	39.0696	42.0040	44.7956	47.4514	49.9783	52.3825	54.6701	56.8469	58.9184	60.8896	81
82	35.9634	39.0474	41.9813	44.7725	47.4282	49.9550	52.3592	54.6470	56.8241	58.8958	82
83	32.7018	35.9429	39.0263	41.9597	44.7506	47.4061	49.9328	52.3371	54.6250	56.8023	83
84	29.2772	32.6833	35.9235	39.0062	41.9391	44.7297	47.3850	49.9117	52.3160	54.6041	84
85	25.6814	29.2606	32.6656	35.9050	38.9871	41.9195	44.7098	47.3650	49.8916	52.2960	85
86	21.9058	25.6668	29.2448	32.6488	35.8874	38.9689	41.9009	44.6909	47.3459	49.8725	86
87	17.9414	21.8934	25.6530	29.2297	32.6328	35.8707	38.9516	41.8832	44.6730	47.3278	87
88	13.7788	17.9312	21.8815	25.6397	29.2154	32.6176	35.8547	38.9351	41.8664	44.6558	88
89	9.4080	13.7710	17.9215	21.8703	25.6272	29.2018	32.6031	35.8396	38.9194	41.8503	89
90	4.8188	9.4027	13.7635	17.9123	21.8595	25.6152	29.1888	32.5893	35.8252	38.9045	90
91	0.0000	4.8160	9.3976	13.7564	17.9035	21.8493	25.6039	29.1765	32.5762	35.8114	91
92		0.0000	4.8134	9.3928	13.7497	17.8952	21.8396	25.5930	29.1647	32.5637	92
93			0.0000	4.8109	9.3882	13.7433	17.8872	21.8304	25.5827	29.1536	93
94				0.0000	4.8086	9.3838	13.7372	17.8797	21.8216	25.5729	94
95					0.0000	4.8063	9.3796	13.7314	17.8725	21.8133	95
96						0.0000	4.8042	9.3757	13.7258	17.8656	96
97							0.0000	4.8022	9.3719	13.7206	97
98								0.0000	4.8002	9.3683	98
99									0.0000	4.7984	99
100										0.0000	100

6% INTEREST RATE

1–10 Years Probable Life

Age, years	Condition-percent, %										Age, years
	1	2	3	4	5	6	7	8	9	10	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	0.0000	51.4563	68.5890	77.1409	82.2604	85.6637	88.0865	89.8964	91.2978	92.4132	1
2		0.0000	35.2934	52.9102	63.4563	70.4673	75.4582	79.1866	82.0734	84.3712	2
3			0.0000	27.2256	43.5241	54.3591	62.0722	67.8342	72.2956	75.8467	3
4				0.0000	22.3959	37.2844	47.8830	55.8007	61.9311	66.8107	4
5					0.0000	19.1852	32.9425	43.0451	50.9448	57.2325	5
6						0.0000	16.8995	29.5242	39.2992	47.0797	6
7							0.0000	15.1921	26.9549	36.3177	7
8								0.0000	13.8700	24.9099	8
9									0.0000	12.8177	9
10										0.0000	10

11–20 Years Probable Life

Age, years	11	12	13	14	15	16	17	18	19	20	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	93.3207	94.0723	94.7040	95.2415	95.7037	96.1048	96.4555	96.7643	97.0379	97.2815	1
2	86.2407	87.7889	89.0902	90.1975	91.1497	91.9759	92.6984	93.3346	93.8981	94.4000	2
3	78.7358	81.1286	83.1396	84.8509	86.3224	87.5992	88.7158	89.6990	90.5699	91.3455	3
4	70.7807	74.0686	76.8320	79.1834	81.2054	82.9599	84.4943	85.8453	87.0420	88.1078	4
5	62.3482	66.5850	70.1459	73.1759	75.7815	78.0423	80.0194	81.7603	83.3024	84.6758	5
6	53.4098	58.6524	63.0586	66.8080	70.0321	72.8296	75.2761	77.4303	79.3385	81.0379	6
7	43.9351	50.2438	55.5461	60.0580	63.9378	67.3042	70.2482	72.8404	75.1367	77.1817	7
8	33.8919	41.3307	47.5829	52.9030	57.4777	61.4472	64.9186	67.9752	70.6828	73.0942	8
9	23.2461	31.8829	39.1419	45.3187	50.6301	55.2389	59.2693	62.8181	65.9617	68.7614	9
10	11.9616	21.8682	30.1944	37.2793	43.3717	48.6580	53.2809	57.3515	60.9574	64.1686	10
11	0.0000	11.2526	20.7100	28.7576	35.6777	41.6822	46.9333	51.5569	55.6527	59.3002	11
12		0.0000	10.6566	19.7245	27.5221	34.2880	40.2048	45.4147	50.0298	54.1398	12
13			0.0000	10.1495	18.8771	26.4500	33.0726	38.9039	44.0695	48.6697	13
14				0.0000	9.7135	18.1418	25.5125	32.0025	37.7516	42.8715	14
15					0.0000	9.3351	17.4988	24.6870	31.0546	36.7253	15
16						0.0000	9.0042	16.9326	23.9558	30.2104	16
17							0.0000	8.7129	16.4310	23.3045	17
18								0.0000	8.4548	15.9843	18
19									0.0000	8.2250	19
20										0.0000	20

21–30 Years Probable Life

Age, years	21	22	23	24	25	26	27	28	29	30	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	97.4995	97.6954	97.8722	98.0321	98.1773	98.3096	98.4303	98.5407	98.6420	98.7351	1
2	94.8491	95.2526	95.6166	95.9461	96.2453	96.5177	96.7664	96.9939	97.2026	97.3943	2
3	92.0396	92.6632	93.2258	93.7350	94.1973	94.6183	95.0026	95.3543	95.6768	95.9731	3
4	89.0615	89.9184	90.6915	91.3912	92.0265	92.6050	93.1331	93.6163	94.0594	94.4666	4
5	85.9047	87.0090	88.0051	88.9068	89.7254	90.4709	91.1514	91.7740	92.3450	92.8697	5
6	82.5585	83.9250	85.1576	86.2733	87.2863	88.2087	89.0507	89.8212	90.5278	91.1770	6
7	79.0116	80.6559	82.1392	83.4818	84.7008	85.8108	86.8241	87.7513	88.6015	89.3827	7
8	75.2518	77.1907	78.9397	80.5228	81.9602	83.2690	84.4638	85.5571	86.5596	87.4808	8
9	71.2665	73.5176	75.5482	77.3862	79.0551	80.5747	81.9619	83.2312	84.3952	85.4647	9
10	67.0420	69.6241	71.9533	74.0615	75.9757	77.7187	79.3099	80.7659	82.1010	83.3277	10
11	62.5641	65.4970	68.1426	70.5373	72.7116	74.6914	76.4988	78.1526	79.6691	81.0625	11
12	57.8175	61.1223	64.1033	66.8016	69.2516	71.4825	73.5190	75.3825	77.0913	78.6614	12
13	52.7861	56.4850	59.8217	62.8418	65.5841	68.0810	70.3604	72.4461	74.3588	76.1162	13
14	47.4528	51.5696	55.2831	58.6445	61.6964	64.4754	67.0123	69.3337	71.4624	73.4182	14
15	41.7995	46.3592	50.4723	54.1952	57.5755	60.6535	63.4633	66.0344	68.3921	70.5584	15

6% INTEREST RATE

21–30 Years Probable Life

Age, years	Condition-percent, %										Age, years
	21	22	23	24	25	26	27	28	29	30	
16	35.8070	40.8362	45.3728	49.4790	53.2074	56.6023	59.7014	62.5372	65.1377	67.5271	16
17	29.4550	34.9818	39.9673	44.4799	48.5772	52.3080	55.7138	58.8302	61.6880	64.3138	17
18	22.7218	28.7762	34.2375	39.1808	43.6691	47.7560	51.4869	54.9008	58.0313	60.9077	18
19	15.5847	22.1982	28.1639	33.5637	38.4666	42.9309	47.0064	50.7356	54.1552	57.2973	19
20	8.0193	15.2255	21.7259	27.6096	32.9519	37.8163	42.2570	46.3204	50.0466	53.4702	20
21	0.0000	7.8345	14.9016	21.2983	27.1064	32.3949	37.2227	41.6404	45.6914	49.4136	21
22		0.0000	7.6678	14.6083	20.9101	26.6482	31.8864	36.6796	41.0750	45.1135	22
23			0.0000	7.5169	14.3420	20.5566	26.2299	31.4211	36.1815	40.5554	23
24				0.0000	7.3799	14.0996	20.2339	25.8471	30.9944	35.7238	24
25					0.0000	7.2551	13.8782	19.9387	25.4961	30.6024	25
26						0.0000	7.1412	13.6758	19.6679	25.1736	26
27							0.0000	7.0370	13.4901	19.4192	27
28								0.0000	6.9415	13.3194	28
29									0.0000	6.8537	29
30										0.0000	30

31–40 Years Probable Life

Age, years	31	32	33	34	35	36	37	38	39	40	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.8208	98.8998	98.9727	99.0402	99.1026	99.1605	99.2143	99.2642	99.3106	99.3538	1
2	97.5708	97.7335	97.8838	98.0227	98.1514	98.2707	98.3814	98.4842	98.5799	98.6689	2
3	96.2458	96.4973	96.7295	96.9442	97.1431	97.3274	97.4985	97.6575	97.8053	97.9429	3
4	94.8414	95.1869	95.5060	95.8011	96.0743	96.3276	96.5627	96.7811	96.9842	97.1733	4
5	93.3526	93.7979	94.2091	94.5893	94.9414	95.2678	95.5707	95.8522	96.1139	96.3576	5
6	91.7746	92.3255	92.8343	93.3048	93.7404	94.1443	94.5192	94.8675	95.1914	95.4929	6
7	90.1018	90.7648	91.3771	91.9432	92.4675	92.9535	93.4046	93.8237	94.2135	94.5763	7
8	88.3287	89.1105	89.8324	90.5000	91.1182	91.6912	92.2231	92.7173	93.1769	93.6047	8
9	86.4492	87.3569	88.1951	88.9701	89.6879	90.3532	90.9708	91.5446	92.0782	92.5748	9
10	84.4569	85.4980	86.4595	87.3485	88.1717	88.9349	89.6433	90.3014	90.9135	91.4832	10
11	82.3451	83.5277	84.6197	85.6296	86.5647	87.4316	88.2361	88.9837	89.6789	90.3260	11
12	80.1066	81.4391	82.6696	83.8075	84.8612	85.8380	86.7446	87.5869	88.3703	89.0994	12
13	77.7338	79.2252	80.6025	81.8761	83.0554	84.1488	85.1635	86.1063	86.9831	87.7992	13
14	75.2186	76.8785	78.4114	79.8288	81.1414	82.3582	83.4876	84.5369	85.5127	86.4210	14
15	72.5525	74.3910	76.0888	77.6587	79.1125	80.4602	81.7111	82.8733	83.9541	84.9601	15
16	69.7264	71.7542	73.6268	75.3584	76.9618	78.4483	79.8280	81.1098	82.3019	83.4116	16
17	66.7308	68.9592	71.0171	72.9201	74.6822	76.3158	77.8319	79.2406	80.5507	81.7701	17
18	63.5554	65.9966	68.2508	70.3354	72.2657	74.0552	75.7161	77.2592	78.6944	80.0302	18
19	60.1895	62.8561	65.3186	67.5957	69.7043	71.6591	73.4733	75.1590	76.7266	78.1859	19
20	56.6216	59.5272	62.2104	64.6916	66.9891	69.1191	71.0960	72.9327	74.6409	76.2308	20
21	52.8397	55.9986	58.9158	61.6133	64.1111	66.4268	68.5760	70.5729	72.4299	74.1585	21
22	48.8309	52.2583	55.4234	58.3502	61.0604	63.5729	65.9048	68.0714	70.0864	71.9619	22
23	44.5815	48.2936	51.7215	54.8914	57.8266	60.5478	63.0734	65.4199	67.6022	69.6335	23
24	40.0772	44.0910	47.7975	51.2250	54.3988	57.3412	60.0720	62.6093	64.9689	67.1653	24
25	35.3026	39.6362	43.6381	47.3387	50.7654	53.9422	56.8906	59.6300	62.1777	64.5491	25
26	30.2415	34.9141	39.2291	43.2192	46.9139	50.3392	53.5183	56.4720	59.2190	61.7759	26
27	24.8768	29.9088	34.5555	38.8525	42.8313	46.5201	49.9436	53.1245	56.0827	58.8363	27
28	19.1901	24.6030	29.6016	34.2238	38.5038	42.4718	46.1545	49.5762	52.7583	55.7203	28
29	13.1623	18.9790	24.3504	29.3174	33.9167	38.1806	42.1380	45.8150	49.2344	52.4174	29
30	6.7728	13.0175	18.7841	24.1166	29.0543	33.6320	37.8806	41.8280	45.4991	48.9162	30
31	0.0000	6.6983	12.8838	18.6037	23.9002	28.8104	33.3677	37.6019	41.5397	45.2051	31
32		0.0000	6.6295	12.7601	18.4368	23.6996	28.5840	33.1222	37.3427	41.2712	32
33			0.0000	6.5659	12.6456	18.2821	23.5133	28.3737	32.8939	37.1013	33
34				0.0000	6.5070	12.5395	18.1383	23.3403	28.1781	32.6813	34
35					0.0000	6.4524	12.4409	18.0050	23.1794	27.9960	35
36						0.0000	6.4016	12.3494	17.8808	23.0296	36
37							0.0000	6.3545	12.2643	17.7652	37
38								0.0000	6.3107	12.1850	38
39									0.0000	6.2700	39
40										0.0000	40

6% INTEREST RATE

41–50 Years Probable Life

Age, years	Condition-percent, % 41	42	43	44	45	46	47	48	49	50	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.3941	99.4317	99.4667	99.4994	99.5300	99.5585	99.5852	99.6102	99.6336	99.6556	1
2	98.7519	98.8292	98.9014	98.9688	99.0317	99.0905	99.1456	99.1971	99.2453	99.2905	2
3	98.0711	98.1906	98.3021	98.4063	98.5035	98.5945	98.6795	98.7591	98.8337	98.9035	3
4	97.3495	97.5137	97.6670	97.8100	97.9437	98.0687	98.1855	98.2949	98.3973	98.4933	4
5	96.5846	96.7962	96.9937	97.1780	97.3503	97.5113	97.6619	97.8029	97.9348	98.0584	5
6	95.7738	96.0356	96.2800	96.5081	96.7213	96.9205	97.1069	97.2813	97.4446	97.5975	6
7	94.9143	95.2294	95.5235	95.7980	96.0545	96.2942	96.5185	96.7284	96.9249	97.1089	7
8	94.0033	94.3749	94.7216	95.0453	95.3477	95.6304	95.8948	96.1423	96.3740	96.5910	8
9	93.0376	93.4690	93.8715	94.2474	94.5985	94.9268	95.2338	95.5211	95.7901	96.0421	9
10	92.0140	92.5088	92.9705	93.4016	93.8044	94.1809	94.5330	94.8626	95.1711	95.4602	10
11	90.9289	91.4910	92.0154	92.5051	92.9626	93.3902	93.7902	94.1646	94.5150	94.8433	11
12	89.7787	90.4121	91.0031	91.5548	92.0703	92.5522	93.0029	93.4247	93.8196	94.1895	12
13	88.5596	89.2685	89.9299	90.5475	91.1245	91.6638	92.1683	92.6404	93.0824	93.4964	13
14	87.2673	88.0563	88.7924	89.4797	90.1219	90.7222	91.2836	91.8090	92.3010	92.7618	14
15	85.8974	86.7713	87.5866	88.3479	89.0591	89.7240	90.3459	90.9278	91.4727	91.9831	15
16	84.4454	85.4092	86.3085	87.1482	87.9326	88.6660	89.3519	89.9937	90.5947	91.1576	16
17	82.9062	83.9654	84.9537	85.8765	86.7385	87.5444	88.2982	89.0036	89.6640	90.2827	17
18	81.2747	82.4350	83.5176	84.5284	85.4728	86.3556	87.1813	87.9540	88.6775	89.3552	18
19	79.5453	80.8128	81.9954	83.0995	84.1311	85.0055	85.9974	86.8415	87.6318	88.3721	19
20	77.7121	79.0932	80.3818	81.5849	82.7089	83.7597	84.7425	85.6622	86.5234	87.3300	20
21	75.7690	77.2705	78.6714	79.9794	81.2014	82.3438	83.4123	84.4122	85.3484	86.2253	21
22	73.7092	75.3383	76.8584	78.2776	79.6035	80.8430	82.0023	83.0872	84.1030	85.0544	22
23	71.5259	73.2903	74.9366	76.4736	77.9096	79.2520	80.5076	81.6826	82.7828	83.8133	23
24	69.2116	71.1194	72.8994	74.5614	76.1142	77.5657	78.9233	80.1938	81.3834	82.4976	24
25	66.7584	68.8182	70.7401	72.5345	74.2109	75.7781	77.2440	78.6157	79.9001	81.1031	25
26	64.1580	66.3789	68.4512	70.3860	72.1935	73.8833	75.4638	76.9429	78.3277	79.6248	26
27	61.4016	63.7933	66.0250	68.1085	70.0551	71.8749	73.5769	75.1697	76.6610	78.0579	27
28	58.4798	61.0526	63.4531	65.6944	67.7884	69.7459	71.5767	73.2901	74.8943	76.3969	28
29	55.3827	58.1474	60.7270	63.1355	65.3856	67.4891	69.4566	71.2977	73.0216	74.6363	29
30	52.0998	55.0679	57.8373	60.4230	62.8387	65.0970	67.2092	69.1858	71.0366	72.7701	30
31	48.6199	51.8037	54.7742	57.5478	60.1390	62.5613	64.8270	66.9472	68.9324	70.7918	31
32	44.9312	48.3435	51.5274	54.5000	57.2773	59.8735	62.3018	64.5743	66.7020	68.6949	32
33	41.0212	44.6758	48.0857	51.2695	54.2438	57.0245	59.6252	62.0589	64.3377	66.4722	33
34	36.8766	40.7880	44.4376	47.8450	51.0285	54.0044	56.7879	59.3927	61.8317	64.1161	34
35	32.4833	36.6670	40.5705	44.2151	47.6201	50.8032	53.7804	56.5665	59.1752	61.6186	35
36	27.8264	32.2987	36.4714	40.3674	44.0073	47.4099	50.5925	53.5707	56.3594	58.9713	36
37	22.8901	27.6682	32.1264	36.2889	40.1777	43.8130	47.2133	50.3953	53.3746	56.1652	37
38	17.6576	22.7600	27.5207	31.9656	36.1183	40.0003	43.6313	47.0292	50.2107	53.1907	38
39	12.1112	17.5573	22.6386	27.3829	31.8153	35.9589	39.8344	43.4612	46.8570	50.0377	39
40	6.2320	12.0424	17.4636	22.5253	27.2542	31.6749	35.8097	39.6791	43.3020	46.6955	40
41	0.0000	6.1965	11.9781	17.3762	22.4194	27.1339	31.5435	35.6701	39.5338	43.1528	41
42		0.0000	6.1635	11.9182	17.2945	22.3205	27.0213	31.4206	35.5395	39.3975	42
43			0.0000	6.1326	11.8621	17.2182	22.2279	26.9160	31.3055	35.4170	43
44				0.0000	6.1038	11.8098	17.1468	22.1412	26.8174	31.1976	44
45					0.0000	6.0769	11.7608	17.0799	22.0601	26.7250	45
46						0.0000	6.0517	11.7150	17.0173	21.9841	46
47							0.0000	6.0281	11.6720	16.9587	47
48								0.0000	6.0060	11.6318	48
49									0.0000	5.9853	49
50										0.0000	50

6% INTEREST RATE

51–60 Years Probable Life

Age, years	Condition-percent, % 51	52	53	54	55	56	57	58	59	60	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.6761	99.6954	99.7134	99.7304	99.7463	99.7612	99.7753	99.7884	99.8008	99.8124	1
2	99.3328	99.3725	99.4097	99.4446	99.4774	99.5081	99.5370	99.5642	99.5896	99.6136	2
3	98.9689	99.0302	99.0877	99.1417	99.1923	99.2399	99.2845	99.3264	99.3658	99.4028	3
4	98.5831	98.6674	98.7464	98.8206	98.8902	98.9555	99.0168	99.0744	99.1286	99.1794	4
5	98.1743	98.2828	98.3847	98.4802	98.5699	98.6541	98.7331	98.8073	98.8771	98.9426	5
6	97.7408	97.8752	98.0012	98.1194	98.2304	98.3345	98.4323	98.5242	98.6105	98.6916	6
7	97.2814	97.4431	97.5947	97.7370	97.8705	97.9958	98.1135	98.2241	98.3279	98.4256	7
8	96.7944	96.9851	97.1639	97.3316	97.4891	97.6368	97.7756	97.9060	98.0284	98.1435	8
9	96.2782	96.4995	96.7072	96.9019	97.0847	97.2563	97.4174	97.5687	97.7109	97.8446	9
10	95.7310	95.9849	96.2230	96.4464	96.6561	96.8529	97.0377	97.2113	97.3744	97.5277	10
11	95.1510	95.4394	95.7099	95.9636	96.2018	96.4253	96.6352	96.8324	97.0176	97.1917	11
12	94.5362	94.8611	95.1659	95.4518	95.7202	95.9720	96.2086	96.4308	96.6395	96.8357	12
13	93.8845	94.2482	94.5893	94.9093	95.2097	95.4916	95.7564	96.0050	96.2387	96.4582	13
14	93.1936	93.5985	93.9781	94.3343	94.6686	94.9823	95.2770	95.5538	95.8138	96.0582	14
15	92.4614	92.9097	93.3302	93.7248	94.0950	94.4425	94.7689	95.0754	95.3634	95.6341	15
16	91.6852	92.1797	92.6435	93.0786	93.4870	93.8703	94.2303	94.5684	94.8860	95.1846	16
17	90.8624	91.4059	91.9156	92.3938	92.8425	93.2638	93.6593	94.0309	94.3800	94.7081	17
18	89.9903	90.5856	91.1440	91.6678	92.1594	92.6208	93.0541	93.4612	93.8436	94.2030	18
19	89.0658	89.7161	90.3260	90.8982	91.4352	91.9393	92.4126	92.8573	93.2750	93.6676	19
20	88.0859	88.7945	89.4591	90.0825	90.6676	91.2169	91.7327	92.2171	92.6723	93.1001	20
21	87.0472	87.8176	88.5400	89.2179	89.8540	90.4511	91.0119	91.5386	92.0334	92.4985	21
22	85.9461	86.7820	87.5659	88.3014	88.9915	89.6394	90.2478	90.8193	91.3563	91.8608	22
23	84.7790	85.6843	86.5333	87.3298	88.0773	88.7790	89.4380	90.0569	90.6384	91.1849	23
24	83.5419	84.5207	85.4387	86.3000	87.1083	87.8670	88.5795	89.2487	89.8775	90.4684	24
25	82.2305	83.2873	84.2785	85.2084	86.0811	86.9003	87.6695	88.3921	89.0710	89.7090	25
26	80.8404	81.9800	83.0487	84.0513	84.9923	85.8755	86.7050	87.4840	88.2160	88.9039	26
27	79.3670	80.5941	81.7450	82.8248	83.8381	84.7893	85.6825	86.5215	87.3098	88.0506	27
28	77.8051	79.1252	80.3632	81.5247	82.6147	83.6379	84.5987	85.5012	86.3492	87.1460	28
29	76.1495	77.5681	78.8984	80.1466	81.3179	82.4174	83.4499	84.4197	85.3310	86.1872	29
30	74.3946	75.9175	77.3458	78.6858	79.9433	81.1237	82.2321	83.2733	84.2516	85.1709	30
31	72.5344	74.1680	75.7000	77.1373	78.4862	79.7523	80.9413	82.0582	83.1075	84.0936	31
32	70.5626	72.3134	73.9555	75.4959	76.9416	78.2987	79.5731	80.7701	81.8948	82.9516	32
33	68.4725	70.3476	72.1062	73.7561	75.3044	76.7579	78.1227	79.4047	80.6092	81.7411	33
34	66.2570	68.2639	70.1460	71.9119	73.5690	75.1246	76.5853	77.9574	79.2466	80.4580	34
35	63.9085	66.0551	68.0683	69.9570	71.7294	73.3933	74.9557	76.4233	77.8022	79.0980	35
36	61.4191	63.7138	65.8658	67.8848	69.7795	71.5581	73.2284	74.7971	76.2711	77.6563	36
37	58.7804	61.2320	63.5321	65.6883	67.7126	69.6128	71.3973	73.0734	74.6482	76.1281	37
38	55.9833	58.6013	61.0565	63.3600	65.5216	67.5508	69.4564	71.2463	72.9279	74.5082	38
39	53.0185	55.8127	58.4334	60.8919	63.1992	65.3651	67.3991	69.3095	71.1044	72.7911	39
40	49.8757	52.8569	55.6528	58.2759	60.7375	63.0483	65.2183	67.2565	69.1714	70.9710	40
41	46.5444	49.7237	52.7054	55.5028	58.1281	60.5924	62.9066	65.0803	67.1225	69.0417	41
42	43.0131	46.4025	49.5812	52.5634	55.3621	57.9892	60.4563	62.7735	64.9507	66.9966	42
43	39.2700	42.8820	46.2695	49.4476	52.4301	55.2298	57.8589	60.3284	62.6485	64.8288	43
44	35.3024	39.1503	42.7591	46.1448	49.3221	52.3048	55.1057	57.7365	60.2082	62.5310	44
45	31.0966	35.1947	39.0381	42.6439	46.0278	49.2044	52.1873	54.9891	57.6215	60.0953	45
46	26.6385	31.0018	35.0939	38.9330	42.5358	45.9179	49.0938	52.0769	54.8796	57.5134	46
47	21.9129	26.5573	30.9130	34.9993	38.8342	42.4342	45.8147	48.9899	51.9732	54.7767	47
48	16.9038	21.8462	26.4812	30.8297	34.9105	38.7414	42.3388	45.7177	48.8923	51.8757	48
49	11.5942	16.8523	21.7836	26.4098	30.7515	34.8272	38.6544	42.2492	45.6267	48.8006	49
50	5.9659	11.5588	16.8040	21.7248	26.3428	30.6780	34.7489	38.5726	42.1651	45.5411	50
51	0.0000	5.9478	11.5257	16.7587	21.6697	26.2799	30.6091	34.6754	38.4958	42.0860	51
52		0.0000	5.9307	11.4946	16.7162	21.6180	26.2209	30.5443	34.6063	38.4235	52
53			0.0000	5.9147	11.4655	16.6763	21.5 94	26.1654	30.4835	34.5414	53
54				0.0000	5.8997	11.4381	16.6388	21.5238	26.1133	30.4263	54
55					0.0000	5.8856	11.4124	16.6036	21.4809	26.0643	55
56						0.0000	5.8724	11.3883	16.5705	21.4406	56
57							0.0000	5.8600	11.3656	16.5395	57
58								0.0000	5.8483	11.3442	58
59									0.0000	5.8373	59
60										0.0000	60

6% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, %										Age, years
	61	62	63	64	65	66	67	68	69	70	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.8234	99.8336	99.8433	99.8524	99.8609	99.8690	99.8765	99.8837	99.8904	99.8967	1
2	99.6361	99.6573	99.6772	99.6959	99.7135	99.7301	99.7457	99.7604	99.7742	99.7872	2
3	99.4376	99.4704	99.5011	99.5301	99.5573	99.5829	99.6070	99.6297	99.6510	99.6711	3
4	99.2273	99.2722	99.3145	99.3542	99.3916	99.4268	99.4599	99.4911	99.5204	99.5480	4
5	99.0043	99.0622	99.1166	99.1679	99.2161	99.2614	99.3041	99.3442	99.3820	99.4176	5
6	98.7679	98.8395	98.9069	98.9703	99.0300	99.0861	99.1389	99.1886	99.2353	99.2794	6
7	98.5173	98.6036	98.6847	98.7609	98.8327	98.9002	98.9638	99.0235	99.0798	99.1328	7
8	98.2517	98.3534	98.4490	98.5390	98.6236	98.7032	98.7781	98.8486	98.9150	98.9775	8
9	97.9702	98.0882	98.1993	98.3037	98.4019	98.4944	98.5814	98.6632	98.7403	98.8128	9
10	97.6717	97.8072	97.9345	98.0543	98.1670	98.2730	98.3728	98.4667	98.5551	98.6383	10
11	97.3554	97.5092	97.6539	97.7900	97.9180	98.0384	98.1517	98.2583	98.3587	98.4532	11
12	97.0201	97.1934	97.3564	97.5097	97.6540	97.7897	97.9173	98.0375	98.1506	98.2571	12
13	96.6646	96.8587	97.0411	97.2127	97.3741	97.5260	97.6689	97.8034	97.9300	98.0492	13
14	96.2879	96.5038	96.7069	96.8979	97.0775	97.2466	97.4056	97.5553	97.6962	97.8289	14
15	95.8885	96.1277	96.3526	96.5641	96.7631	96.9503	97.1265	97.2923	97.4484	97.5953	15
16	95.4652	95.7290	95.9770	96.2103	96.4298	96.6363	96.8306	97.0135	97.1856	97.3477	16
17	95.0164	95.3063	95.5789	95.8354	96.0765	96.3035	96.5170	96.7180	96.9072	97.0852	17
18	94.5408	94.8584	95.1570	95.4379	95.7021	95.9507	96.1846	96.4047	96.6120	96.8070	18
19	94.0366	94.3835	94.7097	95.0165	95.3051	95.5767	95.8322	96.0727	96.2991	96.5122	19
20	93.5021	93.8801	94.2356	94.5699	94.8844	95.1803	95.4587	95.7207	95.9674	96.1996	20
21	92.9356	93.3465	93.7330	94.0965	94.4384	94.7601	95.0627	95.3476	95.6158	95.8682	21
22	92.3351	92.7810	93.2003	93.5946	93.9656	94.3147	94.6430	94.9522	95.2431	95.5170	22
23	91.6986	92.1815	92.6356	93.0627	93.4645	93.8425	94.1982	94.5330	94.8481	95.1447	23
24	91.0238	91.5460	92.0370	92.4989	92.9333	93.3420	93.7266	94.0886	94.4293	94.7501	24
25	90.3086	90.8724	91.4025	91.9012	92.3702	92.8115	93.2268	93.6176	93.9855	94.3318	25
26	89.5505	90.1584	90.7300	91.2676	91.7734	92.2492	92.6969	93.1183	93.5150	93.8884	26
27	88.7469	89.4015	90.0171	90.5961	91.1407	91.6531	92.1353	92.5891	93.0163	93.4184	27
28	87.8951	88.5993	89.2614	89.8842	90.4701	91.0213	91.5399	92.0281	92.4876	92.9202	28
29	86.9921	87.7488	88.4603	89.1297	89.7592	90.3516	90.9089	91.4335	91.9272	92.3921	29
30	86.0350	86.8474	87.6113	88.3298	89.0057	89.6416	90.2400	90.8032	91.3332	91.8323	30
31	85.0205	85.8918	86.7112	87.4820	88.2070	88.8891	89.5309	90.1350	90.7036	91.2389	31
32	83.9450	84.8790	85.7573	86.5833	87.3603	88.0914	88.7793	89.4268	90.0362	90.6099	32
33	82.8051	83.8054	84.7459	85.6307	86.4629	87.2459	87.9826	88.6761	89.3288	89.9432	33
34	81.5968	82.6674	83.6740	84.6209	85.5116	86.3496	87.1381	87.8803	88.5789	89.2365	34
35	80.3159	81.4610	82.5377	83.5506	84.5032	85.3995	86.2429	87.0368	87.7840	88.4873	35
36	78.9583	80.1823	81.3333	82.4160	83.4343	84.3925	85.2941	86.1427	86.9414	87.6933	36
37	77.5191	78.8269	80.0566	81.2133	82.3013	83.3250	84.2883	85.1949	86.0482	86.8515	37
38	75.9936	77.3001	78.7033	79.9385	81.1003	82.1935	83.2222	84.1903	85.1015	85.9593	38
39	74.3766	75.8671	77.2688	78.5872	79.8273	80.9941	82.0920	83.1253	84.0980	85.0136	39
40	72.6625	74.2528	75.7483	77.1548	78.4779	79.7227	80.8941	81.9965	83.0342	84.0111	40
41	70.8456	72.5416	74.1365	75.6365	77.0475	78.3750	79.6243	80.8000	81.9066	82.9484	41
42	68.9197	70.7277	72.4279	74.0271	75.5313	76.9465	78.2783	79.5317	80.7114	81.8220	42
43	66.8782	68.8050	70.6169	72.3210	73.9241	75.4323	76.8515	78.1872	79.4445	80.6280	43
44	64.7143	66.7670	68.6972	70.5127	72.2204	73.8272	75.3392	76.7621	78.1015	79.3624	44
45	62.4205	64.6066	66.6623	68.5959	70.4146	72.1258	73.7361	75.2515	76.6780	78.0208	45
46	59.9891	62.3166	64.5054	66.5640	68.5004	70.3223	72.0368	73.6503	75.1690	76.5988	46
47	57.4118	59.8893	62.2190	64.4102	66.4714	68.4107	70.2355	71.9530	73.5696	75.0914	47
48	54.6799	57.3163	59.7954	62.1272	64.3206	66.3843	68.3262	70.1538	71.8741	73.4935	48
49	51.7840	54.5889	57.2265	59.7072	62.0408	64.2363	66.3023	68.2467	70.0769	71.7998	49
50	48.7144	51.6979	54.5033	57.1420	59.6241	61.9595	64.1570	66.2252	68.1719	70.0045	50
51	45.4606	48.6334	51.6168	54.4229	57.0625	59.5460	61.8830	64.0824	66.1526	68.1015	51
52	42.0116	45.3850	48.5572	51.5406	54.3472	56.9878	59.4725	61.8110	64.0121	66.0843	52
53	38.3557	41.9417	45.3139	48.4855	51.4690	54.2760	56.9174	59.4033	61.7433	63.9460	53
54	34.4804	38.2919	41.8760	45.2470	48.4180	51.4015	54.2090	56.8512	59.3382	61.6795	54
55	30.3726	34.4230	38.2319	41.8142	45.1841	48.3546	51.3381	54.1459	56.7889	59.2769	55
56	26.0183	30.3220	34.3691	38.1754	41.7561	45.1249	48.2949	51.2784	54.0866	56.7302	56
57	21.4027	25.9750	30.2745	34.3183	38.1223	41.7013	45.0692	48.2387	51.2222	54.0307	57
58	16.5102	21.3671	25.9343	30.2298	34.2706	38.0724	41.6498	45.0167	48.1859	51.1692	58
59	11.3242	16.4828	21.3336	25.8960	30.1878	34.2257	38.0254	41.6014	44.9674	48.1361	59
60	5.8270	11.3054	16.4569	21.3021	25.8600	30.1482	34.1835	37.9811	41.5558	44.9209	60

6% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, % 61	62	63	64	65	66	67	68	69	70	Age, years
61	0.0000	5.8173	11.2877	16.4326	21.2725	25.8261	30.1110	34.1437	37.9390	41.5123	61
62		0.0000	5.8082	11.2710	16.4098	21.2446	25.7942	30.0760	34.1063	37.9003	62
63			0.0000	5.7996	11.2553	16.3883	21.2184	25.7642	30.0430	34.0710	63
64				0.0000	5.7916	11.2406	16.3681	21.1937	25.7360	30.0120	64
65					0.0000	5.7840	11.2267	16.3490	21.1705	25.7094	65
66						0.0000	5.7768	11.2136	16.3311	21.1486	66
67							0.0000	5.7701	11.2013	16.3142	67
68								0.0000	5.7638	11.1898	68
69									0.0000	5.7578	69
70										0.0000	70

71–80 Years Probable Life

Age, years	71	72	73	74	75	76	77	78	79	80	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9026	99.9082	99.9135	99.9185	99.9231	99.9275	99.9317	99.9356	99.9393	99.9427	1
2	99.7994	99.8109	99.8218	99.8320	99.8417	99.8507	99.8593	99.8673	99.8749	99.8821	2
3	99.6900	99.7078	99.7246	99.7404	99.7553	99.7693	99.7825	99.7950	99.8067	99.8177	3
4	99.5740	99.5985	99.6216	99.6433	99.6637	99.6830	99.7011	99.7182	99.7344	99.7495	4
5	99.4511	99.4827	99.5124	99.5403	99.5667	99.5915	99.6149	99.6369	99.6577	99.6773	5
6	99.3208	99.3598	99.3966	99.4312	99.4638	99.4945	99.5235	99.5507	99.5764	99.6006	6
7	99.1827	99.2297	99.2739	99.3155	99.3548	99.3918	99.4266	99.4594	99.4903	99.5194	7
8	99.0363	99.0917	99.1438	99.1929	99.2392	99.2828	99.3239	99.3625	99.3990	99.4333	8
9	98.8811	98.9454	99.0059	99.0630	99.1167	99.1673	99.2150	99.2599	99.3022	99.3421	9
10	98.7166	98.7903	98.8598	98.9252	98.9868	99.0449	99.0996	99.1511	99.1996	99.2453	10
11	98.5422	98.6260	98.7049	98.7792	98.8492	98.9151	98.9772	99.0357	99.0909	99.1428	11
12	98.3574	98.4518	98.5407	98.6244	98.7033	98.7776	98.8475	98.9135	98.9756	99.0341	12
13	98.1614	98.2671	98.3666	98.4603	98.5486	98.6317	98.7101	98.7839	98.8534	98.9189	13
14	97.9538	98.0714	98.1821	98.2864	98.3846	98.4772	98.5644	98.6465	98.7239	98.7968	14
15	97.7336	97.8639	97.9865	98.1020	98.2108	98.3133	98.4099	98.5009	98.5866	98.6674	15
16	97.5003	97.6439	97.7792	97.9066	98.0266	98.1397	98.2462	98.3465	98.4411	98.5301	16
17	97.2529	97.4108	97.5594	97.6995	97.8314	97.9556	98.0726	98.1829	98.2868	98.3847	17
18	96.9907	97.1637	97.3265	97.4799	97.6244	97.7605	97.8887	98.0095	98.1233	98.2305	18
19	96.7128	96.9017	97.0796	97.2471	97.4050	97.5536	97.6937	97.8256	97.9500	98.0671	19
20	96.4182	96.6240	96.8179	97.0004	97.1724	97.3344	97.4870	97.6308	97.7662	97.8939	20
21	96.1059	96.3297	96.5404	96.7389	96.9259	97.1020	97.2679	97.4242	97.5715	97.7103	21
22	95.7749	96.0177	96.2464	96.4617	96.6646	96.8557	97.0357	97.2052	97.3650	97.5156	22
23	95.4240	95.6870	95.9346	96.1679	96.3876	96.5945	96.7895	96.9731	97.1462	97.3093	23
24	95.0521	95.3365	95.6042	95.8564	96.0940	96.3178	96.5285	96.7271	96.9143	97.0906	24
25	94.6578	94.9649	95.2539	95.5262	95.7827	96.0243	96.2519	96.4663	96.6684	96.8588	25
26	94.2399	94.5710	94.8827	95.1763	95.4528	95.7134	95.9587	96.1899	96.4078	96.6131	26
27	93.7970	94.1535	94.4891	94.8053	95.1031	95.3837	95.6479	95.8969	96.1315	96.3526	27
28	93.3274	93.7109	94.0720	94.4121	94.7324	95.0342	95.3185	95.5863	95.8387	96.0765	28
29	92.8297	93.2418	93.6298	93.9953	94.3395	94.6638	94.9693	95.2571	95.5283	95.7838	29
30	92.3021	92.7445	93.1611	93.5534	93.9230	94.2712	94.5991	94.9081	95.1993	95.4736	30
31	91.7429	92.2174	92.6642	93.0851	93.4815	93.8550	94.2068	94.5382	94.8505	95.1448	31
32	91.1501	91.6587	92.1376	92.5887	93.0136	93.4139	93.7909	94.1461	94.4808	94.7962	32
33	90.5217	91.0664	91.5793	92.0625	92.5175	92.9462	93.3500	93.7304	94.0889	94.4267	33
34	89.8556	90.4386	90.9876	91.5046	91.9917	92.4505	92.8827	93.2898	93.6735	94.0351	34
35	89.1496	89.7731	90.3603	90.9134	91.4343	91.9250	92.3873	92.8228	93.2332	93.6199	35
36	88.4012	89.0677	89.6954	90.2866	90.8435	91.3681	91.8623	92.3278	92.7665	93.1799	36
37	87.6079	88.3200	88.9907	89.6223	90.2173	90.7778	91.3057	91.8031	92.2718	92.7134	37
38	86.7670	87.5275	88.2436	88.9181	89.5535	90.1520	90.7157	91.2468	91.7473	92.2189	38
39	85.8757	86.6873	87.4517	88.1716	88.8498	89.4886	90.0903	90.6572	91.1915	91.6948	39
40	84.9308	85.7968	86.6124	87.3804	88.1039	88.7854	89.4274	90.0322	90.6022	91.1393	40
41	83.9293	84.8528	85.7226	86.5417	87.3133	88.0401	88.7248	89.3698	89.9776	90.5504	41
42	82.8677	83.8522	84.7795	85.6526	86.4752	87.2500	87.9799	88.6676	89.3156	89.9261	42
43	81.7424	82.7916	83.7797	84.7103	85.5869	86.4125	87.1904	87.9233	88.6138	89.2644	43
44	80.5496	81.6673	82.7200	83.7114	84.6452	85.5248	86.3535	87.1343	87.8699	88.5631	44
45	79.2852	80.4756	81.5967	82.6525	83.6470	84.5839	85.4664	86.2979	87.0814	87.8196	45

6% INTEREST RATE

71–80 Years Probable Life

Age, years	Condition-percent, % 71	72	73	74	75	76	77	78	79	80	Age, years
46	77.9449	79.2123	80.4060	81.5302	82.5890	83.5864	84.5261	85.4114	86.2455	87.0315	46
47	76.5242	77.8733	79.1438	80.3404	81.4675	82.5292	83.5293	84.4716	85.3595	86.1961	47
48	75.0183	76.4539	77.8060	79.0793	80.2787	81.4084	82.4728	83.4755	84.4203	85.3107	48
49	73.4220	74.9494	76.3878	77.7425	79.0185	80.2205	81.3528	82.4197	83.4249	84.3720	49
50	71.7300	73.3546	74.8846	76.3255	77.6828	78.9612	80.1657	81.3004	82.3696	83.3771	50
51	69.9364	71.6641	73.2911	74.8235	76.2668	77.6265	78.9073	80.1141	81.2511	82.3225	51
52	68.0352	69.8722	71.6021	73.2314	74.7660	76.2116	77.5735	78.8565	80.0654	81.2046	52
53	66.0199	67.9727	69.8117	71.5437	73.1751	74.7118	76.1595	77.5235	78.8086	80.0196	53
54	63.8837	65.9593	67.9139	69.7548	71.4887	73.1221	74.6608	76.1105	77.4764	78.7635	54
55	61.6194	63.8251	65.9023	67.8586	69.7012	71.4369	73.0721	74.6127	76.0643	77.4321	55
56	59.2192	61.5628	63.7699	65.8485	67.8064	69.6507	71.3881	73.0251	74.5674	76.0207	56
57	56.6750	59.1648	61.5096	63.7179	65.7979	67.7572	69.6031	71.3421	72.9808	74.5247	57
58	53.9781	56.6229	59.1137	61.4594	63.6689	65.7502	67.7110	69.5583	71.2988	72.9390	58
59	51.1194	53.9285	56.5740	59.0655	61.4122	63.6228	65.7053	67.6674	69.5161	71.2580	59
60	48.0892	51.0725	53.8819	56.5278	59.0200	61.3677	63.5793	65.6630	67.6263	69.4763	60
61	44.8772	48.0451	51.0283	53.8380	56.4844	58.9773	61.3258	63.5384	65.6232	67.5876	61
62	41.4724	44.8360	48.0035	50.9867	53.7966	56.4434	58.9370	61.2863	63.4998	65.5856	62
63	37.8634	41.4344	44.7972	47.9644	50.9475	53.7576	56.4049	58.8990	61.2490	63.4635	63
64	34.0378	37.8287	41.3985	44.7607	47.9275	50.9106	53.7209	56.3686	58.8633	61.2140	64
65	29.9827	34.0066	37.7959	41.3648	44.7263	47.8928	50.8759	53.6863	56.3343	58.8296	65
66	25.6843	29.9552	33.9772	37.7651	41.3330	44.6939	47.8600	50.8430	53.6537	56.3021	66
67	21.1280	25.6608	29.9293	33.9495	37.7361	41.3030	44.6634	47.8292	50.8122	53.6229	67
68	16.2983	21.1086	25.6386	29.9049	33.9234	37.7087	41.2749	44.6346	47.8002	50.7831	68
69	11.1789	16.2834	21.0904	25.6177	29.8819	33.8988	37.6830	41.2482	44.6075	47.7728	69
70	5.7522	11.1686	16.2693	21.0732	25.5980	29.8603	33.8757	37.6587	41.2232	44.5819	70
71	0.0000	5.7470	11.1589	16.2560	21.0570	25.5794	29.8399	33.8538	37.6358	41.1996	71
72		0.0000	5.7420	11.1498	16.2435	21.0417	25.5620	29.8207	33.8333	37.6143	72
73			0.0000	5.7373	11.1413	16.2318	21.0274	25.5455	29.8025	33.8139	73
74				0.0000	5.7329	11.1332	16.2207	21.0138	25.5300	29.7855	74
75					0.0000	5.7287	11.1256	16.2102	21.0010	25.5154	75
76						0.0000	5.7248	11.1184	16.2004	20.9890	76
77							0.0000	5.7211	11.1117	16.1911	77
78								0.0000	5.7177	11.1053	78
79									0.0000	5.7144	79
80										0.0000	80

81–90 Years Probable Life

Age, years	81	82	83	84	85	86	87	88	89	90	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9460	99.9491	99.9520	99.9547	99.9573	99.9598	99.9620	99.9642	99.9662	99.9682	1
2	99.8888	99.8951	99.9011	99.9068	99.9121	99.9171	99.9218	99.9263	99.9305	99.9344	2
3	99.8281	99.8379	99.8472	99.8559	99.8641	99.8719	99.8792	99.8860	99.8925	99.8986	3
4	99.7638	99.7773	99.7900	99.8020	99.8133	99.8239	99.8340	99.8434	99.8523	99.8607	4
5	99.6957	99.7131	99.7294	99.7449	99.7594	99.7731	99.7860	99.7982	99.8097	99.8205	5
6	99.6234	99.6449	99.6652	99.6843	99.7023	99.7193	99.7352	99.7503	99.7645	99.7779	6
7	99.5469	99.5727	99.5971	99.6201	99.6417	99.6622	99.6814	99.6995	99.7166	99.7328	7
8	99.4657	99.4962	99.5249	99.5520	99.5776	99.6016	99.6243	99.6457	99.6659	99.6849	8
9	99.3797	99.4151	99.4484	99.4799	99.5095	99.5375	99.5638	99.5887	99.6121	99.6342	9
10	99.2884	99.3291	99.3673	99.4034	99.4374	99.4695	99.4997	99.5282	99.5551	99.5804	10
11	99.1918	99.2379	99.2814	99.3224	99.3610	99.3974	99.4317	99.4641	99.4946	99.5234	11
12	99.0893	99.1413	99.1903	99.2364	99.2800	99.3210	99.3597	99.3961	99.4305	99.4629	12
13	98.9807	99.0388	99.0937	99.1454	99.1941	99.2400	99.2833	99.3241	99.3626	99.3989	13
14	98.8655	98.9303	98.9913	99.0488	99.1031	99.1542	99.2024	99.2478	99.2906	99.3310	14
15	98.7435	98.8152	98.8828	98.9465	99.0066	99.0632	99.1165	99.1668	99.2143	99.2590	15
16	98.6141	98.6932	98.7678	98.8380	98.9043	98.9667	99.0256	99.0811	99.1334	99.1827	16
17	98.4770	98.5639	98.6458	98.7231	98.7958	98.8645	98.9292	98.9901	99.0476	99.1018	17
18	98.3316	98.4268	98.5166	98.6012	98.6809	98.7561	98.8269	98.8937	98.9567	99.0161	18
19	98.1775	98.2815	98.3796	98.4720	98.5591	98.6412	98.7186	98.7916	98.8604	98.9252	19
20	98.0142	98.1275	98.2344	98.3350	98.4300	98.5194	98.6038	98.6833	98.7582	98.8289	20

6% INTEREST RATE

81–90 Years Probable Life

Age, years	Condition-percent, %										Age, years
	81	82	83	84	85	86	87	88	89	90	
21	97.8410	97.9643	98.0804	98.1899	98.2931	98.3903	98.4820	98.5685	98.6500	98.7268	21
22	97.6575	97.7912	97.9173	98.0360	98.1480	98.2535	98.3530	98.4468	98.5352	98.6186	22
23	97.4630	97.6078	97.7443	97.8729	97.9942	98.1085	98.2162	98.3178	98.4136	98.5038	23
24	97.2568	97.4133	97.5610	97.7000	97.8312	97.9547	98.0712	98.1811	98.2846	98.3822	24
25	97.0382	97.2072	97.3666	97.5168	97.6583	97.7918	97.9176	98.0361	98.1479	98.2533	25
26	96.8065	96.9888	97.1606	97.3225	97.4751	97.6190	97.7547	97.8825	98.0031	98.1167	26
27	96.5609	96.7572	96.9423	97.1166	97.2810	97.4359	97.5820	97.7197	97.8495	97.9719	27
28	96.3006	96.5117	96.7108	96.8983	97.0752	97.2418	97.3989	97.5470	97.6867	97.8183	28
29	96.0246	96.2515	96.4654	96.6670	96.8570	97.0361	97.2049	97.3641	97.5141	97.6556	29
30	95.7321	95.9757	96.2054	96.4217	96.6257	96.8180	96.9993	97.1701	97.3312	97.4831	30
31	95.4221	95.6834	95.9297	96.1618	96.3806	96.5868	96.7813	96.9645	97.1373	97.3003	31
32	95.0934	95.3734	95.6375	95.8862	96.1207	96.3418	96.5502	96.7466	96.9318	97.1064	32
33	94.7450	95.0449	95.3277	95.5942	95.8453	96.0820	96.3052	96.5156	96.7140	96.9010	33
34	94.3757	94.6967	94.9994	95.2845	95.5534	95.8067	96.0456	96.2707	96.4831	96.6832	34
35	93.9843	94.3276	94.6514	94.9564	95.2439	95.5149	95.7704	96.0112	96.2383	96.4524	35
36	93.5694	93.9364	94.2824	94.6085	94.9158	95.2055	95.4787	95.7361	95.9788	96.2077	36
37	93.1296	93.5217	93.8914	94.2397	94.5681	94.8776	95.1694	95.4445	95.7038	95.9483	37
38	92.6633	93.0821	93.4769	93.8488	94.1995	94.5300	94.8416	95.1353	95.4123	95.6734	38
39	92.1692	92.6162	93.0375	93.4345	93.8088	94.1615	94.4942	94.8076	95.1033	95.3819	39
40	91.6453	92.1222	92.5717	92.9953	93.3947	93.7710	94.1259	94.4603	94.7757	95.0730	40
41	91.0901	91.5987	92.0780	92.5298	92.9557	93.3571	93.7355	94.0921	94.4285	94.7455	41
42	90.5015	91.0437	91.5547	92.0363	92.4903	92.9182	93.3216	93.7019	94.0604	94.3984	42
43	89.8776	90.4554	91.0000	91.5133	91.9971	92.4531	92.8830	93.2882	93.6703	94.0305	43
44	89.2163	89.8318	90.4120	90.9588	91.4742	91.9600	92.4180	92.8497	93.2568	93.6405	44
45	88.5153	89.1708	89.7887	90.3711	90.9200	91.4374	91.9251	92.3849	92.8184	93.2271	45
46	87.7722	88.4702	89.1281	89.7481	90.3325	90.8834	91.4027	91.8922	92.3538	92.7889	46
47	86.9845	87.7271	88.4277	89.0877	89.7098	90.2961	90.8489	91.3700	91.8612	92.3244	47
48	86.1496	86.9402	87.6854	88.3877	89.0497	89.6737	90.2618	90.8164	91.3391	91.8320	48
49	85.2646	86.1057	86.8985	87.6453	88.3500	89.0138	89.6396	90.2296	90.7857	91.3100	49
50	84.3265	85.2212	86.0645	86.8592	87.6079	88.3144	88.9800	89.6075	90.1991	90.7569	50
51	83.3321	84.2835	85.1803	86.0255	86.8221	87.5727	88.2809	88.9482	89.5773	90.1704	51
52	82.2781	83.2897	84.2431	85.1417	85.9887	86.7872	87.5398	88.2493	88.9182	89.5488	52
53	81.1608	82.2361	83.2497	84.2050	85.1054	85.9541	86.7542	87.5084	88.2195	88.8898	53
54	79.9764	81.1194	82.1967	83.2120	84.1690	85.0711	85.9215	86.7232	87.4789	88.1914	54
55	78.7210	79.9357	81.0805	82.1595	83.1765	84.1351	85.0388	85.8907	86.6939	87.4510	55
56	77.3903	78.6809	79.8973	81.0438	82.1244	83.1430	84.1032	85.0084	85.8617	86.6663	56
57	75.9797	77.3508	78.6431	79.8632	81.0092	82.0913	83.1114	84.0731	84.9797	85.8344	57
58	74.4845	75.9410	77.3159	78.6075	79.8270	80.9766	82.0601	83.0817	84.0447	84.9526	58
59	72.8995	74.4466	75.9045	77.2787	78.5740	79.7949	80.9458	82.0308	83.0536	84.0180	59
60	71.2195	72.8666	74.4108	75.8702	77.2457	78.5479	79.7646	80.9169	82.0031	83.0272	60
61	69.4387	71.1833	72.8295	74.3771	75.8378	77.2147	78.5159	79.7361	80.8895	81.9770	61
62	67.5511	69.4034	71.1491	72.7945	74.3454	75.8073	77.1853	78.4842	79.7092	80.8638	62
63	65.5495	67.5167	69.3700	71.1169	72.7634	74.3155	75.7785	77.1577	78.4579	79.6838	63
64	63.4292	65.5162	67.4843	69.3387	71.0866	72.7342	74.2872	75.7514	77.1317	78.4330	64
65	61.1809	63.3969	65.4847	67.4537	69.3091	71.0580	72.7065	74.2607	75.7258	77.1071	65
66	58.7978	61.1498	63.3664	65.4551	67.4249	69.2812	71.0310	72.6805	74.2356	75.7017	66
67	56.2717	58.7679	61.1204	63.3378	65.4271	67.3978	69.2548	71.0056	72.6536	74.2120	67
68	53.5940	56.2430	58.7397	61.0928	63.3107	65.4008	67.3722	69.2301	70.9816	72.6328	68
69	50.7557	53.5658	56.2160	58.7131	61.0667	63.2853	65.3760	67.3481	69.2067	70.9590	69
70	47.7470	50.7298	53.5401	56.1906	58.6880	61.0421	63.2612	65.3526	67.3254	69.1847	70
71	44.5578	47.7227	50.7055	53.5168	56.1666	58.6644	61.0190	63.2386	65.3305	67.3039	71
72	41.1773	44.5352	47.6998	50.6825	53.4939	56.1440	58.6421	60.9971	63.2172	65.3097	72
73	37.5940	41.1564	44.5138	47.6782	50.6609	53.4724	56.1227	58.6211	60.9765	63.1971	73
74	33.7956	37.5749	41.1366	44.4936	47.6579	50.6405	53.4521	56.1026	58.6014	60.9571	74
75	29.7694	33.7784	37.5568	41.1180	44.4747	47.6387	50.6213	53.4330	56.0837	58.5827	75
76	25.5016	29.7543	33.7622	37.5398	41.1005	44.4568	47.6206	50.6032	53.4149	56.0658	76
77	20.9777	25.4886	29.7400	33.7470	37.5238	41.0839	44.4399	47.6036	50.5861	53.3979	77
78	16.1824	20.9670	25.4764	29.7265	33.7326	37.5087	41.0683	44.4240	47.5875	50.5700	78
79	11.0993	16.1741	20.9570	25.4648	29.7138	33.7190	37.4945	41.0536	44.4090	47.5723	79
80	5.7113	11.0937	16.1664	20.9475	25.4540	29.7019	33.7062	37.4810	41.0398	44.3948	80

6% INTEREST RATE

81–90 Years Probable Life

Age, years	Condition-percent, %										Age, years
	81	82	83	84	85	86	87	88	89	90	
81	0.0000	5.7084	11.0884	16.1591	20.9385	25.4437	29.6906	33.6941	37.4684	41.0267	81
82		0.0000	5.7057	11.0833	16.1522	20.9301	25.4341	29.6800	33.6827	37.4565	82
83			0.0000	5.7031	11.0786	16.1457	20.9222	25.4250	29.6699	33.6720	83
84				0.0000	5.7006	11.0741	16.1395	20.9147	25.4164	29.6605	84
85					0.0000	5.6983	11.0699	16.1338	20.9076	25.4083	85
86						0.0000	5.6962	11.0660	16.1283	20.9009	86
87							0.0000	5.6941	11.0622	16.1232	87
88								0.0000	5.6922	11.0587	88
89									0.0000	5.6904	89
90										0.0000	90

91–100 Years Probable Life

Age, years	91	92	93	94	95	96	97	98	99	100	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9700	99.9717	99.9733	99.9748	99.9762	99.9776	99.9789	99.9801	99.9812	99.9823	1
2	99.9381	99.9417	99.9450	99.9481	99.9511	99.9538	99.9565	99.9589	99.9613	99.9635	2
3	99.9044	99.9098	99.9150	99.9198	99.9244	99.9287	99.9327	99.9365	99.9401	99.9435	3
4	99.8687	99.8761	99.8832	99.8898	99.8961	99.9020	99.9075	99.9128	99.9177	99.9224	4
5	99.8307	99.8404	99.8494	99.8580	99.8661	99.8737	99.8809	99.8876	99.8940	99.9000	5
6	99.7906	99.8025	99.8137	99.8243	99.8343	99.8437	99.8526	99.8609	99.8688	99.8763	6
7	99.7480	99.7623	99.7758	99.7886	99.8006	99.8119	99.8226	99.8327	99.8422	99.8511	7
8	99.7028	99.7197	99.7357	99.7507	99.7649	99.7782	99.7908	99.8027	99.8139	99.8245	8
9	99.6550	99.6746	99.6931	99.7105	99.7270	99.7425	99.7571	99.7709	99.7839	99.7962	9
10	99.6042	99.6267	99.6480	99.6680	99.6869	99.7046	99.7214	99.7372	99.7522	99.7662	10
11	99.5505	99.5760	99.6001	99.6229	99.6443	99.6645	99.6836	99.7015	99.7185	99.7345	11
12	99.4935	99.5223	99.5495	99.5750	99.5992	99.6220	99.6435	99.6637	99.6828	99.7008	12
13	99.4331	99.4653	99.4957	99.5244	99.5514	99.5769	99.6009	99.6236	99.6450	99.6651	13
14	99.3690	99.4049	99.4387	99.4706	99.5007	99.5291	99.5558	99.5811	99.6049	99.6273	14
15	99.3011	99.3409	99.3783	99.4137	99.4470	99.4784	99.5081	99.5360	99.5623	99.5872	15
16	99.2292	99.2730	99.3143	99.3533	99.3901	99.4247	99.4574	99.4882	99.5173	99.5447	16
17	99.1529	99.2011	99.2465	99.2893	99.3297	99.3678	99.4037	99.4376	99.4695	99.4996	17
18	99.0721	99.1248	99.1746	99.2215	99.2657	99.3075	99.3468	99.3839	99.4189	99.4519	18
19	98.9864	99.0440	99.0983	99.1496	99.1979	99.2435	99.2865	99.3270	99.3652	99.4013	19
20	98.8955	98.9583	99.0175	99.0734	99.1260	99.1757	99.2225	99.2667	99.3083	99.3476	20
21	98.7992	98.8675	98.9319	98.9926	99.0499	99.1038	99.1548	99.2027	99.2480	99.2907	21
22	98.6971	98.7712	98.8411	98.9069	98.9691	99.0277	99.0829	99.1350	99.1841	99.2304	22
23	98.5890	98.6692	98.7448	98.8162	98.8835	98.9469	99.0067	99.0631	99.1163	99.1665	23
24	98.4743	98.5610	98.6428	98.7200	98.7927	98.8613	98.9260	98.9870	99.0445	99.0987	24
25	98.3527	98.4463	98.5347	98.6180	98.6965	98.7706	98.8404	98.9063	98.9684	99.0269	25
26	98.2238	98.3248	98.4201	98.5099	98.5946	98.6744	98.7497	98.8207	98.8877	98.9508	26
27	98.0872	98.1960	98.2985	98.3953	98.4865	98.5725	98.6536	98.7300	98.8021	98.8701	27
28	97.9424	98.0594	98.1698	98.2738	98.3719	98.4644	98.5517	98.6339	98.7115	98.7846	28
29	97.7890	97.9147	98.0332	98.1450	98.2505	98.3499	98.4436	98.5320	98.6154	98.6940	29
30	97.6263	97.7612	97.8885	98.0085	98.1217	98.2285	98.3291	98.4240	98.5135	98.5978	30
31	97.4538	97.5986	97.7351	97.8638	97.9853	98.0998	98.2077	98.3095	98.4055	98.4960	31
32	97.2710	97.4262	97.5726	97.7105	97.8406	97.9633	98.0790	98.1881	98.2910	98.3880	32
33	97.0773	97.2434	97.4002	97.5480	97.6873	97.8187	97.9427	98.0595	98.1697	98.2735	33
34	96.8719	97.0497	97.2175	97.3756	97.5248	97.6655	97.7981	97.9231	98.0410	98.1522	34
35	96.6542	96.8444	97.0238	97.1930	97.3525	97.5029	97.6448	97.7785	97.9047	98.0236	35
36	96.4234	96.6267	96.8185	96.9994	97.1699	97.3307	97.4823	97.6253	97.7602	97.8873	36
37	96.1788	96.3960	96.6009	96.7942	96.9764	97.1482	97.3102	97.4629	97.6070	97.7428	37
38	95.9195	96.1515	96.3703	96.5767	96.7712	96.9547	97.1277	97.2907	97.4446	97.5896	38
39	95.6446	95.8922	96.1258	96.3461	96.5537	96.7495	96.9342	97.1083	97.2725	97.4273	39
40	95.3532	95.6175	95.8667	96.1016	96.3232	96.5321	96.7291	96.9148	97.0900	97.2551	40
41	95.0444	95.3263	95.5920	95.8425	96.0788	96.3016	96.5117	96.7098	96.8966	97.0727	41
42	94.7171	95.0174	95.3008	95.5679	95.8198	96.0573	96.2812	96.4924	96.6916	96.8793	42
43	94.3701	94.6902	94.9921	95.2768	95.5452	95.7983	96.0370	96.2620	96.4743	96.6744	43
44	94.0023	94.3433	94.6650	94.9682	95.2541	95.5238	95.7780	96.0178	96.2439	96.4572	44
45	93.6124	93.9756	94.3181	94.6411	94.9456	95.2328	95.5036	95.7590	95.9998	96.2269	45

6% INTEREST RATE
91–100 Years Probable Life

Age, years	Condition-percent, % 91	92	93	94	95	96	97	98	99	100	Age, years
46	93.1991	93.5858	93.9505	94.2944	94.6186	94.9244	95.2127	95.4846	95.7410	95.9827	46
47	92.7610	93.1727	93.5608	93.9269	94.2720	94.5974	94.9043	95.1937	95.4666	95.7240	47
48	92.2967	92.7347	93.1478	93.5373	93.9045	94.2508	94.5774	94.8854	95.1758	95.4497	48
49	91.8044	92.2705	92.7099	93.1243	93.5151	93.8835	94.2309	94.5586	94.8676	95.1589	49
50	91.2826	91.7784	92.2458	92.6866	93.1022	93.4941	93.8636	94.2122	94.5408	94.8507	50
51	90.7296	91.2568	91.7539	92.2226	92.6646	93.0813	93.4743	93.8449	94.1945	94.5240	51
52	90.1434	90.7039	91.2324	91.7308	92.2007	92.6438	93.0617	93.4557	93.8273	94.1777	52
53	89.5219	90.1178	90.6796	91.2094	91.7090	92.1800	92.6242	93.0431	93.4382	93.8107	53
54	88.8632	89.4965	90.0937	90.6568	91.1877	91.6884	92.1605	92.6058	93.0256	93.4216	54
55	88.1649	88.8380	89.4726	90.0710	90.6353	91.1673	91.6690	92.1422	92.5884	93.0091	55
56	87.4248	88.1400	88.8143	89.4501	90.0496	90.6149	91.1480	91.6508	92.1249	92.5719	56
57	86.6403	87.4000	88.1164	88.7919	89.4288	90.0294	90.5958	91.1299	91.6336	92.1085	57
58	85.8086	86.6157	87.3767	88.0942	88.7708	89.4088	90.0104	90.5777	91.1127	91.6173	58
59	84.9271	85.7843	86.5926	87.3547	88.0733	88.7509	89.3899	89.9925	90.5607	91.0966	59
60	83.9927	84.9031	85.7614	86.5708	87.3339	88.0535	88.7321	89.3721	89.9756	90.5446	60
61	83.0023	83.9690	84.8804	85.7398	86.5502	87.3144	88.0349	88.7145	89.3553	89.9596	61
62	81.9524	82.9788	83.9465	84.8590	85.7195	86.5308	87.2959	88.0174	88.6978	89.3394	62
63	80.8395	81.9292	82.9566	83.9254	84.8389	85.7002	86.5125	87.2785	88.0008	88.6821	63
64	79.6599	80.8166	81.9073	82.9357	83.9054	84.8199	85.6821	86.4953	87.2621	87.9852	64
65	78.4094	79.6373	80.7950	81.8867	82.9160	83.8866	84.8019	85.6651	86.4790	87.2466	65
66	77.0840	78.3872	79.6160	80.7747	81.8672	82.8974	83.8689	84.7851	85.6489	86.4637	66
67	75.6790	77.0621	78.3663	79.5960	80.7555	81.8489	82.8799	83.8522	84.7691	85.6338	67
68	74.1897	75.6575	77.0416	78.3465	79.5771	80.7374	81.8316	82.8634	83.8364	84.7541	68
69	72.6110	74.1687	75.6373	77.0221	78.3279	79.5592	80.7203	81.8153	82.8478	83.8216	69
70	70.9377	72.5905	74.1489	75.6183	77.0039	78.3104	79.5424	80.7043	81.7999	82.8331	70
71	69.1639	70.9176	72.5711	74.1302	75.6003	76.9866	78.2938	79.5266	80.6891	81.7854	71
72	67.2837	69.1443	70.8987	72.5528	74.1126	75.5834	76.9703	78.2783	79.5116	80.6748	72
73	65.2907	67.2647	69.1259	70.8808	72.5356	74.0960	75.5674	76.9550	78.2635	79.4975	73
74	63.1781	65.2722	67.2467	69.1085	70.8640	72.5193	74.0803	75.5524	76.9405	78.2496	74
75	60.9388	63.1603	65.2548	67.2298	69.0921	70.8481	72.5040	74.0656	75.5381	76.9269	75
76	58.5651	60.9216	63.1434	65.2384	67.2138	69.0765	70.8331	72.4896	74.0516	75.5247	76
77	56.0490	58.5485	60.9053	63.1275	65.2229	67.1987	69.0619	70.8190	72.4759	74.0385	77
78	53.3819	56.0331	58.5329	60.8899	63.1125	65.2082	67.1845	69.0482	70.8057	72.4631	78
79	50.5548	53.3668	56.0182	58.5181	60.8755	63.0983	65.1945	67.1711	69.0352	70.7931	79
80	47.5581	50.5405	53.3525	56.0040	58.5043	60.8618	63.0850	65.1815	67.1585	69.0230	80
81	44.3815	47.5446	50.5270	53.3391	55.9908	58.4911	60.8490	63.0724	65.1692	67.1466	81
82	41.0144	44.3689	47.5319	50.5143	53.3265	55.9782	58.4788	60.8368	63.0606	65.1577	82
83	37.4452	41.0028	44.3571	47.5199	50.5023	53.3145	55.9664	58.4671	60.8254	63.0494	83
84	33.6619	37.4346	40.9918	44.3459	47.5086	50.4909	53.3032	55.9552	58.4561	60.8146	84
85	29.6516	33.6524	37.4246	40.9815	44.3354	47.4980	50.4803	53.2926	55.9447	58.4458	85
86	25.4007	29.6432	33.6434	37.4152	40.9718	44.3254	47.4879	50.4702	53.2826	55.9348	86
87	20.8947	25.3935	29.6353	33.6349	37.4064	40.9626	44.3161	47.4785	50.4607	53.2731	87
88	16.1183	20.8888	25.3867	29.6278	33.6270	37.3979	40.9539	44.3072	47.4695	50.4518	88
89	11.0554	16.1138	20.8832	25.3803	29.6208	33.6194	37.3900	40.9457	44.2989	47.4611	89
90	5.6887	11.0523	16.1095	20.8779	25.3743	29.6141	33.6123	37.3826	40.9380	44.2911	90
91	0.0000	5.6871	11.0493	16.1054	20.8730	25.3686	29.6079	33.6056	37.3755	40.9308	91
92		0.0000	5.6856	11.0465	16.1016	20.8683	25.3632	29.6020	33.5993	37.3689	92
93			0.0000	5.6841	11.0439	16.0980	20.8639	25.3582	29.5964	33.5933	93
94				0.0000	5.6828	11.0414	16.0946	20.8597	25.3534	29.5912	94
95					0.0000	5.6815	11.0391	16.0914	20.8558	25.3489	95
96						0.0000	5.6803	11.0369	16.0883	20.8521	96
97							0.0000	5.6792	11.0348	16.0855	97
98								0.0000	5.6781	11.0329	98
99									0.0000	5.6771	99
100										0.0000	100

7% INTEREST RATE

1–10 Years Probable Life

Age, years	Condition-percent, % 1	2	3	4	5	6	7	8	9	10	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	0.0000	51.6908	68.8948	77.4772	82.6109	86.0204	88.4447	90.2532	91.6514	92.7623	1
2		0.0000	35.6123	53.3778	64.0046	71.0623	76.0805	79.8242	82.7183	85.0179	2
3			0.0000	27.5914	44.0959	55.0570	62.8508	68.6651	73.1599	76.7314	3
4				0.0000	22.7935	37.9315	48.6950	56.7249	62.9325	67.8648	4
5					0.0000	19.6071	33.5484	43.9488	51.9891	58.3776	5
6						0.0000	17.3414	30.2785	40.2797	48.2263	6
7							0.0000	15.6512	27.7506	37.3644	7
8								0.0000	14.3445	25.7421	8
9									0.0000	13.3063	9
10										0.0000	10

11–20 Years Probable Life

Age, years	11	12	13	14	15	16	17	18	19	20	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	93.6643	94.4098	95.0349	95.5655	96.0205	96.4142	96.7575	97.0587	97.3247	97.5607	1
2	86.8851	88.4283	89.7223	90.8206	91.7625	92.5775	93.2880	93.9116	94.4621	94.9507	2
3	79.6314	82.0281	84.0378	85.7435	87.2064	88.4721	89.5756	90.5441	91.3992	92.1579	3
4	71.8699	75.1798	77.9553	80.3111	82.3314	84.0794	85.6034	86.9410	88.1218	89.1697	4
5	63.5651	67.8522	71.4471	74.4984	77.1152	79.3792	81.3531	83.0856	84.6150	85.9723	5
6	54.6790	60.0117	64.4833	68.2788	71.5338	74.3500	76.8053	78.9603	80.8628	82.5510	6
7	45.1708	51.6223	57.0320	61.6238	65.5617	68.9687	71.9392	74.5463	76.8479	78.8903	7
8	34.9971	42.6457	49.0592	54.5030	59.1715	63.2108	66.7324	69.8233	72.5519	74.9733	8
9	24.1112	33.0407	40.5283	46.8837	52.3340	57.0497	61.1611	64.7696	67.9553	70.7822	9
10	12.4633	22.7633	31.4002	38.7310	45.0180	50.4575	55.1999	59.3622	63.0368	66.2976	10
11	0.0000	11.7665	21.6331	30.0077	37.1898	43.4037	48.8214	53.5763	57.7741	61.4992	11
12		0.0000	11.1823	20.6738	28.8136	35.8562	41.9964	47.3854	52.1430	56.3648	12
13			0.0000	10.6864	19.8511	27.7804	34.6936	40.7611	46.1177	50.8711	13
14				0.0000	10.2612	19.1392	26.8796	33.6732	39.6707	44.9928	14
15					0.0000	9.8932	18.5187	26.0890	32.7723	38.7030	15
16						0.0000	9.5725	17.9740	25.3911	31.9729	16
17							0.0000	9.2909	17.4931	24.7717	17
18								0.0000	9.0423	17.0664	18
19									0.0000	8.8218	19
20										0.0000	20

21–30 Years Probable Life

Age, years	21	22	23	24	25	26	27	28	29	30	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	97.7711	97.9594	98.1286	98.2811	98.4189	98.5439	98.6574	98.7608	98.8551	98.9414	1
2	95.3862	95.7760	96.1262	96.4419	96.7272	96.9859	97.2208	97.4349	97.6301	97.8086	2
3	92.8343	93.4397	93.9837	94.4739	94.9171	95.3188	95.6838	96.0161	96.3194	96.5966	3
4	90.1038	90.9400	91.6911	92.3682	92.9802	93.5350	94.0391	94.4981	94.9169	95.2997	4
5	87.1822	88.2652	89.2381	90.1150	90.9078	91.6263	92.2792	92.8737	93.4162	93.9120	5
6	84.0560	85.4032	86.6134	87.7042	88.6903	89.5841	90.3962	91.1357	91.8104	92.4272	6
7	80.7111	82.3408	83.8049	85.1246	86.3175	87.3989	88.3813	89.2760	90.0923	90.8385	7
8	77.1319	79.0641	80.7999	82.3644	83.7787	85.0607	86.2255	87.2861	88.2539	89.1386	8
9	73.3023	75.5580	77.5845	79.4110	81.0622	82.5588	83.9187	85.1570	86.2868	87.3196	9
10	69.2045	71.8065	74.1440	76.2509	78.1555	79.8818	81.4504	82.8788	84.1820	85.3734	10
11	64.8199	67.7924	70.4627	72.8695	75.0453	77.0175	78.8094	80.4411	81.9299	83.2909	11
12	60.1284	63.4972	66.5237	69.2515	71.7174	73.9526	75.9834	77.8328	79.5201	81.0626	12
13	55.1085	58.9015	62.3089	65.3802	68.1566	70.6731	72.9597	75.0419	76.9417	78.6783	13
14	49.7372	53.9840	57.7992	61.2379	64.3465	67.1642	69.7243	72.0556	74.1827	76.1272	14
15	43.9899	48.7223	52.9737	56.8057	60.2697	63.4096	66.2625	68.8603	71.2307	73.3974	15

7% INTEREST RATE

21–30 Years Probable Life

Age, years	Condition-percent, %										Age, years
	21	22	23	24	25	26	27	28	29	30	
16	37.8403	43.0923	47.8105	52.0632	55.9075	59.3921	62.5582	65.4413	68.0719	70.4766	16
17	31.2602	37.0682	42.2858	46.9887	51.2400	55.0935	58.5948	61.7830	64.6921	67.3513	17
18	24.2196	30.6224	36.3744	41.5590	46.2458	50.4939	54.3538	57.8686	61.0757	64.0073	18
19	16.6860	23.7253	30.0493	35.7492	40.9019	45.5724	49.8160	53.6802	57.2061	60.4291	19
20	8.6251	16.3455	23.2813	29.5328	35.1840	40.3063	44.9606	49.1987	53.0657	56.6005	20
21	0.0000	8.4491	16.0396	22.8812	29.0658	34.6717	39.7652	44.4034	48.6354	52.5039	21
22		0.0000	8.2910	15.7639	22.5194	28.6426	34.2062	39.2724	43.8950	48.1206	22
23			0.0000	8.1485	15.5147	22.1915	28.2581	33.7823	38.8228	43.4304	23
24				0.0000	8.0197	15.2888	21.8936	27.9079	33.3956	38.4118	24
25					0.0000	7.9029	15.0835	21.6222	27.5884	33.0420	25
26						0.0000	7.7968	14.8966	21.3747	27.2963	26
27							0.0000	7.7002	14.7261	21.1484	27
28								0.0000	7.6120	14.5702	28
29									0.0000	7.5314	29
30										0.0000	30

31–40 Years Probable Life

Age, years	31	32	33	34	35	36	37	38	39	40	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.0203	99.0927	99.1592	99.2203	99.2766	99.3285	99.3763	99.4205	99.4613	99.4991	1
2	97.9720	98.1219	98.2595	98.3861	98.5026	98.6099	98.7090	98.8004	98.8849	98.9631	2
3	96.8504	97.0831	97.2969	97.4934	97.6744	97.8411	97.9949	98.1369	98.2682	98.3896	3
4	95.6502	95.9717	96.2669	96.5383	96.7882	97.0184	97.2309	97.4270	97.6083	97.7760	4
5	94.3661	94.7824	95.1647	95.5163	95.8399	96.1382	96.4134	96.6674	96.9022	97.1194	5
6	92.9920	93.5099	93.9855	94.4228	94.8253	95.1963	95.5386	95.8546	96.1467	96.4168	6
7	91.5217	92.1483	92.7236	93.2527	93.7397	94.1886	94.6026	94.9850	95.3383	95.6651	7
8	89.9486	90.6914	91.3735	92.0007	92.5781	93.1102	93.6011	94.0544	94.4733	94.8607	8
9	88.2653	89.1325	89.9288	90.6611	91.3352	91.9564	92.5295	93.0587	93.5477	94.0001	9
10	86.4642	87.4644	88.3830	89.2277	90.0052	90.7218	91.3829	91.9933	92.5574	93.0792	10
11	84.5369	85.6797	86.7290	87.6939	88.5822	89.4008	90.1560	90.8533	91.4978	92.0938	11
12	82.4748	83.7700	84.9593	86.0528	87.0596	87.9874	88.8432	89.6336	90.3639	91.0394	12
13	80.2684	81.7266	83.0656	84.2969	85.4303	86.4749	87.4386	88.3284	89.1507	89.9113	13
14	77.9075	79.5401	81.0394	82.4180	83.6871	84.8566	85.9356	86.9319	87.8526	88.7041	14
15	75.3813	77.2006	78.8713	80.4076	81.8218	83.1251	84.3274	85.4376	86.4636	87.4125	15
16	72.6783	74.6974	76.5515	78.2564	79.8259	81.2723	82.6066	83.8387	84.9774	86.0305	16
17	69.7861	72.0189	74.0693	75.9547	77.6903	79.2898	80.7654	82.1279	83.3871	84.5517	17
18	66.6915	69.1529	71.4134	73.4918	75.4052	77.1686	78.7953	80.2974	81.6855	82.9694	18
19	63.3802	66.0864	68.5715	70.8566	72.9602	74.8989	76.6873	78.3387	79.8648	81.2764	19
20	59.8371	62.8051	65.5307	68.0369	70.3440	72.4703	74.4317	76.2429	77.9167	79.4648	20
21	56.0460	59.2942	62.2770	65.0198	67.5447	69.8716	72.0183	74.0004	75.8322	77.5264	21
22	51.9895	55.5375	58.7957	61.7915	64.5494	67.0911	69.4359	71.6009	73.6018	75.4523	22
23	47.6491	51.5178	55.0705	58.3372	61.3445	64.1160	66.6727	69.0335	71.2152	73.2331	23
24	43.0048	47.2168	51.0846	54.6412	57.9152	60.9326	63.7161	66.2863	68.6616	70.8585	24
25	38.0355	42.6147	46.8198	50.6864	54.2459	57.5263	60.5525	63.3469	65.9293	68.3177	25
26	32.7183	37.6904	42.2563	46.4547	50.3197	53.8816	57.1675	60.2016	63.0056	65.5990	26
27	27.0289	32.4214	37.3735	41.9269	46.1187	49.9818	53.5456	56.8362	59.8774	62.6900	27
28	20.9412	26.7836	32.1488	37.0821	41.6236	45.8090	49.6701	53.2353	56.5301	59.5774	28
29	14.4274	20.7512	26.5584	31.8982	36.8139	41.3441	45.5233	49.3822	52.9485	56.2469	29
30	7.4577	14.2965	20.5767	26.3514	31.6674	36.5667	41.0862	45.2595	49.1162	52.6833	30
31	0.0000	7.3900	14.1763	20.4163	26.1608	31.4548	36.3386	40.8482	45.0157	48.8702	31
32		0.0000	7.3279	14.0658	20.2686	25.9851	31.2586	36.1280	40.6281	44.7902	32
33			0.0000	7.2707	13.9640	20.1325	25.8230	31.0775	35.9334	40.4246	33
34				0.0000	7.2181	13.8703	20.0069	25.6734	30.9101	35.7534	34
35					0.0000	7.1697	13.7837	19.8910	25.5351	30.7552	35
36						0.0000	7.1249	13.7039	19.7839	25.4072	36
37							0.0000	7.0837	13.6301	19.6848	37
38								0.0000	7.0455	13.5618	38
39									0.0000	7.0102	39
40										0.0000	40

7% INTEREST RATE

41–50 Years Probable Life

Age, years	Condition-percent, %										Age, years
	41	42	43	44	45	46	47	48	49	50	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.5340	99.5664	99.5964	99.6242	99.6500	99.6740	99.6963	99.7169	99.7361	99.7540	1
2	99.0355	99.1025	99.1646	99.2222	99.2756	99.3252	99.3712	99.4140	99.4538	99.4908	2
3	98.5020	98.6060	98.7025	98.7919	98.8749	98.9520	99.0235	99.0900	99.1517	99.2092	3
4	97.9312	98.0749	98.2081	98.3316	98.4462	98.5526	98.6514	98.7432	98.8285	98.9078	4
5	97.3204	97.5065	97.6791	97.8391	97.9875	98.1253	98.2532	98.3721	98.4827	98.5854	5
6	96.6668	96.8984	97.1130	97.3120	97.4967	97.6681	97.8272	97.9751	98.1126	98.2404	6
7	95.9676	96.2477	96.5073	96.7481	96.9715	97.1788	97.3714	97.5503	97.7166	97.8712	7
8	95.2193	95.5514	95.8592	96.1447	96.4095	96.6553	96.8836	97.0958	97.2929	97.4762	8
9	94.4187	94.8065	95.1658	95.4990	95.8082	96.0952	96.3618	96.6094	96.8396	97.0536	9
10	93.5621	94.0093	94.4238	94.8082	95.1648	95.4959	95.8033	96.0890	96.3545	96.6014	10
11	92.6454	93.1564	93.6299	94.0690	94.4764	94.8546	95.2058	95.5321	95.8354	96.1175	11
12	91.6647	92.2437	92.7804	93.2781	93.7398	94.1684	94.5665	94.9363	95.2801	95.5997	12
13	90.6152	91.2672	91.8714	92.4318	92.9516	93.4342	93.8824	94.2988	94.6858	95.0457	13
14	89.4923	90.2223	90.8988	91.5262	92.1083	92.6486	93.1504	93.6166	94.0500	94.4529	14
15	88.2908	89.1043	89.8582	90.5573	91.2059	91.8080	92.3672	92.8867	93.3696	93.8186	15
16	87.0052	87.9080	88.7447	89.5205	90.2404	90.9086	91.5292	92.1057	92.6417	93.1400	16
17	85.6296	86.6280	87.5532	88.4112	89.2073	89.9462	90.6325	91.2701	91.8627	92.4138	17
18	84.1577	85.2583	86.2783	87.2242	88.1018	88.9165	89.6730	90.3759	91.0293	91.6367	18
19	82.5828	83.7928	84.9142	85.9541	86.9190	87.8146	88.6464	89.4191	90.1374	90.8053	19
20	80.8976	82.2247	83.4546	84.5952	85.6533	86.6356	87.5479	88.3954	89.1832	89.9157	20
21	79.0945	80.5469	81.8929	83.1411	84.2991	85.3741	86.3725	87.3000	88.1622	88.9638	21
22	77.1652	78.7516	80.2218	81.5852	82.8501	84.0243	85.1148	86.1279	87.0697	87.9453	22
23	75.1008	76.8306	78.4337	79.9204	81.2996	82.5800	83.7691	84.8738	85.9007	86.8555	23
24	72.8919	74.7751	76.5205	78.1390	79.6407	81.0346	82.3292	83.5319	84.6499	85.6894	24
25	70.5284	72.5758	74.4733	76.2330	77.8656	79.3810	80.7885	82.0961	83.3116	84.4417	25
26	67.9994	70.2225	72.2829	74.1935	75.9662	77.6117	79.1399	80.5598	81.8795	83.1066	26
27	65.2934	67.7045	69.9391	72.0113	73.9339	75.7186	77.3760	78.9159	80.3472	81.6781	27
28	62.3979	65.0102	67.4312	69.6763	71.7593	73.6929	75.4886	77.1569	78.7077	80.1496	28
29	59.2998	62.1273	64.7478	67.1779	69.4325	71.5254	73.4690	75.2748	76.9534	78.5141	29
30	55.9848	59.0427	61.8766	64.5046	66.9428	69.2062	71.3081	73.2610	75.0762	76.7641	30
31	52.4378	55.7421	58.8043	61.6441	64.2788	66.7246	68.9959	71.1062	73.0677	74.8916	31
32	48.6425	52.2104	55.5171	58.5834	61.4284	64.0693	66.5219	68.8005	70.9186	72.8880	32
33	44.5815	48.4316	51.9997	55.3085	58.3784	61.2282	63.8747	66.3335	68.6191	70.7442	33
34	40.2362	44.3882	48.2361	51.8043	55.1149	58.1882	61.0422	63.6938	66.1585	68.4503	34
35	35.5868	40.0618	44.2090	48.0549	51.6230	54.9353	58.0114	60.8693	63.5258	65.9958	35
36	30.6119	35.4325	39.9001	44.0429	47.8867	51.4547	54.7684	57.8471	60.7087	63.3695	36
37	25.2888	30.4792	35.2895	39.7502	43.8888	47.7306	51.2985	54.6134	57.6945	60.5594	37
38	19.5930	25.1791	30.3562	35.1569	39.6111	43.7457	47.5856	51.1532	54.4693	57.5526	38
39	13.4986	19.5081	25.0775	30.2421	35.0339	39.4819	43.6129	47.4509	51.0183	54.3353	39
40	6.9775	13.4401	19.4294	24.9833	30.1363	34.9197	39.3620	43.4894	47.3257	50.8928	40
41	0.0000	6.9473	13.3858	19.3563	24.8959	30.0381	34.8136	39.2506	43.3746	47.2093	41
42		0.0000	6.9192	13.3355	19.2886	24.8147	29.9468	34.7150	39.1470	43.2679	42
43			0.0000	6.8932	13.2889	19.2257	24.7393	29.8620	34.6234	39.0507	43
44				0.0000	6.8691	13.2455	19.1673	24.6693	29.7832	34.5383	44
45					0.0000	6.8467	13.2053	19.1131	24.6042	29.7100	45
46						0.0000	6.8259	13.1679	19.0626	24.5437	46
47							0.0000	6.8066	13.1332	19.0158	47
48								0.0000	6.7886	13.1009	48
49									0.0000	6.7719	49
50										0.0000	50

7% INTEREST RATE
51–60 Years Probable Life

Age, years	Condition-percent, %										Age, years
	51	52	53	54	55	56	57	58	59	60	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.7706	99.7861	99.8005	99.8139	99.8264	99.8380	99.8488	99.8589	99.8683	99.8771	1
2	99.5252	99.5572	99.5870	99.6148	99.6406	99.6646	99.6871	99.7079	99.7274	99.7455	2
3	99.2626	99.3123	99.3586	99.4017	99.4418	99.4792	99.5140	99.5464	99.5766	99.6048	3
4	98.9816	99.0503	99.1142	99.1737	99.2291	99.2807	99.3288	99.3736	99.4153	99.4542	4
5	98.6810	98.7699	98.8527	98.9297	99.0015	99.0683	99.1306	99.1886	99.2427	99.2931	5
6	98.3593	98.4699	98.5729	98.6687	98.7580	98.8411	98.9185	98.9907	99.0580	99.1207	6
7	98.0151	98.1489	98.2734	98.3894	98.4974	98.5980	98.6917	98.7790	98.8604	98.9362	7
8	97.6468	97.8054	97.9531	98.0905	98.2186	98.3378	98.4489	98.5524	98.6489	98.7388	8
9	97.2527	97.4379	97.6103	97.7708	97.9202	98.0594	98.1891	98.3100	98.4226	98.5276	9
10	96.8310	97.0446	97.2435	97.4286	97.6010	97.7616	97.9112	98.0506	98.1805	98.3017	10
11	96.3798	96.6239	96.8510	97.0625	97.2594	97.4429	97.6138	97.7730	97.9215	98.0598	11
12	95.8970	96.1736	96.4311	96.6708	96.8940	97.1019	97.2956	97.4761	97.6443	97.8011	12
13	95.3804	95.6919	95.9818	96.2516	96.5029	96.7370	96.9551	97.1583	97.3477	97.5243	13
14	94.8277	95.1764	95.5010	95.8031	96.0845	96.3466	96.5907	96.8183	97.0303	97.2280	14
15	94.2363	94.6249	94.9865	95.3232	95.6368	95.9288	96.2009	96.4545	96.6908	96.9111	15
16	93.6035	94.0347	94.4361	94.8098	95.1577	95.4818	95.7838	96.0652	96.3275	96.5719	16
17	92.9263	93.4032	93.8471	94.2603	94.6451	95.0035	95.3375	95.6487	95.9387	96.2090	17
18	92.2018	92.7276	93.2169	93.6724	94.0966	94.4918	94.8599	95.2030	95.5227	95.8207	18
19	91.4266	92.0046	92.5426	93.0434	93.5098	93.9442	94.3489	94.7261	95.0776	95.4053	19
20	90.5971	91.2310	91.8210	92.3703	92.8818	93.3583	93.8022	94.2158	94.6013	94.9607	20
21	89.7095	90.4033	91.0490	91.6501	92.2099	92.7313	93.2171	93.6698	94.0917	94.4850	21
22	88.7598	89.5176	90.2229	90.8795	91.4910	92.0605	92.5912	93.0856	93.5465	93.9761	22
23	87.7436	88.5700	89.3390	90.0550	90.7217	91.3428	91.9214	92.4605	92.9630	93.4315	23
24	86.6563	87.5560	88.3932	89.1727	89.8986	90.5747	91.2047	91.7917	92.3388	92.8487	24
25	85.4929	86.4710	87.3812	88.2287	89.0179	89.7530	90.4378	91.0760	91.6708	92.2252	25
26	84.2480	85.3100	86.2984	87.2186	88.0755	88.8737	89.6173	90.3102	90.9561	91.5581	26
27	82.9160	84.0678	85.1398	86.1378	87.0672	87.9328	88.7393	89.4909	90.1913	90.8442	27
28	81.4908	82.7387	83.9001	84.9814	85.9882	86.9261	87.7999	88.6141	89.3730	89.9804	28
29	79.9658	81.3165	82.5736	83.7439	84.8338	85.8489	86.7947	87.6760	88.4974	89.2631	29
30	78.3340	79.7947	81.1542	82.4199	83.5985	84.6963	85.7191	86.6722	87.5606	88.3886	30
31	76.5880	78.1664	79.6355	81.0032	82.2768	83.4630	84.5683	85.5982	86.5581	87.4529	31
32	74.7196	76.4240	78.0105	79.4871	80.8625	82.1433	83.3367	84.4488	85.4853	86.4517	32
33	72.7208	74.5600	76.2717	77.8653	79.3493	80.7315	82.0193	83.2193	84.3378	85.3804	33
34	70.5819	72.5653	74.4112	76.1298	77.7301	79.2208	80.6095	81.9036	83.1097	84.2341	34
35	68.2933	70.4309	72.4205	74.2727	75.9976	77.6042	79.1009	80.4957	81.7957	83.0076	35
36	65.8445	68.1472	70.2904	72.2857	74.1438	75.8744	77.4868	78.9894	80.3897	81.6952	36
37	63.2242	65.7036	68.0112	70.1596	72.1602	74.0236	75.7597	77.3775	78.8853	80.2909	37
38	60.4205	63.0889	65.5725	67.8847	70.0378	72.0433	73.9117	75.6528	77.2756	78.7884	38
39	57.4206	60.2913	62.9631	65.4505	67.7668	69.9243	71.9343	73.8075	75.5532	77.1806	39
40	54.2107	57.2978	60.1710	62.8459	65.3368	67.6570	69.8186	71.8329	73.7103	75.4603	40
41	50.7761	54.0947	57.1835	60.0590	62.7368	65.2310	67.5547	69.7201	71.7383	73.6196	41
42	47.1010	50.6674	53.9868	57.0770	59.9547	62.6351	65.1323	67.4594	69.6283	71.6501	42
43	43.1687	47.0002	50.5664	53.8863	56.9779	59.8576	62.5404	65.0405	67.3706	69.5427	43
44	38.9611	43.0764	46.9065	50.4722	53.7927	56.8856	59.7671	62.4522	64.9548	67.2878	44
45	34.4591	38.8778	42.9904	46.8192	50.3846	53.7056	56.7996	59.6828	62.3700	64.8750	45
46	29.6418	34.3853	38.8003	42.9104	46.7379	50.3030	53.6244	56.7195	59.6042	62.2933	46
47	24.4874	29.5784	34.3168	38.7280	42.8359	46.6622	50.2269	53.5487	56.6448	59.5309	47
48	18.9721	24.4350	29.5194	34.2529	38.6608	42.7665	46.5916	50.1561	53.4782	56.5751	48
49	13.0708	18.9315	24.3863	29.4645	34.1934	38.5981	42.7018	46.5259	50.0900	53.4125	49
50	6.7564	13.0428	18.8938	24.3409	29.4133	34.1380	38.5398	42.6416	46.4646	50.0284	50
51	0.0000	6.7420	13.0168	18.8586	24.2986	29.3657	34.0864	38.4854	42.5854	46.4075	51
52		0.0000	6.7285	12.9926	18.8259	24.2592	29.3213	34.0383	38.4347	42.5331	52
53			0.0000	6.7160	12.9701	18.7954	24.2226	29.2799	33.9935	38.3875	53
54				0.0000	6.7043	12.9490	18.7670	24.1884	29.2413	33.9517	54
55					0.0000	6.6935	12.9295	18.7405	24.1565	29.2054	55
56						0.0000	6.6833	12.9112	18.7158	24.1268	56
57							0.0000	6.6739	12.8942	18.6928	57
58								0.0000	6.6651	12.8784	58
59									0.00 0	6.6569	59
60										0.0000	60

7% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, %										Age, years
	61	62	63	64	65	66	67	68	69	70	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.8853	99.8929	99.9000	99.9066	99.9128	99.9186	99.9240	99.9290	99.9337	99.9380	1
2	99.7625	99.7782	99.7930	99.8067	99.8195	99.8314	99.8426	99.8530	99.8627	99.8718	2
3	99.6311	99.6556	99.6785	99.6998	99.7197	99.7382	99.7555	99.7717	99.7868	99.8008	3
4	99.4905	99.5244	99.5560	99.5854	99.6107	99.6363	99.6624	99.6847	99.7055	99.7249	4
5	99.3401	99.3839	99.4248	99.4629	99.4985	99.5317	99.5627	99.5916	99.6185	99.6437	5
6	99.1792	99.2337	99.2845	99.3320	99.3762	99.4175	99.4560	99.4920	99.5255	99.5568	6
7	99.0070	99.0729	99.1344	99.1918	99.2453	99.2953	99.3419	99.3854	99.4260	99.4639	7
8	98.8227	98.9009	98.9738	99.0419	99.1053	99.1645	99.2198	99.2713	99.3195	99.3644	8
9	98.6255	98.7168	98.8020	98.8814	98.9555	99.0246	99.0891	99.1493	99.2055	99.2579	9
10	98.4146	98.5199	98.6181	98.7097	98.7952	98.8749	98.9493	99.0187	99.0835	99.1440	10
11	98.1888	98.3091	98.4213	98.5260	98.6236	98.7147	98.7997	98.8790	98.9531	99.0222	11
12	97.9473	98.0837	98.2108	98.3294	98.4401	98.5433	98.6397	98.7295	98.8134	98.8918	12
13	97.6889	97.8424	97.9856	98.1191	98.2437	98.3599	98.4684	98.5696	98.6641	98.7522	13
14	97.4124	97.5842	97.7445	97.8941	98.0335	98.1637	98.2851	98.3984	98.5042	98.6029	14
15	97.1165	97.3080	97.4866	97.6533	97.8087	97.9537	98.0890	98.2153	98.3332	98.4432	15
16	96.7999	97.0124	97.2107	97.3956	97.5681	97.7290	97.8792	98.0194	98.1502	98.2723	16
17	96.4611	96.6962	96.9154	97.1199	97.3106	97.4887	97.6547	97.8097	97.9543	98.0894	17
18	96.0986	96.3578	96.5995	96.8249	97.0352	97.2314	97.4145	97.5854	97.7448	97.8937	18
19	95.7108	95.9957	96.2614	96.5093	96.7404	96.9562	97.1575	97.3453	97.5206	97.6843	19
20	95.2958	95.6083	95.8997	96.1715	96.4251	96.6617	96.8825	97.0885	97.2807	97.4602	20
21	94.8518	95.1937	95.5126	95.8101	96.0876	96.3466	96.5882	96.8136	97.0240	97.2205	21
22	94.3766	94.7502	95.0985	95.4235	95.7266	96.0094	96.2733	96.5196	96.7494	96.9639	22
23	93.8683	94.2755	94.6554	95.0097	95.3402	95.6486	95.9364	96.2049	96.4555	96.6895	23
24	93.3243	93.7677	94.1812	94.5670	94.9269	95.2626	95.5759	95.8682	96.1411	96.3958	24
25	92.7422	93.2243	93.6739	94.0933	94.4845	94.8495	95.1902	95.5080	95.8046	96.0815	25
26	92.1194	92.6429	93.1310	93.5864	94.0112	94.4076	94.7774	95.1225	95.4446	95.7453	26
27	91.4531	92.0208	92.5502	93.0441	93.5048	93.9347	94.3358	94.7101	95.0594	95.3855	27
28	90.7400	91.3551	91.9287	92.4638	92.9629	93.4287	93.8633	94.2688	94.6473	95.0005	28
29	89.9778	90.6428	91.2637	91.8429	92.3831	92.8873	93.3576	93.7966	94.2062	94.5886	29
30	89.1607	89.8807	90.5521	91.1785	91.7627	92.3080	92.8166	93.2913	93.7343	94.1479	30
31	88.2872	89.0652	89.7908	90.4676	91.0989	91.6881	92.2378	92.7507	93.2294	93.6763	31
32	87.3526	88.1927	88.9761	89.7069	90.3886	91.0248	91.6184	92.1722	92.6891	93.1717	32
33	86.3525	87.2590	88.1044	88.8930	89.6286	90.3151	90.9556	91.5532	92.1110	92.6318	33
34	85.2824	86.2600	87.1717	88.0222	88.8155	89.5558	90.2464	90.8909	91.4925	92.0540	34
35	84.1375	85.1911	86.1737	87.0903	87.9454	88.7432	89.4876	90.1823	90.8306	91.4358	35
36	82.9123	84.0473	85.1058	86.0933	87.0144	87.8738	88.6757	89.4240	90.1224	90.7744	36
37	81.6014	82.8235	83.9632	85.0264	86.0182	86.9435	87.8069	88.6127	89.3647	90.0667	37
38	80.1988	81.5141	82.7406	83.8849	84.9522	85.9481	86.8774	87.7446	88.5539	89.3094	38
39	78.6980	80.1129	81.4324	82.6634	83.8117	84.8831	85.8828	86.8157	87.6864	88.4991	39
40	77.0921	78.6137	80.0327	81.3564	82.5913	83.7434	84.8185	85.8218	86.7581	87.6321	40
41	75.3738	77.0095	78.5350	79.9580	81.2855	82.5240	83.6798	84.7582	85.7648	86.7044	41
42	73.5352	75.2931	76.9324	78.4617	79.8883	81.2193	82.4613	83.6203	84.7021	85.7117	42
43	71.5679	73.4564	75.2177	76.8606	78.3933	79.8232	81.1575	82.4027	83.5649	84.6496	43
44	69.4629	71.4913	73.3829	75.1475	76.7936	78.3294	79.7625	81.0999	82.3481	83.5131	44
45	67.2106	69.3885	71.4197	73.3144	75.0819	76.7310	78.2698	79.7059	81.0461	82.2971	45
46	64.8005	67.1385	69.3191	71.3531	73.2505	75.0208	76.6727	78.2143	79.6530	80.9959	46
47	62.2218	64.7311	67.0714	69.2544	71.2908	73.1908	74.9637	76.6182	78.1624	79.6037	47
48	59.4626	62.1552	64.6664	67.0088	69.1939	71.2327	73.1351	74.9105	76.5674	78.1140	48
49	56.5102	59.3989	62.0930	64.6060	66.9503	69.1376	71.1786	73.0832	74.8608	76.5200	49
50	53.3512	56.4497	59.3395	62.0350	64.5496	66.8958	69.0850	71.1280	73.0347	74.8144	50
51	49.9710	53.2940	56.3932	59.2841	61.9809	64.4971	66.8449	69.0360	71.0808	72.9895	51
52	46.3543	49.9175	53.2407	56.3406	59.2324	61.9304	64.4480	66.7975	68.9902	71.0368	52
53	42.4843	46.3046	49.8676	53.1910	56.2914	59.1841	61.8833	64.4023	66.7531	68.9474	53
54	38.3435	42.4388	46.2583	49.8210	53.1446	56.2456	59.1391	61.8394	64.3595	66.7118	54
55	33.9128	38.3024	42.3963	46.2151	49.7775	53.1013	56.2028	59.0971	61.7984	64.3197	55
56	29.1719	33.8764	38.2641	42.3567	46.1748	49.7370	53.0610	56.1629	59.0579	61.7601	56
57	24.0992	29.1406	33.8425	38.2283	42.3198	46.1372	49.6992	53.0233	56.1257	59.0213	57
58	18.6713	24.0733	29.1115	33.8109	38.1950	42.2853	46.1021	49.6639	52.9881	56.0909	58
59	12.8636	18.6513	24.0493	29.0843	33.7814	38.1639	42.2532	46.0693	49.6310	52.9553	59
60	6.6493	12.8498	18.6327	24.0268	29.0589	33.7539	38.1349	42.2232	46.0388	49.6002	60

7% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, %										Age, years
	61	62	63	64	65	66	67	68	69	70	
61	0.0000	6.6422	12.8370	18.6153	24.0059	29.0353	33.7283	38.1078	42.1952	46.0103	61
62		0.0000	6.6355	12.8250	18.5991	23.9863	29.0132	33.7043	38.0825	42.1690	62
63			0.0000	6.6293	12.8138	18.5839	23.9681	28.9926	33.6819	38.0589	63
64				0.0000	6.6236	12.8034	18.5698	23.9510	28.9734	33.6611	64
65					0.0000	6.6182	12.7936	18.5566	23.9352	28.9554	65
66						0.0000	6.6131	12.7845	18.5443	23.9203	66
67							0.0000	6.6084	12.7761	18.5328	67
68								0.0000	6.6040	12.7681	68
69									0.0000	6.6000	69
70										0.0000	70

71–80 Years Probable Life

Age, years	71	72	73	74	75	76	77	78	79	80	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9421	99.9459	99.9495	99.9528	99.9559	99.9588	99.9615	99.9641	99.9664	99.9686	1
2	99.8802	99.8881	99.8955	99.9024	99.9088	99.9148	99.9204	99.9256	99.9305	99.9351	2
3	99.8140	99.8262	99.8377	99.8484	99.8584	99.8677	99.8764	99.8845	99.8921	99.8992	3
4	99.7431	99.7600	99.7758	99.7906	99.8044	99.8173	99.8293	99.8405	99.8510	99.8608	4
5	99.6672	99.6892	99.7096	99.7288	99.7466	99.7633	99.7789	99.7934	99.8070	99.8197	5
6	99.5861	99.6134	99.6388	99.6626	99.6848	99.7056	99.7249	99.7430	99.7599	99.7757	6
7	99.4992	99.5322	99.5631	99.5918	99.6187	99.6438	99.6672	99.6891	99.7095	99.7286	7
8	99.4063	99.4454	99.4820	99.5161	99.5480	99.5777	99.6055	99.6314	99.6557	99.6783	8
9	99.3069	99.3526	99.3952	99.4351	99.4722	99.5070	99.5394	99.5697	99.5980	99.6244	9
10	99.2005	99.2532	99.3024	99.3484	99.3912	99.4313	99.4687	99.5037	99.5363	99.5668	10
11	99.0867	99.1469	99.2031	99.2556	99.3046	99.3503	99.3931	99.4330	99.4703	99.5051	11
12	98.9649	99.0331	99.0968	99.1563	99.2118	99.2637	99.3121	99.3574	99.3996	99.4391	12
13	98.8345	98.9114	98.9831	99.0501	99.1126	99.1710	99.2255	99.2765	99.3240	99.3684	13
14	98.6951	98.7811	98.8614	98.9364	99.0064	99.0718	99.1329	99.1899	99.2431	99.2929	14
15	98.5459	98.6418	98.7312	98.8148	98.8928	98.9657	99.0337	99.0973	99.1566	99.2120	15
16	98.3862	98.4926	98.5919	98.6847	98.7713	98.8521	98.9276	98.9981	99.0640	99.1255	16
17	98.2154	98.3331	98.4429	98.5455	98.6412	98.7306	98.8141	98.8921	98.9649	99.0329	17
18	98.0326	98.1623	98.2834	98.3965	98.5020	98.6006	98.6926	98.7786	98.8589	98.9339	18
19	97.8370	97.9796	98.1127	98.2371	98.3531	98.4615	98.5627	98.6572	98.7454	98.8279	19
20	97.6277	97.7842	97.9301	98.0665	98.1937	98.3126	98.4236	98.5273	98.6241	98.7145	20
21	97.4038	97.5750	97.7348	97.8840	98.0232	98.1533	98.2748	98.3883	98.4942	98.5932	21
22	97.1642	97.3512	97.5257	97.6887	97.8408	97.9829	98.1156	98.2395	98.3552	98.4633	22
23	96.9079	97.1117	97.3020	97.4797	97.6496	97.8006	97.9452	98.0804	98.2065	98.3244	23
24	96.6335	96.8555	97.0627	97.2562	97.4368	97.6054	97.7629	97.9101	98.0474	98.1758	24
25	96.3400	96.5813	96.8066	97.0169	97.2132	97.3967	97.5679	97.7278	97.8772	98.0167	25
26	96.0260	96.2880	96.5325	96.7069	96.9741	97.1733	97.3592	97.5329	97.6950	97.8465	26
27	95.6899	95.9741	96.2393	96.4870	96.7182	96.9342	97.1359	97.3242	97.5001	97.6644	27
28	95.3303	95.6382	95.9256	96.1940	96.4445	96.6785	96.8969	97.1010	97.2916	97.4696	28
29	94.9456	95.2788	95.5899	95.8804	96.1515	96.4048	96.6413	96.8622	97.0684	97.2611	29
30	94.5339	94.8943	95.2307	95.5449	95.8381	96.1120	96.3677	96.6066	96.8296	97.0380	30
31	94.0934	94.4828	94.8463	95.1858	95.5027	95.7987	96.0750	96.3331	96.5741	96.7993	31
32	93.6221	94.0426	94.4351	94.8017	95.1438	95.4634	95.7618	96.0405	96.3007	96.5439	32
33	93.1178	93.5715	93.9951	94.3906	94.7599	95.1047	95.4267	95.7274	96.0082	96.2706	33
34	92.5782	93.0674	93.5242	93.9508	94.3490	94.7209	95.0681	95.3924	95.6953	95.9782	34
35	92.0008	92.5281	93.0204	93.4802	93.9094	94.3102	94.6844	95.0340	95.3604	95.6653	35
36	91.3830	91.9510	92.4814	92.9766	93.4390	93.8707	94.2739	94.6504	95.0020	95.3305	36
37	90.7219	91.3335	91.9046	92.4378	92.9356	93.4005	93.8345	94.2400	94.6186	94.9723	37
38	90.0146	90.6728	91.2874	91.8612	92.3971	92.8974	93.3645	93.8009	94.2083	94.5890	38
39	89.2577	89.9659	90.6270	91.2444	91.8208	92.3590	92.8616	93.3311	93.7694	94.1788	39
40	88.4479	89.2094	89.9204	90.5843	91.2042	91.7830	92.3235	92.8283	93.2997	93.7400	40
41	87.5814	88.4001	89.1644	89.8780	90.5444	91.1666	91.7477	92.2904	92.7971	93.2705	41
42	86.6542	87.5340	88.3554	89.1223	89.8385	90.5071	91.1316	91.7148	92.2594	92.7681	42
43	85.6622	86.6073	87.4898	88.3137	89.0831	89.8015	90.4723	91.0989	91.6840	92.2305	43
44	84.6007	85.6158	86.5636	87.4485	88.2749	89.0464	89.7670	90.4399	91.0683	91.6552	44
45	83.4648	84.5549	85.5725	86.5228	87.4100	88.2385	89.0122	89.7347	90.4095	91.0398	45

7% INTEREST RATE

71–80 Years Probable Life

Age, years	Condition-percent, % 71	72	73	74	75	76	77	78	79	80	Age, years
46	82.2494	83.4196	84.5121	85.5322	86.4847	87.3740	88.2046	88.9802	89.7046	90.3812	46
47	80.9490	82.2050	83.3775	84.4723	85.4945	86.4491	87.3405	88.1729	88.9503	89.6765	47
48	79.5576	80.9053	82.1635	83.3382	84.4351	85.4593	86.4158	87.3091	88.1433	88.9225	48
49	78.0688	79.5146	80.8644	82.1247	83.3015	84.4003	85.4265	86.3848	87.2798	88.1157	49
50	76.4757	78.0266	79.4744	80.8263	82.0885	83.2672	84.3679	85.3959	86.3558	87.2524	50
51	74.7711	76.4344	77.9872	79.4370	80.7907	82.0547	83.2352	84.3376	85.3671	86.3287	51
52	72.9472	74.7307	76.3958	77.9504	79.4020	80.7574	82.0232	83.2053	84.3093	85.3404	52
53	70.9957	72.9078	74.6930	76.3597	77.9160	79.3693	80.7264	81.9937	83.1774	84.2828	53
54	68.9075	70.9573	72.8710	74.6578	76.3261	77.8840	79.3388	80.6974	81.9662	83.1513	54
55	66.6732	68.8703	70.9215	72.8366	74.6249	76.2947	77.8540	79.3103	80.6703	81.9405	55
56	64.2824	66.6371	68.8355	70.8880	72.8046	74.5941	76.2653	77.8260	79.2836	80.6450	56
57	61.7243	64.2477	66.6035	68.8030	70.8568	72.7746	74.5655	76.2379	77.7999	79.2588	57
58	58.9872	61.6910	64.2153	66.5721	68.7727	70.8276	72.7466	74.5387	76.2124	77.7755	58
59	56.0584	58.9553	61.6598	64.1850	66.5428	68.7444	70.8004	72.7205	74.5137	76.1885	59
60	52.9246	56.0281	58.9255	61.6307	64.1567	66.5154	68.7180	70.7750	72.6961	74.4903	60
61	49.5715	52.8960	55.9998	57.8977	60.6036	63.1303	66.4898	68.6933	70.7512	72.6733	61
62	45.9836	49.5447	52.8693	55.9734	58.8718	61.5782	64.1056	66.4659	68.6702	70.7290	62
63	42.1446	45.9588	49.5197	52.8444	55.9488	58.8476	61.5546	64.0826	66.4426	68.6487	63
64	38.0369	42.1218	45.9356	49.4963	52.8211	55.9257	58.8249	61.5324	64.0611	66.4228	64
65	33.6416	38.0163	42.1006	45.9139	49.4745	52.7994	55.9042	58.8038	61.5118	64.0410	65
66	28.9386	33.6234	37.9971	42.0807	45.8937	49.4542	52.7791	55.8841	58.7841	61.4925	66
67	23.9065	28.9230	33.6064	37.9792	42.0622	45.8748	49.4351	52.7601	55.8654	58.7656	67
68	18.5221	23.8936	28.9084	33.5906	37.9625	42.0449	45.8572	49.4174	52.7424	55.8479	68
69	12.7607	18.5121	23.8815	28.8948	33.5758	37.9468	42.0287	45.8407	49.4008	52.7259	69
70	6.5961	12.7539	18.5027	23.8702	28.8820	33.5620	37.9323	42.0136	45.8253	49.3853	70
71	0.0000	6.5926	12.7474	18.4940	23.8597	28.8701	33.5491	37.9186	41.9995	45.8109	71
72		0.0000	6.5892	12.7414	18.4858	23.8499	28.8590	33.5370	37.9059	41.9863	72
73			0.0000	6.5861	12.7358	18.4782	23.8407	28.8487	33.5258	37.8940	73
74				0.0000	6.5832	12.7305	18.4711	23.8322	28.8390	33.5152	74
75					0.0000	6.5805	12.7257	18.4645	23.8242	28.8300	75
76						0.0000	6.5780	12.7211	18.4583	23.8167	76
77							0.0000	6.5756	12.7168	18.4525	77
78								0.0000	6.5734	12.7128	78
79									0.0000	6.5714	79
80										0.0000	80

81–90 Years Probable Life

Age, years	81	82	83	84	85	86	87	88	89	90	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9707	99.9726	99.9744	99.9761	99.9777	99.9791	99.9805	99.9818	99.9830	99.9841	1
2	99.9394	99.9433	99.9471	99.9505	99.9538	99.9568	99.9596	99.9623	99.9648	99.9671	2
3	99.9058	99.9120	99.9178	99.9232	99.9282	99.9329	99.9373	99.9414	99.9453	99.9489	3
4	99.8699	99.8785	99.8864	99.8939	99.9009	99.9074	99.9134	99.9191	99.9244	99.9294	4
5	99.8315	99.8426	99.8529	99.8626	99.8716	99.8800	99.8879	99.8952	99.9021	99.9085	5
6	99.7904	99.8042	99.8170	99.8291	99.8403	99.8508	99.8605	99.8697	99.8782	99.8862	6
7	99.7465	99.7631	99.7787	99.7932	99.8068	99.8195	99.8313	99.8424	99.8527	99.8624	7
8	99.6994	99.7192	99.7376	99.7548	99.7709	99.7859	99.8000	99.8131	99.8254	99.8368	8
9	99.6491	99.6721	99.6937	99.7138	99.7325	99.7501	99.7665	99.7818	99.7961	99.8095	9
10	99.5952	99.6218	99.6466	99.6698	99.6915	99.7117	99.7306	99.7483	99.7648	99.7802	10
11	99.5376	99.5680	99.5963	99.6228	99.6476	99.6707	99.6923	99.7125	99.7313	99.7490	11
12	99.4759	99.5103	99.5425	99.5725	99.6006	99.6268	99.6513	99.6741	99.6955	99.7155	12
13	99.4099	99.4487	99.4849	99.5187	99.5503	99.5798	99.6074	99.6331	99.6572	99.6797	13
14	99.3393	99.3827	99.4232	99.4611	99.4965	99.5295	99.5604	99.5892	99.6162	99.6413	14
15	99.2638	99.3121	99.3573	99.3995	99.4389	99.4757	99.5101	99.5422	99.5723	99.6003	15
16	99.1830	99.2366	99.2867	99.3336	99.3773	99.4182	99.4563	99.4920	99.5253	99.5564	16
17	99.0965	99.1558	99.2112	99.2630	99.3114	99.3566	99.3988	99.4382	99.4751	99.5095	17
18	99.0039	99.0693	99.1304	99.1875	99.2408	99.2907	99.3372	99.3807	99.4213	99.4592	18
19	98.9049	98.9768	99.0440	99.1068	99.1654	99.2202	99.2713	99.3191	99.3638	99.4055	19
20	98.7990	98.8778	98.9515	99.0203	99.0846	99.1447	99.2008	99.2532	99.3022	99.3480	20

7% INTEREST RATE

81–90 Years Probable Life

Age, years	Condition-percent, %										Age, years
	81	82	83	84	85	86	87	88	89	90	
21	98.6856	98.7719	98.8525	98.9279	98.9982	99.0640	99.1254	99.1827	99.2363	99.2864	21
22	98.5643	98.6586	98.7466	98.8289	98.9058	98.9776	99.0446	99.1073	99.1659	99.2206	22
23	98.4345	98.5373	98.6333	98.7230	98.8068	98.8852	98.9583	99.0266	99.0905	99.1501	23
24	98.2956	98.4075	98.5121	98.6098	98.7010	98.7863	98.8658	98.9403	99.0098	99.0747	24
25	98.1470	98.2687	98.3823	98.4885	98.5877	98.6804	98.7670	98.8479	98.9234	98.9940	25
26	97.9880	98.1201	98.2435	98.3588	98.4665	98.5672	98.6612	98.7490	98.8311	98.9077	26
27	97.8178	97.9612	98.0950	98.2201	98.3369	98.4460	98.5479	98.6432	98.7322	98.8153	27
28	97.6358	97.7911	97.9361	98.0716	98.1981	98.3164	98.4268	98.5300	98.6264	98.7165	28
29	97.4410	97.6091	97.7660	97.9127	98.0497	98.1777	98.2972	98.4089	98.5133	98.6107	29
30	97.2326	97.4143	97.5841	97.7427	97.8908	98.0293	98.1585	98.2793	98.3922	98.4976	30
31	97.0096	97.2060	97.3894	97.5608	97.7209	97.8705	98.0101	98.1406	98.2626	98.3765	31
32	96.7709	96.9830	97.1811	97.3661	97.5390	97.7005	97.8513	97.9923	98.1240	98.2470	32
33	96.5156	96.7444	96.9582	97.1579	97.3444	97.5186	97.6814	97.8335	97.9756	98.1084	33
34	96.2424	96.4892	96.7197	96.9350	97.1362	97.3241	97.4996	97.6636	97.8169	97.9600	34
35	95.9501	96.2160	96.4645	96.6966	96.9134	97.1159	97.3051	97.4819	97.6471	97.8013	35
36	95.6373	95.9238	96.1914	96.4414	96.6750	96.8931	97.0970	97.2874	97.4653	97.6315	36
37	95.3026	95.6111	95.8992	96.1684	96.4199	96.6548	96.8742	97.0793	97.2708	97.4498	37
38	94.9445	95.2765	95.5866	95.8763	96.1470	96.3998	96.6359	96.8566	97.0628	97.2554	38
39	94.5613	94.9185	95.2521	95.5638	95.8549	96.1269	96.3809	96.6184	96.8401	97.0473	39
40	94.1513	94.5354	94.8941	95.2293	95.5424	95.8349	96.1081	96.3634	96.6019	96.8248	40
41	93.7125	94.1255	94.5111	94.8715	95.2081	95.5225	95.8162	96.0907	96.3470	96.5866	41
42	93.2431	93.6869	94.1014	94.4886	94.8503	95.1882	95.5038	95.7988	96.0743	96.3317	42
43	92.7408	93.2176	93.6629	94.0789	94.4675	94.8305	95.1696	95.4865	95.7825	96.0590	43
44	92.2034	92.7155	93.1938	93.6405	94.0579	94.4478	94.8120	95.1523	95.4702	95.7672	44
45	91.6283	92.1782	92.6918	93.1714	93.6196	94.0383	94.4294	94.7948	95.1361	95.4550	45
46	91.0130	91.6033	92.1546	92.6696	93.1507	93.6001	94.0200	94.4122	94.7786	95.1210	46
47	90.3546	90.9881	91.5798	92.1326	92.6489	93.1312	93.5819	94.0028	94.3961	94.7636	47
48	89.6502	90.3299	90.9649	91.5580	92.1120	92.6296	93.1131	93.5648	93.9868	94.3811	48
49	88.8964	89.6257	90.3068	90.9431	91.5375	92.0928	92.6115	93.0962	93.5489	93.9719	49
50	88.0898	88.8721	89.6027	90.2852	90.9228	91.5184	92.0748	92.5947	93.0803	93.5340	50
51	87.2268	88.0657	88.8493	89.5813	90.2651	90.9038	91.5006	92.0581	92.5789	93.0655	51
52	86.3034	87.2030	88.0432	88.8281	89.5613	90.2462	90.8861	91.4839	92.0424	92.5642	52
53	85.3154	86.2798	87.1807	88.0222	88.8083	89.5426	90.2287	90.8696	91.4683	92.0277	53
54	84.2581	85.2920	86.2577	87.1598	88.0025	88.7897	89.5252	90.2122	90.8541	91.4538	54
55	83.1269	84.2351	85.2702	86.2371	87.1404	87.9842	88.7724	89.5089	90.1969	90.8396	55
56	81.9165	83.1042	84.2135	85.2498	86.2179	87.1222	87.9670	88.7563	89.4936	90.1825	56
57	80.6214	81.8941	83.0829	84.1934	85.2308	86.1999	87.1052	87.9510	88.7411	89.4794	57
58	79.2356	80.5993	81.8731	83.0631	84.1746	85.2130	86.1831	87.0894	87.9360	88.7270	58
59	77.7527	79.2139	80.5787	81.8536	83.0445	84.1570	85.1964	86.1674	87.0745	87.9220	59
60	76.1661	77.7315	79.1936	80.5594	81.8353	83.0272	84.1406	85.1809	86.1527	87.0607	60
61	74.4685	76.1453	77.7116	79.1747	80.5414	81.8182	83.0110	84.1253	85.1664	86.1390	61
62	72.6520	74.4481	76.1258	77.6930	79.1570	80.5246	81.8023	82.9959	84.1110	85.1528	62
63	70.7083	72.6321	74.4291	76.1076	77.6757	79.1405	80.5089	81.7874	82.9817	84.0976	63
64	68.6286	70.6890	72.6135	74.4113	76.0906	77.6595	79.1251	80.4943	81.7734	82.9685	64
65	66.4033	68.6099	70.6709	72.5962	74.3947	76.0748	77.6443	79.1106	80.4806	81.7604	65
66	64.0222	66.3852	68.5923	70.6540	72.5800	74.3791	76.0599	77.6302	79.0972	80.4678	66
67	61.4745	64.0048	66.3681	68.5759	70.6382	72.5648	74.3646	76.0461	77.6170	79.0846	67
68	58.7484	61.4577	63.9883	66.3523	68.5606	70.6235	72.5507	74.3511	76.0331	77.6046	68
69	55.8315	58.7324	61.4420	63.9730	66.3375	68.5463	70.6097	72.5374	74.3384	76.0210	69
70	52.7104	55.8163	58.7143	61.4273	63.9588	66.3236	68.5329	70.5968	72.5251	74.3266	70
71	49.3708	52.6960	55.8019	58.7033	61.4136	63.9454	66.3107	68.5204	70.5848	72.5136	71
72	45.7975	49.3574	52.6825	55.7886	58.6902	61.4007	63.9330	66.2986	68.5087	70.5736	72
73	41.9740	45.7850	49.3447	52.6699	55.7762	58.6779	61.3888	63.9213	66.2873	68.4978	73
74	37.8829	41.9626	45.7733	49.3329	52.6582	55.7645	58.6665	61.3776	63.9104	66.2768	74
75	33.5054	37.8726	41.9518	45.7623	49.3219	52.6472	55.7536	58.6558	61.3671	63.9003	75
76	28.8215	33.4963	37.8629	41.9418	45.7521	49.3116	52.6369	55.7435	58.6458	61.3574	76
77	23.8097	28.8136	33.4877	37.8538	41.9324	45.7426	49.3020	52.6273	55.7340	58.6365	77
78	18.4471	23.8032	28.8062	33.4797	37.8454	41.9237	45.7336	49.2930	52.6184	55.7251	78
79	12.7091	18.4421	23.7971	28.7994	33.4722	37.8375	41.9155	45.7253	49.2846	52.6100	79
80	6.5694	12.7056	18.4373	23.7914	28.7929	33.4652	37.8301	41.9079	45.7175	49.2768	80

7% INTEREST RATE

81–90 Years Probable Life

Age, years	Condition-percent, %										Age, years
	81	82	83	84	85	86	87	88	89	90	
81	0.0000	6.5676	12.7024	18.4329	23.7861	28.7869	33.4587	37.8232	41.9007	45.7103	81
82		0.0000	6.5660	12.6993	18.4288	23.7811	28.7813	33.4526	37.8168	41.8941	82
83			0.0000	6.5644	12.6965	18.4250	23.7765	28.7761	33.4469	37.8107	83
84				0.0000	6.5629	12.6938	18.4214	23.7722	28.7712	33.4416	84
85					0.0000	6.5616	12.6914	18.4180	23.7681	28.7666	85
86						0.0000	6.5603	12.6891	18.4149	23.7644	86
87							0.0000	6.5591	12.6869	18.4120	87
88								0.0000	6.5580	12.6849	88
89									0.0000	6.5569	89
90										0.0000	90

91–100 Years Probable Life

Age, years	91	92	93	94	95	96	97	98	99	100	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9851	99.9861	99.9870	99.9879	99.9887	99.9894	99.9901	99.9908	99.9914	99.9919	1
2	99.9692	99.9712	99.9731	99.9749	99.9765	99.9781	99.9795	99.9809	99.9821	99.9833	2
3	99.9522	99.9554	99.9583	99.9610	99.9636	99.9660	99.9682	99.9703	99.9722	99.9740	3
4	99.9340	99.9383	99.9424	99.9462	99.9497	99.9530	99.9561	99.9589	99.9616	99.9641	4
5	99.9145	99.9201	99.9254	99.9303	99.9348	99.9391	99.9431	99.9468	99.9503	99.9536	5
6	99.8937	99.9007	99.9072	99.9132	99.9189	99.9242	99.9292	99.9338	99.9382	99.9422	6
7	99.8714	99.8798	99.8877	99.8950	99.9019	99.9084	99.9144	99.9200	99.9252	99.9301	7
8	99.8475	99.8575	99.8668	99.8756	99.8837	99.8913	99.8985	99.9051	99.9113	99.9171	8
9	99.8220	99.8336	99.8445	99.8547	99.8643	99.8732	99.8815	99.8892	99.8965	99.9033	9
10	99.7947	99.8081	99.8207	99.8324	99.8434	99.8537	99.8633	99.8722	99.8806	99.8884	10
11	99.7654	99.7808	99.7952	99.8086	99.8211	99.8329	99.8438	99.8540	99.8636	99.8725	11
12	99.7341	99.7516	99.7678	99.7830	99.7973	99.8106	99.8230	99.8346	99.8454	99.8555	12
13	99.7006	99.7203	99.7386	99.7557	99.7717	99.7867	99.8007	99.8137	99.8259	99.8373	13
14	99.6648	99.6868	99.7073	99.7265	99.7444	99.7612	99.7768	99.7915	99.8051	99.8179	14
15	99.6265	99.6510	99.6739	99.6952	99.7152	99.7339	99.7513	99.7676	99.7828	99.7971	15
16	99.5855	99.6127	99.6381	99.6618	99.6839	99.7047	99.7240	99.7421	99.7590	99.7748	16
17	99.5416	99.5717	99.5998	99.6260	99.6505	99.6734	99.6948	99.7148	99.7334	99.7509	17
18	99.4947	99.5278	99.5588	99.5877	99.6147	99.6400	99.6635	99.6856	99.7062	99.7254	18
19	99.4444	99.4809	99.5149	99.5467	99.5764	99.6042	99.6301	99.6543	99.6769	99.6981	19
20	99.3907	99.4307	99.4680	99.5028	99.5354	99.5659	99.5943	99.6209	99.6457	99.6689	20
21	99.3332	99.3769	99.4177	99.4559	99.4915	99.5249	99.5560	99.5851	99.6122	99.6377	21
22	99.2716	99.3194	99.3640	99.4057	99.4446	99.4810	99.5150	99.5468	99.5764	99.6042	22
23	99.2058	99.2579	99.3065	99.3520	99.3944	99.4341	99.4711	99.5058	99.5382	99.5684	23
24	99.1353	99.1920	99.2450	99.2944	99.3407	99.3839	99.4242	99.4620	99.4972	99.5302	24
25	99.0599	99.1216	99.1792	99.2329	99.2832	99.3302	99.3740	99.4151	99.4533	99.4892	25
26	98.9793	99.0462	99.1087	99.1671	99.2217	99.2727	99.3203	99.3649	99.4064	99.4454	26
27	98.8929	98.9656	99.0333	99.0967	99.1559	99.2112	99.2628	99.3112	99.3562	99.3985	27
28	98.8006	98.8793	98.9527	99.0213	99.0855	99.1454	99.2013	99.2537	99.3026	99.3483	28
29	98.7018	98.7869	98.8664	98.9407	99.0101	99.0750	99.1355	99.1922	99.2451	99.2945	29
30	98.5961	98.6881	98.7741	98.8544	98.9295	98.9997	99.0651	99.1264	99.1836	99.2371	30
31	98.4829	98.5824	98.6753	98.7621	98.8432	98.9190	98.9898	99.0560	99.1178	99.1757	31
32	98.3618	98.4693	98.5696	98.6633	98.7509	98.8328	98.9092	98.9807	99.0474	99.1099	32
33	98.2323	98.3483	98.4565	98.5576	98.6521	98.7405	98.8229	98.9001	98.9721	99.0394	33
34	98.0937	98.2187	98.3355	98.4445	98.5465	98.6417	98.7306	98.8138	98.8915	98.9641	34
35	97.9454	98.0802	98.2060	98.3235	98.4334	98.5360	98.6319	98.7215	98.8053	98.8835	35
36	97.7868	97.9318	98.0674	98.1940	98.3124	98.4229	98.5263	98.6228	98.7130	98.7973	36
37	97.6170	97.7732	97.9191	98.0555	98.1829	98.3020	98.4132	98.5171	98.6143	98.7050	37
38	97.4353	97.6034	97.7605	97.9073	98.0444	98.1725	98.2922	98.4041	98.5086	98.6063	38
39	97.2409	97.4218	97.5908	97.7486	97.8961	98.0340	98.1628	98.2831	98.3956	98.5007	39
40	97.0329	97.2274	97.4091	97.5789	97.7375	97.8858	98.0243	98.1537	98.2747	98.3876	40
41	96.8103	97.0194	97.2148	97.3973	97.5678	97.7272	97.8761	98.0152	98.1452	98.2667	41
42	96.5722	96.7969	97.0068	97.2030	97.3862	97.5575	97.7175	97.8670	98.0068	98.1373	42
43	96.3174	96.5588	96.7844	96.9951	97.1919	97.3759	97.5479	97.7085	97.8586	97.9988	43
44	96.0448	96.3040	96.5463	96.7726	96.9841	97.1817	97.3663	97.5388	97.7001	97.8507	44
45	95.7530	96.0314	96.2915	96.5346	96.7616	96.9738	97.1721	97.3573	97.5304	97.6922	45

7% INTEREST RATE
91–100 Years Probable Life

Age, years	Condition-percent, %										Age, years
	91	92	93	94	95	96	97	98	99	100	
46	95.4409	95.7397	96.0190	96.2798	96.5236	96.7514	96.9642	97.1631	97.3489	97.5225	46
47	95.1069	95.4276	95.7273	96.0073	96.2689	96.5134	96.7418	96.9552	97.1547	97.3410	47
48	94.7495	95.0936	95.4153	95.7157	95.9964	96.2587	96.5038	96.7329	96.9469	97.1468	48
49	94.3671	94.7363	95.0814	95.4037	95.7048	95.9862	96.2492	96.4949	96.7245	96.9390	49
50	93.9579	94.3540	94.7241	95.0698	95.3928	95.6947	95.9768	96.2403	96.4866	96.7167	50
51	93.5201	93.9449	94.3418	94.7125	95.0590	95.3827	95.6852	95.9679	96.2320	96.4788	51
52	93.0517	93.5071	93.9327	94.3303	94.7018	95.0489	95.3733	95.6764	95.9596	96.2242	52
53	92.5504	93.0387	93.4950	93.9213	94.3196	94.6918	95.0395	95.3645	95.6681	95.9518	53
54	92.0141	92.5375	93.0267	93.4836	93.9106	94.3096	94.6824	95.0307	95.3562	95.6604	54
55	91.4402	92.0013	92.5255	93.0154	93.4731	93.9007	94.3003	94.6736	95.0225	95.3485	55
56	90.8261	91.4275	91.9893	92.5143	93.0048	93.4631	93.8914	94.2916	94.6655	95.0149	56
57	90.1691	90.8135	91.4156	91.9782	92.5038	92.9950	93.4539	93.8827	94.2834	94.6578	57
58	89.4661	90.1566	90.8017	91.4045	91.9678	92.4940	92.9858	93.4453	93.8746	94.2758	58
59	88.7139	89.4537	90.1449	90.7907	91.3942	91.9580	92.4849	92.9772	93.4372	93.8670	59
60	87.9090	88.7015	89.4420	90.1340	90.7804	91.3845	91.9489	92.4763	92.9691	93.4296	60
61	87.0477	87.8967	88.6900	89.4312	90.1237	90.7708	91.3755	91.9404	92.4683	92.9616	61
62	86.1262	87.0356	87.8853	88.6793	89.4211	90.1142	90.7618	91.3670	91.9325	92.4609	62
63	85.1402	86.1142	87.0243	87.8747	88.6692	89.4116	90.1053	90.7534	91.3591	91.9250	63
64	84.0851	85.1283	86.1031	87.0138	87.8647	88.6598	89.4028	90.0969	90.7456	91.3517	64
65	82.9562	84.0734	85.1173	86.0926	87.0039	87.8554	88.6511	89.3945	90.0892	90.7383	65
66	81.7483	82.9447	84.0625	85.1070	86.0829	86.9947	87.8467	88.6428	89.3868	90.0819	66
67	80.4558	81.7369	82.9339	84.0523	85.0973	86.0737	86.9861	87.8386	88.6352	89.3795	67
68	79.0729	80.4446	81.7263	82.9239	84.0428	85.0883	86.0652	86.9780	87.8310	88.6280	68
69	77.5931	79.0619	80.4342	81.7164	82.9145	84.0339	85.0799	86.0573	86.9705	87.8239	69
70	76.0097	77.5823	79.0516	80.4244	81.7072	82.9057	84.0256	85.0720	86.0498	86.9635	70
71	74.3156	75.9992	77.5722	79.0420	80.4153	81.6985	82.8975	84.0178	85.0647	86.0429	71
72	72.5028	74.3052	75.9893	77.5628	79.0331	80.4068	81.6904	82.8898	84.0106	85.0578	72
73	70.5631	72.4927	74.2956	75.9801	77.5540	79.0247	80.3988	81.6829	82.8827	84.0038	73
74	68.4877	70.5533	72.4833	74.2866	75.9715	77.5458	79.0169	80.3914	81.6758	82.8769	74
75	66.2669	68.4782	70.5442	72.4745	74.2782	75.9634	77.5382	79.0096	80.3845	81.6692	75
76	63.8908	66.2577	68.4693	70.5356	72.4663	74.2703	75.9559	77.5310	79.0027	80.3780	76
77	61.3483	63.8819	66.2491	68.4610	70.5276	72.4586	74.2629	75.9489	77.5243	78.9963	77
78	58.6278	61.3397	63.8736	66.2411	68.4532	70.5201	72.4515	74.2561	75.9423	77.5180	78
79	55.7169	58.6196	61.3318	63.8659	66.2336	68.4460	70.5132	72.4448	74.2497	75.9362	79
80	52.6022	55.7091	58.6120	61.3243	63.8586	66.2266	68.4392	70.5066	72.4385	74.2437	80
81	49.2695	52.5949	55.7019	58.6049	61.3174	63.8519	66.2200	68.4329	70.5005	72.4326	81
82	45.7035	49.2626	52.5880	55.6951	58.5983	61.3109	63.8455	66.2139	68.4269	70.4948	82
83	41.8878	45.6971	49.2562	52.5817	55.6888	58.5921	61.3048	63.8396	66.2082	68.4214	83
84	37.8051	41.8820	45.6912	49.2503	52.5757	55.6829	58.5863	61.2992	63.8341	66.2028	84
85	33.4366	37.7999	41.8766	45.6856	49.2447	52.5701	55.6774	58.5808	61.2939	63.8290	85
86	28.7623	33.4320	37.7950	41.8715	45.6805	49.2395	52.5649	55.6723	58.5758	61.2889	86
87	23.7608	28.7583	33.4276	37.7904	41.8668	45.6756	49.2346	52.5601	55.6675	58.5711	87
88	18.4092	23.7575	28.7546	33.4236	37.7861	41.8623	45.6711	49.2300	52.5555	55.6629	88
89	12.6830	18.4067	23.7544	28.7511	33.4198	37.7821	41.8582	45.6669	49.2258	62.5513	89
90	6.5559	12.6812	18.4043	23.7516	28.7478	33.4163	37.7784	41.8543	45.6629	49.2218	90
91	0.0000	6.5550	12.6796	18.4020	23.7489	28.7448	33.4130	37.7949	41.8474	45.6592	91
92		0.0000	6.5542	12.6781	18.4000	23.7463	28.7420	33.4099	37.8507	41.8473	92
93			0.0000	6.5534	12.6766	18.3980	23.7440	28.7393	33.4070	37.7685	93
94				0.0000	6.5526	12.6753	18.3962	23.7418	28.7368	33.4043	94
95					0.0000	6.5520	12.6740	18.3945	23.7398	28.7345	95
96						0.0000	6.5513	12.6728	18.3929	23.7378	96
97							0.0000	6.5507	12.6718	18.3914	97
98								0.0000	6.5501	12.6707	98
99									0.0000	6.5496	99
100										0.0000	100

8% INTEREST RATE

1–10 Years Probable Life

Age, years	Condition-percent, %										Age, years
	1	2	3	4	5	6	7	8	9	10	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	0.0000	51.9231	69.1966	77.8079	82.9544	86.3685	88.7928	90.5985	91.9920	93.0971	1
2		0.0000	35.9290	53.8405	64.5451	71.6464	76.6889	80.4449	83.3434	85.6419	2
3			0.0000	27.9556	44.6630	55.7466	63.6168	69.4790	74.0029	77.5903	3
4				0.0000	23.1904	38.5748	49.4989	57.6359	63.9152	68.8945	4
5					0.0000	20.0292	34.2516	44.8453	53.0204	59.5032	5
6						0.0000	17.7845	31.0314	41.2541	49.3605	6
7							0.0000	16.1125	28.5464	38.4063	7
8								0.0000	14.8222	26.5759	8
9									0.0000	13.7990	9
10										0.0000	10

11–20 Years Probable Life

Age, years	11	12	13	14	15	16	17	18	19	20	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	93.9924	94.7305	95.3478	95.8703	96.3170	96.7023	97.0371	97.3298	97.5872	97.8148	1
2	87.5041	89.0394	90.3235	91.4103	92.3395	93.1408	93.8371	94.4460	94.9815	95.4547	2
3	80.4968	82.8931	84.8972	86.5934	88.0437	89.2944	90.3811	91.3314	92.1672	92.9059	3
4	72.9289	76.2550	79.0368	81.3912	83.4042	85.1403	86.6486	87.9677	89.1278	90.1531	4
5	64.7556	69.0859	72.7075	75.7728	78.3936	80.6538	82.6176	84.3349	85.8453	87.1802	5
6	55.9284	61.3433	65.8719	69.7049	72.9821	75.8084	78.2641	80.4115	82.3001	83.9694	6
7	46.3951	52.9813	58.4895	63.1516	67.1377	70.5754	73.5622	76.1742	78.4714	80.5017	7
8	36.0990	43.9503	50.5165	56.0741	60.8258	64.9237	68.4843	71.5980	74.3363	76.7566	8
9	24.9793	34.1968	41.9056	48.4303	54.0089	58.8199	63.0001	66.6556	69.8705	72.7119	9
10	12.9700	23.6630	32.6059	40.1751	46.6467	52.2279	57.0771	61.3178	65.0474	68.3437	10
11	0.0000	12.2866	22.5622	31.2594	38.6954	45.1084	50.6804	55.5531	59.8384	63.6259	11
12		0.0000	11.7150	21.6304	30.1081	37.4194	43.7719	49.3271	54.2127	58.5308	12
13			0.0000	11.2312	20.8338	29.1152	36.3107	42.6030	48.1369	53.0280	13
14				0.0000	10.8176	20.1468	28.2526	35.3411	41.5751	47.0850	14
15					0.0000	10.4608	19.5498	27.4982	34.4884	40.6666	15
16						0.0000	10.1509	19.0278	26.8347	33.7347	16
17							0.0000	9.8798	18.5687	26.2483	17
18								0.0000	9.6414	18.1629	18
19									0.0000	9.4308	19
20										0.0000	20

21–30 Years Probable Life

Age, years	21	22	23	24	25	26	27	28	29	30	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	98.0168	98.1968	98.3578	98.5022	98.6321	98.7493	98.8552	98.9511	99.0381	99.1173	1
2	95.8749	96.2493	96.5842	96.8846	97.1548	97.3985	97.6188	97.8183	97.9993	98.1639	2
3	93.5617	94.1461	94.6687	95.1376	95.5593	95.9397	96.2835	96.5949	96.8774	97.1343	3
4	91.0634	91.8745	92.6000	93.2508	93.8362	94.3641	94.8414	95.2736	95.6658	96.0223	4
5	88.3652	89.4213	90.3658	91.2130	91.9752	92.6626	93.2839	93.8466	94.3572	94.8213	5
6	85.4512	86.7718	87.9528	89.0123	89.9653	90.8249	91.6018	92.3054	92.9439	93.5243	6
7	82.3041	83.9103	85.3468	86.6355	87.7947	88.8401	89.7851	90.6410	91.4176	92.1235	7
8	78.9052	80.8200	82.5323	84.0685	85.4504	86.6966	87.8231	88.8433	89.7691	90.6106	8
9	75.2344	77.4823	79.4927	81.2962	82.9185	84.3816	85.7041	86.9019	87.9888	88.9767	9
10	71.2699	73.8777	76.2099	78.3021	80.1841	81.8815	83.4157	84.8052	86.0660	87.2121	10
11	66.9883	69.9847	72.6645	75.0685	77.2310	79.1813	80.9441	82.5407	83.9895	85.3063	11
12	62.3641	65.7803	68.8354	71.5761	74.0416	76.2651	78.2748	80.0951	81.7468	83.2481	12
13	57.3700	61.2395	64.7001	67.8044	70.5971	73.1156	75.3920	77.4538	79.3247	81.0252	13
14	51.9764	56.3355	60.2338	63.7310	66.8769	69.7141	72.2785	74.6012	76.7088	78.6244	14
15	46.1512	51.0391	55.4103	59.3317	62.8592	66.0405	68.9160	71.5204	73.8836	76.0317	15

8% INTEREST RATE

21–30 Years Probable Life

Age, years	Condition-percent, % 21	22	23	24	25	26	27	28	29	30	Age, years
16	39.8601	45.3190	50.2009	54.5804	58.5201	62.0730	65.2845	68.1931	70.8325	73.2315	16
17	33.0657	39.1414	44.5748	49.4490	53.8338	57.7882	61.3624	64.5997	67.5372	70.2072	17
18	25.7277	32.4695	38.4986	43.9072	48.7726	53.1605	57.1266	60.7188	63.9784	66.9411	18
19	17.8027	25.2638	31.9362	37.9220	43.3066	48.1626	52.5519	56.5274	60.1347	63.4136	19
20	9.2437	17.4817	24.8489	31.4579	37.4032	42.7649	47.6113	52.0007	55.9837	59.6039	20
21	0.0000	9.0770	17.1946	24.4767	31.0276	36.9354	42.2754	47.1119	51.5005	55.4895	21
22		0.0000	8.9280	16.9371	24.1419	30.6395	36.5126	41.8319	46.6587	51.0459	22
23			0.0000	8.7943	16.7054	23.8400	30.2888	36.1296	41.4295	46.2469	23
24				0.0000	8.6740	16.4964	23.5671	29.9710	35.7820	41.0639	24
25					0.0000	8.5655	16.3077	23.3198	29.6828	35.4662	25
26						0.0000	8.4674	16.1365	23.0955	29.4208	26
27							0.0000	8.3786	15.9813	22.8917	27
28								0.0000	8.2980	15.8403	28
29									0.0000	8.2248	29
30										0.0000	30

31–40 Years Probable Life

Age, years	31	32	33	34	35	36	37	38	39	40	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.1893	99.2549	99.3148	99.3696	99.4197	99.4655	99.5076	99.5461	99.5815	99.6140	1
2	98.3137	98.4502	98.5749	98.6887	98.7929	98.8883	98.9757	99.0559	99.1295	99.1971	2
3	97.3681	97.5812	97.7757	97.9534	98.1160	98.2649	98.4013	98.5265	98.6413	98.7468	3
4	96.3468	96.6426	96.9126	97.1593	97.3850	97.5916	97.7810	97.9547	98.1141	98.2606	4
5	95.2438	95.6289	95.9804	96.3016	96.5955	96.8645	97.1111	97.3372	97.5448	97.7354	5
6	94.0526	94.5341	94.9737	95.3753	95.7428	96.0792	96.3875	96.6703	96.9298	97.1682	6
7	92.7660	93.3518	93.8864	94.3750	94.8219	95.2311	95.6061	95.9500	96.2657	96.5557	7
8	91.3766	92.0749	92.7122	93.2946	93.8273	94.3151	94.7621	95.1721	95.5484	95.8941	8
9	89.8760	90.6958	91.4440	92.1277	92.7531	93.3258	93.8506	94.3320	94.7738	95.1796	9
10	88.2553	89.2063	90.0743	90.8675	91.5931	92.2574	92.8662	93.4246	93.9372	94.4080	10
11	86.5050	87.5978	88.5951	89.5065	90.3402	91.1035	91.8031	92.4447	93.0336	93.5746	11
12	84.6147	85.8605	86.9976	88.0366	88.9871	89.8574	90.6549	91.3864	92.0578	92.6745	12
13	82.5732	83.9843	85.2722	86.4491	87.5257	88.5115	89.4149	90.2434	91.0039	91.7025	13
14	80.3683	81.9579	83.4088	84.7346	85.9474	87.0579	88.0756	89.0090	89.8657	90.6527	14
15	77.9870	79.7695	81.3964	82.8830	84.2429	85.4881	86.6292	87.6758	88.6365	89.5188	15
16	75.4153	77.4060	79.2229	80.8832	82.4020	83.7927	85.0671	86.2360	87.3089	88.2943	16
17	72.6377	74.8533	76.8756	78.7235	80.4139	81.9616	83.3800	84.6810	85.8751	86.9719	17
18	69.6380	72.0965	74.3405	76.3910	78.2666	79.9841	81.5580	83.0016	84.3266	85.5436	18
19	66.3984	69.1192	71.6026	73.8718	75.9477	77.8483	79.5902	81.1878	82.6542	84.0011	19
20	62.8995	65.9036	68.6456	71.1512	73.4431	75.5417	77.4650	79.2289	80.8480	82.3351	20
21	59.1207	62.4308	65.4521	68.2128	70.7383	73.0506	75.1697	77.1133	78.8974	80.5359	21
22	55.0397	58.6802	62.0031	65.0395	67.8170	70.3602	72.6909	74.8285	76.7906	78.5928	22
23	50.6321	54.6296	58.2782	61.6122	64.6620	67.4545	70.0137	72.3609	74.5154	76.4942	23
24	45.8719	50.2548	54.2553	57.9108	61.2546	64.3164	67.1224	69.6959	72.0581	74.2277	24
25	40.7310	45.5302	49.9105	53.9132	57.5747	60.9273	63.9997	66.7177	69.4042	71.7799	25
26	35.1787	40.4275	45.2182	49.5959	53.6004	57.2670	60.6272	63.7092	66.5380	69.1363	26
27	29.1823	34.9166	40.1505	44.9331	49.3081	53.3139	56.9850	60.3520	63.4426	66.2812	27
28	22.7061	28.9648	34.6774	39.8974	44.6724	49.0445	53.0513	56.7263	60.0995	63.1977	28
29	15.7119	22.5370	28.7664	34.4587	39.6659	44.4336	48.8030	52.8105	56.4889	59.8675	29
30	8.1581	15.5948	22.3825	28.5850	34.2588	39.4539	44.2148	48.5814	52.5895	56.2709	30
31	0.0000	8.0973	15.4880	22.2414	28.4192	34.0757	39.2596	44.0141	48.3782	52.3865	31
32		0.0000	8.0418	15.3903	22.1124	28.2673	33.9079	39.0813	43.8299	48.1914	32
33			0.0000	7.9911	15.3011	21.9942	28.1281	33.7539	38.9178	43.6607	33
34				0.0000	7.9448	15.2193	21.8859	28.0003	33.6127	38.7676	34
35					0.0000	7.9023	15.1443	21.7864	27.8832	33.4830	35
36						0.0000	7.8634	15.0755	21.6953	27.7756	36
37							0.0000	7.8277	15.0124	21.6116	37
38								0.0000	7.7949	14.9545	38
39									0.0000	7.7648	39
40										0.0000	40

8% INTEREST RATE

41–50 Years Probable Life

Age, years	Condition-percent, %										Age, years
	41	42	43	44	45	46	47	48	49	50	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.6439	99.6713	99.6966	99.7198	99.7413	99.7610	99.7792	99.7960	99.8114	99.8257	1
2	99.2592	99.3163	99.3689	99.4173	99.4618	99.5029	99.5407	99.5756	99.6078	99.6375	2
3	98.8438	98.9330	99.0150	99.0905	99.1601	99.2241	99.2832	99.3376	99.3879	99.4342	3
4	98.3952	98.5189	98.6328	98.7376	98.8341	98.9231	99.0051	99.0806	99.1503	99.2146	4
5	97.9106	98.0717	98.2200	98.3565	98.4821	98.5979	98.7047	98.8031	98.8938	98.9775	5
6	97.3873	97.5888	97.7742	97.9448	98.1020	98.2468	98.3802	98.5033	98.6168	98.7215	6
7	96.8222	97.0672	97.2927	97.5003	97.6914	97.8675	98.0299	98.1795	98.3175	98.4449	7
8	96.2118	96.5039	96.7727	97.0201	97.2480	97.4579	97.6514	97.8298	97.9944	98.1462	8
9	95.5526	95.8955	96.2111	96.5016	96.7691	97.0156	97.2428	97.4522	97.6454	97.8236	9
10	94.8406	95.2385	95.6046	95.9416	96.2519	96.5378	96.8014	97.0444	97.2685	97.4752	10
11	94.0717	94.5289	94.9495	95.3367	95.6933	96.0219	96.3247	96.6039	96.8614	97.0989	11
12	93.2413	93.7625	94.2421	94.6835	95.0901	95.4646	95.8099	96.1282	96.4217	96.6926	12
13	92.3445	92.9348	93.4780	93.9781	94.4386	94.8628	95.2539	95.6144	95.9469	96.2537	13
14	91.3759	92.0409	92.6529	93.2161	93.7349	94.2129	94.6534	95.0595	95.4341	95.7797	14
15	90.3298	91.0755	91.7617	92.3933	92.9750	93.5109	94.0048	94.4602	94.8803	95.2678	15
16	89.2000	90.0329	90.7992	91.5046	92.1543	92.7528	93.3044	93.8130	94.2821	94.7149	16
17	87.9799	88.9068	89.7597	90.5448	91.2679	91.9340	92.5480	93.1141	93.6362	94.1178	17
18	86.6621	87.6907	88.6371	89.5082	90.3106	91.0497	91.7310	92.3591	92.9385	93.4729	18
19	85.2390	86.3773	87.4246	88.3887	89.2767	90.0947	90.8487	91.5438	92.1850	92.7765	19
20	83.7019	84.9588	86.1152	87.1797	88.1601	89.0633	89.8958	90.6633	91.3713	92.0243	20
21	82.0419	83.4268	84.7010	85.8739	86.9542	87.9494	88.8667	89.7124	90.4924	91.2120	21
22	80.2491	81.7723	83.1737	84.4637	85.6518	86.7463	87.7552	88.6853	89.5432	90.3347	22
23	78.3129	79.9853	81.5241	82.9407	84.2452	85.4471	86.5548	87.5761	88.5181	89.3872	23
24	76.2218	78.0555	79.7426	81.2957	82.7261	84.0438	85.2584	86.3782	87.4110	88.3638	24
25	73.9634	75.9712	77.8186	79.5192	81.0854	82.5284	83.8583	85.0844	86.2154	87.2587	25
26	71.5243	73.7203	75.7407	77.6006	79.3136	80.8916	82.3461	83.6871	84.9240	86.0651	26
27	68.8901	71.2892	73.4966	75.5285	77.3999	79.1240	80.7130	82.1781	83.5294	84.7760	27
28	66.0452	68.6637	71.0729	73.2906	75.3331	77.2149	78.9493	80.5483	82.0232	83.3838	28
29	62.9726	65.8281	68.4553	70.8737	73.1010	75.1531	77.0444	78.7881	80.3965	81.8802	29
30	59.6543	62.7656	65.6283	68.2635	70.6904	72.9263	74.9872	76.8872	78.6397	80.2563	30
31	56.0705	59.4582	62.5751	65.4444	68.0869	70.5214	72.7654	74.8342	76.7423	78.5025	31
32	52.2000	55.8862	59.2778	62.3998	65.2751	67.9242	70.3658	72.6169	74.6931	76.6085	32
33	48.0198	52.0284	55.7165	59.1117	62.2384	65.1191	67.7742	70.2222	72.4800	74.5629	33
34	43.5053	47.8620	51.8704	55.5604	58.9588	62.0897	64.9754	67.6359	70.0898	72.3536	34
35	38.6295	43.3622	47.7167	51.7251	55.4167	58.8179	61.9526	64.8428	67.5085	69.9676	35
36	33.3638	38.5025	43.2306	47.5830	51.5913	55.2843	58.6880	61.8262	64.7205	67.3907	36
37	27.6767	33.2540	38.3856	43.1094	47.4599	51.4680	55.1623	58.5683	61.7096	64.6077	37
38	21.5347	27.5857	33.1531	38.2780	42.9980	47.3465	51.3545	55.0497	58.4578	61.6020	38
39	14.9013	21.4638	27.5019	33.0601	38.1791	42.8952	47.2420	51.2496	54.9459	58.3559	39
40	7.7372	14.8523	21.3986	27.4248	32.9746	38.0878	42.8006	47.1456	51.1530	54.8501	40
41	0.0000	7.7117	14.8071	21.3386	27.3540	32.8959	38.0038	42.7132	47.0567	51.0638	41
42		0.0000	7.6883	14.7656	21.2836	27.2886	32.8233	37.9262	42.6327	46.9747	42
43			0.0000	7.6668	14.7275	21.2327	27.2284	32.7563	37.8547	42.5584	43
44				0.0000	7.6470	14.6923	21.1859	27.1728	32.6945	37.7887	44
45					0.0000	7.6287	14.6599	21.1426	27.1215	32.6376	45
46						0.0000	7.6119	14.6300	21.1027	27.0743	46
47							0.0000	7.5963	14.6024	21.0659	47
48								0.0000	7.5820	14.5769	48
49									0.0000	7.5688	49
50										0.0000	50

8% INTEREST RATE
51–60 Years Probable Life

Age, years	Condition-percent, %										Age, years
	51	52	53	54	55	56	57	58	59	60	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.8389	99.8510	99.8623	99.8726	99.8822	99.8910	99.8992	99.9068	99.9138	99.9202	1
2	99.6649	99.6902	99.7135	99.7351	99.7550	99.7734	99.7904	99.8061	99.8206	99.8340	2
3	99.4770	99.5164	99.5529	99.5865	99.6176	99.6463	99.6728	99.6973	99.7200	99.7410	3
4	99.2740	99.3288	99.3794	99.4261	99.4692	99.5091	99.5459	99.5799	99.6114	99.6404	4
5	99.0548	99.1261	99.1920	99.2528	99.3089	99.3608	99.4088	99.4531	99.4940	99.5319	5
6	98.8181	98.9072	98.9896	99.0656	99.1359	99.2007	99.2607	99.3161	99.3673	99.4146	6
7	98.5624	98.6709	98.7710	98.8635	98.9489	99.0278	99.1008	99.1682	99.2304	99.2880	7
8	98.2863	98.4156	98.5350	98.6452	98.7470	98.8411	98.9280	99.0084	99.0826	99.1513	8
9	97.9881	98.1399	98.2800	98.4095	98.5290	98.6395	98.7415	98.8358	98.9230	99.0036	9
10	97.6660	97.8421	98.0047	98.1548	98.2935	98.4217	98.5400	98.6495	98.7506	98.8440	10
11	97.3181	97.5205	97.7073	97.8799	98.0392	98.1864	98.3225	98.4482	98.5644	98.6718	11
12	96.9425	97.1732	97.3862	97.5829	97.7646	97.9324	98.0875	98.2308	98.3633	98.4857	12
13	96.5368	96.7981	97.0393	97.2621	97.4679	97.6580	97.8337	97.9961	98.1461	98.2848	13
14	96.0986	96.3930	96.6648	96.9157	97.1476	97.3617	97.5596	97.7425	97.9115	98.0678	14
15	95.6254	95.9554	96.2602	96.5416	96.8016	97.0417	97.2636	97.4687	97.6582	97.8334	15
16	95.1143	95.4829	95.8233	96.1376	96.4279	96.6961	96.9439	97.1730	97.3846	97.5803	16
17	94.5623	94.9726	95.3514	95.7012	96.0244	96.3228	96.5987	96.8536	97.0891	97.3069	17
18	93.9662	94.4214	94.8418	95.2300	95.5885	95.9197	96.2258	96.5086	96.7700	97.0117	18
19	93.3223	93.8262	94.2914	94.7210	95.1178	95.4843	95.8230	96.1361	96.4254	96.6928	19
20	92.6270	93.1833	93.6970	94.1713	94.6094	95.0141	95.3881	95.7337	96.0532	96.3484	20
21	91.8761	92.4891	93.0550	93.5776	94.0604	94.5063	94.9184	95.2992	95.6512	95.9765	21
22	91.0650	91.7392	92.3617	92.9365	93.4674	93.9579	94.4111	94.8299	95.2170	95.5748	22
23	90.1891	90.9294	91.6129	92.2440	92.8270	93.3656	93.8632	94.3231	94.7481	95.1410	23
24	89.2431	90.0548	90.8042	91.4962	92.1354	92.7258	93.2714	93.7757	94.2417	94.6725	24
25	88.2215	89.1102	89.9308	90.6885	91.3884	92.0350	92.6324	93.1846	93.6948	94.1665	25
26	87.1181	88.0901	88.9875	89.8162	90.5817	91.2888	91.9422	92.5461	93.1042	93.6200	26
27	85.9264	86.9883	87.9687	88.8741	89.7104	90.4830	91.1968	91.8565	92.4663	93.0298	27
28	84.6394	85.7984	86.8685	87.8567	88.7695	89.6126	90.3918	91.1118	91.7773	92.3924	28
29	83.2494	84.5133	85.6803	86.7579	87.7532	88.6727	89.5223	90.3076	91.0332	91.7041	29
30	81.7483	83.1254	84.3969	85.5711	86.6557	87.6576	88.5833	89.4389	90.2297	90.9606	30
31	80.1270	81.6265	83.0110	84.2894	85.4704	86.5612	87.5692	88.5008	89.3618	90.1576	31
32	78.3761	80.0077	81.5141	82.9052	84.1902	85.3772	86.4740	87.4876	88.4245	89.2904	32
33	76.4850	78.2593	79.8975	81.4103	82.8076	84.0984	85.2912	86.3934	87.4122	88.3539	33
34	74.4427	76.3711	78.1516	79.7957	81.3144	82.7173	84.0137	85.2116	86.3189	87.3424	34
35	72.2371	74.3319	76.2659	78.0520	79.7017	81.2258	82.6340	83.9353	85.1381	86.2500	35
36	69.8549	72.1295	74.2295	76.1688	77.9601	79.6148	81.1439	82.5569	83.8629	85.0702	36
37	67.2822	69.7508	72.0301	74.1349	76.0791	77.8751	79.5347	81.0683	82.4857	83.7960	37
38	64.5036	67.1820	69.6548	71.9384	74.0476	75.9961	77.7966	79.4605	80.9983	82.4199	38
39	61.5028	64.4076	67.0894	69.5661	71.8536	73.9669	75.9196	77.7241	79.3920	80.9337	39
40	58.2619	61.4112	64.3189	67.0040	69.4841	71.7753	73.8924	75.8488	77.6571	79.3286	40
41	54.7618	58.1751	61.3266	64.2369	66.9250	69.4084	71.7030	73.8235	75.7834	77.5951	41
42	50.9816	54.6802	58.0950	61.2485	64.1612	66.8521	69.3385	71.6361	73.7598	75.7229	42
43	46.8990	50.9056	54.6049	58.0210	61.1763	64.0913	66.7848	69.2738	71.5744	73.7009	43
44	42.4898	46.8291	50.8355	54.5353	57.9527	61.1097	64.0268	66.7225	69.2141	71.5172	44
45	37.7279	42.4265	46.7646	50.7708	54.4711	57.8895	61.0481	63.9671	66.6649	69.1588	45
46	32.5850	37.6717	42.3681	46.7051	50.7110	54.4117	57.8312	60.9912	63.9119	66.6117	46
47	27.0307	32.5364	37.6198	42.3141	46.6500	50.6557	54.3569	57.7773	60.9386	63.8609	47
48	21.0320	26.9904	32.4916	37.5719	42.2643	46.5992	50.6047	54.3062	57.7274	60.8900	48
49	14.5534	21.0007	26.9532	32.4502	37.5276	42.2182	46.5523	50.5575	54.2594	57.6814	49
50	7.5566	14.5318	20.9717	26.9189	32.4120	37.4867	42.1757	46.5089	50.5139	54.2161	50
51	0.0000	7.5453	14.5117	20.9450	26.8872	32.3767	37.4489	42.1363	46.4687	50.4736	51
52		0.0000	7.5349	14.4933	20.9203	26.8579	32.3441	37.4140	42.1000	46.4317	52
53			0.0000	7.5253	14.4762	20.8976	26.8308	32.3139	37.3817	42.0664	53
54				0.0000	7.5165	14.4604	20.8765	26.8058	32.2860	37.3519	54
55					0.0000	7.5083	14.4458	20.8570	26.7827	32.2603	55
56						0.0000	7.5007	14.4324	20.8391	26.7613	56
57							0.0000	7.4937	14.4199	20.8224	57
58								0.0000	7.4873	14.4084	58
59									0.0000	7.4813	59
60										0.0000	60

8% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, %										Age, years
	61	62	63	64	65	66	67	68	69	70	
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9262	99.9317	99.9368	99.9415	99.9459	99.9499	99.9536	99.9571	99.9603	99.9632	1
2	99.8464	99.8579	99.8685	99.8783	99.8874	99.8958	99.9036	99.9107	99.9174	99.9235	2
3	99.7603	99.7782	99.7948	99.8101	99.8243	99.8374	99.8495	99.8607	99.8711	99.8806	3
4	99.6673	99.6922	99.7152	99.7364	99.7561	99.7742	99.7911	99.8066	99.8210	99.8343	4
5	99.5669	99.5992	99.6291	99.6568	99.6824	99.7061	99.7280	99.7482	99.7670	99.7843	5
6	99.4584	99.4989	99.5363	99.5709	99.6029	99.6325	99.6599	99.6852	99.7086	99.7303	6
7	99.3412	99.3904	99.4360	99.4780	99.5170	99.5530	99.5863	99.6171	99.6456	99.6720	7
8	99.2147	99.2734	99.3276	99.3778	99.4242	99.4671	99.5068	99.5435	99.5775	99.6090	8
9	99.0780	99.1469	99.2106	99.2695	99.3240	99.3744	99.4210	99.4641	99.5040	99.5409	9
10	98.9305	99.0104	99.0842	99.1526	99.2158	99.2742	99.3283	99.3783	99.4246	99.4674	10
11	98.7711	98.8629	98.9478	99.0263	99.0989	99.1661	99.2282	99.2857	99.3389	99.3880	11
12	98.5989	98.7036	98.8004	98.8899	98.9727	99.0492	99.1201	99.1856	99.2462	99.3023	12
13	98.4130	98.5316	98.6412	98.7426	98.8364	98.9231	99.0033	99.0776	99.1462	99.2097	13
14	98.2122	98.3458	98.4693	98.5835	98.6891	98.7868	98.8772	98.9608	99.0382	99.1098	14
15	97.9954	98.1451	98.2836	98.4117	98.5301	98.6397	98.7410	98.8348	98.9215	99.0018	15
16	97.7612	97.9284	98.0831	98.2261	98.3584	98.4808	98.5940	98.6987	98.7955	98.8852	16
17	97.5082	97.6944	97.8665	98.0257	98.1729	98.3091	98.4351	98.5516	98.6595	98.7592	17
18	97.2350	97.4416	97.6326	97.8093	97.9726	98.1237	98.2635	98.3929	98.5125	98.6232	18
19	96.9400	97.1686	97.3800	97.5755	97.7563	97.9235	98.0783	98.2214	98.3538	98.4763	19
20	96.6214	96.8738	97.1072	97.3231	97.5227	97.7073	97.8782	98.0362	98.1824	98.3176	20
21	96.2773	96.5554	96.8126	97.0504	97.2704	97.4738	97.6620	97.8362	97.9973	98.1463	21
22	95.9056	96.2115	96.4944	96.7559	96.9978	97.2216	97.4286	97.6201	97.7973	97.9612	22
23	95.5042	95.8401	96.1507	96.4379	96.7035	96.9492	97.1766	97.3868	97.5814	97.7613	23
24	95.0708	95.4390	95.7795	96.0945	96.3857	96.6551	96.9043	97.1349	97.3482	97.5455	24
25	94.6026	95.0058	95.3787	95.7235	96.0424	96.3374	96.6103	96.8627	97.0963	97.3123	25
26	94.0970	94.5380	94.9458	95.3229	95.6717	95.9943	96.2928	96.5688	96.8243	97.0606	26
27	93.5509	94.0327	94.4782	94.8902	95.2713	95.6238	95.9498	96.2514	96.5305	96.7886	27
28	92.9611	93.4870	93.9733	94.4230	94.8388	95.2236	95.5794	95.9086	96.2132	96.4950	28
29	92.3242	92.8977	93.4279	93.9183	94.3718	94.7914	95.1794	95.5384	95.8706	96.1778	29
30	91.6364	92.2612	92.8389	93.3733	93.8674	94.3246	94.7474	95.1386	95.5005	95.8353	30
31	90.8934	91.5738	92.2028	92.7846	93.3227	93.8204	94.2808	94.7067	95.1008	95.4654	31
32	90.0910	90.8313	91.5159	92.1489	92.7344	93.2760	93.7769	94.2404	94.6692	95.0658	32
33	89.2245	90.0295	90.7739	91.4623	92.0990	92.6880	93.2327	93.7367	94.2030	94.6343	33
34	88.2887	89.1636	89.9726	90.7208	91.4128	92.0529	92.6450	93.1927	93.6995	94.1683	34
35	87.2779	88.2283	89.1072	89.9200	90.6717	91.3670	92.0102	92.6052	93.1557	93.6650	35
36	86.1863	87.2183	88.1725	89.0551	89.8713	90.6263	91.3247	91.9707	92.5684	93.1215	36
37	85.0074	86.1275	87.1631	88.1210	89.0069	89.8263	90.5842	91.2855	91.9342	92.5344	37
38	83.7342	84.9493	86.0730	87.1122	88.0733	88.9623	89.7846	90.5454	91.2492	91.9004	38
39	82.3591	83.6770	84.8956	86.0227	87.0650	88.0292	88.9210	89.7461	90.5094	91.2157	39
40	80.8740	82.3028	83.6241	84.8460	85.9761	87.0214	87.9883	88.8828	89.7104	90.4761	40
41	79.2701	80.8187	82.2508	83.5752	84.8000	85.9330	86.9810	87.9506	88.8476	89.6775	41
42	77.5378	79.2159	80.7676	82.2027	83.5299	84.7576	85.8931	86.9437	87.9157	88.8149	42
43	75.6670	77.4849	79.1658	80.7204	82.1582	83.4881	84.7182	85.8563	86.9092	87.8833	43
44	73.6466	75.6153	77.4359	79.1195	80.6767	82.1170	83.4493	84.6819	85.8222	86.8772	44
45	71.4644	73.5963	75.5675	77.3906	79.0767	80.6363	82.0789	83.4135	84.6483	85.7907	45
46	69.1078	71.4156	73.5497	75.5233	77.3487	79.0371	80.5989	82.0437	83.3804	84.6171	46
47	66.5626	69.0606	71.3705	73.5067	75.4824	77.3099	78.0004	80.5643	82.0111	83.3498	47
48	63.8137	66.5171	69.0169	71.3287	73.4669	75.4446	77.2740	78.9665	80.5323	81.9810	48
49	60.8450	63.7702	66.4750	68.9765	71.2901	73.4301	75.4096	77.2409	78.9351	80.5027	49
50	57.6388	60.8035	63.7298	66.4362	68.9392	71.2544	73.3960	75.3773	77.2102	78.9061	50
51	54.1761	57.5994	60.7650	63.6926	66.4002	68.9047	71.2214	73.3646	75.3473	77.1819	51
52	50.4363	54.1391	57.5630	60.7295	63.6581	66.3669	68.8727	71.1908	73.3354	75.3196	52
53	46.3974	50.4019	54.1048	57.5293	60.6966	63.6262	66.3362	68.8432	71.1625	73.3085	53
54	42.0353	46.3657	50.3700	54.0732	57.4982	60.6662	63.5967	66.3077	68.8158	71.1364	54
55	37.3243	42.0067	46.3364	50.3405	54.0439	57.4694	60.6380	63.5694	66.2813	68.7905	55
56	32.2365	37.2988	41.9801	46.3093	50.3133	54.0168	57.4427	60.6120	63.5441	66.2570	56
57	26.7415	32.2144	37.2753	41.9555	46.2842	50.2881	53.9918	57.4181	60.5879	63.5208	57
58	20.8070	26.7233	32.1941	37.2535	41.9328	46.2610	50.2648	53.9686	57.3953	60.5657	58
59	14.3978	20.7928	26.7064	32.1753	37.2333	41.9118	46.2396	50.2432	53.9472	57.3742	59
60	7.4758	14.3879	20.7797	26.6908	32.1578	37.2146	41.8924	46.2197	50.2232	53.9273	60

8% INTEREST RATE

61–70 Years Probable Life

Age, years	Condition-percent, % 61	62	63	64	65	66	67	68	69	70	Age, years
61	0.0000	7.4707	14.3788	20.7675	26.6763	32.1417	37.1974	41.8744	46.2014	50.2048	61
62		0.0000	7.4659	14.3704	20.7563	26.6630	32.1268	37.1814	41.8578	46.1844	62
63			0.0000	7.4616	14.3627	20.7459	26.6506	32.1130	37.1667	41.8424	63
64				0.0000	7.4575	14.3555	20.7363	26.6392	32.1003	37.1530	64
65					0.0000	7.4538	14.3488	20.7274	26.6286	32.0885	65
66						0.0000	7.4503	14.3426	20.7191	26.6188	66
67							0.0000	7.4471	14.3369	20.7115	67
68								0.0000	7.4442	14.3317	68
69									0.0000	7.4414	69
70										0.0000	70

71–80 Years Probable Life

Age, years	71	72	73	74	75	76	77	78	79	80	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9660	99.9685	99.9708	99.9730	99.9750	99.9769	99.9786	99.9802	99.9817	99.9830	1
2	99.9292	99.9345	99.9394	99.9439	99.9480	99.9519	99.9555	99.9588	99.9618	99.9647	2
3	99.8895	99.8977	99.9054	99.9124	99.9189	99.9249	99.9305	99.9357	99.9404	99.9449	3
4	99.8467	99.8581	99.8686	99.8784	99.8874	99.8958	99.9035	99.9107	99.9173	99.9235	4
5	99.8004	99.8152	99.8289	99.8417	99.8534	99.8643	99.8744	99.8837	99.8924	99.9003	5
6	99.7504	99.7689	99.7861	99.8020	99.8167	99.8303	99.8429	99.8546	99.8654	99.8754	6
7	99.6964	99.7189	99.7398	99.7592	99.7771	99.7936	99.8090	99.8232	99.8363	99.8484	7
8	99.6380	99.6650	99.6899	99.7129	99.7343	99.7540	99.7723	99.7892	99.8048	99.8193	8
9	99.5751	99.6067	99.6359	99.6630	99.6880	99.7112	99.7326	99.7525	99.7709	99.7879	9
10	99.5070	99.5437	99.5776	99.6090	99.6381	99.6650	99.6898	99.7129	99.7342	99.7539	10
11	99.4336	99.4757	99.5147	99.5508	99.5841	99.6150	99.6436	99.6701	99.6946	99.7172	11
12	99.3542	99.4023	99.4467	99.4878	99.5259	99.5611	99.5937	99.6239	99.6518	99.6776	12
13	99.2685	99.3229	99.3733	99.4199	99.4630	99.5029	99.5398	99.5740	99.6056	99.6349	13
14	99.1760	99.2373	99.2940	99.3465	99.3950	99.4400	99.4816	99.5201	99.5557	99.5887	14
15	99.0760	99.1448	99.2083	99.2672	99.3216	99.3720	99.4187	99.4618	99.5018	99.5388	15
16	98.9681	99.0448	99.1158	99.1816	99.2424	99.2987	99.3508	99.3990	99.4436	99.4849	16
17	98.8515	98.9369	99.0159	99.0891	99.1568	99.2194	99.2774	99.3311	99.3807	99.4267	17
18	98.7256	98.8204	98.9081	98.9893	99.0643	99.1339	99.1982	99.2577	99.3128	99.3638	18
19	98.5896	98.6945	98.7916	98.8814	98.9645	99.0414	99.1126	99.1785	99.2395	99.2960	19
20	98.4428	98.5586	98.6657	98.7649	98.8567	98.9416	99.0202	99.0930	99.1603	99.2226	20
21	98.2842	98.4118	98.5298	98.6391	98.7402	98.8338	98.9204	99.0006	99.0748	99.1435	21
22	98.1129	98.2532	98.3831	98.5033	98.6145	98.7174	98.8127	98.9008	98.9824	99.0580	22
23	97.9279	98.0820	98.2245	98.3565	98.4786	98.5917	98.6963	98.7931	98.8827	98.9656	23
24	97.7281	97.8970	98.0533	98.1981	98.3319	98.4559	98.5706	98.6767	98.7749	98.8659	24
25	97.5123	97.6973	97.8685	98.0269	98.1735	98.3092	98.4348	98.5510	98.6586	98.7581	25
26	97.2792	97.4816	97.6688	97.8421	98.0024	98.1508	98.2882	98.4153	98.5329	98.6418	26
27	97.0275	97.2486	97.4532	97.6425	97.8176	97.9798	98.1298	98.2687	98.3972	98.5162	27
28	96.7557	96.9970	97.2203	97.4269	97.6181	97.7950	97.9588	98.1104	98.2506	98.3805	28
29	96.4621	96.7253	96.9687	97.1941	97.4025	97.5955	97.7741	97.9394	98.0923	98.2340	29
30	96.1451	96.4318	96.6971	96.9426	97.1697	97.3800	97.5746	97.7547	97.9214	98.0757	30
31	95.8027	96.1148	96.4037	96.6710	96.9183	97.1473	97.3592	97.5553	97.7367	97.9047	31
32	95.4328	95.7725	96.0868	96.3776	96.6468	96.8959	97.1265	97.3399	97.5373	97.7201	32
33	95.0334	95.4028	95.7446	96.0609	96.3536	96.6245	96.8752	97.1073	97.3220	97.5207	33
34	94.6021	95.0036	95.3750	95.7187	96.0369	96.3313	96.6038	96.8560	97.0894	97.3054	34
35	94.1362	94.5724	94.9758	95.3493	95.6948	96.0147	96.3107	96.5847	96.8382	97.0729	35
36	93.6331	94.1067	94.5448	94.9502	95.3254	95.6727	95.9941	96.2916	96.5669	96.8217	36
37	93.0897	93.6037	94.0792	94.5192	94.9265	95.3034	95.6522	95.9751	96.2739	96.5505	37
38	92.5029	93.0605	93.5764	94.0538	94.4956	94.9045	95.2830	95.6333	95.9575	96.2575	38
39	91.8691	92.4738	93.0333	93.5511	94.0303	94.4738	94.8842	95.2641	95.6157	95.9412	39
40	91.1846	91.8402	92.4468	93.0082	93.5277	94.0085	94.4535	94.8654	95.2466	95.5995	40
41	90.4454	91.1559	91.8134	92.4219	92.9850	93.5061	93.9884	94.4348	94.8480	95.2304	41
42	89.6470	90.4169	91.1293	91.7886	92.3988	92.9635	93.4861	93.9698	94.4175	94.8319	42
43	88.7847	89.6187	90.3905	91.1047	91.7657	92.3774	92.9435	93.4675	93.9525	94.4015	43
44	87.8534	88.7567	89.5926	90.3661	91.0820	91.7445	92.3576	92.9251	93.4504	93.9366	44
45	86.8477	87.8258	88.7308	89.5684	90.3435	91.0609	91.7248	92.3393	92.9081	93.4345	45

8% INTEREST RATE

71–80 Years Probable Life

Age, years	Condition-percent, %										Age, years
	71	72	73	74	75	76	77	78	79	80	
46	85.7615	86.8203	87.8002	88.7069	89.5460	90.3226	91.0414	91.7066	92.3224	92.8923	46
47	84.5883	85.7344	86.7950	87.7764	88.6847	89.5253	90.3033	91.0233	91.6898	92.3067	47
48	83.3214	84.5617	85.7095	86.7716	87.7545	88.6642	89.5061	90.2854	91.0067	91.6742	48
49	81.9531	83.2951	84.5371	85.6863	86.7499	87.7342	88.6452	89.4884	90.2688	90.9912	49
50	80.4753	81.9273	83.2709	84.5142	85.6649	86.7298	87.7154	88.6277	89.4720	90.2535	50
51	78.8793	80.4499	81.9034	83.2484	84.4931	85.6451	86.7113	87.6981	88.6114	89.4568	51
52	77.1556	78.8544	80.4265	81.8813	83.2276	84.4736	85.6268	86.6941	87.6320	88.5964	52
53	75.2940	77.1313	78.8314	80.4048	81.8608	83.2083	84.4555	85.6098	86.6782	87.6671	53
54	73.2835	75.2703	77.1088	78.8102	80.3847	81.8419	83.1905	84.4387	85.5941	86.6635	54
55	71.1122	73.2604	75.2484	77.0880	78.7905	80.3661	81.8243	83.1740	84.4233	85.5795	55
56	68.7671	71.0898	73.2391	75.2280	77.0688	78.7723	80.3489	81.8081	83.1588	84.4089	56
57	66.2344	68.7454	71.0690	73.2193	75.2093	77.0509	78.7554	80.3330	81.7931	83.1446	57
58	63.4992	66.2136	68.7254	71.0498	73.2010	75.1919	77.0344	78.7398	80.3182	81.7792	58
59	60.5451	63.4792	66.1943	68.7068	71.0321	73.1841	75.1757	77.0191	78.7253	80.3046	59
60	57.3546	60.5260	63.4606	66.1764	68.6897	71.0157	73.1684	75.1608	77.0050	78.7120	60
61	53.9090	57.3366	60.5084	63.4435	66.1599	68.6738	71.0005	73.1539	75.1471	76.9919	61
62	50.1877	53.8920	57.3199	60.4920	63.4277	66.1446	68.6591	70.9864	73.1405	75.1343	62
63	46.1687	50.1719	53.8763	57.3044	60.4770	63.4130	66.1304	68.6455	70.9734	73.1281	63
64	41.8281	46.1541	50.1572	53.8617	57.2901	60.4629	63.3994	66.1173	68.6329	70.9613	64
65	37.1403	41.8150	46.1407	50.1437	53.8483	57.2768	60.4500	63.3869	66.1052	68.6212	65
66	32.0775	37.1286	41.8028	46.1282	50.1312	53.8359	57.2646	60.4380	63.3752	66.0939	66
67	26.6097	32.0674	37.1178	41.7915	46.1167	50.1196	53.8243	57.2532	60.4269	63.3645	67
68	20.7045	26.6013	32.0581	37.1078	41.7810	46.1060	50.1088	53.8136	57.2427	60.4166	68
69	14.3268	20.6979	26.5936	32.0494	37.0985	41.7714	46.0961	50.0989	53.8038	57.2330	69
70	7.4389	14.3223	20.6919	26.5864	32.0414	37.0900	41.7624	46.0870	50.0897	53.7946	70
71	0.0000	7.4366	14.3181	20.6863	26.5798	32.0340	37.0820	41.7542	46.0786	50.0812	71
72		0.0000	7.4344	14.3142	20.6812	26.5736	32.0272	37.0747	41.7465	46.0707	72
73			0.0000	7.4324	14.3107	20.6764	26.5679	32.0208	37.0679	41.7394	73
74				0.0000	7.4305	14.3074	20.6720	26.5627	32.0149	37.0616	74
75					0.0000	7.4288	14.3043	20.6679	26.5578	32.0095	75
76						0.0000	7.4272	14.3015	20.6641	26.5533	76
77							0.0000	7.4258	14.2988	20.6606	77
78								0.0000	7.4244	14.2964	78
79									0.0000	7.4231	79
80										0.0000	80

81–90 Years Probable Life

Age, years	81	82	83	84	85	86	87	88	89	90	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9843	99.9854	99.9865	99.9875	99.9884	99.9893	99.9901	99.9908	99.9915	99.9921	1
2	99.9673	99.9697	99.9720	99.9740	99.9760	99.9778	99.9794	99.9809	99.9823	99.9837	2
3	99.9489	99.9527	99.9562	99.9595	99.9625	99.9653	99.9679	99.9702	99.9724	99.9745	3
4	99.9291	99.9344	99.9393	99.9438	99.9479	99.9518	99.9554	99.9587	99.9617	99.9646	4
5	99.9077	99.9146	99.9209	99.9268	99.9322	99.9373	99.9419	99.9462	99.9502	99.9539	5
6	99.8846	99.8932	99.9011	99.9085	99.9152	99.9215	99.9274	99.9327	99.9377	99.9423	6
7	99.8597	99.8701	99.8797	99.8887	99.8969	99.9046	99.9116	99.9182	99.9243	99.9299	7
8	99.8327	99.8451	99.8566	99.8673	99.8771	99.8862	99.8947	99.9025	99.9097	99.9164	8
9	99.8036	99.8182	99.8317	99.8442	99.8557	99.8664	99.8763	99.8855	99.8940	99.9019	9
10	99.7722	99.7891	99.8047	99.8192	99.8326	99.8451	99.8565	99.8672	99.8770	99.8862	10
11	99.7382	99.7577	99.7756	99.7923	99.8077	99.8220	99.8352	99.8474	99.8587	99.8692	11
12	99.7016	99.7237	99.7442	99.7632	99.7808	99.7970	99.8121	99.8260	99.8389	99.8509	12
13	99.6620	99.6870	99.7103	99.7318	99.7517	99.7701	99.7871	99.8029	99.8175	99.8311	13
14	99.6192	99.6475	99.6736	99.6978	99.7202	99.7410	99.7602	99.7780	99.7944	99.8097	14
15	99.5730	99.6047	99.6340	99.6612	99.6863	99.7096	99.7311	99.7511	99.7695	99.7866	15
16	99.5231	99.5585	99.5913	99.6216	99.6497	99.6757	99.6997	99.7220	99.7426	99.7617	16
17	99.4692	99.5086	99.5451	99.5788	99.6101	99.6390	99.6658	99.6906	99.7135	99.7348	17
18	99.4111	99.4548	99.4952	99.5327	99.5673	99.5994	99.6291	99.6567	99.6821	99.7057	18
19	99.3482	99.3966	99.4413	99.4828	99.5212	99.5567	99.5896	99.6200	99.6482	99.6743	19
20	99.2803	99.3338	99.3832	99.4289	99.4713	99.5105	99.5468	99.5805	99.6115	99.6404	20

8% INTEREST RATE

81–90 Years Probable Life

Age, years	Condition-percent, % 81	82	83	84	85	86	87	88	89	90	Age, years
21	99.2070	99.2659	99.3203	99.3708	99.4175	99.4607	99.5007	99.5377	99.5720	99.6037	21
22	99.1279	99.1926	99.2525	99.3080	99.3593	99.4068	99.4508	99.4916	99.5292	99.5642	22
23	99.0424	99.1135	99.1792	99.2401	99.2965	99.3487	99.3970	99.4417	99.4831	99.5214	23
24	98.9501	99.0280	99.1001	99.1668	99.2287	99.2859	99.3388	99.3879	99.4332	99.4753	24
25	98.8503	98.9357	99.0146	99.0877	99.1554	99.2181	99.2760	99.3298	99.3794	99.4255	25
26	98.7426	98.8359	98.9223	99.0022	99.0763	99.1448	99.2082	99.2670	99.3213	99.3716	26
27	98.6263	98.7283	98.8226	98.9099	98.9908	99.0657	99.1350	99.1991	99.2585	99.3135	27
28	98.5007	98.6120	98.7149	98.8103	98.8986	98.9803	99.0559	99.1259	99.1907	99.2507	28
29	98.3650	98.4864	98.5986	98.7026	98.7989	98.8880	98.9704	99.0468	99.1175	99.1829	29
30	98.2185	98.3507	98.4731	98.5863	98.6912	98.7883	98.8782	98.9614	99.0384	99.1097	30
31	98.0603	98.2042	98.3374	98.4608	98.5750	98.6807	98.7785	98.8691	98.9530	99.0306	31
32	97.8894	98.0460	98.1910	98.3251	98.4494	98.5645	98.6709	98.7695	98.8607	98.9452	32
33	97.7048	97.8751	98.0327	98.1787	98.3138	98.4389	98.5547	98.6619	98.7611	98.8529	33
34	97.5054	97.6906	97.8619	98.0205	98.1674	98.3033	98.4291	98.5456	98.6535	98.7533	34
35	97.2901	97.4912	97.6773	97.8497	98.0092	98.1569	98.2935	98.4201	98.5373	98.6457	35
36	97.0576	97.2760	97.4781	97.6652	97.8384	97.9988	98.1472	98.2846	98.4118	98.5295	36
37	96.8065	97.0435	97.2628	97.4659	97.6539	97.8280	97.9890	98.1381	98.2762	98.4040	37
38	96.5353	96.7924	97.0304	97.2507	97.4547	97.6435	97.8182	97.9800	98.1298	98.2685	38
39	96.2424	96.5213	96.7794	97.0183	97.2395	97.4443	97.6338	97.8093	97.9717	98.1221	39
40	95.9261	96.2284	96.5082	96.7673	97.0071	97.2291	97.4346	97.6248	97.8010	97.9640	40
41	95.5844	95.9121	96.2154	96.4962	96.7561	96.9968	97.2195	97.4257	97.6166	97.7933	41
42	95.2155	95.5705	95.8992	96.2034	96.4851	96.7458	96.9871	97.2105	97.4174	97.6089	42
43	94.8170	95.2016	95.5576	95.8872	96.1923	96.4748	96.7362	96.9782	97.2023	97.4097	43
44	94.3866	94.8032	95.1888	95.5457	95.8761	96.1821	96.4652	96.7273	96.9700	97.1947	44
45	93.9218	94.3729	94.7904	95.1768	95.5347	95.8659	96.1725	96.4563	96.7191	96.9624	45
46	93.4198	93.9081	94.3601	94.7786	95.1659	95.5245	95.8564	96.1637	96.4482	96.7115	46
47	92.8777	93.4062	93.8955	94.3484	94.7676	95.1557	95.5150	95.8476	96.1555	96.4406	47
48	92.2922	92.8642	93.3936	93.8838	94.3375	94.7575	95.1463	95.5062	95.8395	96.1480	48
49	91.6598	92.2787	92.8516	93.3820	93.8729	94.3274	94.7481	95.1376	95.4981	95.8319	49
50	90.9769	91.6465	92.2663	92.8401	93.3712	93.8629	94.3180	94.7394	95.1295	95.4906	50
51	90.2393	90.9636	91.6341	92.2548	92.8293	93.3612	93.8536	94.3094	94.7314	95.1220	51
52	89.4427	90.2262	90.9514	91.6227	92.2441	92.8194	93.3520	93.8450	94.3014	94.7239	52
53	88.5824	89.4297	90.2140	90.9400	91.6121	92.2343	92.8102	93.3434	93.8370	94.2940	53
54	87.6533	88.5695	89.4177	90.2028	90.9295	91.6023	92.2251	92.8017	93.3355	93.8296	54
55	86.6498	87.6405	88.5576	89.4065	90.1923	90.9198	91.5932	92.2167	92.7938	93.3281	55
56	85.5661	86.6372	87.6287	88.5465	89.3962	90.1827	90.9108	91.5848	92.2088	92.7865	56
57	84.3957	85.5536	86.6255	87.6178	88.5363	89.3866	90.1738	90.9025	91.5771	92.2016	57
58	83.1316	84.3834	85.5421	86.6147	87.6077	88.5268	89.3777	90.1655	90.8947	91.5699	58
59	81.7664	83.1195	84.3720	85.5314	86.6047	87.5983	88.5181	89.3695	90.1578	90.8876	59
60	80.2920	81.7545	83.1083	84.3615	85.5215	86.5954	87.5896	88.5099	89.3620	90.1507	60
61	78.6996	80.2803	81.7434	83.0979	84.3517	85.5124	86.5869	87.5816	88.5024	89.3549	61
62	76.9798	78.6881	80.2694	81.7332	83.0883	84.3427	85.5039	86.5789	87.5741	88.4955	62
63	75.1225	76.9686	78.6775	80.2594	81.7238	83.0794	84.3343	85.4961	86.5716	87.5673	63
64	73.1166	75.1116	76.9582	78.6677	80.2501	81.7151	83.0712	84.3266	85.4888	86.5648	64
65	70.9502	73.1059	75.1014	76.9486	78.6586	80.2416	81.7070	83.0636	84.3194	85.4821	65
66	68.6105	70.9398	73.0961	75.0920	76.9397	78.6502	80.2336	81.6995	83.0565	84.3128	66
67	66.0836	68.6005	70.9303	73.0869	75.0834	76.9315	78.6424	80.2263	81.6925	83.0500	67
68	63.3545	66.0739	68.5912	70.9214	73.0785	75.0753	76.9239	78.6352	80.2195	81.6861	68
69	60.4072	63.3453	66.0650	68.5826	70.9132	73.0707	75.0679	76.9168	78.6285	80.2131	69
70	57.2240	60.3984	63.3367	66.0568	68.5747	70.9056	73.0634	75.0610	76.9103	78.6223	70
71	53.7862	57.2157	60.3902	63.3288	66.0491	68.5672	70.8986	73.0567	75.0547	76.9043	71
72	50.0734	53.7784	57.2079	60.3827	63.3215	66.0421	68.5606	70.8921	73.0505	75.0488	72
73	46.0635	50.0661	53.7711	57.2008	60.3757	63.3148	66.0355	68.5543	70.8861	73.0448	73
74	41.7328	46.0568	50.0593	53.7644	57.1942	60.3692	63.3085	66.0295	68.5485	70.8805	74
75	37.0557	41.7268	46.0506	50.0531	53.7582	57.1881	60.3633	63.3027	66.0238	68.5431	75
76	32.0045	37.0504	41.7211	46.0448	50.0473	53.7524	57.1824	60.3577	63.2973	66.0187	76
77	26.5491	31.9998	37.0453	41.7159	46.0395	50.0419	53.7471	57.1772	60.3526	63.2923	77
78	20.6573	26.5453	31.9955	37.0407	41.7111	46.0346	50.0370	53.7422	57.1723	60.3479	78
79	14.2942	20.6543	26.5417	31.9915	37.0364	41.7066	46.0300	50.0324	53.7376	57.1678	79
80	7.4220	14.2921	20.6515	26.5383	31.9878	37.0325	41.7025	46.0258	50.0281	53.7334	80

8% INTEREST RATE

81–90 Years Probable Life

Age, years	Condition-percent, % 81	82	83	84	85	86	87	88	89	90	Age, years
81	0.0000	7.4209	14.2902	20.6489	26.5353	31.9844	37.0288	41.6987	46.0219	50.0242	81
82		0.0000	7.4199	14.2884	20.6465	26.5324	31.9812	37.0254	41.6952	46.0183	82
83			0.0000	7.4190	14.2867	20.6443	26.5298	31.9783	37.0223	41.6919	83
84				0.0000	7.4181	14.2852	20.6423	26.5274	31.9756	37.0194	84
85					0.0000	7.4173	14.2838	20.6404	26.5251	31.9731	85
86						0.0000	7.4166	14.2825	20.6387	26.5230	86
87							0.0000	7.4159	14.2813	20.6370	87
88								0.0000	7.4153	14.2801	88
89									0.0000	7.4147	89
90										0.0000	90

91–100 Years Probable Life

Age, years	91	92	93	94	95	96	97	98	99	100	Age, years
0	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	0
1	99.9927	99.9933	99.9938	99.9942	99.9947	99.9950	99.9954	99.9958	99.9961	99.9964	1
2	99.9849	99.9860	99.9870	99.9880	99.9889	99.9897	99.9905	99.9912	99.9918	99.9924	2
3	99.9764	99.9781	99.9798	99.9813	99.9826	99.9839	99.9851	99.9862	99.9872	99.9882	3
4	99.9672	99.9696	99.9719	99.9740	99.9759	99.9777	99.9793	99.9809	99.9823	99.9836	4
5	99.9573	99.9605	99.9634	99.9661	99.9686	99.9710	99.9731	99.9751	99.9769	99.9787	5
6	99.9466	99.9506	99.9542	99.9576	99.9608	99.9637	99.9664	99.9689	99.9712	99.9733	6
7	99.9351	99.9399	99.9443	99.9485	99.9523	99.9558	99.9591	99.9621	99.9649	99.9675	7
8	99.9226	99.9283	99.9336	99.9386	99.9431	99.9473	99.9512	99.9549	99.9582	99.9613	8
9	99.9091	99.9159	99.9221	99.9279	99.9332	99.9382	99.9428	99.9470	99.9509	99.9546	9
10	99.8946	99.9024	99.9096	99.9163	99.9225	99.9283	99.9336	99.9385	99.9431	99.9473	10
11	99.8789	99.8879	99.8962	99.9039	99.9110	99.9176	99.9237	99.9294	99.9346	99.9394	11
12	99.8619	99.8722	99.8816	99.8904	99.8985	99.9060	99.9130	99.9195	99.9254	99.9310	12
13	99.8436	99.8552	99.8659	99.8759	99.8851	99.8936	99.9015	99.9088	99.9155	99.9218	13
14	99.8238	99.8369	99.8489	99.8602	99.8705	99.8801	99.8890	99.8972	99.9048	99.9119	14
15	99.8024	99.8171	99.8306	99.8432	99.8548	99.8656	99.8755	99.8848	99.8933	99.9012	15
16	99.7793	99.7957	99.8108	99.8249	99.8379	99.8499	99.8610	99.8713	99.8808	99.8897	16
17	99.7544	99.7726	99.7895	99.8051	99.8195	99.8329	99.8453	99.8568	99.8674	99.8772	17
18	99.7275	99.7477	99.7664	99.7837	99.7998	99.8146	99.8283	99.8411	99.8528	99.8638	18
19	99.6984	99.7208	99.7415	99.7606	99.7784	99.7948	99.8100	99.8241	99.8371	99.8492	19
20	99.6670	99.6917	99.7145	99.7357	99.7553	99.7734	99.7902	99.8058	99.8202	99.8335	20
21	99.6331	99.6603	99.6855	99.7088	99.7304	99.7504	99.7689	99.7860	99.8018	99.8166	21
22	99.5964	99.6264	99.6541	99.6797	99.7035	99.7254	99.7458	99.7646	99.7821	99.7982	22
23	99.5569	99.5898	99.6201	99.6483	99.6744	99.6985	99.7209	99.7416	99.7607	99.7785	23
24	99.5142	99.5502	99.5835	99.6144	99.6430	99.6694	99.6940	99.7167	99.7376	99.7571	24
25	99.4681	99.5075	99.5440	99.5778	99.6091	99.6381	99.6649	99.6897	99.7127	99.7340	25
26	99.4182	99.4613	99.5012	99.5383	99.5725	99.6041	99.6335	99.6607	99.6858	99.7091	26
27	99.3644	99.4115	99.4551	99.4955	99.5329	99.5675	99.5996	99.6293	99.6567	99.6822	27
28	99.3063	99.3577	99.4053	99.4494	99.4902	99.5280	99.5630	99.5954	99.6254	99.6531	28
29	99.2435	99.2996	99.3515	99.3996	99.4441	99.4853	99.5234	99.5588	99.5914	99.6217	29
30	99.1757	99.2368	99.2933	99.3458	99.3943	99.4391	99.4807	99.5192	99.5548	99.5878	30
31	99.1025	99.1690	99.2306	99.2877	99.3405	99.3893	99.4346	99.4765	99.5153	99.5512	31
32	99.0234	99.0958	99.1628	99.2249	99.2824	99.3355	99.3848	99.4304	99.4726	99.5117	32
33	98.9380	99.0167	99.0896	99.1571	99.2196	99.2774	99.3310	99.3806	99.4265	99.4690	33
34	98.8458	98.9313	99.0106	99.0839	99.1518	99.2146	99.2729	99.3268	99.3767	99.4228	34
35	98.7461	98.8391	98.9252	99.0048	99.0786	99.1469	99.2101	99.2687	99.3229	99.3730	35
36	98.6386	98.7395	98.8329	98.9194	98.9996	99.0737	99.1423	99.2059	99.2648	99.3192	36
37	98.5224	98.6319	98.7333	98.8272	98.9142	98.9946	99.0691	99.1381	99.2020	99.2611	37
38	98.3969	98.5157	98.6258	98.7276	98.8219	98.9092	98.9901	99.0649	99.1342	99.1984	38
39	98.2613	98.3903	98.5096	98.6201	98.7223	98.8170	98.9047	98.9859	99.0611	99.1306	39
40	98.1150	98.2547	98.3841	98.5039	98.6148	98.7174	98.8125	98.9005	98.9820	99.0574	40
41	97.9569	98.1084	98.2486	98.3784	98.4986	98.6099	98.7129	98.8083	98.8966	98.9784	41
42	97.7862	97.9503	98.1022	98.2429	98.3732	98.4937	98.6053	98.7087	98.8044	98.8930	42
43	97.6018	97.7796	97.9442	98.0966	98.2377	98.3683	98.4892	98.6012	98.7049	98.8008	43
44	97.4027	97.5952	97.7735	97.9385	98.0913	98.2328	98.3638	98.4850	98.5973	98.7013	44
45	97.1876	97.3961	97.5891	97.7678	97.9333	98.0865	98.2283	98.3596	98.4812	98.5937	45

8% INTEREST RATE
91–100 Years Probable Life

Age, years	Condition-percent, % 91	92	93	94	95	96	97	98	99	100	Age, years
46	96.9553	97.1811	97.3900	97.5835	97.7626	97.9284	98.0820	98.2241	98.3557	98.4776	46
47	96.7045	96.9488	97.1750	97.3844	97.5783	97.7578	97.9240	98.0778	98.2203	98.3522	47
48	96.4336	96.6980	96.9427	97.1694	97.3792	97.5734	97.7533	97.9198	98.0740	98.2167	48
49	96.1410	96.4271	96.6919	96.9371	97.1642	97.3744	97.5690	97.7491	97.9160	98.0704	49
50	95.8250	96.1345	96.4210	96.6863	96.9320	97.1594	97.3699	97.5648	97.7453	97.9124	50
51	95.4837	95.8185	96.1285	96.4155	96.6812	96.9272	97.1549	97.3658	97.5610	97.7417	51
52	95.1151	95.4773	95.8125	96.1229	96.4103	96.6764	96.9227	97.1508	97.3619	97.5574	52
53	94.7170	95.1087	95.4713	95.8070	96.1178	96.4055	96.6720	96.9186	97.1470	97.3584	53
54	94.2871	94.7107	95.1028	95.4658	95.8019	96.1130	96.4011	96.6678	96.9148	97.1434	54
55	93.8228	94.2808	94.7047	95.0973	95.4607	95.7971	96.1086	96.3970	96.6641	96.9113	55
56	93.3213	93.8165	94.2749	94.6993	95.0922	95.4559	95.7927	96.1045	96.3932	96.6605	56
57	92.7798	93.3151	93.8106	94.2694	94.6942	95.0875	95.4516	95.7887	96.1008	96.3897	57
58	92.1949	92.7735	93.3092	93.8052	94.2644	94.6895	95.0831	95.4475	95.7849	96.0973	58
59	91.5632	92.1887	92.7677	93.3039	93.8002	94.2597	94.6852	95.0791	95.4438	95.7814	59
60	90.8810	91.5570	92.1829	92.7624	93.2989	93.7956	94.2554	94.6812	95.0753	95.4403	60
61	90.1442	90.8749	91.5513	92.1776	92.7574	93.2942	93.7913	94.2514	94.6774	95.0719	61
62	89.3484	90.1381	90.8692	91.5460	92.1727	92.7528	93.2900	93.7873	94.2477	94.6740	62
63	88.4890	89.3424	90.1325	90.8639	91.5411	92.1681	92.7486	93.2860	93.7836	94.2443	63
64	87.5609	88.4831	89.3369	90.1273	90.8591	91.5366	92.1639	92.7446	93.2823	93.7802	64
65	86.5585	87.5550	88.4776	89.3317	90.1225	90.8546	91.5324	92.1600	92.7410	93.2789	65
66	85.4759	86.5527	87.5495	88.4724	89.3269	90.1180	90.8504	91.5285	92.1564	92.7376	66
67	84.3067	85.4701	86.5473	87.5445	88.4677	89.3225	90.1139	90.8466	91.5249	92.1530	67
68	83.0439	84.3010	85.4648	86.5423	87.5398	88.4633	89.3184	90.1100	90.8430	91.5216	68
69	81.6802	83.0383	84.2957	85.4599	86.5376	87.5355	88.4593	89.3146	90.1065	90.8397	69
70	80.2073	81.6747	83.0332	84.2909	85.4553	86.5333	87.5314	88.4555	89.2311	90.1032	70
71	78.6166	80.2019	81.6696	83.0284	84.2864	85.4511	86.5294	87.5277	88.4521	89.3078	71
72	76.8987	78.6113	80.1969	81.6649	83.0239	84.2822	85.4471	86.5257	87.5243	88.4488	72
73	75.0433	76.8935	78.6064	80.1923	81.6605	83.0198	84.2783	85.4435	86.5223	87.5211	73
74	73.0395	75.0382	76.8887	78.6019	80.1880	81.6565	83.0160	84.2748	85.4402	86.5192	74
75	70.8754	73.0346	75.0336	76.8843	78.5977	80.1840	81.6527	83.0125	84.2714	85.4370	75
76	68.5381	70.8706	73.0300	75.0292	76.8801	78.5938	80.1803	81.6492	83.0092	84.2684	76
77	66.0139	68.5335	70.8662	73.0258	75.0252	76.8763	78.5902	80.1769	81.6460	83.0062	77
78	63.2877	66.0094	68.5292	70.8621	73.0219	75.0215	76.8728	78.5868	80.1738	81.6431	78
79	60.3435	63.2835	66.0053	68.5253	70.8583	73.0183	75.0181	76.8696	78.5838	80.1709	79
80	57.1637	60.3394	63.2795	66.0015	68.5216	70.8548	73.0149	75.0149	76.8665	78.5809	80
81	53.7295	57.1598	60.3356	63.2759	65.9980	68.5182	70.8515	73.0118	75.0119	76.8637	81
82	50.0206	53.7259	57.1562	60.3321	63.2725	65.9947	68.5151	70.8485	73.0090	75.0092	82
83	46.0149	50.0172	53.7225	57.1529	60.3289	63.2693	65.9917	68.5122	70.8457	73.0063	83
84	41.6888	46.0118	50.0141	53.7194	57.1499	60.3259	63.2664	65.9889	68.5095	70.8432	84
85	37.0167	41.6860	46.0090	50.0112	53.7165	57.1471	60.3232	63.2638	65.9863	68.5070	85
86	31.9723	37.0142	41.6834	46.0063	50.0085	53.7139	57.1444	60.3206	63.2613	65.9839	86
87	26.5211	31.9686	37.0119	41.6810	46.0038	50.0060	53.7114	57.1420	60.3182	63.2590	87
88	20.6355	26.5193	31.9666	37.0097	41.6788	46.0016	50.0037	53.7091	57.1398	60.3160	88
89	14.2791	20.6341	26.5177	31.9647	37.0078	41.6767	45.9995	50.0016	53.7070	57.1377	89
90	7.4141	14.2781	20.6329	26.5161	31.9630	37.0059	41.6748	45.9975	49.9997	53.7051	90
91	0.0000	7.4136	14.2772	20.6317	26.5147	31.9614	37.0042	41.6731	45.9957	49.9978	91
92		0.0000	7.4132	14.2764	20.6306	26.5134	31.9600	37.0027	41.6714	45.9940	92
93			0.0000	7.4128	14.2757	20.6295	26.5122	31.9586	37.0012	41.6699	93
94				0.0000	7.4124	14.2749	20.6286	26.5111	31.9574	36.9999	94
95					0.0000	7.4120	14.2743	20.6277	26.5100	31.9562	95
96						0.0000	7.4117	14.2737	20.6269	26.5091	96
97							0.0000	7.4113	14.2731	20.6262	97
98								0.0000	7.4110	14.2726	98
99									0.0000	7.4108	99
100										0.0000	100

APPENDIX D[1]

TABLE D.1. COMPOUND AMOUNT OF 1.

TABLE D.2. THE AMOUNT OF 1 PER ANNUM AT COMPOUND INTEREST. FOR DERIVATION OF FORMULA EMPLOYED SEE SEC. 5.6.

[1] Tables D.1 and D.2 are taken from "Mathematical Principles of Finance" by Frederick Charles Kent, with the permission of the author.

TABLE D.1 COMPOUND AMOUNT OF 1 $s = (1 + i)^n$

n	2 per cent	$2\frac{1}{4}$ per cent	$2\frac{1}{2}$ per cent	$2\frac{3}{4}$ per cent	n
1	1.0200 000 000	1.0225 000 000	1.0250 000 000	1.0275 000 000	1
2	1.0404 000 000	1.0455 062 500	1.0506 250 000	1.0557 562 500	2
3	1.0612 080 000	1.0690 301 406	1.0768 906 250	1.0847 895 469	3
4	1.0824 321 600	1.0930 833 188	1.1038 128 906	1.1146 212 594	4
5	1.1040 808 032	1.1176 776 935	1.1314 082 129	1.1452 733 440	5
6	1.1261 624 193	1.1428 254 416	1.1596 934 182	1.1767 683 610	6
7	1.1486 856 676	1.1685 390 140	1.1886 857 537	1.2091 294 909	7
8	1.1716 593 810	1.1948 311 418	1.2184 028 975	1.2423 805 519	8
9	1.1950 925 686	1.2217 148 425	1.2488 629 699	1.2765 460 171	9
10	1.2189 944 200	1.2492 034 265	1.2800 845 442	1.3116 510 326	10
11	1.2433 743 084	1.2773 105 036	1.3120 866 578	1.3477 214 360	11
12	1.2682 417 946	1.3060 499 899	1.3448 888 242	1.3847 837 755	12
13	1.2936 066 305	1.3354 361 147	1.3785 110 449	1.4228 653 293	13
14	1.3194 787 631	1.3654 834 272	1.4129 738 210	1.4619 941 259	14
15	1.3458 683 383	1.3962 068 044	1.4482 981 665	1.5021 989 643	15
16	1.3727 857 051	1.4276 214 575	1.4845 056 207	1.5435 094 358	16
17	1.4002 414 192	1.4597 429 402	1.5216 182 612	1.5859 559 453	17
18	1.4282 462 476	1.4925 871 564	1.5596 587 177	1.6295 697 338	18
19	1.4568 111 725	1.5261 703 674	1.5986 501 856	1.6743 829 015	19
20	1.4859 473 960	1.5605 092 007	1.6386 164 403	1.7204 284 313	20
21	1.5156 663 439	1.5956 206 577	1.6795 818 513	1.7677 402 131	21
22	1.5459 796 708	1.6315 221 225	1.7215 713 976	1.8163 530 690	22
23	1.5768 992 642	1.6682 313 703	1.7646 106 825	1.8663 027 784	23
24	1.6084 372 495	1.7057 665 761	1.8087 259 496	1.9176 261 048	24
25	1.6406 059 945	1.7441 463 240	1.8539 440 983	1.9703 608 227	25
26	1.6734 181 144	1.7833 896 163	1.9002 927 008	2.0245 457 453	26
27	1.7068 864 766	1.8235 158 827	1.9478 000 183	2.0802 207 533	27
28	1.7410 242 062	1.8645 449 901	1.9964 950 188	2.1374 268 240	28
29	1.7758 446 903	1.9064 972 523	2.0464 073 942	2.1962 060 617	29
30	1.8113 615 841	1.9493 934 405	2.0975 675 791	2.2566 017 284	30
31	1.8475 888 158	1.9932 547 929	2.1500 067 686	2.3186 582 759	31
32	1.8845 405 921	2.0381 030 258	2.2037 569 378	2.3824 213 785	32
33	1.9222 314 039	2.0839 603 439	2.2588 508 612	2.4479 379 664	33
34	1.9606 760 320	2.1308 494 516	2.3153 221 327	2.5152 562 605	34
35	1.9998 895 527	2.1787 935 643	2.3732 051 861	2.5844 258 077	35
36	2.0398 873 437	2.2278 164 194	2.4325 353 157	2.6554 975 174	36
37	2.0806 850 906	2.2779 422 889	2.4933 486 986	2.7285 236 991	37
38	2.1222 987 924	2.3291 959 904	2.5556 824 161	2.8035 581 008	38
39	2.1647 447 682	2.3816 029 002	2.6195 744 765	2.8806 559 486	39
40	2.2080 396 636	2.4351 889 654	2.6850 638 384	2.9598 739 872	40
41	2.2522 004 569	2.4899 807 171	2.7521 904 343	3.0412 705 218	41
42	2.2972 444 660	2.5460 052 833	2.8209 951 952	3.1249 054 612	42
43	2.3431 893 553	2.6032 904 022	2.8915 200 751	3.2108 403 614	43
44	2.3900 531 425	2.6618 644 362	2.9638 080 770	3.2991 384 713	44
45	2.4378 542 053	2.7217 563 860	3.0379 032 789	3.3898 647 793	45
46	2.4866 112 894	2.7829 959 047	3.1138 508 609	3.4830 860 607	46
47	2.5363 435 152	2.8456 133 126	3.1916 971 324	3.5788 709 274	47
48	2.5870 703 855	2.9096 396 121	3.2714 895 607	3.6772 898 779	48
49	2.6388 117 932	2.9751 065 034	3.3532 767 997	3.7784 153 495	49
50	2.6915 880 291	3.0420 463 997	3.4371 087 197	3.8823 217 716	50

TABLE D.1 COMPOUND AMOUNT OF 1 $s = (1 + i)^n$

n	2 per cent	$2\frac{1}{4}$ per cent	$2\frac{1}{2}$ per cent	$2\frac{3}{4}$ per cent	n
51	2.7454 197 897	3.1104 924 437	3.5230 364 377	3.9890 856 203	51
52	2.8003 281 854	3.1804 785 237	3.6111 123 486	4.0987 854 749	52
53	2.8563 347 492	3.2520 392 904	3.7013 901 574	4.2115 020 754	53
54	2.9134 614 441	3.3252 101 745	3.7939 249 113	4.3273 183 825	54
55	2.9717 306 730	3.4000 274 034	3.8887 730 341	4.4463 196 380	55
56	3.0311 652 865	3.4765 280 200	3.9859 923 599	4.5685 934 281	56
57	3.0917 885 922	3.5547 499 004	4.0856 421 689	4.6942 297 474	57
58	3.1536 243 641	3.6347 317 732	4.1877 832 231	4.8233 210 654	58
59	3.2166 968 513	3.7165 132 381	4.2924 778 037	4.9559 623 947	59
60	3.2810 307 884	3.8001 347 859	4.3997 897 488	5.0922 513 606	60
61	3.3466 514 041	3.8856 378 186	4.5097 844 925	5.2322 882 730	61
62	3.4135 844 322	3.9730 646 695	4.6225 291 048	5.3761 762 005	62
63	3.4818 561 209	4.0624 586 246	4.7380 923 325	5.5240 210 460	63
64	3.5514 932 433	4.1538 639 437	4.8565 446 408	5.6759 316 248	64
65	3.6225 231 081	4.2473 258 824	4.9779 582 568	5.8320 197 444	65
66	3.6949 735 703	4.3428 907 148	5.1024 072 132	5.9924 002 874	66
67	3.7688 730 417	4.4406 057 558	5.2299 673 936	6.1571 912 953	67
68	3.8442 505 025	4.5405 193 853	5.3607 165 784	6.3265 140 559	68
69	3.9211 355 126	4.6426 810 715	5.4947 344 929	6.5004 931 925	69
70	3.9995 582 228	4.7471 413 956	5.6321 028 552	6.6792 567 553	70
71	4.0795 493 873	4.8539 520 770	5.7729 054 266	6.8629 363 160	71
72	4.1611 403 751	4.9631 659 988	5.9172 280 622	7.0516 670 647	72
73	4.2443 631 826	5.0748 372 337	6.0651 587 638	7.2455 879 090	73
74	4.3292 504 462	5.1890 210 715	6.2167 877 329	7.4448 415 765	74
75	4.4158 354 551	5.3057 740 456	6.3722 074 262	7.6495 747 199	75
76	4.5041 521 642	5.4251 539 616	6.5315 126 118	7.8599 380 247	76
77	4.5942 352 075	5.5472 199 258	6.6948 004 271	8.0760 863 203	77
78	4.6861 199 117	5.6720 323 741	6.8621 704 378	8.2981 786 942	78
79	4.7798 423 099	5.7996 531 025	7.0337 246 988	8.5263 786 082	79
80	4.8754 391 561	5.9301 452 973	7.2095 678 162	8.7608 540 200	80
81	4.9729 479 392	6.0635 735 665	7.3898 070 116	9.0017 775 055	81
82	5.0724 068 980	6.2000 039 717	7.5745 521 869	9.2493 263 869	82
83	5.1738 550 360	6.3395 040 611	7.7639 159 916	9.5036 828 626	83
84	5.2773 321 367	6.4821 429 025	7.9580 138 914	9.7650 341 413	84
85	5.3828 787 794	6.6279 911 178	8.1569 642 387	10.0335 725 802	85
86	5.4905 363 550	6.7771 209 179	8.3608 883 446	10.3094 958 261	86
87	5.6003 470 821	6.9296 061 386	8.5699 105 533	10.5930 069 613	87
88	5.7123 540 237	7.0855 222 767	8.7841 583 171	10.8843 146 528	88
89	5.8266 011 042	7.2449 465 279	9.0037 622 750	11.1836 333 057	89
90	5.9431 331 263	7.4079 578 248	9.2288 563 319	11.4911 832 216	90
91	6.0619 957 888	7.5746 368 759	9.4595 777 402	11.8071 907 602	91
92	6.1832 357 046	7.7450 662 056	9.6960 671 837	12.1318 885 061	92
93	6.3069 004 187	7.9193 301 952	9.9384 688 633	12.4655 154 401	93
94	6.4330 384 271	8.0975 151 246	10.1869 305 849	12.8083 171 147	94
95	6.5616 991 956	8.2797 092 149	10.4416 038 495	13.1605 458 353	95
96	6.6929 331 795	8.4660 026 723	10.7026 439 457	13.5224 608 458	96
97	6.8267 918 431	8.6564 877 324	10.9702 100 444	13.8943 285 190	97
98	6.9633 276 800	8.8512 587 064	11.2444 652 955	14.2764 225 533	98
99	7.1025 942 336	9.0504 120 272	11.5255 769 279	14.6690 241 735	99
100	7.2446 461 183	9.2540 462 979	11.8137 163 511	15.0724 223 383	100

TABLE D.1 COMPOUND AMOUNT OF 1 $s = (1 + i)^n$

n	3 per cent	$3\frac{1}{2}$ per cent	4 per cent	$4\frac{1}{2}$ per cent	n
1	1.0300 000 000	1.0350 000 000	1.0400 000 000	1.0450 000 000	1
2	1.0609 000 000	1.0712 250 000	1.0816 000 000	1.0920 250 000	2
3	1.0927 270 000	1.1087 178 750	1.1248 640 000	1.1411 661 250	3
4	1.1255 088 100	1.1475 230 006	1.1698 585 600	1.1925 186 006	4
5	1.1592 740 743	1.1876 863 056	1.2166 529 024	1.2461 819 377	5
6	1.1940 522 965	1.2292 553 263	1.2653 190 185	1.3022 601 248	6
7	1.2298 738 654	1.2722 792 628	1.3159 317 792	1.3608 618 305	7
8	1.2667 700 814	1.3168 090 370	1.3685 690 504	1.4221 006 128	8
9	1.3047 731 838	1.3628 973 533	1.4233 118 124	1.4860 951 404	9
10	1.3439 163 793	1.4105 987 606	1.4802 442 849	1.5529 694 217	10
11	1.3842 338 707	1.4599 697 172	1.5394 540 563	1.6228 530 457	11
12	1.4257 608 868	1.5110 686 573	1.6010 322 186	1.6958 814 328	12
13	1.4685 337 135	1.5639 560 604	1.6650 735 073	1.7721 960 972	13
14	1.5125 897 249	1.6186 945 225	1.7316 764 476	1.8519 449 216	14
15	1.5579 674 166	1.6753 488 308	1.8009 435 055	1.9352 824 431	15
16	1.6047 064 391	1.7339 860 398	1.8729 812 457	2.0223 701 530	16
17	1.6528 476 323	1.7946 755 512	1.9479 004 956	2.1133 768 099	17
18	1.7024 330 612	1.8574 891 955	2.0258 165 154	2.2084 787 664	18
19	1.7535 060 531	1.9225 013 174	2.1068 491 760	2.3078 603 108	19
20	1.8061 112 347	1.9897 888 635	2.1911 231 430	2.4117 140 248	20
21	1.8602 945 717	2.0594 314 737	2.2787 680 688	2.5202 411 560	21
22	1.9161 034 089	2.1315 115 753	2.3699 187 915	2.6336 520 080	22
23	1.9735 865 111	2.2061 144 804	2.4647 155 432	2.7521 663 483	23
24	2.0327 941 065	2.2833 284 877	2.5633 041 649	2.8760 138 340	24
25	2.0937 779 297	2.3632 449 843	2.6658 363 315	3.0054 344 565	25
26	2.1565 912 675	2.4459 585 587	2.7724 697 847	3.1406 790 071	26
27	2.2212 890 056	2.5315 671 083	2.8833 685 761	3.2820 095 624	27
28	2.2879 276 757	2.6201 719 571	2.9987 033 192	3.4296 999 927	28
29	2.3565 655 060	2.7118 779 756	3.1186 514 519	3.5840 364 924	29
30	2.4272 624 712	2.8067 937 047	3.2433 975 100	3.7453 181 345	30
31	2.5000 803 453	2.9050 314 844	3.3731 334 104	3.9138 574 506	31
32	2.5750 827 557	3.0067 075 863	3.5080 587 468	4.0899 810 359	32
33	2.6523 352 384	3.1119 423 518	3.6483 810 967	4.2740 301 825	33
34	2.7319 052 955	3.2208 603 342	3.7943 163 406	4.4663 615 407	34
35	2.8138 624 544	3.3335 904 459	3.9460 889 942	4.6673 478 100	35
36	2.8982 783 280	3.4502 661 115	4.1039 325 540	4.8773 784 615	36
37	2.9852 266 778	3.5710 254 254	4.2680 898 561	5.0968 604 922	37
38	3.0747 834 782	3.6960 113 152	4.4388 134 504	5.3262 192 144	38
39	3.1670 269 825	3.8253 717 113	4.6163 659 884	5.5658 990 790	39
40	3.2620 377 920	3.9592 597 212	4.8010 206 279	5.8163 645 376	40
41	3.3598 989 258	4.0978 338 114	4.9930 614 531	6.0781 009 418	41
42	3.4606 958 935	4.2412 579 948	5.1927 839 112	6.3516 154 842	42
43	3.5645 167 703	4.3897 020 246	5.4004 952 676	6.6374 381 810	43
44	3.6714 522 734	4.5433 415 955	5.6165 150 783	6.9361 228 991	44
45	3.7815 958 417	4.7023 585 513	5.8411 756 815	7.2482 484 296	45
46	3.8950 437 169	4.8669 411 006	6.0748 227 087	7.5744 196 089	46
47	4.0118 950 284	5.0372 840 392	6.3178 156 171	7.9152 684 913	47
48	4.1322 518 793	5.2135 889 805	6.5705 282 418	8.2714 555 734	48
49	4.2562 194 356	5.3960 645 948	6.8333 493 714	8.6436 710 742	49
50	4.3839 060 187	5.5849 268 557	7.1066 833 463	9.0326 362 725	50

TABLE D.1 COMPOUND AMOUNT OF 1 $s = (1 + i)^n$

n	3 per cent	$3\frac{1}{2}$ per cent	4 per cent	$4\frac{1}{2}$ per cent	n
51	4.5154 231 993	5.7803 992 956	7.3909 506 801	9.4391 049 048	51
52	4.6508 858 952	5.9827 132 710	7.6865 887 073	9.8638 646 255	52
53	4.7904 124 721	6.1921 082 354	7.9940 522 556	10.3077 385 337	53
54	4.9341 248 463	6.4088 320 237	8.3138 143 459	10.7715 867 677	54
55	5.0821 485 917	6.6331 411 445	8.6463 669 197	11.2563 081 722	55
56	5.2346 130 494	6.8653 010 846	8.9922 215 965	11.7628 420 400	56
57	5.3916 514 409	7.1055 866 225	9.3519 104 603	12.2921 699 318	57
58	5.5534 009 841	7.3542 821 543	9.7259 868 787	12.8453 175 787	58
59	5.7200 030 136	7.6116 820 297	10.1150 263 539	13.4233 568 698	59
60	5.8916 031 040	7.8780 909 008	10.5196 274 081	14.0274 079 289	60
61	6.0683 511 972	8.1538 240 823	10.9404 125 044	14.6586 412 857	61
62	6.2504 017 331	8.4392 079 252	11.3780 290 045	15.3182 801 435	62
63	6.4379 137 851	8.7345 802 025	11.8331 501 647	16.0076 027 500	63
64	6.6310 511 986	9.0402 905 096	12.3064 761 713	16.7279 448 738	64
65	6.8299 827 346	9.3567 006 775	12.7987 352 182	17.4807 023 931	65
66	7.0348 822 166	9.6841 852 012	13.3106 846 269	18.2673 340 008	66
67	7.2459 286 831	10.0231 316 832	13.8431 120 120	19.0893 640 308	67
68	7.4633 065 436	10.3739 412 921	14.3968 364 925	19.9483 854 122	68
69	7.6872 057 399	10.7370 292 374	14.9727 099 522	20.8460 627 557	69
70	7.9178 219 121	11.1128 252 607	15.5716 183 502	21.7841 355 797	70
71	8.1553 565 695	11.5017 741 448	16.1944 830 843	22.7644 216 808	71
72	8.4000 172 666	11.9043 362 399	16.8422 624 076	23.7888 206 564	72
73	8.6520 177 846	12.3209 880 083	17.5159 529 039	24.8593 175 860	73
74	8.9115 783 181	12.7522 225 885	18.2165 910 201	25.9779 868 774	74
75	9.1789 256 676	13.1985 503 791	18.9452 546 609	27.1469 962 869	75
76	9.4542 934 377	13.6604 996 424	19.7030 648 473	28.3686 111 198	76
77	9.7379 222 408	14.1386 171 299	20.4911 874 412	29.6451 986 202	77
78	10.0300 599 080	14.6334 687 294	21.3108 349 389	30.9792 325 581	78
79	10.3309 617 053	15.1456 401 350	22.1632 683 364	32.3732 980 232	79
80	10.6408 905 564	15.6757 375 397	23.0497 990 699	33.8300 964 342	80
81	10.9601 172 731	16.2243 883 536	23.9717 910 327	35.3524 507 738	81
82	11.2889 207 913	16.7922 419 460	24.9306 626 740	36.9433 110 586	82
83	11.6275 884 151	17.3799 704 141	25.9278 891 809	38.6057 600 562	83
84	11.9764 160 675	17.9882 693 786	26.9650 047 482	40.3430 192 587	84
85	12.3357 085 495	18.6178 588 068	28.0436 049 381	42.1584 551 254	85
86	12.7057 798 060	19.2694 838 651	29.1653 491 356	44.0555 856 060	86
87	13.0869 532 002	19.9439 158 003	30.3319 631 010	46.0380 869 583	87
88	13.4795 617 962	20.6419 528 533	31.5452 416 251	48.1098 008 714	88
89	13.8839 486 501	21.3644 212 032	32.8070 512 901	50.2747 419 106	89
90	14.3004 671 096	22.1121 759 453	34.1193 333 417	52.5371 052 966	90
91	14.7294 811 229	22.8861 021 034	35.4841 066 754	54.9012 750 350	91
92	15.1713 655 566	23.6871 156 770	36.9034 709 424	57.3718 324 115	92
93	15.6265 065 233	24.5161 647 257	38.3796 097 801	59.9535 648 701	93
94	16.0953 017 190	25.3742 304 911	39.9147 941 713	62.6514 752 892	94
95	16.5781 607 705	26.2623 285 583	41.5113 859 381	65.4707 916 772	95
96	17.0755 055 936	27.1815 100 579	43.1718 413 757	68.4169 773 027	96
97	17.5877 707 615	28.1328 629 099	44.8987 150 307	71.4957 412 813	97
98	18.1154 038 843	29.1175 131 117	46.6946 636 319	74.7130 496 390	98
99	18.6588 660 008	30.1366 260 706	48.5624 501 772	78.0751 368 727	99
100	19.2186 319 809	31.1914 079 831	50.5049 481 843	81.5885 180 320	100

TABLE D.1 COMPOUND AMOUNT OF 1 $s = (1 + i)^n$

n	5 per cent	$5\frac{1}{2}$ per cent	6 per cent	$6\frac{1}{2}$ per cent	n
1	1.0500 000 000	1.0550 000 000	1.0600 000 000	1.0650 000 000	1
2	1.1025 000 000	1.1130 250 000	1.1236 000 000	1.1342 250 000	2
3	1.1576 250 000	1.1742 413 750	1.1910 160 000	1.2079 496 250	3
4	1.2155 062 500	1.2388 246 506	1.2624 769 600	1.2864 663 506	4
5	1.2762 815 625	1.3069 600 064	1.3382 255 776	1.3700 866 634	5
6	1.3400 956 406	1.3788 428 068	1.4185 191 123	1.4591 422 965	6
7	1.4071 004 227	1.4546 791 611	1.5036 302 590	1.5539 865 458	7
8	1.4774 554 438	1.5346 865 150	1.5938 480 745	1.6549 956 713	8
9	1.5513 282 160	1.6190 942 733	1.6894 789 590	1.7625 703 899	9
10	1.6288 946 268	1.7081 444 584	1.7908 476 965	1.8771 374 653	10
11	1.7103 393 581	1.8020 924 036	1.8982 985 583	1.9991 514 005	11
12	1.7958 563 260	1.9012 074 858	2.0121 964 718	2.1290 962 415	12
13	1.8856 491 423	2.0057 738 975	2.1329 282 601	2.2674 874 972	13
14	1.9799 315 994	2.1160 914 618	2.2609 039 558	2.4148 741 846	14
15	2.0789 281 794	2.2324 764 922	2.3965 581 931	2.5718 410 066	15
16	2.1828 745 884	2.3552 626 993	2.5403 516 847	2.7390 106 720	16
17	2.2920 183 178	2.4848 021 478	2.6927 727 858	2.9170 463 657	17
18	2.4066 192 337	2.6214 662 659	2.8543 391 529	3.1066 543 794	18
19	2.5269 501 954	2.7656 469 105	3.0255 995 021	3.3085 869 141	19
20	2.6532 977 051	2.9177 574 906	3.2071 354 722	3.5236 450 635	20
21	2.7859 625 904	3.0782 341 526	3.3995 636 005	3.7526 819 926	21
22	2.9252 607 199	3.2475 370 310	3.6035 374 166	3.9966 063 222	22
23	3.0715 237 559	3.4261 515 677	3.8197 496 616	4.2563 857 331	23
24	3.2250 999 437	3.6145 899 039	4.0489 346 413	4.5330 508 058	24
25	3.3863 549 409	3.8133 923 486	4.2918 707 197	4.8276 991 081	25
26	3.5556 726 879	4.0231 289 278	4.5493 829 629	5.1414 995 502	26
27	3.7334 563 223	4.2444 010 188	4.8223 459 407	5.4756 970 209	27
28	3.9201 291 385	4.4778 430 749	5.1116 866 971	5.8316 173 273	28
29	4.1161 355 954	4.7241 244 440	5.4183 878 990	6.2106 724 536	29
30	4.3219 423 752	4.9839 512 884	5.7434 911 729	6.6143 661 630	30
31	4.5380 394 939	5.2580 686 093	6.0881 006 433	7.0442 999 636	31
32	4.7649 414 686	5.5472 623 828	6.4533 866 819	7.5021 794 613	32
33	5.0031 885 420	5.8523 618 138	6.8405 898 828	7.9898 211 263	33
34	5.2533 479 691	6.1742 417 136	7.2510 252 758	8.5091 594 995	34
35	5.5160 153 676	6.5138 250 078	7.6860 867 923	9.0622 548 669	35
36	5.7918 161 360	6.8720 853 833	8.1472 519 999	9.6513 014 333	36
37	6.0814 069 428	7.2500 500 793	8.6360 871 198	10.2786 360 264	37
38	6.3854 772 899	7.6488 028 337	9.1542 523 470	10.9467 473 682	38
39	6.7047 511 544	8.0694 869 896	9.7035 074 879	11.6582 859 471	39
40	7.0399 887 121	8.5133 087 740	10.2857 179 371	12.4160 745 337	40
41	7.3919 881 477	8.9815 407 565	10.9028 610 134	13.2231 193 783	41
42	7.7615 875 551	9.4755 254 982	11.5570 326 742	14.0826 221 379	42
43	8.1496 669 329	9.9966 794 006	12.2504 546 346	14.9979 925 769	43
44	8.5571 502 795	10.5464 967 676	12.9854 819 127	15.9728 620 944	44
45	8.9850 077 935	11.1265 540 898	13.7646 108 274	17.0110 981 305	45
46	9.4342 581 832	11.7385 145 647	14.5904 874 771	18.1168 195 090	46
47	9.9059 710 923	12.3841 328 658	15.4659 167 257	19.2944 127 771	47
48	10.4012 696 469	13.0652 601 734	16.3938 717 293	20.5485 496 076	48
49	10.9213 331 293	13.7838 494 830	17.3775 040 330	21.8842 053 321	49
50	11.4673 997 858	14.5419 612 045	18.4201 542 750	23.3066 786 787	50

Table D.1 — Compound Amount of 1 — $s = (1 + i)^n$

n	5 per cent	5½ per cent	6 per cent	6½ per cent	n
51	12.0407 697 750	15.3417 690 708	19.5253 635 315	24.8216 127 928	51
52	12.6428 082 638	16.1855 663 697	20.6968 853 434	26.4350 176 243	52
53	13.2749 486 770	17.0757 725 200	21.9386 984 640	28.1532 937 699	53
54	13.9386 961 108	18.0149 400 086	23.2550 203 718	29.9832 578 650	54
55	14.6356 309 164	19.0057 617 091	24.6503 215 941	31.9321 696 262	55
56	15.3674 124 622	20.0510 786 031	26.1293 408 898	34.0077 606 519	56
57	16.1357 830 853	21.1538 879 262	27.6971 013 432	36.2182 650 943	57
58	16.9425 722 396	22.3173 517 622	29.3589 274 238	38.5724 523 254	58
59	17.7897 008 515	23.5448 061 091	31.1204 630 692	41.0796 617 266	59
60	18.6791 858 941	24.8397 704 451	32.9876 908 533	43.7498 397 388	60
61	19.6131 451 888	26.2059 578 196	34.9669 523 045	46.5935 793 218	61
62	20.5938 024 483	27.6472 854 997	37.0649 694 428	49.6221 619 777	62
63	21.6234 925 707	29.1678 862 021	39.2888 676 094	52.8476 025 063	63
64	22.7046 671 992	30.7721 199 432	41.6461 996 659	56.2826 966 692	64
65	23.8399 005 592	32.4645 865 401	44.1449 716 459	59.9410 719 527	65
66	25.0318 955 871	34.2501 387 998	46.7936 699 446	63.8372 416 296	66
67	26.2834 903 665	36.1338 964 338	49.6012 901 413	67.9866 623 355	67
68	27.5976 648 848	38.1212 607 377	52.5773 675 498	72.4057 953 873	68
69	28.9775 481 291	40.2179 300 783	55.7320 096 028	77.1121 720 875	69
70	30.4264 255 355	42.4299 162 326	59.0759 301 790	82.1244 632 732	70
71	31.9477 468 123	44.7635 616 254	62.6204 859 897	87.4625 533 860	71
72	33.5451 341 529	47.2255 575 147	66.3777 151 491	93.1476 193 560	72
73	35.2223 908 605	49.8229 631 781	70.3603 780 580	99.2022 146 142	73
74	36.9835 104 036	52.5632 261 529	74.5820 007 415	105.6503 585 641	74
75	38.8326 859 238	55.4542 035 913	79.0569 207 860	112.5176 318 708	75
76	40.7743 202 199	58.5041 847 888	83.8003 360 332	119.8312 779 424	76
77	42.8130 362 309	61.7219 149 522	88.8283 561 951	127.6203 110 086	77
78	44.9536 880 425	65.1166 202 745	94.1580 575 669	135.9156 312 242	78
79	47.2013 724 446	68.6980 343 896	99.8075 410 209	144.7501 472 538	79
80	49.5614 410 668	72.4764 262 811	105.7959 934 821	154.1589 068 252	80
81	52.0395 313 202	76.4626 297 265	112.1437 530 910	164.1792 357 689	81
82	54.6414 887 762	80.6680 743 615	118.8723 782 765	174.8508 860 939	82
83	57.3735 632 150	85.1048 184 514	126.0047 209 731	186.2161 936 900	83
84	60.2422 413 758	89.7855 834 662	133.5650 042 315	198.3202 462 798	84
85	63.2543 534 445	94.7237 905 568	141.5789 044 854	211.2110 622 880	85
86	66.4170 711 168	99.9335 990 374	150.0736 387 545	224.9397 813 367	86
87	69.7379 246 726	105.4299 469 845	159.0780 570 798	239.5608 671 236	87
88	73.2248 209 062	111.2285 940 687	168.6227 405 045	255.1323 234 866	88
89	76.8860 619 515	117.3461 667 424	178.7401 049 348	271.7159 245 133	89
90	80.7303 650 491	123.8002 059 133	189.4645 112 309	239.3774 596 066	90
91	84.7668 833 016	130.6092 172 385	200.8323 819 048	308.1869 944 811	91
92	89.0052 274 667	137.7927 241 866	212.8823 248 190	328.2191 491 223	92
93	93.4554 888 400	145.3713 240 169	225.6552 643 082	349.5533 938 153	93
94	98.1282 632 820	153.3667 468 378	239.1945 801 667	372.2743 644 133	94
95	103.0346 764 461	161.8019 179 139	253.5462 549 767	396.4721 981 002	95
96	108.1864 102 684	170.7010 233 991	268.7590 302 753	422.2428 909 767	96
97	113.5957 307 818	180.0895 796 861	284.8845 720 918	449.6686 788 901	97
98	119.2755 173 209	189.9945 065 688	301.9776 464 173	478.9184 430 180	98
99	125.2392 931 870	200.4442 044 301	320.0963 052 023	510.0481 418 142	99
100	131.5012 578 463	211.4686 356 738	339.3020 835 145	543.2012 710 321	100

TABLE D.1 COMPOUND AMOUNT OF 1 $s = (1 + i)^n$

n	7 per cent	7½ per cent	8 per cent	8½ per cent	n
1	1.0700 000 000	1.0750 000 000	1.0800 000 000	1.0850 000 000	1
2	1.1449 000 000	1.1556 250 000	1.1664 000 000	1.1772 250 000	2
3	1.2250 430 000	1.2422 968 750	1.2597 120 000	1.2772 891 250	3
4	1.3107 960 100	1.3354 691 406	1.3604 889 600	1.3858 587 006	4
5	1.4025 517 307	1.4356 293 262	1.4693 280 768	1.5036 566 902	5
6	1.5007 303 518	1.5433 015 256	1.5868 743 229	1.6314 675 088	6
7	1.6057 814 765	1.6590 491 401	1.7138 242 688	1.7701 422 471	7
8	1.7181 861 798	1.7834 778 256	1.8509 302 103	1.9206 043 381	8
9	1.8384 592 124	1.9172 386 625	1.9990 046 271	2.0838 557 068	9
10	1.9671 513 573	2.0610 315 622	2.1589 249 973	2.2609 834 419	10
11	2.1048 519 523	2.2156 089 293	2.3316 389 971	2.4531 670 345	11
12	2.2521 915 890	2.3817 795 990	2.5181 701 168	2.6616 862 324	12
13	2.4098 450 002	2.5604 130 690	2.7196 237 262	2.8879 295 622	13
14	2.5785 341 502	2.7524 440 491	2.9371 936 243	3.1334 035 750	14
15	2.7590 315 407	2.9588 773 528	3.1721 691 142	3.3997 428 788	15
16	2.9521 637 486	3.1807 931 543	3.4259 426 433	3.6887 210 235	16
17	3.1588 152 110	3.4193 526 408	3.7000 180 548	4.0022 623 105	17
18	3.3799 322 757	3.6758 040 889	3.9960 194 992	4.3424 546 069	18
19	3.6165 275 350	3.9514 893 956	4.3157 010 591	4.7115 632 485	19
20	3.8696 844 625	4.2478 511 002	4.6609 571 438	5.1120 461 246	20
21	4.1405 623 749	4.5664 399 328	5.0338 337 154	5.5465 700 452	21
22	4.4304 017 411	4.9089 229 277	5.4365 404 126	6.0180 284 991	22
23	4.7405 298 630	5.2770 921 473	5.8714 636 456	6.5295 609 215	23
24	5.0723 669 534	5.6728 740 583	6.3411 807 372	7.0845 735 998	24
25	5.4274 326 401	6.0983 396 127	6.8484 751 962	7.6867 623 558	25
26	5.8073 529 249	6.5557 150 837	7.3963 532 119	8.3401 371 560	26
27	6.2138 676 297	7.0473 937 149	7.9880 614 689	9.0490 488 143	27
28	6.6488 383 638	7.5759 482 436	8.6271 063 864	9.8182 179 635	28
29	7.1142 570 492	8.1441 443 618	9.3172 748 973	10.6527 664 904	29
30	7.6122 550 427	8.7549 551 890	10.0626 568 891	11.5582 516 421	30
31	8.1451 128 956	9.4115 768 281	10.8676 694 402	12.5407 030 317	31
32	8.7152 707 983	10.1174 450 903	11.7370 829 954	13.6066 627 894	32
33	9.3253 397 542	10.8762 534 720	12.6760 496 350	14.7632 291 265	33
34	9.9781 135 370	11.6919 724 824	13.6901 336 059	16.0181 036 022	34
35	10.6765 814 846	12.5688 704 186	14.7853 442 943	17.3796 424 084	35
36	11.4239 421 885	13.5115 357 000	15.9681 718 379	18.8569 120 131	36
37	12.2236 181 417	14.5249 008 775	17.2456 255 849	20.4597 495 342	37
38	13.0792 714 117	15.6142 684 433	18.6252 756 317	22.1988 282 446	38
39	13.9948 204 105	16.7853 385 766	20.1152 976 822	24.0857 286 454	39
40	14.9744 578 392	18.0442 389 698	21.7245 214 968	26.1330 155 803	40
41	16.0226 698 880	19.3975 568 925	23.4624 832 165	28.3543 219 046	41
42	17.1442 567 801	20.8523 736 595	25.3394 818 739	30.7644 392 665	42
43	18.3443 547 547	22.4163 016 839	27.3666 404 238	33.3794 166 042	43
44	19.6284 595 875	24.0975 243 102	29.5559 716 577	36.2166 670 155	44
45	21.0024 517 587	25.9048 386 335	31.9204 493 903	39.2950 837 118	45
46	22.4726 233 818	27.8477 015 310	34.4740 853 415	42.6351 658 273	46
47	24.0457 070 185	29.9362 791 458	37.2320 121 688	46.2591 549 227	47
48	25.7289 065 098	32.1815 000 818	40.2105 731 423	50.1911 830 911	48
49	27.5299 299 655	34.5951 125 879	43.4274 189 937	54.4574 336 538	49
50	29.4570 250 631	37.1897 460 320	46.9016 125 132	59.0863 155 144	50

TABLE D.1 COMPOUND AMOUNT OF 1 $s = (1 + i)^n$

n	7 per cent	7½ per cent	8 per cent	8½ per cent	n
51	31.5190 168 175	39.9789 769 844	50.6537 415 143	64.1086 523 331	51
52	33.7253 479 947	42.9774 002 582	54.7060 408 354	69.5578 877 815	52
53	36.0861 223 543	46.2007 052 776	59.0825 241 023	75.4703 082 429	53
54	38.6121 509 191	49.6657 581 734	63.8091 260 304	81.8852 844 435	54
55	41.3150 014 835	53.3906 900 364	68.9138 561 129	88.8455 336 212	55
56	44.2070 515 873	57.3949 917 892	74.4269 646 019	96.3974 039 790	56
57	47.3015 451 984	61.6996 161 734	80.3811 217 701	104.5911 833 172	57
58	50.6126 533 623	66.3270 873 864	86.8116 115 117	113.4814 338 992	58
59	54.1555 390 977	71.3016 189 403	93.7565 404 326	123.1273 557 806	59
60	57.9464 268 345	76.6492 403 609	101.2570 636 672	133.5931 810 220	60
61	62.0026 767 130	82.3979 333 879	109.3576 287 606	144.9486 014 089	61
62	66.3428 640 829	88.5777 783 920	118.1062 390 614	157.2692 325 286	62
63	70.9868 645 687	95.2211 117 714	127.5547 381 864	170.6371 172 935	63
64	75.9559 450 885	102.3626 951 543	137.7591 172 413	185.1412 722 635	64
65	81.2728 612 447	110.0398 972 908	148.7798 466 206	200.8782 804 059	65
66	86.9619 615 318	118.2928 895 876	160.6822 343 502	217.9529 342 404	66
67	93.0492 988 390	127.1648 563 067	173.5368 130 982	236.4789 336 508	67
68	99.5627 497 577	136.7022 205 297	187.4197 581 461	256.5796 430 111	68
69	106.5321 422 408	146.9548 870 695	202.4133 387 978	278.3889 126 671	69
70	113.9893 921 976	157.9765 035 997	218.6064 059 016	302.0519 702 438	70
71	121.9686 496 515	169.8247 413 696	236.0949 183 737	327.7263 877 145	71
72	130.5064 551 271	182.5615 969 724	254.9825 118 436	355.5831 306 702	72
73	139.6419 069 860	196.2537 167 453	275.3811 127 911	385.8076 967 772	73
74	149.4168 404 750	210.9727 455 012	297.4116 018 144	418.6013 510 033	74
75	159.8760 193 082	226.7957 014 138	321.2045 299 596	454.1824 658 386	75
76	171.0673 406 598	243.8053 790 198	346.9008 923 563	492.7879 754 348	76
77	183.0420 545 060	262.0907 824 463	374.6529 637 448	534.6749 533 468	77
78	195.8549 983 214	281.7475 911 298	404.6252 008 444	580.1223 243 813	78
79	209.5648 482 039	302.8786 604 645	436.9952 169 120	629.4327 219 537	79
80	224.2343 875 782	325.5945 599 993	471.9548 342 649	682.9345 033 198	80
81	239.9307 947 087	350.0141 519 993	509.7112 210 061	740.9839 361 019	81
82	256.7259 503 383	376.2652 133 992	550.4881 186 866	803.9675 706 706	82
83	274.6967 668 619	404.4851 044 042	594.5271 681 815	872.3048 141 776	83
84	293.9255 405 423	434.8214 872 345	642.0893 416 361	946.4507 233 827	84
85	314.5003 283 802	467.4330 987 771	693.4564 889 669	1026.8990 348 702	85
86	336.5153 513 669	502.4905 811 854	748.9330 080 843	1114.1854 528 342	86
87	360.0714 259 625	540.1773 747 743	808.8476 487 310	1208.8912 163 251	87
88	385.2764 257 799	580.6906 778 823	873.5554 606 295	1311.6469 697 127	88
89	412.2457 755 845	624.2424 787 235	943.4398 974 799	1423.1369 621 383	89
90	441.1029 798 754	671.0606 646 278	1018.9150 892 783	1544.1036 039 201	90
91	471.9801 884 667	721.3902 144 749	1100.4282 964 205	1675.3524 102 533	91
92	505.0188 016 594	775.4944 805 605	1188.4625 601 342	1817.7573 651 248	92
93	540.3701 177 755	833.6565 666 025	1283.5395 649 449	1972.2667 411 604	93
94	578.1960 260 198	896.1808 090 977	1386.2227 301 405	2139.9094 141 591	94
95	618.6697 478 412	963.3943 697 800	1497.1205 485 517	2321.8017 143 626	95
96	661.9766 301 901	1035.6489 475 135	1616.8901 924 359	2519.1548 600 834	96
97	708.3149 943 034	1113.3226 185 770	1746.2414 078 307	2733.2830 231 905	97
98	757.8970 439 046	1196.8218 149 703	1885.9407 204 572	2965.6120 801 617	98
99	810.9498 369 780	1286.5834 510 931	2036.8159 780 938	3217.6891 069 754	99
100	867.7163 255 664	1383.0772 099 251	2199.7612 563 413	3491.1926 810 683	100

Table D.2 The Amount of 1 per Annum at Compound Interest* $f_n = \dfrac{[(1 + i)^n - 1]}{i}$

n	2 per cent	2¼ per cent	2½ per cent	2¾ per cent	n
1	1.0000 000 000	1.0000 000 000	1.0000 000 000	1.0000 000 000	1
2	2.0200 000 000	2.0225 000 000	2.0250 000 000	2.0275 000 000	2
3	3.0604 000 000	3.0680 062 500	3.0756 250 000	3.0832 562 500	3
4	4.1216 080 000	4.1370 363 906	4.1525 156 250	4.1680 457 969	4
5	5.2040 401 600	5.2301 197 094	5.2563 285 156	5.2826 670 563	5
6	6.3081 209 632	6.3477 974 029	6.3877 367 285	6.4279 404 003	6
7	7.4342 833 825	7.4906 228 444	7.5474 301 467	7.6047 087 613	7
8	8.5829 690 501	8.6591 618 584	8.7361 159 004	8.8138 382 523	8
9	9.7546 284 311	9.8539 930 003	9.9545 187 979	10.0562 188 042	9
10	10.9497 209 997	11.0757 078 428	11.2033 817 679	11.3327 648 213	10
11	12.1687 154 197	12.3249 112 692	12.4834 663 121	12.6444 158 539	11
12	13.4120 897 281	13.6022 217 728	13.7955 529 699	13.9921 372 899	12
13	14.6803 315 227	14.9082 717 627	15.1404 417 941	15.3769 210 654	13
14	15.9739 381 531	16.2437 078 773	16.5189 528 390	16.7997 863 947	14
15	17.2934 169 162	17.6091 913 046	17.9319 266 599	18.2617 805 205	15
16	18.6392 852 545	19.0053 981 089	19.3802 248 264	19.7639 794 848	16
17	20.0120 709 596	20.4330 195 664	20.8647 304 471	21.3074 889 207	17
18	21.4123 123 788	21.8927 625 066	22.3863 487 083	22.8934 448 660	18
19	22.8405 586 264	23.3853 496 630	23.9460 074 260	24.5230 145 998	19
20	24.2973 697 989	24.9115 200 304	25.5446 576 116	26.1973 975 013	20
21	25.7833 171 949	26.4720 292 311	27.1832 740 519	27.9178 259 326	21
22	27.2989 835 388	28.0676 498 888	28.8628 559 032	29.6855 661 457	22
23	28.8449 632 096	29.6991 720 113	30.5844 273 008	31.5019 192 147	23
24	30.4218 624 738	31.3674 033 816	32.3490 379 833	33.3682 219 932	24
25	32.0302 997 232	33.0731 699 577	34.1577 639 329	35.2858 480 980	25
26	33.6709 057 177	34.8173 162 817	36.0117 080 312	37.2562 089 207	26
27	35.3443 238 321	36.6007 058 980	37.9120 007 320	39.2807 546 660	27
28	37.0512 103 087	38.4242 217 807	39.8598 007 503	41.3609 754 193	28
29	38.7922 345 149	40.2887 667 708	41.8562 957 690	43.4984 022 433	29
30	40.5680 792 052	42.1952 640 232	43.9027 031 633	45.6946 083 050	30
31	42.3794 407 893	44.1446 574 637	46.0002 707 424	47.9512 100 334	31
32	44.2270 296 051	46.1379 122 566	48.1502 775 109	50.2698 683 093	32
33	46.1115 701 972	48.1760 152 824	50.3540 344 487	52.6522 896 878	33
34	48.0338 016 011	50.2599 756 262	52.6128 853 099	55.1002 276 542	34
35	49.9944 776 331	52.3908 250 778	54.9282 074 426	57.6154 839 147	35
36	51.9943 671 858	54.5696 186 421	57.3014 126 287	60.1999 097 224	36
37	54.0342 545 295	56.7974 350 615	59.7339 479 444	62.8554 072 398	37
38	56.1149 396 201	59.0753 773 504	62.2272 966 430	65.5839 309 388	38
39	58.2372 384 125	61.4045 733 408	64.7829 790 591	68.3874 890 397	39
40	60.4019 831 807	63.7861 762 410	67.4025 535 356	71.2681 449 883	40
41	62.6100 228 444	66.2213 652 064	70.0876 173 740	74.2280 189 754	41
42	64.8622 233 012	68.7113 459 235	72.8398 078 083	77.2692 894 973	42
43	67.1594 677 673	71.2573 512 068	75.6608 030 035	80.3941 949 584	43
44	69.5026 571 226	73.8606 416 090	78.5523 230 786	83.6050 353 198	44
45	71.8927 102 651	76.5225 060 452	81.5161 311 556	86.9041 737 911	45
46	74.3305 644 704	79.2442 624 312	84.5540 344 345	90.2940 385 703	46
47	76.8171 757 598	82.0272 583 359	87.6678 852 954	93.7771 246 310	47
48	79.3535 192 750	84.8728 716 484	90.8595 824 277	97.3559 955 584	48
49	81.9405 896 605	87.7825 112 605	94.1310 719 884	101.0332 854 362	49
50	84.5794 014 537	90.7576 177 639	97.4843 487 881	104.8117 007 857	50

* From Kent, "Mathematical Principles of Finance."

Table D.2 The Amount of 1 per Annum at Compound Interest $f_n = \frac{[(1+i)^n - 1]}{i}$

n	2 per cent	2¼ per cent	2½ per cent	2¾ per cent	n
51	87.2709 894 828	93.7996 641 636	100.9214 575 078	108.6940 225 573	51
52	90.0164 092 724	96.9101 566 073	104.4444 939 455	112.6831 081 777	52
53	92.8167 374 579	100.0906 351 309	108.0556 062 942	116.7818 936 525	53
54	95.6730 722 070	103.3426 744 214	111.7569 964 515	120.9933 957 280	54
55	98.5865 336 512	106.6678 845 958	115.5509 213 628	125.3207 141 105	55
56	101.5582 643 242	110.0679 119 993	119.4396 943 969	129.7670 337 485	56
57	104.5894 296 107	113.5444 400 192	123.4256 867 568	134.3356 271 766	57
58	107.6812 182 029	117.0991 899 197	127.5113 289 257	139.0298 569 240	58
59	110.8348 425 669	120.7339 216 929	131.6991 121 489	143.8531 779 894	59
60	114.0515 394 183	124.4504 349 309	135.9915 899 526	148.8091 403 841	60
61	117.3325 702 066	128.2505 697 169	140.3913 797 014	153.9013 917 447	61
62	120.6792 216 108	132.1362 075 355	144.9011 641 940	159.1336 800 177	62
63	124.0928 060 430	136.1092 722 051	149.5236 932 988	164.5098 562 181	63
64	127.5746 621 638	140.1717 308 297	154.2617 856 313	170.0338 772 641	64
65	131.1261 554 071	144.3255 947 734	159.1183 302 721	175.7098 088 889	65
66	134.7486 785 153	148.5729 206 558	164.0962 885 289	181.5418 286 333	66
67	138.4436 520 856	152.9158 113 705	169.1986 957 421	187.5342 289 208	67
68	142.2125 251 273	157.3564 171 263	174.4286 631 356	193.6914 202 161	68
69	146.0567 756 298	161.8969 365 117	179.7893 797 140	200.0179 342 720	69
70	149.9779 111 424	166.5396 175 832	185.2841 142 069	206.5184 274 645	70
71	153.9774 693 653	171.2867 589 788	190.9162 170 620	213.1976 842 198	71
72	158.0570 187 526	176.1407 110 558	196.6891 224 886	220.0606 205 358	72
73	162.2181 591 276	181.1038 770 546	202.6063 505 508	227.1122 876 006	73
74	166.4625 223 102	186.1787 142 883	208.6715 093 146	234.3578 755 096	74
75	170.7917 727 564	191.3677 353 598	214.8882 970 474	241.8027 170 861	75
76	175.2076 082 115	196.6735 094 054	221.2605 044 736	249.4522 918 060	76
77	179.7117 603 757	202.0986 633 670	227.7920 170 855	257.3122 298 306	77
78	184.3059 955 833	207.6458 832 928	234.4868 175 126	265.3883 161 510	78
79	188.9921 154 949	213.3179 156 669	241.3489 879 504	273.6864 948 451	79
80	193.7719 578 048	219 1175 687 694	248.3827 126 492	282.2128 734 533	80
81	198.6473 969 609	225.0477 140 667	255.5922 804 654	290.9737 274 733	81
82	203.6203 449 001	231.1112 876 332	262.9820 874 770	299.9755 049 788	82
83	208.6927 517 981	237.3112 916 049	270.5566 396 640	309.2248 313 657	83
84	213.8666 068 341	243.6507 956 661	278.3205 556 556	318.7285 142 283	84
85	219.1439 389 708	250.1329 385 685	286.2785 695 470	328.4935 483 696	85
86	224.5268 177 502	256.7609 296 863	294.4355 337 856	338.5271 209 497	86
87	230.0173 541 052	263.5380 506 043	302.7964 221 303	348.8366 167 759	87
88	235.6177 011 873	270.4676 567 429	311.3663 326 835	359.4296 237 372	88
89	241.3300 552 110	277.5531 790 196	320.1504 910 006	370.3139 383 900	89
90	247.1566 563 153	284.7981 255 475	329.1542 532 756	381.4975 716 957	90
91	253.0997 894 416	292.2060 833 724	338.3831 096 075	392.9887 549 173	91
92	259.1617 852 304	299.7807 202 482	347.8426 873 477	404.7959 456 776	92
93	265.3450 209 350	307.5257 864 538	357.5387 545 314	416.9278 341 837	93
94	271.6519 213 537	315.4451 166 490	367.4772 233 947	429.3933 496 237	94
95	278.0849 597 808	323.5426 317 736	377.6641 539 796	442.2016 667 384	95
96	284.6466 589 764	331.8223 409 885	388.1057 578 290	455.3622 125 737	96
97	291.3395 921 559	340.2883 436 608	398.8084 017 748	468.8846 734 195	97
98	298.1663 839 991	348.9448 313 932	409.7786 118 191	482.7790 019 385	98
99	305.1297 116 790	357.7960 900 995	421.0230 771 146	497.0554 244 918	99
100	312.2323 059 126	366.8465 021 268	432.5486 540 425	511.7244 486 653	100

TABLE D.2 THE AMOUNT OF 1 PER ANNUM AT COMPOUND INTEREST $f_n = \frac{[(1+i)^n - 1]}{i}$

n	3 per cent	$3\frac{1}{2}$ per cent	4 per cent	$4\frac{1}{2}$ per cent	n
1	1.0000 000 000	1.0000 000 000	1.0000 000 000	1.0000 000 000	1
2	2.0300 000 000	2.0350 000 000	2.0400 000 000	2.0450 000 000	2
3	3.0909 000 000	3.1062 250 000	3.1216 000 000	3.1370 250 000	3
4	4.1836 270 000	4.2149 428 750	4.2464 640 000	4.2781 911 250	4
5	5.3091 358 100	5.3624 658 756	5.4163 225 600	5.4707 097 256	5
6	6.4684 098 843	6.5501 521 813	6.6329 754 624	6.7168 916 633	6
7	7.6624 621 808	7.7794 075 076	7.8982 944 809	8.0191 517 881	7
8	8.8923 360 463	9.0516 867 704	9.2142 262 601	9.3800 136 186	8
9	10.1591 061 276	10.3684 958 073	10.5827 953 105	10.8021 142 314	9
10	11.4638 793 115	11.7313 931 606	12.0061 071 230	12.2882 093 718	10
11	12.8077 956 908	13.1419 919 212	13.4863 514 079	13.8411 787 936	11
12	14.1920 295 615	14.6019 616 385	15.0258 054 642	15.4640 318 393	12
13	15.6177 904 484	16.1130 302 958	16.6268 376 828	17.1599 132 721	13
14	17.0863 241 618	17.6769 863 562	18.2919 111 901	18.9321 093 693	14
15	18.5989 138 867	19.2956 808 786	20.0235 876 377	20.7840 542 909	15
16	20.1568 813 033	20.9710 297 094	21.8245 311 432	22.7193 367 340	16
17	21.7615 877 424	22.7050 157 492	23.6975 123 889	24.7417 068 870	17
18	23.4144 353 747	24.4996 913 004	25.6454 128 845	26.8550 836 970	18
19	25.1168 684 359	26.3571 804 960	27.6712 293 998	29.0635 624 633	19
20	26.8703 744 890	28.2796 818 133	29.7780 785 758	31.3714 227 742	20
21	28.6764 857 236	30.2694 706 768	31.9692 017 189	33.7831 367 990	21
22	30.5367 802 954	32.3289 021 505	34.2479 697 876	36.3033 779 550	22
23	32.4528 837 042	34.4604 137 257	36.6178 885 791	38.9370 299 629	23
24	34.4264 702 153	36.6665 282 061	39.0826 041 223	41.6891 963 113	24
25	36.4592 643 218	38.9498 566 933	41.6459 082 872	44.5652 101 453	25
26	38.5530 422 515	41.3131 016 776	44.3117 446 187	47.5706 446 018	26
27	40.7096 335 190	43.7590 602 363	47.0842 144 034	50.7113 236 089	27
28	42.9309 225 246	46.2906 273 446	49.9675 829 796	53.9933 331 713	28
29	45.2188 502 003	48.9107 993 017	52.9662 862 987	57.4230 331 640	29
30	47.5754 157 063	51.6226 772 772	56.0849 377 507	61.0070 696 564	30
31	50.0026 781 775	54.4294 709 819	59.3283 352 607	64.7523 877 909	31
32	52.5027 585 228	57.3345 024 663	62.7014 686 711	68.6662 452 415	32
33	55.0778 412 785	60.3412 100 526	66.2095 274 180	72.7562 262 774	33
34	57.7301 765 169	63.4531 524 044	69.8579 085 147	77.0302 564 599	34
35	60.4620 818 124	66.6740 127 386	73.6522 248 553	81.4966 180 005	35
36	63.2759 442 668	70.0076 031 844	77.5983 138 495	86.1639 658 106	36
37	66.1742 225 948	73.4578 692 959	81.7022 464 035	91.0413 442 720	37
38	69.1594 492 726	77.0288 947 212	85.9703 362 596	96.1382 047 643	38
39	72.2342 327 508	80.7249 060 365	90.4091 497 100	101.4644 239 787	39
40	75.4012 597 333	84.5502 777 478	95.0255 156 984	107.0303 230 577	40
41	78.6632 975 253	88.5095 374 689	99.8265 363 264	112.8466 875 953	41
42	82.0231 964 511	92.6073 712 804	104.8195 977 794	118.9247 885 371	42
43	85.4838 923 446	96.8486 292 752	110.0123 816 906	125.2764 040 213	43
44	89.0484 091 149	101.2383 312 998	115.4128 769 582	131.9138 422 022	44
45	92.7198 613 884	105.7816 728 953	121.0293 920 365	138.8499 651 013	45
46	96.5014 572 300	110.4840 314 466	126.8705 677 180	146.0982 135 309	46
47	100.3965 009 469	115.3509 725 473	132.9453 904 267	153.6726 331 398	47
48	104.4083 959 753	120.3882 565 864	139.2632 060 438	161.5879 016 311	48
49	108.5406 478 546	125.6018 455 669	145.8337 342 855	169.8593 572 045	49
50	112.7968 672 902	130.9979 101 618	152.6670 836 570	178.5030 282 787	50

TABLE D.2 THE AMOUNT OF 1 PER ANNUM AT COMPOUND INTEREST $f_n = \frac{[(1+i)^n - 1]}{i}$

n	3 per cent	3½ per cent	4 per cent	4½ per cent	n
51	117.1807 733 089	136.5828 370 175	159.7737 670 032	187.5356 645 512	51
52	121.6961 965 082	142.3632 363 131	167.1647 176 834	196.9747 694 560	52
53	126.3470 824 035	148.3459 495 840	174.8513 063 907	206.8386 340 815	53
54	131.1374 948 756	154.5380 578 195	182.8453 586 463	217.1463 726 152	54
55	136.0716 197 218	160.9468 898 431	191.1591 729 922	227.9179 593 829	55
56	141.1537 683 135	167.5800 309 877	199.8055 399 119	239.1742 675 551	56
57	146.3883 813 629	174.4453 320 722	208.7977 615 083	250.9371 095 951	57
58	151.7800 328 038	181.5509 186 948	218.1496 719 687	263.2292 795 269	58
59	157.3334 337 879	188.9052 008 491	227.8756 588 474	276.0745 971 056	59
60	163.0534 368 015	196.5168 828 788	237.9906 852 013	289.4979 539 753	60
61	168.9450 399 056	204.3949 737 795	248.5103 126 094	303.5253 619 042	61
62	175.0133 911 027	212.5487 978 618	259.4507 251 137	318.1840 031 899	62
63	181.2637 928 358	220.9880 057 870	270.8287 541 183	333.5022 833 335	63
64	187.7017 066 209	229.7225 859 895	282.6619 042 830	349.5098 860 835	64
65	194.3327 578 195	238.7628 764 992	294.9683 804 543	366.2378 309 572	65
66	201.1627 405 541	248.1195 771 766	307.7671 156 725	383.7185 333 503	66
67	208.1976 227 707	257.8037 623 778	321.0778 002 994	401.9858 673 511	67
68	215.4435 514 538	267.8268 940 611	334.9209 123 114	421.0752 313 819	68
69	222.9068 579 975	278.2008 353 532	349.3177 488 039	441.0236 167 941	69
70	230.5940 637 374	288.9378 645 906	364.2904 587 560	461.8696 795 498	70
71	238.5118 856 495	300.0506 898 512	379.8620 771 063	483.6538 151 295	71
72	246.6672 422 190	311.5524 639 960	396.0565 601 905	506.4182 368 104	72
73	255.0672 594 856	323.4568 002 359	412.8988 225 981	530.2070 574 668	73
74	263.7192 772 701	335.7777 882 441	430.4147 755 020	555.0663 750 528	74
75	272.6308 555 882	348.5300 108 327	448.6313 665 221	581.0443 619 302	75
76	281.8097 812 559	361.7285 612 118	467.5766 211 830	608.1913 582 171	76
77	291.2640 746 936	375.3890 608 542	487.2796 860 303	636.5599 693 368	77
78	301.0019 969 344	389.5276 779 841	507.7708 734 715	666.2051 679 570	78
79	311.0320 568 424	404.1611 467 136	529.0817 084 104	697.1844 005 151	79
80	321.3630 185 477	419.3067 868 486	551.2449 767 468	729.5576 985 382	80
81	332.0039 091 041	434.9825 243 883	574.2947 758 167	763.3877 949 725	81
82	342.9640 263 772	451.2069 127 419	598.2665 668 494	798.7402 457 462	82
83	354.2529 471 685	467.9991 546 878	623.1972 295 233	835.6835 568 048	83
84	365.8805 355 836	485.3791 251 019	649.1251 187 043	874.2893 168 610	84
85	377.8569 516 511	503.3673 944 805	676.0901 234 524	914.6323 361 198	85
86	390.1926 602 006	521.9852 532 873	704.1337 283 905	956.7907 912 452	86
87	402.8984 400 067	541.2547 371 523	733.2990 775 262	1000.8463 768 512	87
88	415.9853 932 069	561.1986 529 527	763.6310 406 272	1046.8844 638 095	88
89	429.4649 550 031	581.8406 058 060	795.1762 822 523	1094.9942 646 809	89
90	443.3489 036 532	603.2050 270 092	827.9833 335 424	1145.2690 065 916	90
91	457.6493 707 627	625.3172 029 545	862.1026 668 841	1197.8061 118 882	91
92	472.3788 518 856	648.2033 050 580	897.5867 735 595	1252.7073 869 231	92
93	487.5502 174 422	671.8904 207 350	934.4902 445 018	1310.0792 193 347	93
94	503.1767 239 655	696.4065 854 607	972.8698 542 819	1370.0327 842 047	94
95	519.2720 256 844	721.7808 159 518	1012.7846 484 532	1432.6842 594 940	95
96	535.8501 864 550	748.0431 445 101	1054.2960 343 913	1498.1550 511 712	96
97	552.9256 920 486	775.2246 545 680	1097.4678 757 670	1566.5720 284 739	97
98	570.5134 628 101	803.3575 174 779	1142.3665 907 976	1638.0677 697 552	98
99	588.6288 666 944	832.4750 305 896	1189.0612 544 295	1712.7808 193 942	99
100	607.2877 326 952	862.6116 566 602	1237.6237 046 067	1790.8559 562 669	100

Table D.2 The Amount of 1 per Annum at Compound Interest $f_n = \frac{[(1+i)^n - 1]}{i}$

n	5 per cent	5½ per cent	6 per cent	6½ per cent	n
1	1.0000 000 000	1.0000 000 000	1.0000 000 000	1.0000 000 000	1
2	2.0500 000 000	2.0550 000 000	2.0600 000 000	2.0650 000 000	2
3	3.1525 000 000	3.1680 250 000	3.1836 000 000	3.1992 250 000	3
4	4.3101 250 000	4.3422 663 750	4.3746 160 000	4.4071 746 250	4
5	5.5256 312 500	5.5810 910 256	5.6370 929 600	5.6936 409 756	5
6	6.8019 128 125	6.8880 510 320	6.9753 185 376	7.0637 276 390	6
7	8.1420 084 531	8.2668 938 388	8.3938 376 499	8.5228 699 356	7
8	9.5491 088 758	9.7215 729 999	9.8974 679 088	10.0768 564 814	8
9	11.0265 643 196	11.2562 595 149	11.4913 159 834	11.7318 521 527	9
10	12.5778 925 355	12.8753 537 882	13.1807 949 424	13.4944 225 426	10
11	14.2067 871 623	14.5834 982 466	14.9716 426 389	15.3715 600 079	11
12	15.9171 265 204	16.3855 906 501	16.8699 411 973	17.3707 114 084	12
13	17.7129 828 465	18.2867 981 359	18.8821 376 691	19.4998 076 499	13
14	19.5986 319 888	20.2925 720 334	21.0150 659 292	21.7672 951 472	14
15	21.5785 635 882	22.4086 634 952	23.2759 698 850	24.1821 693 317	15
16	23.6574 917 676	24.6411 399 875	25.6725 280 781	26.7540 103 383	16
17	25.8403 663 560	26.9964 026 868	28.2128 797 628	29.4930 210 103	17
18	28.1323 846 738	29.4812 048 345	30.9056 525 485	32.4100 673 760	18
19	30.5390 039 075	32.1026 711 004	33.7599 917 015	35.5167 217 554	19
20	33.0659 541 029	34.8683 180 110	36.7855 912 035	38.8253 086 695	20
21	35.7192 518 080	37.7860 755 016	39.9927 266 758	42.3489 537 330	21
22	38.5052 143 984	40.8643 096 542	43.3922 902 763	46.1016 357 257	22
23	41.4304 751 184	44.1118 466 851	46.9958 276 929	50.0982 420 478	23
24	44.5019 988 743	47.5379 982 528	50.8155 773 545	54.3546 277 809	24
25	47.7270 988 180	51.1525 881 567	54.8645 119 957	58.8876 785 867	25
26	51.1134 537 589	54.9659 805 053	59.1563 827 155	63.7153 776 948	26
27	54.6691 264 468	58.9891 094 331	63.7057 656 784	68.8568 772 450	27
28	58.4025 827 692	63.2335 104 520	68.5281 116 191	74.3325 742 659	28
29	62.3227 119 076	67.7113 535 268	73.6397 983 162	80.1641 915 932	29
30	66.4388 475 030	72.4354 779 708	79.0581 862 152	86.3748 640 468	30
31	70.7607 898 782	77.4194 292 592	84.8016 773 881	92.9892 302 098	31
32	75.2988 293 721	82.6774 978 685	90.8897 780 314	100.0335 301 735	32
33	80.0637 708 407	88.2247 602 512	97.3431 647 133	107.5357 096 347	33
34	85.0669 593 827	94.0771 220 650	104.1837 545 961	115.5255 307 610	34
35	90.3203 073 518	100.2513 637 786	111.4347 798 719	124.0346 902 605	35
36	95.8363 227 194	106.7651 887 864	119.1208 666 642	133.0969 451 274	36
37	101.6281 388 554	113.6372 741 697	127.2681 186 640	142.7482 465 607	37
38	107.7095 457 982	120.8873 242 490	135.9042 057 839	153.0268 825 871	38
39	114.0950 230 881	128.5361 270 827	145.0584 580 309	163.9736 299 553	39
40	120.7997 742 425	136.6056 140 723	154.7619 656 188	175.6319 159 024	40
41	127.8397 629 546	145.1189 228 463	165.0476 835 559	188.0479 904 360	41
42	135.2317 511 023	154.1004 636 028	175.9505 445 692	201.2711 098 144	42
43	142.9933 386 575	163.5759 891 010	187.5075 772 434	215.3537 319 523	43
44	151.1430 055 903	173.5726 685 015	199.7580 318 780	230.3517 245 292	44
45	159.7001 558 699	184.1191 652 691	212.7435 137 907	246.3245 866 236	45
46	168.6851 636 633	195.2457 193 589	226.5081 246 181	263.3356 847 541	46
47	178.1194 218 465	206.9842 339 236	241.0986 120 952	281.4525 042 631	47
48	188.0253 929 388	219.3683 667 894	256.5645 288 209	300.7469 170 402	48
49	198.4266 625 858	232.4336 269 629	272.9584 005 502	321.2954 666 479	49
50	209.3479 957 151	246.2174 764 458	290.3359 045 832	343.1796 719 800	50

Table D.2 The Amount of 1 per Annum at Compound Interest $f_n = \frac{[(1+i)^n - 1]}{i}$

n	5 per cent	5½ per cent	6 per cent	6½ per cent	n
51	220.8153 955 008	260.7594 376 503	308.7560 588 582	366.4863 506 587	51
52	232.8561 652 759	276.1012 067 211	328.2814 223 897	391.3079 634 515	52
53	245.4989 735 397	292.2867 730 908	348.9783 077 331	417.7429 810 758	53
54	258.7739 222 166	309.3625 456 108	370.9170 061 970	445.8962 748 458	54
55	272.7126 183 275	327.3774 856 193	394.1720 265 689	475.8795 327 107	55
56	287.3482 492 438	346.3832 473 284	418.8223 481 630	507.8117 023 369	56
57	302.7156 617 060	366.4343 259 315	444.9516 890 528	541.8194 629 888	57
58	318.8514 447 913	387.5882 138 577	472.6487 903 959	578.0377 280 831	58
59	335.7940 170 309	409.9055 656 199	502.0077 178 197	616.6101 804 085	59
60	353.5837 178 825	433.4503 717 290	533.1281 808 889	657.6898 421 351	60
61	372.2629 037 766	458.2901 421 741	566.1158 717 422	701.4396 818 738	61
62	391.8760 489 654	484.4960 999 936	601.0828 240 467	748.0332 611 956	62
63	412.4698 514 137	512.1433 854 933	638.1477 934 895	797.6554 231 734	63
64	434.0933 439 844	541.3112 716 954	677.4366 610 989	850.5030 256 796	64
65	456.7980 111 836	572.0833 916 387	719.0828 607 648	906.7857 223 488	65
66	480.6379 117 428	604.5479 781 788	763.2278 324 107	966.7267 943 015	66
67	505.6698 073 299	638.7981 169 786	810.0215 023 554	1030.5640 359 311	67
68	531.9532 976 964	674.9320 134 125	859.6227 924 967	1098.5506 982 666	68
69	559.5509 625 812	713.0532 741 502	912.2001 600 465	1170.9564 936 539	69
70	588.5285 107 103	753.2712 042 284	967.9321 696 493	1248.0686 657 414	70
71	618.9549 362 458	795.7011 204 610	1027.0080 998 283	1330.1931 290 146	71
72	650.9026 830 581	840.4646 820 863	1089.6285 858 180	1417.6556 824 006	72
73	684.4478 172 110	887.6902 396 011	1156.0063 009 670	1510.8033 017 566	73
74	719.6702 080 715	937.5132 027 791	1226.3666 790 250	1610.0055 163 708	74
75	756.6537 184 751	990.0764 289 320	1300.9486 797 666	1715.6558 749 349	75
76	795.4864 043 989	1045.5306 325 233	1380.0056 005 525	1828.1735 068 057	76
77	836.2607 246 188	1104.0348 173 120	1463.8059 365 857	1948.0047 847 480	77
78	879.0737 608 497	1165.7567 322 642	1552.6342 927 808	2075.6250 957 566	78
79	924.0274 488 922	1230.8733 525 387	1646.7923 503 477	2211.5407 269 808	79
80	971.2288 213 368	1299.5713 869 284	1746.5998 913 686	2356.2908 742 346	80
81	1020.7902 624 037	1372.0478 132 094	1852.3958 848 507	2510.4497 810 598	81
82	1072.8297 755 239	1448.5104 429 359	1964.5396 379 417	2674.6290 168 287	82
83	1127.4712 643 001	1529.1785 172 974	2083.4120 162 182	2849.4799 029 226	83
84	1184.8448 275 151	1614.2833 357 488	2209.4167 371 913	3035.6960 966 126	84
85	1245.0870 688 908	1704.0689 192 150	2342.9817 414 228	3234.0163 428 924	85
86	1308.3414 223 354	1798.7927 097 718	2484.5606 459 081	3445.2274 051 804	86
87	1374.7584 934 521	1898.7263 088 092	2634.6342 846 626	3670.1671 865 171	87
88	1444.4964 181 247	2004.1562 557 937	2793.7123 417 424	3909.7280 536 407	88
89	1517.7212 390 310	2115.3848 498 624	2962.3350 822 469	4164.8603 771 274	89
90	1594.6073 009 825	2232.7310 166 048	3141.0751 871 817	4436.5763 016 406	90
91	1675.3376 660 316	2356.5312 225 181	3330.5396 984 127	4725.9537 612 473	91
92	1760.1045 493 332	2487.1404 397 566	3531.3720 803 174	5034.1407 557 284	92
93	1849.1097 767 999	2624.9331 639 432	3744.2544 051 365	5362.3599 048 507	93
94	1942.5652 656 399	2770.3044 879 601	3969.9096 694 446	5711.9132 986 660	94
95	2040.6935 289 219	2923.6712 347 979	4209.1042 496 113	6084.1876 630 793	95
96	2143.7282 053 680	3085.4731 527 118	4462.6505 045 880	6480.6598 611 794	96
97	2251.9146 156 364	3256.1741 761 109	4731.4095 348 633	6902.9027 521 561	97
98	2365.5103 464 182	3436.2637 557 970	5016.2941 069 551	7352.5914 310 462	98
99	2484.7858 637 391	3626.2582 623 659	5318.2717 533 724	7831.5098 740 642	99
100	2610.0251 569 260	3826.7024 667 960	5638.3680 585 747	8341.5580 158 784	100

Table D.2 The Amount of 1 per Annum at Compound Interest $f_n = \frac{[(1+i)^n - 1]}{i}$

n	7 per cent	7½ per cent	8 per cent	8½ per cent	n
1	1.0000 000 000	1.0000 000 000	1.0000 000 000	1.0000 000 000	1
2	2.0700 000 000	2.0750 000 000	2.0800 000 000	2.0850 000 000	2
3	3.2149 000 000	3.2306 250 000	3.2464 000 000	3.2622 250 000	3
4	4.4399 430 000	4.4729 218 750	4.5061 120 000	4.5395 141 250	4
5	5.7507 390 100	5.8083 910 156	5.8666 009 600	5.9253 728 256	5
6	7.1532 907 407	7.2440 203 418	7.3359 290 368	7.4290 295 158	6
7	8.6540 210 925	8.7873 218 674	8.9228 033 597	9.0604 970 246	7
8	10.2598 025 690	10.4463 710 075	10.6366 276 285	10.8306 392 717	8
9	11.9779 887 489	12.2298 488 331	12.4875 578 388	12.7512 436 098	9
10	13.8164 479 613	14.1470 874 955	14.4865 624 659	14.8350 993 167	10
11	15.7835 993 186	16.2081 190 577	16.6454 874 632	17.0960 827 586	11
12	17.8884 512 709	18.4237 279 870	18.9771 264 602	19.5492 497 931	12
13	20.1406 428 598	20.8055 075 860	21.4952 965 771	22.2109 360 255	13
14	22.5504 878 600	23.3659 206 550	24.2149 203 032	25.0988 655 877	14
15	25.1290 220 102	26.1183 647 041	27.1521 139 275	28.2322 691 626	15
16	27.8880 535 509	29.0772 420 569	30.3242 830 417	31.6320 120 414	16
17	30.8402 172 995	32.2580 352 112	33.7502 256 850	35.3207 330 649	17
18	33.9990 325 105	35.6773 878 520	37.4502 437 398	39.3229 953 755	18
19	37.3789 647 862	39.3531 919 410	41.4462 632 390	43.6654 499 824	19
20	40.9954 923 212	43.3046 813 365	45.7619 642 981	48.3770 132 309	20
21	44.8651 767 837	47.5525 324 368	50.4229 214 420	53.4890 593 555	21
22	49.0057 391 586	52.1189 723 695	55.4567 551 573	59.0356 294 007	22
23	53.4361 408 997	57.0278 952 972	60.8932 955 699	65.0536 578 998	23
24	58.1766 707 627	62.3049 874 445	66.7647 592 155	71.5832 188 213	24
25	63.2490 377 160	67.9778 615 029	73.1059 399 527	78.6677 924 211	25
26	68.6764 703 562	74.0762 011 156	79.9544 151 490	86.3545 547 769	26
27	74.4838 232 811	80.6319 161 992	87.3507 683 609	94.6946 919 329	27
28	80.6976 909 108	87.6793 099 142	95.3388 298 297	103.7437 407 472	28
29	87.3465 292 745	95.2552 581 578	103.9659 362 161	113.5619 587 107	29
30	94.4607 863 237	103.3994 025 196	113.2832 111 134	124.2147 252 011	30
31	102.0730 413 664	112.1543 577 086	123.3458 680 025	135.7729 768 432	31
32	110.2181 542 621	121.5659 345 367	134.2135 374 427	148.3136 798 749	32
33	118.9334 250 604	131.6833 796 269	145.9506 204 381	161.9203 426 642	33
34	128.2587 648 146	142.5596 330 990	158.6266 700 732	176.6835 717 907	34
35	138.2368 783 516	154.2516 055 814	172.3168 036 790	192.7016 753 929	35
36	148.9134 598 363	166.8204 760 000	187.1021 479 733	210.0813 178 013	36
37	160.3374 020 248	180.3320 117 000	203.0703 198 112	228.9382 298 144	37
38	172.5610 201 665	194.8569 125 775	220.3159 453 961	249.3979 793 487	38
39	185.6402 915 782	210.4711 810 208	238.9412 210 278	271.5968 075 933	39
40	199.6351 119 887	227.2565 195 974	259.0565 187 100	295.6825 362 387	40
41	214.6095 698 279	245.3007 585 672	280.7810 402 068	321.8155 518 190	41
42	230.6322 397 158	264.6983 154 597	304.2435 234 233	350.1698 737 236	42
43	247.7764 964 959	285.5506 891 192	329.5830 052 972	380.9343 129 901	43
44	266.1208 512 507	307.9669 908 031	356.9496 457 210	414.3137 295 943	44
45	285.7493 108 382	332.0645 151 134	386.5056 173 787	450.5303 966 098	45
46	306.7517 625 969	357.9693 537 469	418.4260 667 690	489.8254 803 216	46
47	329.2243 859 787	385.8170 552 779	452.9001 521 105	532.4606 461 490	47
48	353.2700 929 972	415.7533 344 237	490.1321 642 793	578.7198 010 717	48
49	378.9989 995 070	447.9348 345 055	530.3427 374 217	628.9109 841 627	49
50	406.5289 294 724	482.5299 470 934	573.7701 564 154	683.3684 178 166	50

TABLE D.2 THE AMOUNT OF 1 PER ANNUM AT COMPOUND INTEREST $f_n = \frac{[(1+i)^n - 1]}{i}$

n	7 per cent	7½ per cent	8 per cent	8½ per cent	n
51	435.9859 545 355	519.7196 931 254	620.6717 689 286	742.4547 333 310	51
52	467.5049 713 530	559.6986 701 098	671.3255 104 429	806.5633 856 641	52
53	501.2303 193 477	602.6760 703 681	726.0315 512 783	876.1212 734 456	53
54	537.3164 417 021	648.8767 756 457	785.1140 753 806	951.5915 816 884	54
55	575.9285 926 212	698.5425 338 191	848.9232 014 111	1033.4768 661 320	55
56	617.2435 941 047	751.9332 238 555	917.8370 575 239	1122.3223 997 532	56
57	661.4506 456 920	809.3282 156 447	992.2640 221 259	1218.7198 037 322	57
58	708.7521 908 904	871.0278 318 180	1072.6451 438 959	1323.3109 870 494	58
59	759.3648 442 528	937.3549 192 044	1159.4567 554 076	1436.7924 209 486	59
60	813.5203 833 505	1008.6565 381 447	1253.2132 958 402	1559.9197 767 293	60
61	871.4668 101 850	1085.3057 785 056	1354.4703 595 074	1693.5129 577 513	61
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63	999.8123 509 808	1256.2814 902 855	1581.9342 273 295	1995.7307 916 887	63
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66	1228.0280 218 826	1563 9051 945 020	1996.0279 293 776	2552.3874 616 516	66
67	1314.9899 834 144	1682.1980 840 897	2156.7101 637 279	2770.3403 958 920	67
68	1408.0392 822 534	1809.3629 403 964	2330.2469 768 261	3006.8193 295 429	68
69	1507.6020 320 111	1946.0651 609 261	2517 6667 349 722	3263 3989 725 540	69
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72	1850.0922 161 010	2420.8212 929 649	3174.7813 980 453	4171.5662 431 794	72
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84	4184.6505 791 754	5784.2864 964 599	8013.6167 704 506	11122.9496 868 552	84
85	4478.5761 197 178	6219.1079 836 944	8655.7061 120 867	12069.4004 102 379	85
86	4793.0764 480 980	6686.5410 824 715	9349.1626 010 536	13096.2994 451 081	86
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88	5489.6632 254 274	7729.2090 384 311	10906.9432 578 689	15419.3761 142 674	88
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90	6287.1854 267 918	8934.1421 950 369	12723.9386 159 783	18154.1600 461 185	90
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97	10104.4999 186 200	14830.9682 476 937	21815.5175 978 842	32144.5061 551 822	97
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99	11570.7119 568 280	17141.1126 812 411	25447.6997 261 721	37843.4012 585 343	99
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